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MASTER THESIS

Interest rate risk management in the new Dutch pension contract

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Abstract

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In June 2020, the Dutch Minister of Social Affairs and Employment provided a memo in which the main features of the new Dutch pension system were listed. In the new pension system, pension funds are forced to exchange their top-down strategies by a bottom-up approach. This thesis develops a framework to derive a collective investment strategy in a bottom-up procedure. The main features include bond portfolios with an age-specific maturity, and aggregation across age cohorts. Additionally, the illiquidity and unavailability of bonds corresponding to long-term maturities will be discussed, and the use of swaps as a partial solution will be considered. Next to limitations from the financial market, we focus on policy restrictions implied by the government or regulator. The welfare loss of truncated interest rate risk hedging under borrowing constraints seems to have a similar size as the cost of limited equity investments. It remains to discuss in which way robustness checks and thinking about stress-scenarios can even improve the quality of an investor's interest rate risk management policy. It turns out that implementing an incorrect risk-aversion parameter will result in substantial welfare effects. Therefore, we would advise to emphasize on an adequate research on risk appetite amongst participants while entering the new Dutch pension system.

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Introduction

At the beginning of July 2020, the Dutch government agreed with the labor unions and employers about a switch to a new pension system, including a shift from a Defined Benefit system to a pension system in which all participants accumulate personal pension wealth, and the level of someone's pension benefits depend on investment returns on this accumulated wealth. In the upcoming 2020-2026 period, social partners have to agree upon further details of the new pension pension contract and pension funds have to draft new policies, adjust operations and manage the transition.

The pension agreement focuses on a transition towards a pension system that is more future-proof. To this end, the main change is that the new pension contract does not have pension liabilities. Participants do not accumulate an entitlement to a pension benefit, which they would do in the situation of a Defined Benefit system. Instead, participants acquire a personal 'share' in a collective pool of assets, which is similar to building up individual pension wealth in a Defined Contribution pension system. Then, in the retirement phase, pension payments depend on investment returns on the participant's share of assets, implying that a particular amount of risk is included and therefore the annuity is called variable. An essential similarity to the current pension system is that the shape of our pension benefits still remains a life-long annuity.

In this new contract, the pension assets are part of the collective property of the pension fund. Pension funds have to consider two aspects of the allocation of collective investments risks to age cohorts: the allocation of excess returns and of hedge returns. Excess returns are earned by a portfolio of risky assets, consisting of equities and other types of assets, if its return exceeds the risk-free rate. The new pension contract allows to tailor the exposure to risky assets along participants, which opens the door for the application of life-cycle equity investment. The purpose of a hedge return is to compensate participants for changes in interest rates. The extend to which interest rate risks will be hedged can be determined by pension funds themselves and can also be tailored to the different age-cohorts, creating opportunities to distribute the degree of hedging in accordance with life-cycle investment theory.

To this end, the focus in this thesis will be on interest rate risk management in the new Dutch pension contract. The analysis will start in section 1 with an evaluation on the principles of life-cycle investment for equity risk, including the way along which financial decisions of an individual participant can be optimized. Furthermore, using a financial setting including both equity and interest rate risk, similar to van Bilsen and Mehlkopf (2020) and van Bilsen et al. (2019), the degree of interest rate sensitivity of individual optimal decision-making can be investigated. Notice that, to limit the scope of this thesis and to reduce complexity, we will work with a non-stochastic inflation rate. This is a deviation from the real world, where inflation is uncertain.

As an extension to the earlier mentioned papers papers, the expected future pension benefits in our model will include the probability that the participant will not receive any payments at some point anymore, because of passing away. The survival rates will be calculated on the base of the 2020 actuarial mortality table of the Dutch Actuarial Institute. Furthermore, the calculation of expected pension payments for one participant will include the fact that also other participants of the same age cohort will pass away at a certain moment. In that case, the pension assets of this participant will be divided across the other participants in the same age cohort, increasing the pension benefits of the latter.

In section 2, we will then first focus on the optimal investment policy of an individual participant along the life-cycle, during which financial capital can either be dedicated to stocks, cash or a 30-year real bond. Moreover, we will evaluate the level of interest rate sensitivity in the individual optimal setting, and find to which degree interest rate hedging is important for different age cohorts in a pension funds.

After evaluating the investment mix and interest rate risk hedging policy on individual level, we will step in section 3 towards a collective pension policy. In line with the policy descriptions for the new Dutch pension contract of Centraal PlanBureau (2020), we will work with an age-specific maturity of the bond portfolio. To this end, we will derive the interest rate sensitivity of the expected pension annuity, which is different for each age, based on currently accumulated pension capital. Then, the interest rate sensitivity of the expected pension annuity will be matched to the maturity of the bond portfolio to be invested for each age-cohort. To create a collective policy, we will show in which way to aggregate across age cohorts with respect to their current financial capital. For this, we will start with a simple fictional composition of a fund, and broaden the analysis to a collective investment mix of an actual Dutch pension fund. In addition, we will elaborate on the differences between the optimal hedging policy of a green, average and grey pension fund, and between the optimal policy according to our model and the currently used policy by the fund.

While taking into account an age-specific maturity for the bond portfolio, we face extensive investment horizons, especially for younger age cohorts. To this end, we will discuss the issue of bond illiquidity, signifying that the optimal bond allocation for interest rate risk hedging cannot quickly and easily be bought or exchanged on the financial market, e.g. because the products we would have wanted to invest in are not available. Furthermore, this thesis will investigate in which way to respond to those situations and whether interest rate and inflation-linked swaps can be used to establish optimal allocations for the hedge returns in case physical bonds are not available for all maturities.

In chapter 4, we will discuss complexities in the market which are regularly faced by an investor. As contribution to the existing literature, we will show the welfare loss of borrowing constraints in a financial setting including a time-varying interest rate. While calculating this, we will distinguish between a set-up in which only hedging demand for bonds is present, and a setting in which also speculative demand for bonds will play a role, turning out to result in an even higher welfare loss due to borrowing constraints. Additionally, we will show the difference in investment policy for a young age cohort in case we take into account that we have only bonds available up to a certain maturity. Next to that, we will calculate the welfare loss in case we would set a borrowing constraint on the investment mix including only bonds up to available maturities.

After considering some of the frequently faced market limitations by investors, we will consider two more essential elements in interest rate risk management in section 5. To create a robust investment plan, an investor must be able to handle outstanding financial market scenarios. Accordingly, it is important to consider in which way the optimal asset allocation behaves in response to changes in parameters. In the last section, we will show those differences with respect to changes in the correlation between stocks and interest rates and the level of risk-aversion. Considering this, we will also derive the welfare losses of composing your asset allocation on a parameter that differs from its actual value. As a result, we see that even though different parameter values might result in a substantial different asset allocation, the welfare loss of using the wrong value seems to be acceptable. However, this depends highly on which parameter you consider.

Parameter changes are driven by constant transitions in the financial market. Therefore, the second part of chapter 5 will discuss the importance of stress-scenarios thinking while coming up with an investment plan. Thinking about potential stress-scenarios that might occur in the future helps to strengthen the base of your asset allocation, to make sure that participants will not experience a substantial decrease in their expected future pension benefits in case one of the stress-scenarios materializes. To provide that those benefits cannot drop below a certain threshold, lastly the implementation of a minimum required consumption level in an alternative preference function will be considered.

In summary, we will explain the financial model and parameters that will be used throughout this thesis in section 1. In section 2, we will derive the optimal asset allocation obtained from our model and parameter assumptions, and discuss the interest rate sensitivity associated with this allocation. Then, in chapter 3, we will step from individual level to a collective pension scheme, introduce the age-specific maturity for each age cohort, and explain in which way to aggregate the optimal hedging policy for all age cohorts. In this chapter we also discuss the illiquidity of long-term bonds and the levels of liquidity for interest rate and inflation-linked swaps. Chapter 4 will then include a discussion of particular market limitations and their resulting welfare losses. In chapter 5, we will investigate changes of the asset allocation and welfare losses according to differences in particular parameter values. Furthermore, in this section, the fundamentals of scenario-thinking will be introduced, and we will evaluate how to come up with relevant stress-scenarios.

Chapter 1

Life-Cycle Investment Theory

First of all, we will explore the existing fundamentals for Life-Cycle theory regarding optimal investments. To this end, in this chapter, the model of Brennan and Xia (2002) regarding life-cycle theory for optimal asset allocations will be explained. In our financial setting, both equity risk and interest rate risk are present. Furthermore, we will provide some assumptions about parameter values needed to come up with the results regarding a life-cycle framework for optimal asset allocations.

1.1 Model

1.1.1 Financial Market

The starting point is the classical investment model of Brennan and Xia (2002), implying that the financial market consists of two state variables: real interest rate $r(t)$ and stock price $S(t)$ at age t, which follow the dynamics

$$dr(t) = \kappa_r(\bar{r} - r(t))dt + \sigma_r dZ_r(t) \tag{1.1.1}$$

$$dS(t) = (r(t) - \lambda_S\sigma_S)S(t)dt + \sigma_S S(t)dZ_S(t) \tag{1.1.2}$$

In which $\bar{r}$ represents the expected long-term real interest rate, κ_r the degree of predictability of future interest rates, and λ_S the Sharpe ratio of the stock. Both $Z_r(t)$ and $Z_S(t)$ represent standard Brownian Motions, whereas σ_r and σ_S (≥ 0) are the diffusion coefficients of the interest rate and stock price, respectively. The correlation between $Z_r(t)$ and $Z_S(t)$ is denoted by ρ_{rS}. Inflation is held constant at 2% per year, implying that we disregard inflation rate risk.

The stochastic discount factor $M(t)$ is then defined by

$$dM(t) = -r(t)M(t)dt + \phi' M(t)dZ(t) \tag{1.1.3}$$

Where $\phi = (\phi_r, \phi_S)$ represents a vector of factor coefficients, which describe the constant loadings on stochastic developments in the economy, and are of importance in the definition of the market prices of risk. The market price for interest rate risk λ_r and investment risk λ_S are related to ϕ_r and ϕ_S as follows

$$\lambda_r = -\phi_r - \rho_{rS}\phi_S \tag{1.1.4}$$

$$\lambda_S = -\phi_S - \rho_{rS}\phi_r \tag{1.1.5}$$

An individual invests total wealth - which is the sum of human capital and financial capital - in a bond with time to maturity h, a risky asset and an openly absorbable cash account. We implicitly assume that real bonds and inflation linked swaps are sufficiently available at the financial market. To the extend that real bonds are not available in practice, we will use index-linked swaps as an alternative to approach the optimal interest rate risk hedge. Actually, in a world without inflation uncertainty, there is no difference between a nominal and a real bond, and hence there is only one type of bond in this model. The price at time t of a zero-coupon bond with time to maturity h is denoted by

$$dP(t,h) = (r(t) - \lambda_r \sigma_r B_r(h))P(t,h)dt - B_r(h)\sigma_r P(t,h)dZ_r(t) \tag{1.1.6}$$

Where

$$B_r(h) = \frac{1}{\kappa_r}(1 - e^{-\kappa_r h}) \in [0, h] \tag{1.1.7}$$

models the interest rate sensitivity of the bond. Interest rate sensitivity refers to the change in relative value of a product per percentage point change in $r(t)$. In section 2.3, we will measure the sensitivity of a bond's or fixed income portfolio's price to changes in interest rates. Notice that, in case $B_r(h)$ converges to h, the bond price will demonstrate more sensitivity with respect to fluctuations in the interest rate.

1.1.2 Wealth Dynamics

Variables $w_P(t)$ and $w_S(t)$ are the parts of total wealth invested at age t in the bond and risky asset, respectively. The remaining part of total wealth, equal to $1 - w_P(t) - w_S(t)$, will be invested in the cash account. Note that $w_P(t), w_S(t) \in [-1, 1]$, implying that the cash position can become negative. Total wealth at age t is denoted by $W(t)$ and follows the dynamics

$$dW(t) = (r(t) + w(t)'(\mu(t) - r(t)))W(t)dt + w(t)'\Sigma(t)W(t)dZ(t) - c(t)dt \tag{1.1.8}$$

with

$$\mu(t) = \begin{pmatrix} r(t) - \lambda_r \sigma_r B_r(h) \\ r(t) + \lambda_S \sigma_S \end{pmatrix} \text{ and } \Sigma(t) = \begin{pmatrix} -B_r(h)\sigma_r & 0 \\ 0 & \sigma_S \end{pmatrix} \tag{1.1.9}$$

And $c(t)$ equals consumption at age t. Denote that $W(t) = H(t) + F(t)$, the sum of human capital and financial capital. Human capital is the present value of future income and throughout the life cycle, human capital is used for consumption or converted into financial capital through savings. There is relatively low risk associated with human capital, implying that human capital is weakly correlated with risky assets Broeders et al. (2009). Financial capital is the part of total wealth which will actually be invested in pension assets.

In the first few years of the life cycle, total wealth consists mainly of human capital, and participants have only built up a small part of financial capital yet. To this end, we will see in section 2 that it is optimal for young workers to borrow against human capital, such that it is possible for them to invest more than 100% of financial capital into assets.

We can define the dynamics of human capital by

$$dH(t) = (r(t) - \lambda_r \sigma_r D_H(t))H(t)dt - \sigma_r D_H(t)H(t)dZ_r(t) - dt \tag{1.1.10}$$

In fact, if we assume human capital to be risk-free over the life cycle, we can interpret human capital as a bond also, implying that we can measure its interest rate sensitivity. To this end, we denote by $D_H(t)$ the interest rate sensitivity of human capital at age t by

$$D_H(t) = \int_0^{T-t} \frac{H(t,h)}{H(t)} B_r(h) dh \tag{1.1.11}$$

where $H(t,h)$ represents the discounted value at age t of income that will be received from labor at age $t+h$. In case $t+h$ is above the retirement age, then $H(t,h)$ is the discounted value at age t of the state old age pension (OAP) benefit that the participant will receive at age $t+h$. This implies that both the wage income, including rate for premium payments, before the retirement date and guaranteed state pension after the retirement date are included in 1.1.11.

Variables $\hat{w}_P(t)$ and $\hat{w}_S(t)$ are equal to the parts of financial wealth invested at age t in the bond or risky asset, respectively. The remaining part of financial wealth, which is defined as $1 - \hat{w}_P(t) - \hat{w}_S(t)$, will be invested in the cash account. Real wage income at age t is defined as $y(t)$ and equals 1 for all ages below the retirement age, and $y(t)$ is equal to the old age guaranteed state pension for all ages above the retirement age. Financial wealth at age t is denoted by $F(t)$ and evolves as

$$\begin{aligned} dF(t) = (r(t) + \hat{w}(t)'(\mu(t) - r(t)))F(t)dt + \hat{w}_P(t)B_r(h)\sigma_r F(t)dZ_r(t) \\ + \hat{w}_S(t)\sigma_S F(t)dZ_S(t) + (y(t) - c(t))dt \end{aligned} \tag{1.1.12}$$

with μ as in 1.1.9 and $\hat{w}(t) = (\hat{w}_P(t), \hat{w}_S(t))$.

1.1.3 Optimization Problem

In this thesis, we will extend the classical model of Brennan and Xia (2002) by implementing that the moment of death is unknown in advance. To this end, we will incorporate the probability that someone is still alive at time t, which is a condition to be met in order to actually consume. Based on a participant's age and by the use of the AG2020 projection table, we can calculate for all participants the survival rates for their remaining life cycle. The AG projection table reports one-year mortality rates q_x for ages 0-120 and for the years 2020-2191. These can be converted into one-year survival rates by using the relation

$$p_x = 1 - q_x \tag{1.1.13}$$

Then, to calculate ${}_hp_t$, the probability that someone who is currently age t will survive at least h years, can be calculated using

$${}_hp_t = {}_1p_t \cdot {}_2p_{t+1} \cdot \cdots \cdot {}_{h-1}p_{t+h-1} \cdot {}_hp_{t+h} \tag{1.1.14}$$

With this in mind, we can calculate for each age cohort the survival probabilities for the remaining life cycle. Regarding the optimal consumption decision during the retirement period, there is another extra effect we will need to take into account. Upon the death of one of the participants in a certain age-cohort, the accumulated wealth of this participant will belong to the solidarity circle of this cohort. To be compensated for this transfer after passing away, the participants receive a bio-metric return while being alive (Bovenberg et al. (2014)). This bio-metric return $R^{bio}_{t,h}$ in year h for the cohort with current age t is based on actual mortality rates such that

$$R_{t,h}^{bio} = \frac{1}{1 - {}_hq_t} \tag{1.1.15}$$

In which we implicitly assume that for a pool of participants that is acceptably large, the expected mortality rates will match perfectly to the actual mortality rates of the pool.

By implementing both effects and implicitly assuming that society will exactly behave according to the predicted mortality rates, the individual optimal consumption choice will not be affected, since

$${}_hp_t \cdot R_{t,h}^{bio} = {}_hp_t \cdot \frac{1}{1 - {}_hq_t} = {}_hp_t \cdot \frac{1}{1 - (1 - {}_hp_t)} = 1 \tag{1.1.16}$$

implying that implementation of both the individual survival rate and bio-metric return will not affect the individual optimal consumption decision, which we will derive later. However, the implementation of survival rates will have a substantial impact in determining the interest rate sensitivity of the bond portfolio and the corresponding time to maturity of the bonds to be invested in, which is highly important in a collective pension scheme. This will be discussed further in detail in section 3.1.

Since we have survival rates available up to and including age 120, we will calculate the consumption of an agent at time t, represented by $c(t)$, up to the age of 120. Then, expected utility of this consumption is defined by

$$U = \mathbb{E}\left(\int_0^{120} e^{-\delta t} \frac{1}{1-\gamma} c(t)^{1-\gamma} dt\right) \tag{1.1.17}$$

where $c(t)$ thus represents consumption at age t for all t during the retirement phase, which is equal to the pension payment that a participant receives at age t, regardless of old age pension guaranteed by the state. Furthermore, $\delta \geq 0$ models the rate of time preference, $\gamma > 0$ represents risk aversion of the participant and $\mathbb{E}$ describes the unconditional expected value operation.

An individual maximizes expected utility considering the budget constraint, which is given by the dynamics of financial wealth in formula 1.1.12. Using this, we can define the following maximization problem

$$\max_{c(t),w(t)} \quad \mathbb{E}\left(\int_0^{T} e^{-\delta t} \frac{1}{1-\gamma} c(t)^{1-\gamma} dt\right) \tag{1.1.18a}$$

$$\text{s.t.} \quad dF(t) = (r(t) + \hat{w}(t)'(\mu(t) - r(t)))F(t)dt + \hat{w}_P(t)B_r(h)\sigma_r F(t) dZ_r(t) + \hat{w}_S(t)\sigma_S F(t) dZ_S(t) + (y(t) - c(t))dt \tag{1.1.18b}$$

1.1.4 Optimal Individual Lifecycle Policies

In van Bilsen et al. (2019), it is derived that an individual's optimal consumption choice at time t is given by

$$c^*(t) = c^*(0) exp\left(\frac{1}{\gamma}\left(\int_0^t \left(r(s) + \frac{1}{2}\phi'\rho\phi - \delta\right) ds - \phi' \int_0^t dZ(s)\right)\right) \tag{1.1.19}$$

with

$$\rho = \begin{pmatrix} 1 & \rho_{rS} \\ \rho_{rS} & 1 \end{pmatrix} \text{ and } \phi = \begin{pmatrix} \phi_r \\ \phi_S \end{pmatrix} \tag{1.1.20}$$

and where $c^*(0)$ denotes the optimal consumption choice of the individual at the beginning of the life-cycle. This value is chosen such that total wealth of the individual equals the market-consistent value of the optimal consumption stream. The choice of consumption is of explicit concern to optimize a participant's consumption pattern over the life cycle, but it also has an implicit influence on the level of pension premium that will be paid, which is clear from the dynamics of Financial and Human Capital.

The optimal portfolio weights $w^*(t) = (w_P^*(t), w_S^*(t))$ can be chosen such that fluctuations in total wealth match to fluctuations in the market-consistent value of consumption. In that case, the optimal benchmark portfolio weights in terms of total wealth are defined by (van Bilsen et al. (2019))

$$w_P^*(t) = \left(\frac{1}{\gamma} \frac{\phi_r}{B_r(h)\sigma_r} + \frac{D_{A^*}(t)}{B_r(h)} \right) \tag{1.1.21}$$

$$w_S^*(t) = -\frac{1}{\gamma} \frac{\phi_S}{\sigma_S} \tag{1.1.22}$$

In expression 1.1.21, $D_{A^*}(t)$ denotes the interest rate sensitivity of the optimal annuity factor, which equals $A^*(t) = W^*(t)/c^*(t)$. We can express $D_{A^*}(t)$ in the following way

$$D_{A^*}(t) = \left(1 - \frac{1}{\gamma}\right) \int_0^{T-t} \frac{V^*(t,h)}{V^*(t)} B_r(h) dh \tag{1.1.23}$$

with $V^*(t,h)$ the market-consistent value at time t of the stochastic optimal consumption choice at time $t+h$, and $V^*(t) = \int_0^{T-t} V^*(t,h)dh$.

While actually investing in the financial market, we need to take into account that the participant can only invest with financial capital. As already mentioned, we assume that human capital is risk-free, implying that it can be treated as a bond. Since participants already posses human capital as financial instrument, and human capital acts like a bond, the benchmark portfolio weights change to

$$\hat{w}_P^*(t) = \left(\frac{W(t)}{F(t)} w_P^*(t) - \frac{H(t)}{F(t)} \frac{D_H(t)}{B_r(h)} \right) \tag{1.1.24}$$

$$\hat{w}_S^*(t) = \frac{W(t)}{F(t)} w_S^*(t) \tag{1.1.25}$$

in terms of financial capital, with $F(t) = W(t) - H(t)$ and financial wealth dynamics as described in formula 1.1.12.

1.2 Parameter settings

To obtain some results from the model we described, we first need to set parameter values. We consider an individual that starts working at age 20 and retires at age 68. As we implement the survival rates in the investment decisions, we consider the maximum age to be 120, because the AG2020 projection table provides mortality rates up to and including this age. Therefore, the life cycle has a maximum length of 100 years. The income pattern throughout the working period remains approximately constant and increases with inflation, implying that we define real income to equal 1 each year. The yearly pension savings rate is about 20% of real income. Old age pension benefits that are guaranteed by the state will also be included, and set to 40% of real income. The risk aversion parameter γ equals 5 and the time preference parameter δ is equal to 3%. We assume $\rho_{rS} = 0$, so that $dZ_r(t)$ and $dZ_S(t)$ are not correlated. Inflation is held constant at 2% per year. The volatility of the interest rate σ_r equals 1% and the term premium of interest rate risk will be set to zero, because in the current climate long-term interest rates are not high above short-term interest rates. For risky assets, the market price of risk is $\lambda_S = 20\%$ and the volatility σ_S equals 20%. These parameter values imply a risk premium of 4% on investing in stocks, which correspond to the excess return of the Committee parameters in Dijsselbloem et al. (2019). We suppose that the half-life time of the short term interest rate equals 20 years, which determines the value for κ_r, defined as $-log\left(\frac{1}{2}\right)$ divided by the half-time value of the short term interest rate. Parameter assumptions for the interest rate and their consequences for the term structure will be discussed in section 2.3.

Chapter 2

Optimal Life-Cycle Allocations

In general, life-cycle hypotheses suggest that the optimal fraction invested in equity decreases and the proportion fixed-income investments increases with age. In this way, the risk of immediate substantial losses due to fluctuations in stock prices lowers as the individual approaches the retirement age, because the exposure to equity risk diminishes. In this section, we will first elaborate on the principles of life-cycle theory for equity risk. Secondly, we will include the time-varying interest rate in our model, as described in formula 1.1.1, and investigate life-cycle patterns in interest rate risk hedging. Additionally, this chapter includes a discussion of the level of interest rate sensitivity around some of our model factors in the optimum.

2.1 Life-cycle pattern in optimal equity allocations

At first, it is worth to say that there is no such thing as one optimal life cycle investment policy. The optimal investment policy for pension fund does not only depend on specific characteristics of its participants such as age and risk aversion, but also on other individual characteristics such as accumulated pension wealth, human capital, the risks regarding future income, and many assumptions around financian market parameters, e.g. risk premia, volatility and correlation of stocks to other rates. After implementation of the new pension policy, a pension fund will have to choose the optimal investment mix for its participants. The policy will be partly determined on the base of investment beliefs of the fund, but the driving factor in investment decisions will be based on the characteristics of the fund's participants. Obviously, pension funds are not able to know all details about the characteristics of every single member. Nevertheless, previously performed scientific literature provides multiple insights in various assumptions about model and parameters used to set up a life-cycle framework for equity allocation.

According to formula 1.1.25, we can derive that the model defined in the previous section contributes to the understanding that younger participants should invest a higher fraction of their accumulated pension wealth in risky assets relative to older participants. Since we implicitly assumed that future income is risk-free, $W(t)$ is approximately equal to $H(t)$ in the early years of the life cycle, resulting in a considerably small $F(t)$ for the first years. The fraction $W(t)/F(t)$ becomes substantially large, resulting in giant optimal stock exposures for younger participants. As the participant becomes older, financial capital grows and human capital shrinks, suggesting that the optimal exposure to investment risk lowers with age. This implies that we would obtain a decreasing life-cycle pattern for the optimal fraction of pension wealth invested in risky assets.

To show this, we will display the optimal stock investments regarding the model in 1.1, in a setting in which you can only invest your money in stocks, or you commit it to your cash position. In this setting, the only financial market risk is equity risk, interest rate risk is absent: $\sigma_r = 0$. Furthermore, we assume that bonds are not available in this simplified economic model, and that the future income pattern is flat and riskless. For this case, the life-cycle pattern of optimal stock investment in terms of financial capital is displayed in the next figure

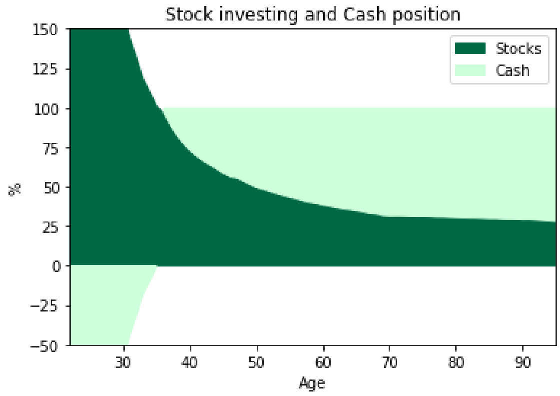


FIGURE 2.1: Optimal equity allocation in terms of Financial Wealth and the corresponding cash position.

In this figure, we certainly see the decreasing life-cycle pattern in optimal stock exposure as described above. Besides that, we see that at the start of the life cycle, there is a short-position in cash. Actually, in the first years, the participant has to borrow money in order to invest the desired fraction of financial capital into stocks. In practice, it often will not be allowed to borrow 50% or more of your financial wealth to invest with, and we will highlight the consequences of this in section 4.

The principles of life-cycle investment risk have been broadly discussed in previous literature, but were not always such straightforward as they are taught nowadays. Another supported view on optimal asset allocation in a pension fund is based on a more risk-averse position, and is called an all-bonds strategy. This approach is based on the main argument that pension liabilities behave naturally as bonds Bodie (1990), and therefore the value of the liabilities equal the value of the replicating bond portfolio that matches the extent and maturity of the pension benefits. Opponents of this strategy argue that investing in risky assets might result in a higher expected return than bonds, yet it comes at the cost of a mismatch risk. In a perfect market, the return of an all-bonds strategy equals the returns on risky assets minus the costs of protection against the mismatch risk coming along with those assets. However, in reality, the financial market turns out often not to correspond to the perfect market setting, because of the existence of a lot of external effects, which we are not able to replicate perfectly in our models.

Bikker et al. (2012) tests the life cycle investment policy for equity risk of Dutch pension funds and reports that assuming a low correlation between wage growth and stock return result in a negative age-dependent equity allocation pattern. Their key finding is supported by the fact that Dutch pension funds with a higher average age amongst participants have significantly lower equity exposures than funds with a lower average age amongst participants. Furthermore, they found that the investment behaviour of larger pension funds is way more related to the principles of life-cycle investing than in smaller funds.

Bovenberg et al. (2007) supports the statement that the behavior of human capital is crucial for saving and investment decisions. In case human capital is assumed to equal the discounted sum of future wage income and state old age pension, and supposed to be risk-free, the life-cycle pattern as we saw in figure 2.3 is genuinely applicable. Viceira (2001) shows that in case human capital is not assumed to be risk-free, the optimal exposure to risky assets will lower. Furthermore, this effect becomes even more important in case of presence of a positive correlation between downwards-moving stock prices and losses in future income. To return to Bovenberg et al. (2007), also welfare losses of suboptimal risk exposure are reported, due to e.g. risk-taking limitations. Those welfare losses are calculated with respect to the optimal life-cycle investment policy, and they find that particular losses are substantial. This emphasizes the importance of differentiating risk-taking across age cohorts and the negative dependence between age and fraction invested in risky assets.

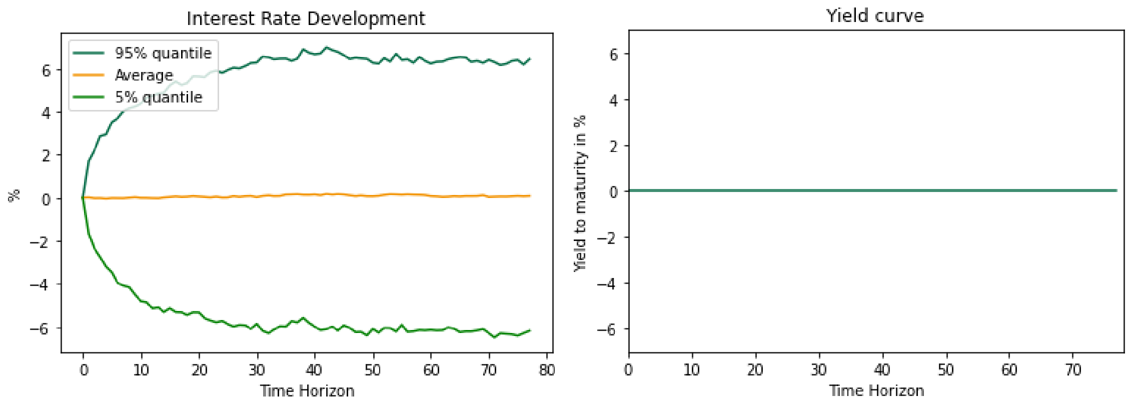
What can be deduced from the discussed papers is that the main principles of live-cycle investment theory are exceedingly supported as far as equity exposure is concerned; a lot of scientific research concluded what was roughly sketched in the first part of this chapter. However, there exists a lot more ambiguity regarding the way to deal with interest rate risk, while this risk is extremely important for pension funds. Since the interest rate is one of the central drivers of the price of a bond, interest rate risk is mainly associated with fixed-income assets. Interest rate risk is the probability of a decline in the value of fixed-income asset as a result of unexpected fluctuations in interest rates. On individual level, contemporary low interest rates result in more expensive pension benefits, especially for younger participants, because they have such a long horizon until their retirement period. This would create a need for higher pension contributions from the employer or employees. From a fund's perspective, interest rate risk is the risk of an increase in the value of pension liabilities due to a decrease in the interest rate. Multiple instruments could be used as a hedge towards this risk, which will be discussed in section 3.3.

In the next part of this section, we will add the time-varying interest rate to our current setting, implying that interest rates become uncertain. As a result, we will see to which extent the optimal asset allocation changes in case wealth can also be invested into bonds. Furthermore, from this resulting optimal asset allocation, we can derive its sensitivity with respect to interest rates, and the optimal percentage of interest rate risk hedging.

2.2 Implementing a time-varying interest rate

The setting in figure 2.1 can be extended to a situation in which the participant can also invest (part of) his money into a 30-year real bond. To perform this extension, we first have to define some extra parameters regarding the initial and long-term interest rates, which will affect the price of the bonds.

Next to the parameters we already defined, $\bar{r}$ is set to 0% because the current nominal long-term interest rate seems to be 0%. Besides this, we set the initial interest rate also equal to 0%: $r(0) = \bar{r}$, implying that there is no term premium on interest rates in this setting and $\phi_r = 0$. In section 1.2 was already stated that we assume that stocks and interest rates are not correlated, by setting $\rho_{rS} = 0$, and together with $\phi_r = 0$ this results in zero market price of risk: $\lambda_r = 0$. Those parameter assumptions result in an average interest rate developments and term structure as displayed in the figures below



(A) Development of interest rates in case $r(0) = 0\%$, $\bar{r} = 0\%$ and market price of interest rate risk $\lambda_r = 0\%$.

(B) Expected yield curve corresponding to the developments of the short rate in figure 2.2a.

In which the orange line in the left figure shows the average interest rate development over a time horizon of around 80 years, and the green lines display the 5% and 95% percentiles, respectively. Notice that, in this thesis, we use the model of Brennan and Xia (2002) with a fixed inflation rate assumption, implying that a nominal bond is exactly the same as a real bond, and therefore we will only work with one type of bonds. In a model including inflation uncertainty, you could introduce another asset and optimize between stocks, nominal bonds, inflation-linked bonds and cash.

Using the developments of the interest rate and the model and parameter assumptions defined in 1.2, we can show optimal asset allocation over the life cycle in terms of financial capital, as exposure to a risky asset and a 30-year real bond. The results are displayed in the figure below

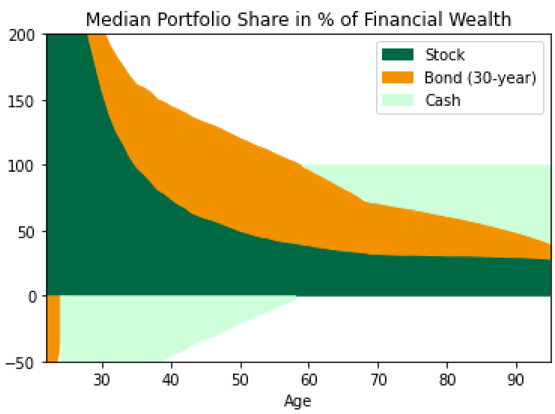


FIGURE 2.3: Optimal portfolio shares in terms of Financial Wealth with $r(0) = \bar{r} = 0\%$ and $\lambda_r = 0$.

Again, we clearly see the life-cycle pattern of optimal stock exposure as before. The cash position equals 100% minus the share of stocks and bonds, and can, as we also see happening in the figure, become negative. In fact, this short position of cash is in line with the construction of a long position in swaps, which is a financial contract in which you exchange the liabilities from two different financial products. In this model, the swap contracts include an exchange of the liabilities from two of the three different financial products stocks, bonds and cash. In practice, the structure would be that in case you go short in cash and more than 100% in equities, this is equal to owning cash, equities and equity futures. On the other hand, if you go short in cash and more than 100% in bonds, this is the same as owning cash, bonds and interest rate swaps.

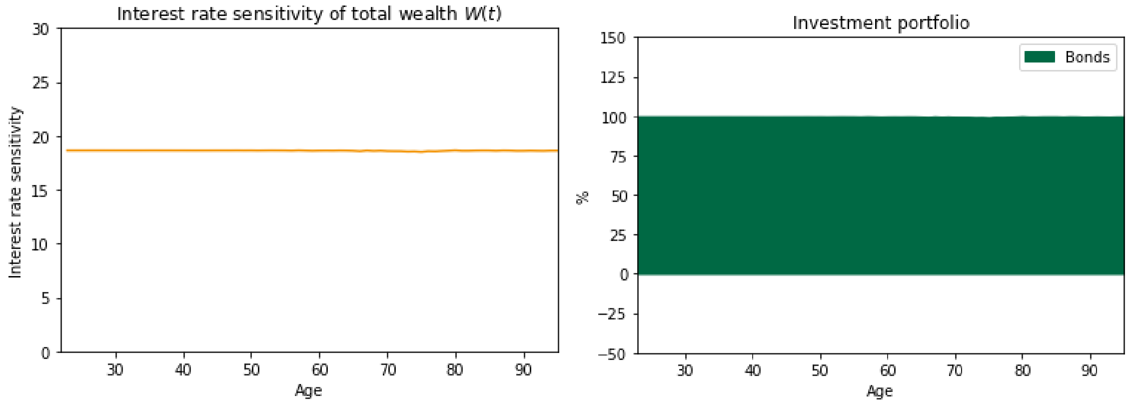
Besides, we see that the fraction of bond exposure seems to be negative for the first few years in this setting. This effect arises because in the median scenario, the interest rate is slightly negative for the first few years, resulting in a negative return on bond investments. As a result, it is optimal to go short in bonds for those years. We will see in figure 2.6 below that this result changes in a situation in which the long-term interest rate will be above the initial interest rate, implying a positive interest rate term premium as well.

Furthermore, we observe that the fraction of bond exposure is highest when approaching the retirement age, and then declines with age. In figure 2.3, bond exposure was expressed in terms of financial capital, and as exposure to a 30-year real bond. In the document of Centraal PlanBureau (2020) regarding the new Dutch pension agreement, it is mentioned that the investment portfolio of each participant will include a fraction to a bond portfolio with maturity specifically for that participant, based on his or her age. The age-specific maturity will be defined in section 3.1 and called natural maturity. For now, we will only discuss the effect of implementing an age-specific maturity on exposure to bonds, to show that bonds could provide full interest rate risk hedging in case the age-specific maturity would be available for each age cohort.

2.2.1 Effects of implementing an age-specific maturity

To show specifically the effect of the implementation of an age-specific maturity, we will have a look at the development of interest rate risk hedging in case we set $r(t) = 0$ and $H(t) = 0$ for all t, $\delta = 0$. Furthermore, we take a highly risk averse agent ($\gamma \to \infty$), and exclude old age pension guaranteed by the state. We can then already derive the the effect we will observe in the next figures from formula 1.1.21. The first component in $w_P^*(t)$ approaches zero due to $\phi_r = 0$. Second, in case of investing in a bond portfolio with an age-specific maturity for each age, the interest rate sensitivity of the optimal annuity factor $D_{A^*}(t)$ fully matches the interest rate sensitivity of the bond. Then, we will see that this setting results in an optimal asset allocation consisting of a portfolio that will be fully invested in bonds over the complete life-cycle, such that $w_P^*(t) = 1$ for all t.

Calculating this numerically, and implicitly assuming we have availability of a bond portfolio with age-specific maturity for all ages, we indeed observe a constant investment allocation of 100% to bonds over the life-cycle, displayed in the figures positioned on the next page.



(A) Interest rate sensitivity of total wealth for a nominal annuity.

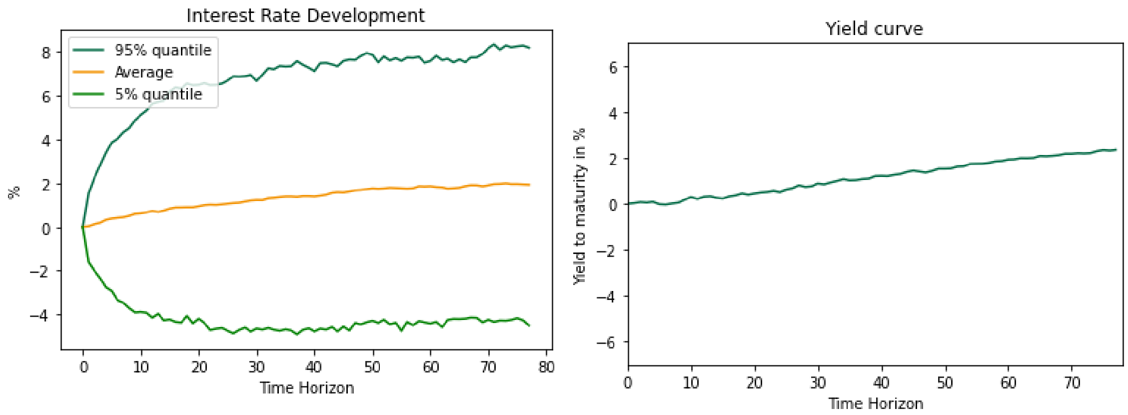
(B) Investment portfolio to provide a constant pattern of expected pension benefits in the setting where $r(t) = 0$ and $H(t) = 0$ for all t, $\delta = 0$, the agent is substantially risk averse and old age pension is excluded.

Hence, in case of absence of human capital, no guaranteed old age pension benefits by the state, and investing in a bond portfolio with maturity adjusted to the age cohort of the participant, we see that over the life-cycle you need to dedicate your total wealth fully to this bond portfolio. While doing this, we implicitly assumed that a bond portfolio with age-specific maturity is available for each age.

In general, we can split demand for bonds into two parts: speculative demand and hedging demand. Speculative demand arises in case the yield for bonds with longer time to maturity is higher than the yield for bonds that expire on a shorter term, implying the existence of a positive term premium. In that situation, one wants to benefit from the term premium by investing in bonds with a maturity date that is far into the future. The hedging demand arises because you would like to use another part of your wealth to form a hedge against interest rate fluctuations. In figure 2.3, as well as in figures 2.4a and 2.4b, the demand for bonds is fully covered by hedging demand. Speculative demand is excluded, since $r(0) = \bar{r} = 0\%$ and $\lambda_r = 0$.

2.2.2 Effect of speculative demand

In case we would partly adjust the parameter setting of figure 2.3 to the committee parameters by setting $\bar{r} = 2\%$ and the interest rate term premium equal to 0.75%, while keeping the initial interest rate at 0%, we would obtain another pattern in interest rate developments. This is displayed below in figure 2.5a. The term structure of interest rates changes to the expected yield curve in figure 2.5b.



(A) Development of interest rates $r(0) = 0\%$, $\bar{r} = 2\%$ and market price of interest rate risk $\lambda_r = 0.75\%$.

(B) Expected yield curve corresponding to the developments of the short rate in figure 2.5a.

Since the term premium becomes positive, the exposure to bonds will consist of both hedging and speculative demand. This will change the investment pattern displayed in figure 2.3 to the portfolio shares in the next figure

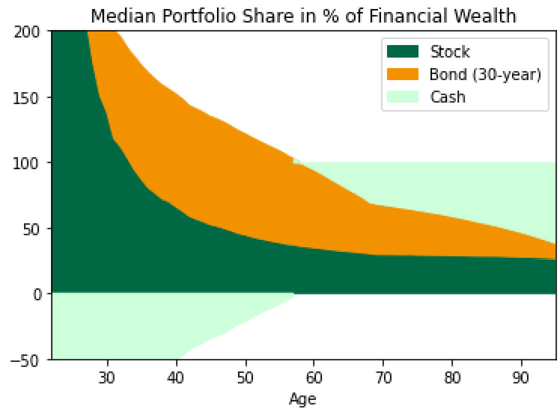


FIGURE 2.6: Optimal portfolio shares in terms of Financial Wealth with $r(0) = 0\%$, $\bar{r} = 2\%$ and market price of interest rate risk $\lambda_r = 0.75\%$.

In fact, the positive term premium makes bonds with longer time to maturity more attractive, and not surprisingly we see a substantial shift upwards in the optimal fractions to invest in bonds for younger participants, with the growth of a short position in cash as a result. Besides, figures 2.3 and 2.6 provides us again with another insight: For younger participants, it is still optimal to invest more than 100% in risky assets, which implies that they have to borrow money to invest in the stock market. This result was already derived in Bodie et al. (1992), and supported various times in later research such as Koijen et al. (2010) and Bovenberg and Mehlkopf (2014). Younger participants can carry a lot of investment risk because they will earn labor income for a longer horizon, which is relatively risk-free.

In the currently used Dutch pension (FTK) policy, it is implicitly possible to invest more than 100% in risky assets for younger participants, because there are particular distribution rules to determine the rate to which different generations are exposed to certain risks. The distribution rules of risk allocation can be adjusted such that pension entitlements never become negative. Exposure to investment risk of more than 100% is in that case not completely allocated to younger participants. Therefore, we call the investment cycle in the FTK contract an implicit life-cycle pattern.

Adversely to the use of the implicit life-cycle contract is that distribution rules do not guarantee the degree of flexibility in risk sharing compared to what would be optimal. However, in explicit life-cycle investing, the investment mix can be adjusted to the needs of every age cohort. Therefore, a pension contract allowing for investing with an explicit life-cycle provides a higher degree of flexibility in handling different investment policies for different ages, implying that risk sharing can be aligned with the desired pattern we obtain from scientific literature.

2.2.3 Changing maturity of the bond

Previously, the percentage exposure to bonds was expressed in terms of exposure to a 30-year real bond. As will be further explained in section 3.1, the model of section 1.1 allows us to interchange between bonds with different maturities to express bond exposure to, but this changes the fraction of bond investing for different age groups. We can display this effect by adjusting the time to maturity of the bond in the parameter setting of figure 2.6 to 10 years to obtain

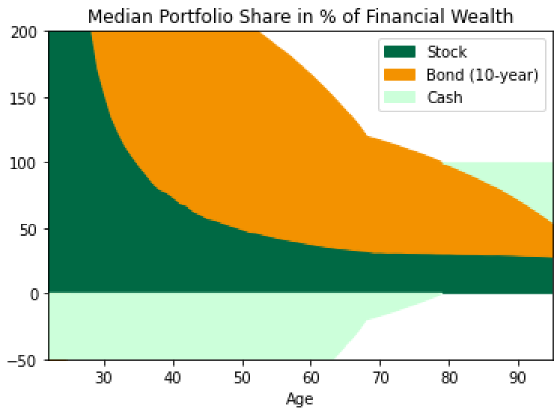


FIGURE 2.7: Optimal portfolio shares in terms of Financial Wealth in the same setting of figure 2.6 with optimal bond fraction expressed in terms of exposure to a 10-year real bond

We see that the fraction invested in bonds grows substantially in case we express it in terms of exposure to a 10-year real bond, resulting also in a considerable increase in the short position in cash. This also implies that the long-position in swaps is significantly larger here.

Because their retirement period is far into the future, younger participants bear more interest rate risk than the elderly. On the other hand, bearing interest rate risk is attractive in case the risk carries a positive risk premium. Therefore, even though younger participants bear more interest rate risk, it could be desirable to hedge a relatively lower part of this risk for these age cohorts compared to older age cohorts. Note also that, according to formula 1.1.10, the interest rate affects the value of human capital, and therefore the optimal exposure to equities by formula 1.1.25. To this end, we will now consider the level of interest rate sensitivity of accumulated pension wealth and human capital in the optimal setting of figure 2.3. Additionally, the percentage interest rate hedging of total wealth which is in line with optimal investment mix in figure 2.3 will be calculated.

2.3 Interest rate sensitivity of the Optimal Investment Mix

As already shortly mentioned before, we can measure the sensitivity of a bond or fixed income portfolio's price to changes in interest rates, which refers to the change of the portfolio in relative value per percentage point change in $r(t)$. In part 1.1, optimal expressions for the investment policy and consumption choices in the new Dutch pension contract were stated. We consider someone who perfectly invests and consumes according to those expressions and investigate the interest rate risk sensitivity of his total wealth in the financial setting where we implement equity risk and interest rate risk, but disregard inflation risk. The asset allocation over the life cycle that the participant follows in the optimal setting was displayed in figure 2.3.

The parameter setting that we will return to now is the one that was described in section 1.2 and used for figure 2.3, in which the fraction of investment in bonds is expressed in terms of exposure to a 30-year real bond. In this setting, we can compare the interest rate sensitivity of total wealth in the optimum with the interest rate sensitivity of total wealth of a nominal guaranteed annuity, to investigate which one demonstrates a higher level of interest rate sensitivity over the life cycle. The interest rate sensitivity of total wealth in the optimum is obtained by multiplying the fractions of exposure to bonds by formula 1.1.7 with the interest rate sensitivity of a 30 year bond, given by formula 1.1.7 with $h = 30$. This is, together with the interest rate sensitivity of a guaranteed annuity, displayed in the following figure.

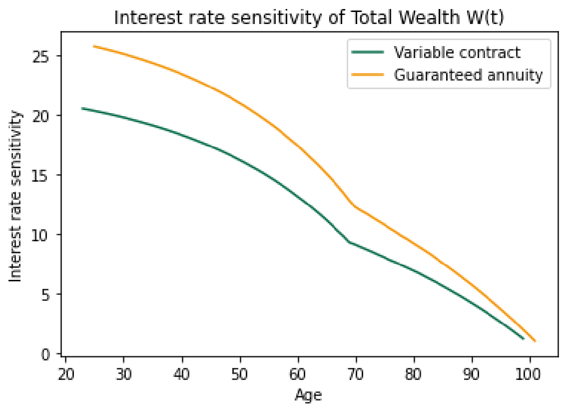


FIGURE 2.8: Interest rate sensitivity of total wealth in the optimal contract and the guaranteed annuity, in the financial market setting from section 1.2, $r(0) = \bar{r} = 0\%$ and the market price of interest rate risk equal to zero.

We now see that interest rate sensitivity shows a decreasing pattern over the life cycle, mainly because the maturity of the bond portfolio in this setting is equal to 30 years, and not adjusted to any form of age-specific maturity. Furthermore, we see that the interest rate sensitivity of the guaranteed annuity is above the interest rate sensitivity of the variable contract over the whole life cycle. In this setting, we have again two effects that can play a role in measuring the interest rates sensitivity: speculative demand and hedging demand. Speculative demand is not age-dependent, and in the parameter setting for figure 2.4a we excluded the speculative demand for both asset risk and interest rate risk, because $\lambda_S = \lambda_r = 0\%$. In the setting for figure 2.8, we only excluded speculative demand for interest rate risk, because $\lambda_r = 0\%$ but $\lambda_S > 0$ here. Including also speculative demand for interest rates would cause the interest rate sensitivity of the variable contract to increase, because long-term interest rates would be more beneficial than short-term interest rates, but investing in long-term products increases interest rate sensitivity of the portfolio. Hedging demand generally equals approximately 80% of the interest rate sensitivity of the fixed annuity, and therefore in case of only taking account hedging demand, the interest rate sensitivity of total wealth in the variable contract is below the interest rate sensitivity of the guaranteed annuity.

If we divide the interest rate sensitivity of total wealth in the variable contract by the interest rate sensitivity of the guaranteed annuity for all ages, we can conclude what the optimal percentage interest rate risk hedging in terms of total wealth should be. This is actually dividing the blue line by the orange line of figure 2.8, or using formula 2.3.1 but then with $D_F(t)$ replaced by the interest rate sensitivity of total wealth in the variable contract. The results for the different ages and corresponding horizons can be found in the figure below

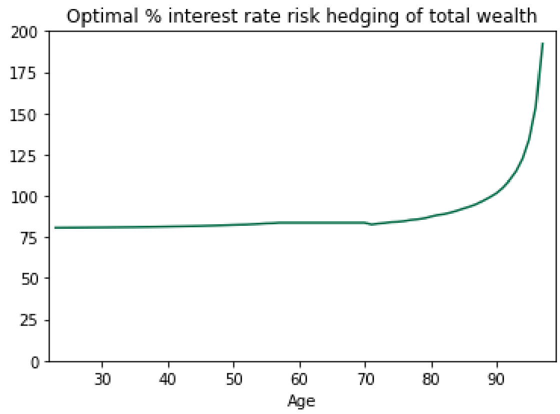


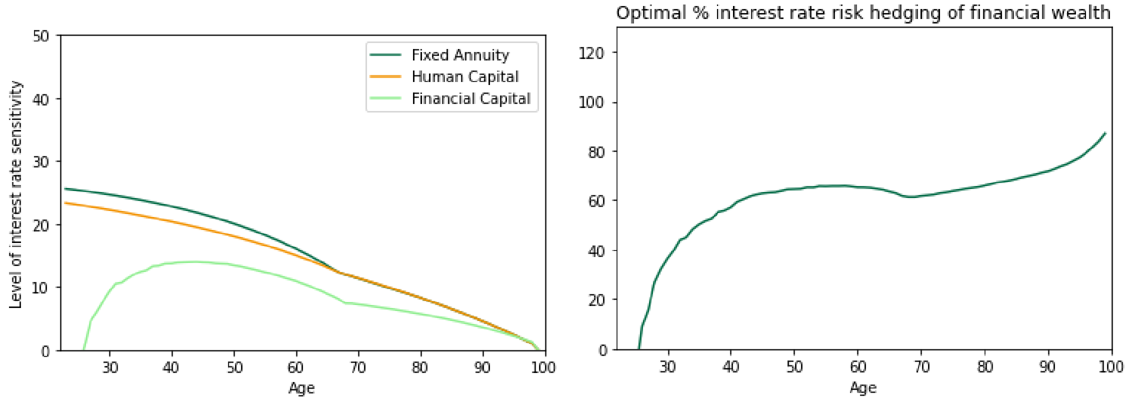
FIGURE 2.9: Optimal % interest rate hedging of total wealth, obtained by dividing the blue line in figure 2.8 by the orange line in the same figure.

The previous results are in terms of total wealth, which is obtained by taking the sum of human capital and financial capital. For human capital, we have already discussed that we can compute its optimal interest rate sensitivity using formula 1.1.11. Furthermore, we can calculate the optimal interest rate sensitivity for financial wealth, using the benchmark portfolio weights. To investigate whether the fact that we can only invests with financial capital results in a higher level of interest rate sensitivity in the optimum, we compare in figure 2.10a the interest rate sensitivity of human capital in combination with old age pension guaranteed by the state (OAP), financial capital and total wealth, under the optimal contract as described before. Next to that, figure 2.10b shows the percentage of interest rate risk hedging a fund should pursue for this participant in the optimal setting, which equals the interest rate sensitivity of financial capital as a percentage of the interest rate sensitivity of the guaranteed annuity.

$$\text{Optimal \% Interest Rate Risk Heding} = \frac{D_F(t)}{D_A(t)} \tag{2.3.1}$$

With

$$D_F(t) = \int_0^{T-t} \frac{F(t,h)}{F(t)} B_r(h) dh \tag{2.3.2}$$



(A) (B)

FIGURE 2.10: Interest rate sensitivity of optimal total wealth, financial capital and human capital plus old age pension for different ages and the corresponding optimal percentages interest rate hedging in terms of financial wealth.

From figure 2.10a, we can conclude that human capital and old age pension show lower interest rate sensitivity than total wealth before retirement. This is due to the fact that premium payments before retirement and guarantees after retirement are included in human capital, whereas the sensitivity of total wealth is also depending on the interest rate sensitivity of financial capital. At the start of the life cycle, financial capital is quite small and can even become negative in case of negative returns. We see that the interest rate sensitivity of financial capital is limited at the start of the life cycle, but increases quite rapidly. Additionally, from figure 2.10b we learn that optimally, the oldest participants should use a large part of their financial wealth as a hedge against interest rate risk, whereas for the younger age cohorts the share of interest rate risk hedging is lower.

Overall, we can conclude that the optimal interest rate sensitivity of the different financial products we considered is quite substantial, which emphasizes the importance of interest rate risk hedging. Note that this thesis is limited to one type of bonds. Under a model with both interest rate risk and inflation rate risk, there is a difference in behaviour of the price of nominal bonds versus real bonds, which would lead to different portfolio results by definition. Furthermore, the percentages of interest rate risk hedging in the figures are based on a situation where only equity risk and interest rate risk are considered, but in reality there are of course more external factors that influence the optimal percentage of interest rate risk hedging. Besides, there are a lot of practical issues, such as borrowing restrictions and illiquidity of products on the financial market, which provide that you cannot always hedge the desired amounts. This will be discussed in detail in section 4. Moreover, one does not only want to hedge against interest rate risk, but also against other type of risks, which will imply that the percentages of interest rate risk hedging in figure 2.10b will not always be realizable.

In the next chapter, we will extend the analysis from a single life-cycle policy to a collective investment policy of a pension fund, and design the collective bond and equity allocation for the fund. In a collective investment policy, it is important to take into account that you will invest for different age cohorts at the same time. Therefore, we will derive the optimal policy per age cohort and aggregate this to obtain the optimal collective investment policy.

Chapter 3

Collective investment in a pension fund

In the background document of Centraal PlanBureau (2020) about the new Dutch pension contract, there is a discussion regarding applying implicit or explicit allocation of risk in the new framework. To each participant with age i, a certain extent of hedging return is allocated, based on parameter x_i. This parameter represents the exposure of the specific participant to a basket of bonds, and the time to maturity of these bonds is determined by the age of the participant and his risk aversion.

This is different from van Bilsen and Mehlkopf (2020) and the setting we used in the previous section, because there the optimal bond allocation was expressed in exposure to a real bond with 30 years to maturity, instead of adjusting the maturity profile of the bonds with respect to age and remaining investment horizon. It is explicitly stated in the CPB document that the particular portfolio of bonds a participant is exposed to with rate x_i depend on the age of the participant. This is also intuitive, because if you are already 65 years old, you might not want to invest in a 40-year bond anymore because you have a relatively small chance of becoming 105, while as a 25-year old you would have invested in a 40-year bond.

To this end, in this section we will derive a method to calculate this age-specific maturity, based on a participant's future expected pension benefits. Furthermore, we will discuss a bottom-up approach to step from individual level to a collective pension scheme, which is something pension funds will have to get used to in the new Dutch pension contract. Currently, most of their strategies are based on top-down plans, which will in a few years not be as straightforward to apply anymore.

3.1 The use of adjusted maturities for different age cohorts

In the model of section 1.1, an equal exposure of all age groups to a real bond with 30 years to maturity infers that it must hold that

$$w_P^{(30)}(t)B_r(30) = w_P^{(h)}B_r(h) \tag{3.1.1}$$

Implying that we can interchange between exposures to bonds with different maturities according to

$$w_P^{(h)} = \frac{w_P^{(30)}(t)B_r(30)}{B_r(h)} \tag{3.1.2}$$

such that the size on total wealth $W(t)$ remains the same for any possible maturity h, and therefore also for the age-dependent time to maturity the Centraal PlanBureau (2020) suggests, which we will further refer to as the 'natural maturity' (NM) for any particular age-cohort.

To assign a natural maturity to each age cohort, we are going to match the interest rate risk that corresponds to the (future) pension annuity of that age cohort with the interest rate risk of the bond portfolio that will be invested in for this age cohort. Each participant can be assigned to a future investment horizon based on its age, an expected future cash-flow pattern depending on the current accumulated pension wealth and survival rates corresponding to its age cohort. If we denote the expected pension payments at age h based on the at age t already accumulated pension wealth by $CF(t,h)$, we can thus use the relation

$$CF(t,h) = {}_h p_t \cdot F(t) \tag{3.1.3}$$

to obtain the cash-flow pattern after retirement of age cohort t. By using the expected future cash-flows for each participant viewed from their current age cohort t, we can calculate the natural maturity of the bond portfolio which will be invested in for that particular age cohort. The natural maturity for age cohorts below the retirement age, which is 68 in this setting, is then defined in the same way as the Macaulay duration[1] and can be written for age group t as

$$NM_t = \frac{\sum_{h=68}^{120} e^{-r(h)} h \cdot CF(t,h)}{\sum_{h=68}^{120} e^{-r(h)} \cdot CF(t,h)} \tag{3.1.4}$$

The natural maturity for age cohorts above the retirement age equals

$$NM_t = \frac{\sum_{h=0}^{120} e^{-r(h)} h \cdot CF(t,h)}{\sum_{h=0}^{120} e^{-r(h)} CF(t,h)} \tag{3.1.5}$$

For those natural maturities, we can compute the interest rate sensitivity using formula 1.1.7. The natural maturities for each age cohort will be used in the next part of this chapter, and are displayed in the figure on the next page. In the remainder of this section, this natural maturity will be used to establish the median horizon of the bond portfolio and the exposure to this portfolio will be derived using formula 1.1.24, such that the interest rate sensitivity of the bond portfolio to invest in equals the interest rate sensitivity of the future expected cash flows.

[1]The Macaulay duration is the weighted average term to maturity of the cash flows from a bond, for which the weight of each cash flow is determined by dividing the present value of the cash flow by its price. In discrete time, the Macaulay duration is equal to

$$D_{Macaulay} = \frac{\sum_{t=1}^{n} \frac{C_t \cdot t}{(1+r)^t}}{\sum_{t=1}^{n} \frac{C_t}{(1+r)^t}}$$

and we translated this to the continuous time version for formulas 3.1.4 and 3.1.5 to stay aligned with the model in section 1.1.

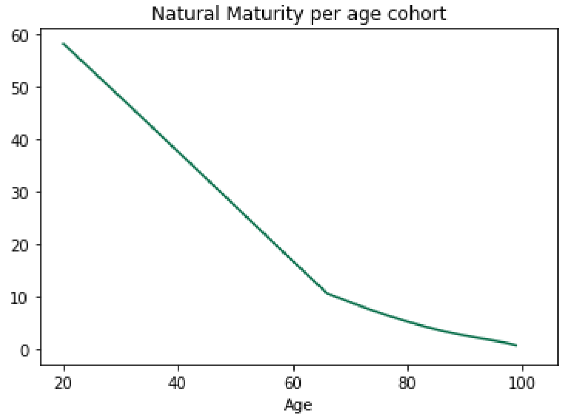


FIGURE 3.1: Natural maturity per age cohort for all age cohorts, using the interest rate developments as shown in figure 2.2a.

3.2 Collective risk management aggregated across age cohorts

3.2.1 Two examples of simplified closed pension funds

In a collective pension scheme, the natural maturity can be computed for all different age cohorts according to formulas 3.1.4 and 3.1.5. To build towards such a collective pension scheme, we will start by showing the collective results for a pension fund where only participants with age 25 are involved, which will be equal to the individual optimal investment pattern of a 25-year old participant. The survival rates for someone who is currently aged 25 can be determined by combining formulas 1.1.13 and 1.1.14, and accordingly we can use formula 3.1.3 to obtain the expected pension benefits pattern after retirement. On the base of those cash-flows, the natural maturity of the bond portfolio for the participants of this age cohort is calculated using 3.1.4.

In this section, we set the parameters to what was discussed in section 1.2, and we use the initially defined interest rate structure as in 2, where $r(0) = \bar{r} = 0$ and $\lambda_r = 0\%$. In case we invest for a particular age cohort in a bond portfolio matched to the natural maturity of the expected pension benefits for this age cohort, we should obtain that the bond exposure as percentage of total wealth should be exactly 100%, as we already observed in figure 2.4b. This result appears because the participant does not have any bonds in the speculative part of the portfolio, as $\lambda_r = 0$. What remains is the hedging portfolio, and you can reach complete hedging by investing your total wealth for 100% in the bond portfolio with the natural maturity.

To show that this result is still valid in case we calculate the natural maturity using the above procedure, take the pension fund where all participants are 25 years old as an example. Their bond exposure for the upcoming investment horizon is 100% of total wealth $W(t)$ to the bond portfolio with NM_{25} in the first year, 100% of total wealth to the bond portfolio with NM_{26} in the second year, and so on. For the remaining investment horizon of a 25-year old, the pattern of exposure to the bond portfolio with adjusted maturity each year is displayed in the next figure

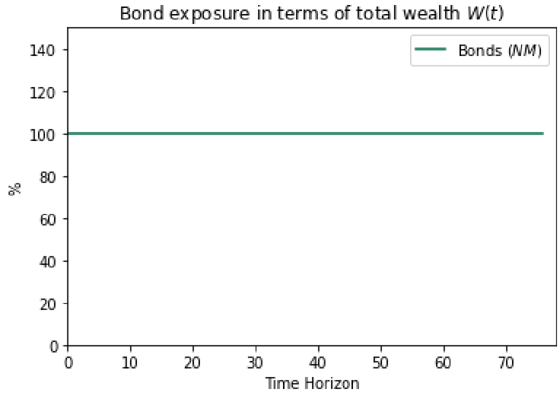


FIGURE 3.2: Investment fractions in terms of total wealth to a bond portfolio for which the interest rate sensitivity of that bond portfolio is matched to the natural maturity of the age cohort. In this figure, all participants start at age 25, and therefore the first bond portfolio has maturity equal to NM_{25}.

The investment pattern above is in terms of total wealth $W(t)$, but can be translated to the exposure to bonds in terms of financial wealth $F(t)$ according to formula 1.1.24. In terms of financial capital, we can, besides the desired fraction to bonds, also show the desired allocation to equities for age cohort that is currently 25, in which we expect to see the life-cycle shape as discussed in section 1 again. By using the parameter setting discussed in section 1.2 and the expressions for determining the optimal investment policy as derived in section 1.1, we obtain the asset allocation over the remaining life-cycle of the participant group that is currently 25, which is displayed in the next figure. Next to the asset allocation, the optimal fraction of interest rate risk hedging along the life-cycle is expressed by the blue line.

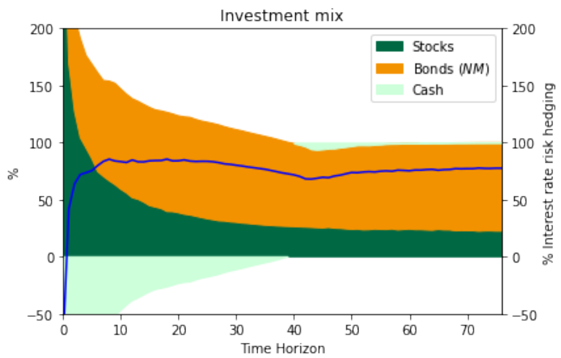


FIGURE 3.3: The collective investment policy of a pension fund in which all participants are currently of age 25, where the natural maturity of the bond portfolio matches the natural maturity of the expected pension benefits at each time point on the remaining investment horizon, starting from NM_{25}.

Again, the figure clearly displays the life-cycle investment pattern for equity investing we were talking about in section 1. Note that the results in figure 3.3 are the optimal investment results in the median expected financial market scenario, the exact future pension benefits will depend on the actual investment returns. We see that currently, the interest rate sensitivity of the pension assets is such small compared to the interest rate sensitivity of a guaranteed pension annuity, that it is not desired for this fund to apply interest rate risk hedging for this group at the first time point. This is in line with the individual results of a 25-year old we observed in figure 2.10b. Currently, the optimal investment portfolio consists for a large part of risky assets, and only a small fraction is allocated to bonds. As a result of the high exposure to risky assets, we see that the cash position, which is equal to $100\% - \hat{w}_P(t) - \hat{w}_S(t)$ for all t, will be negative for almost half of the remaining investment horizon. In practice, a large negative cash exposure might be a reason to consider investing a lower fraction in bonds than optimal theory would suggest, especially in a situation in which the term premium would be positive.

Second, imagine another pension fund that does consist of only 60 year-old participants and will reach the retirement age in a couple of years. Again, we can calculate the optimal investment and hedging policy for the participants of age 60 over their remaining life-cycle, and as a result we obtain the investment mix displayed in the next figure. Additionally, the optimal fraction of interest rate risk hedging along the life-cycle is expressed by the blue line through the figure.

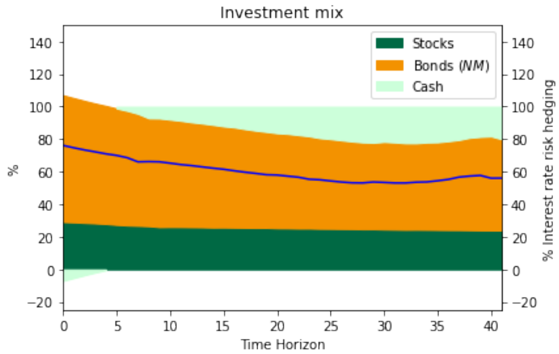


FIGURE 3.4: The collective investment policy of a pension fund in which all participants are of age 60, where the natural maturity of the bond portfolio matches the natural maturity of the expected pension benefits, starting from NM_{60}.

If we compare the results for a cohort with age 60 to the investment mix of the cohort with age 25, we see that the percentages of exposure to risky assets are lower, which is in line with the theory from section 1 that the risk profile of a participant should be less risky as the participant becomes older. Therefore, the portfolio mix of the cohort with age 60 consists mainly of bonds, whereas the investment portfolio for the cohort with age 25 consists mostly of stocks for the first few years. Furthermore, we see that the current desired fraction of interest rate risk hedging for this fund is slightly below 80%.

3.2.2 Combination of simplified funds for a collective investment mix

We can also combine those two age groups to obtain a pension fund that consists of a group of participants with age 25 and a group of participants with age 60, and then aggregate the pension policies of both age cohorts over the remaining investment horizon to obtain the collective investment policy for such a pension fund. If we consider $F(t,h)$ to be the accumulated wealth position of participants in age cohort t we can aggregate the fraction of bond exposure for horizon h by using

$$\hat{w}_P^{(h)} = \frac{F(t,h)}{\sum_t F(t,h)} \hat{w}_P^{(h)}(25) + \frac{F(t,h)}{\sum_t F(t,h)} \hat{w}_P^{(h)}(60) \quad \text{for all } h. \tag{3.2.1}$$

Which can be rewritten in terms of the Centraal PlanBureau (2020) as

$$x_t^{(h)} = \frac{F(t,h)}{\sum_t F(t,h)} \hat{w}_P^{(h)}(t) \quad \text{for all } h. \tag{3.2.2}$$

And the same for exposure to risky assets y_t in relation to $\hat{w}_S(t)$. We will then get a result over the upcoming investment horizon, starting from time point 0, which we consider to be now. In general, the financial capital of a 25 year old equals around 6% of the financial capital of a 60-year old participant. For simplicity, we suppose that half of the participants is 25-year old, and the other half of the participants in the fund is 60 years old. This implies that definitely the largest part of the wealth in the fund belongs to the age cohort with age 60, causing that their investment mix will weight heavier on the collective fund mix, according to formula 3.2.2. The results for the collective pension scheme of this fund are displayed in the next figure.

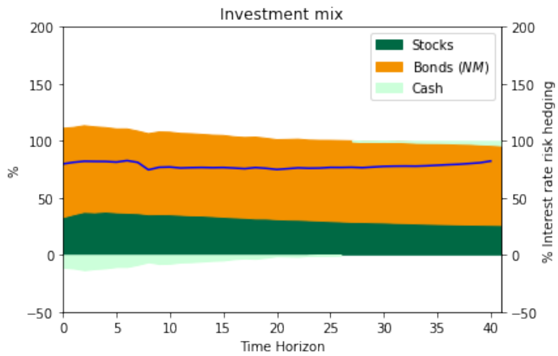


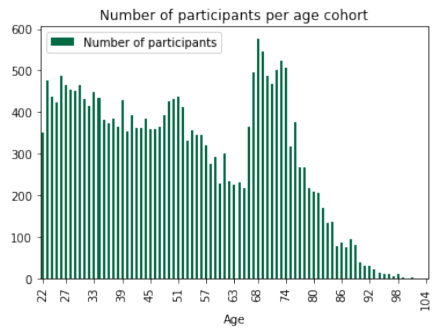
FIGURE 3.5: The collective investment policy of a pension fund in which half of the participants has age 25 and half of the participants is 60 years old, and where the natural maturity of the bond portfolio for each of the age cohorts matches the natural maturity of the expected pension benefits for this age cohort. At the starting point, the total bond exposure consists of 2% invested in the bond with the natural maturity corresponding to age 25, and 98% is invested in the bond with the natural maturity corresponding to age 60. Along the investment horizon, the capital of the young age cohort increases, and the weight on their bond portfolio increases.

The first insight we obtain is that in this figure, the optimal bond exposure is somewhat higher, but the desired fraction of interest rate risk hedging is slightly below the optimal hedging fraction in 3.4. This is the case because the optimal fraction of interest rate risk hedging of the 25 year-old participants is substantially lower than for the 60-year old participants, as we saw in 3.3 and 3.4, and this fund represents an aggregation of the two groups. The aggregation for this figure is performed on financial capital, and the age cohort with age 25 has relatively low accumulated financial capital compared to the 60-year old cohort, therefore the collective investment policy based on the aggregation of the two groups looks more similar to figure 3.4 than to 3.3.

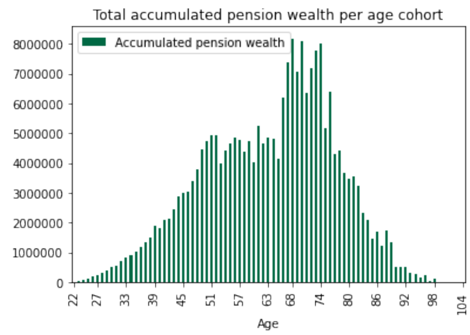
3.2.3 Aggregation across all age cohorts

The procedure that was used in section 3.2.2 for the set-up for the collective investment mix in figure 3.5 can of course also be implemented for all age cohorts within a Dutch pension fund, in general consisting of a lot more different age groups and where all age cohorts include another number of participants. Next, we will actually use the collective data of a Dutch pension fund, and explain how to set up a collective investment policy and show to which extent the collective investment policy depends on the composition of the fund.

From all participants of this fund, we have data such as their age, gender, income and already accumulated pension wealth. Using the participant info, we can group all participants with the same age to one age cohort, and sum their individual accumulated pension wealth to obtain the total accumulated pension wealth for each of the age cohorts. After performing these operations for all age cohorts, we obtain the distribution of participants of participants displayed in figure 3.6a and total accumulated pension wealth per age cohort in figure 3.6b.



(A) Number of participants per age cohort



(B) Accumulated pension wealth per age cohort

FIGURE 3.6: The distribution of participants and their wealth across the different age cohorts in one of the Dutch pension funds. This fund represents an average division in a fund in the Dutch pension sector.

The data displayed in figure 3.6a and 3.6b are for the entire pension fund, including both active and retired people, as well as men and women. According to the AG2020 table, there are different survival rates for men and women. Regarding the calculations for a collective investment policy for this pension fund, we will use the male-female division in the fund as weights for the survival rates of both groups, to come up with a weighted average, gender-neutral survival rate approximation [2].

To set up a collective investment policy for this pension funds, we will start with calculating the optimal asset allocation for each age cohort dependent on age and accumulated pension wealth, in the same way as the example settings before. For all ages, we can start with calculating the corresponding survival rates using formulas 1.1.13 and 1.1.14. According to formula 3.1.3, we can obtain the expected cash-flow pattern after retirement for each age cohort. On the base of this, we use the derivations in 3.1.4 and 3.1.5 to determine the natural maturity corresponding to each age and therefore interest rate sensitivity of their future cash-flows, which is matched to the interest rate sensitivity of the bond portfolio there will be invested in.

After this, we can simulate for all age cohorts in which way total wealth, human capital and financial capital are expected to evolve, based on already accumulated wealth per age cohort. Then, we can use the expressions for the optimal investment policy in formulas 1.1.24 and 1.1.25 to determine the optimal collective investment mix per age cohort. The

[2] It is not yet known whether the upcoming regulations will allow to separate men and women while calculating expected future pension benefits. Therefore, we use a weighted average of the survival rates in this thesis with respect to the division of men and women in the pension fund.

investment mix per age cohort is obtained in a similar way as in the simplified closed funds in section 3.2.1.

In the figures for those simplified funds, we had a look at the current investment decision, as well as at the development of the investment policy over the remaining investment horizon. In general, the development of the investment strategy over the time horizon as was shown in figure 3.5 is especially important for a closed pension fund, implying that the pension plan as displayed in this figure is only open to the current group of employees. Furthermore, the development of the investment strategy is based on the current assumed parameters. Any substantial change to either one the expected parameters for the future, or the composition of the pension fund (implying that a fund is open to enter for new participants) desirably results in an adaption to the prescribed investment strategy.

For any open pension fund, especially the time-zero point of such a figure is of importance ($t = 0$), because that displays the investment decisions that must be made now, and they will constantly rebalance the current investment decision in the future. For an open fund, the collective investment policy as represented in figure 3.5 can be set up at any time and after every change in the composition of the pension fund, to determine what the current ($t = 0$) strategic investment mix should look like. In case the composition in a fund remains quite stable over the years due to gradual inflow and outflow of participants, the time-zero investment policy is in fact the only relevant investment decision, because this is the investment policy the fund should follow as long as they believe that there will be no substantial changes in one of the current financial parameters.

Then, by taking the optimal investment policy for all age cohorts in figures 3.6a and 3.6b together and aggregate according to 3.2.2, we can calculate the collective fund mix of the fund, based on the current composition of participants. Having a look at the resulting current investment mix, which is shown in table 3.1, we observe a desired exposure to equities of around 33%, and the desired fraction of financial capital that must be invested in bonds equals approximately 116%. To show that the current investment decision shows a substantial dependence on the composition of the pension fund, we will compare those results, which are for a fund with an average age composition, to the current investment decision in a 'green' and a 'grey' fund.

In the green fund, most of the participants are in younger age cohorts up to age 35, implying that most of the total collective pension assets of the fund consists of the accumulated pension wealth of the young participants. For the grey fund, we take a grey fund with respect to working participants, implying that most of the people involved are between age 50 and the retirement age and the collective pension assets are mostly formed by their accumulated pension wealth and the pension wealth of retirees. In the next table, the composition of the different funds is expressed, as well as the corresponding optimal allocation to bonds, equities, and the optimal fraction of interest rate risk hedging for each fund[3]. To provide the collective allocation to bonds, the optimal exposure of each age cohort is calculated as exposure to the bond portfolio with natural maturity corresponding to that age cohort, as displayed in figure 3.1. This is then aggregated to come up with the desired investment mix, according to formula 3.2.2.

[3]Optimal level of interest rate risk hedging: The interest rate sensitivity of pension assets divided by the guaranteed interest rate sensitivity. This is similar to the calculations for figure 2.10b.

Fund	Members % Active/% Retirees	Wealth % Active/% Retirees	Actives Average age	Retirees Average age
Green	92.4% / 7.6%	86%/14%	34	75
Average	70.6%/29.4%	52.7%/47.3%	42	75
Grey	61.8%/38.2%	42.3%/57.7%	49	76

Average age of fund	Average Maturity of total fund, in years	Risky Assets in %	Bonds in %	Hedging in %
37	43	47	171	72
52	29	33	116	74
59	20	29	105	77

TABLE 3.1: Statistics of the fond composition and optimal collective investment allocation for three different pension funds: A simulated green and grey fund, and one of the Dutch pension funds, with a composition that represents an average Dutch fund.

For the average fund, we already highlighted the investment mix, in which we found an optimal exposure to risky assets of approximately 33%, and an exposure to bonds of around 116% in terms of financial wealth. While performing the same calculations for a green fund, we find a collective exposure to risky assets of approximately 47%, and an exposure to bonds of around 171%, which is both substantially higher than for the average fund. For the grey fund, we find an optimal collective exposure to risky assets of 29%, and an exposure to bonds of 105%, which is both lower than the results for the average fund. We see substantial differences in the investment mix, especially in the exposure to bonds. Therefore, we can conclude that the collective investment decision and interest rate risk hedging policy in a fund demonstrates a relevant dependence on the age composition of the fund. Furthermore, we learn from figure 3 in Mehlkopf and van Bilsen (2020) that the percentage of desired interest rate risk hedging can vary highly across different parameter settings, especially for younger age cohorts.

In the next part, we will therefore compare the results of the current investment decision of the Dutch fund, that represents the average fund in table 3.1, to the actual investment policy this fund currently approaches, and explain the similarities and differences with respect to our optimal results.

3.2.4 Comparison to current collective investment policy of the fund

In fact, we can compare our optimal results for the Dutch pension fund to the lastly published collective investment mix of the pension fund in this case-study, based on their annual report of 2019 and their strategic investment plan for 2020. We learn that they approach an exposure to stocks of 34%, which is quite close to the optimal exposure according to our model. Notice that this 34% is their exposure to stocks only, and that stocks is the only risky asset we consider in our model. In case we add the investments of the fund in private equity, real estate and infrastructure to obtain the fund's total share in risky assets, we see that they approach a position to equities of almost 50% in total. The fund's exposure to fixed income securities, including cash position, equals 50% in total. The current exposure specifically to bonds is approximately 38%, which is quite far from the optimal exposure of 116% that our model would suggest. Furthermore, the fund hedges only 25% of their total interest rate risk in practice.

There are a couple of explanations for the difference in the desired amount of interest rate risk hedging according to table 3.1 and the fund's currently accomplished allocation for interest rate risk hedging. First of all, a pension fund might want to lower the part of interest rate risk they would like to hedge, based on particular investment beliefs. Especially for younger participants, in case the pension fund's investment beliefs include a certain belief on the development of the real interest rate: if they expect the real interest rate to increase in the upcoming decades, expanding the hedge against interest rate risk will have a higher cost than the benefits of the hedge, and hence they will decide to lower the hedge on interest rate risk. A similar explanation for the difference in investment policy is that the fund's investment strategy is based on another appetite for risk than the assumed risk aversion parameter in our model. A slight decrease in the risk aversion parameter results into a considerable change in optimal investment policy, with much more equities, a smaller level of interest rate risk hedging and a lower position in bonds. These effects can be simulated with a sensitivity analysis on a varying risk-aversion parameter, which we will show in section 5.1.

Furthermore, our model excludes inflation rate risk, and wealth is only allocated to stocks, a bond portfolio with inflation-linked bonds or to the cash position. In practice, the fund bears beside interest rate risk also inflation rate risk for the pension entitlements and therefore also will need part of the pension assets to provide protection against this type of risk. To hedge against inflation rate risk, other financial instruments could be considered, but those are not included in our allocation opportunities. In a world where also inflation rate risk is present, there are multiple scenarios including common relationships between inflation and interest rate in which one should actually prefer a lower hedge, explained by van Bilsen et al. (2019).

Additionally, we did not consider any borrowing restrictions yet, which will be done in section 4. Exclusion of borrowing restrictions implies that the cash position can become infinitely negative, but a large amount of negative cash is often not what is implementable in practice. Together with the other described explanations for the difference in collective bond investing between theory and practice, we can conclude that restrictions to model or parameter assumptions declare a lot of the difference. To this end, in section 4 we will discuss multiple market and model limitations, and investigate the ways to deal with or get around those. As already mentioned, this will include the discussion of borrowing restrictions and illiquidity of products for a hedge against interest rate risk.

Another important note to make here is that the time to maturity of the bonds in which the fund currently invests differs from the natural maturities as calculated in section 3.2. The main reason for this difference is the limited offer of bonds on the financial market with a maturity that correspond to the natural maturity of our younger age cohorts. The natural maturity of the youngest age cohort of this fund equals approximately 58 years and then lowers with age, as visible in figure 3.1. Currently, the financial market covers substantial offer of real bonds with maturities up to 30 years, but for longer maturities this offer is quite limited. Therefore, we need to notice that the mentioned 38% bond allocation of the fund is to bonds that differ in time to maturity from the bonds that we allocate wealth to in table 3.1.

In the remainder of this chapter, we discuss the liquidity of bonds up to certain maturities, and the availability of alternative products in which a pension fund can invest to achieve the desired amount of collective interest rate risk hedging.

3.3 Liquidity of bonds and availability of swap contracts

In the presented figures of this chapter, we expressed the fraction of wealth allocated to bonds in terms of exposure to a bond portfolio with a natural maturity for each age cohort. In fact, the market for inflation-linked bonds does not have the level of liquidity for all financial products that is desired on the investment horizon of a pension fund. Therefore, this section discusses the availability of inflation-linked bonds in different countries and other products that are available to hedge against inflation rate risk, based on studies from European Securities and Markets Authority (2014) and European Systemic Risk Board (2017).

In section 2.3, we expressed bond exposure in terms of exposure to a 30-year and 10-year real bond. In fact, in the Netherlands, the government issues only nominal bonds, which are available up to 30 years, and currently there is no issuance of inflation-linked bonds at all. Broadening the borders to all Euro-countries issuing nominal bonds, we find that the size of the EUR AAA and AA-rated nominal bonds is around 4,500 billion euros[4]. But, holding a nominal bond results in payments of a fixed amount, rather than a fixed inflation-adjusted value. In this thesis, we consider interest rate risk management and discussed that this implies the use of inflation-linked bonds instead of nominal bonds, for which we see a substantial decrease in the issuance.

In the Euro zone, there are other countries in which the government issues also inflation-linked bonds, for example in Germany, France and Great Britain. The size of Euro AAA and AA-rated inflation-linked bonds comes mainly from German and French issuance and equals around 262 billion euros, which is far below the size of the nominal bond market. In Germany, inflation linked bonds are available up to 30 years, and the issued bonds in France are available up to 50 years[5]. What need to be noticed regarding this is that the German real bonds are AAA-rated, whereas the bonds from France are only AA-rated, implying that the German bonds are associated with a lower level of risk than the French ones. Also Spain and Italy provide some inflation-linked bonds, but their credit ratings are substantially less, causing that most institutional investors do not desire to have these in their portfolio for interest rate risk hedging. In Austria, there are some nominal bonds available up to 100 years, but this offer is way too limited to build portfolios on for the Dutch pension sector.

The statement that the offer is too limited to build investment portfolios on does not only hold for the 100-year Austrian real bonds, but in general, the offer of long-term inflation-linked bonds falls short of the demand from the Dutch pension funds. The total level of Dutch pension assets is around 1,550 billion euros. Approximately 20% of this is invested into bonds, implying that the demand from the Dutch pension sector to bonds is around 310 billion euros. Taking into account that the Dutch pension sector is not the only pension sector that preferably invests in inflation-linked bonds, but that there are many more other investors, such as insurers, that also would like to have inflation-linked bonds in their investment portfolios, the demand for inflation-linked bonds is considerably large compared to its issuance. Therefore, a fund has the need to investigate other investment options for interest rate risk hedging.

[4] From Barclays data: https://www.barclays.co.uk/

[5] The details on inflation-linked securities in Germany are available on https://www.deutsche-finanzagentur.de/en/institutional-investors/federal-securities/inflation-linked-securities/ and for France at https://www.aft.gouv.fr/en/encours-detaille-oatei

To achieve the desired interest rate risk hedging position, an investor can then consider the implementation of swaps in the investment portfolio. Inflation-linked swaps are trade-able up to 40 years to maturity, and nominal European interest rate swap contracts show significant trade-ability up to 50 years to maturity. Interest rate swaps offer possibilities to hedge over a longer period of time than other interest rate derivatives, but in case of increasing interest rates the investor will not benefit from holding an interest rate swap, while he would benefit in case of holding a long-term bond.

In the report of European Systemic Risk Board (2017), different measures are used to express the liquidity of swaps. It is first noted that the last deep and liquid point (LLP) of EUR inflation-linked bond is at a maturity of 20 years, and that after 20 years the level liquidity decreases, which we also concluded from the data regarding German and French inflation-linked bonds. Furthermore, their evidence suggests that Euro swap markets are deep and liquid up to 30 years. In the figure that is presented below, we see the relation between average total daily turnover and median price dispersion with respect to time to maturity of a swap contract. Price dispersion measures the variation in swap prices everyday. Variation in prices across the day is driven by different effects, and one of those effects is market impact through trade size. European Systemic Risk Board (2017) reports that a high value of price dispersion for swaps of a certain maturity is related to a lower level of market liquidity.

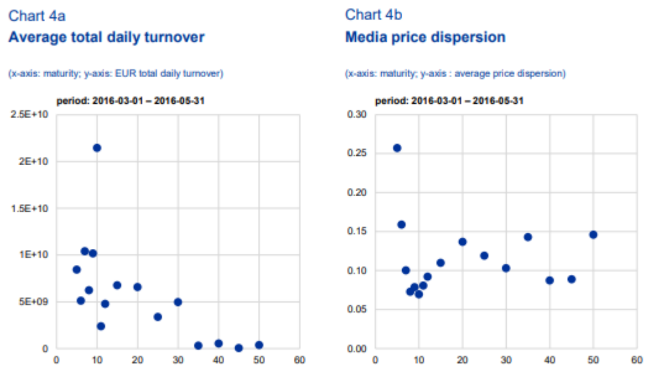


FIGURE 3.7: Results of European Systemic Risk Board (2017) to measure liquidity in the Euro swap markets.

These figures show that in the swap market, there are frequent trades in swap contracts with time to maturity up to and including 50 years, and also the other measures European Systemic Risk Board (2017) uses show support for those findings. Most of the liquidity is in swaps up to 30 years of maturity, and some less liquidity on the 40- and 50 years horizon. This implies that for maturities up to 50 years, there is ability to use swaps to achieve the desired interest risk rate hedging position. In figure 3.1, we saw that only in the youngest age cohorts, the investment horizon goes beyond these 50 years, and for the older age cohorts it is not necessary to invest in such long-term financial products anymore.

Additionally, at the start of this subsection, we discussed that the nominal bond market in the Euro zone covers around 4,500 billion euros. In fact, if a pension fund buys a nominal government bond of e.g. Germany or France, and at the same time an inflation-linked swap, the result is similar to buying an inflation-linked bond. This creates again opportunities for another step in interest rate risk hedging. However, the inflation-linked swap market is also not of greater size than the 262 billion euros of inflation-linked bonds, and therefore still too limited for all Dutch pension funds to cover their interest rate risk, not even talking about all other European pension funds and insurers.

This reduced market liquidity is probably the main reason why the explanatory memorandum for the new Dutch pension contract, published on December 16, 2020 by Ministerie van Sociale Zaken en Werkgelegenheid (2020), hardly makes reference to inflation risk or the use of inflation-linked bonds. Almost all references in the document are made to nominal interest rate risk and the use of nominal bonds and interest rate swaps.

In conclusion, we discussed that unless the market for inflation linked bonds with maturities above 50 years, we can cover a substantial part of interest rate risk hedging with inflation-linked bonds and investing in swap contracts. But, we can definitely not meet the full demand of the Dutch pension sector. Furthermore, especially for the younger age cohorts we cannot trade in financial products with a time to maturity desired across their investment horizon. In the next chapter regarding market limitations, we will discuss investment under borrowing restrictions, and show the difference in investment policy for particular age cohorts in case we can only invest in financial products up to a certain time to maturity.

Chapter 4

Market limitations

In this chapter, we will discuss multiple limitations a pension fund will probably face while investing in the financial market. A particular situation is assigned as a market limitation if it limits the pension fund in performing the optimal investment strategy for a participant. In fact, there are several effects that can serve as a barrier, and therefore a lot of scientific literature has already been committed to market limitations. In section 3.3, we discussed that illiquidity of real bonds with a particular long maturity forms a problem for pension funds, as their investment horizon runs quite far into the future, and the maturity of currently available bonds does often not meet this horizon. Besides this, Goyenko and Ukhov (2009) reports that also monetary policy impacts illiquidity of both bond and stock markets. Their evidence suggests that bond illiquidity seems to form an intermediate route amongst which monetary policy shocks are carried into the stock market. Furthermore, Bao et al. (2011) establishes a relationship between bond illiquidity and their prices. The level of illiquidity of a bond explains a substantial part of the yield spread of that particular bond. Investors that want to access bonds with a higher level of illiquidity will probably face higher bond prices and substantial uncertainty in yield.

Another example of a frequently faced market limitation is addressed by Kearns et al. (2010) and refers to the transaction costs that comes along with investing in the financial market. They highlight the tension between those costs and high-frequency trading, often required to rebalance the fund's portfolio to the optimal investment strategy. Transaction costs are an example of trading costs, which also consist of other pieces such as fees and taxes that can e.g. be imposed by the government.

In this thesis, we will focus on two other market limitations. In 4.1, we will elaborate on borrowing constraints, which is also a limitation that could be imposed by the government or the regulator. We will explore the welfare losses of borrowing constraints with respect to the individual, unrestricted optimal setting. Next to the results from our mathematical model, we will expand a few frequently applied practical investment strategies under borrowing constraints, and investigate their welfare losses with respect to the results in our optimal unrestricted setting. In section 4.2, we will step into the collective investment policy we discussed in section 3, and consider the welfare losses of borrowing constraints in combination with unavailability of long-term bonds.

4.1 Borrowing restrictions

In section 2, we already shortly discussed this policy implication for the execution of the fund's optimal investment allocation: the rules regarding whether it is allowed to invest more than 100% of your wealth in (risky) assets. In this part, we will explore the consequences for the fund's investment and hedging policy in case we would impose borrowing restrictions: you are not allowed to borrow money to invest with, implying that short positions will vanish and you cannot invest more than 100% of your financial capital. Remember that after figures 2.3 and 2.6, we discussed that it would be optimal for a currently 20-year old participant to invest more than 100% of his or her financial capital in stocks and the 30-year real bond until approximately the age of 55. Considering a bond with a shorter time to maturity, we saw that even until a higher age it would be optimal to have a short position in cash, also implying a long position in swap contracts.

In an explicit life-cycle contract, where the risk and returns of a particular cohort are of direct impact to the participant's accumulated pension wealth, it is not always allowed to invest more than 100% in risky assets. During a depression, a fund can face massive declines in stock prices, and a participant's pension entitlements could become negative in case more than 100% of someone's wealth was invested in risky assets. Of course, regulators want to protect upon this, and could therefore decide not to allow investing more than 100% of your financial wealth in risky assets.

In case we impose borrowing restrictions on both stock and bonds, consequences for the optimal investment allocations will follow. A first observation is that, by investing not more than 100%, our short position in cash vanishes, since this position was defined by 100% minus the sum of the exposure to stocks and bonds. The dropout of the short position in cash implies disappearance of the long position in swaps, while the position in swap contracts provided part of the hedge against interest rate risk for the fund. Despite the fact that one does not have to borrow money through a position in swaps, the risk that financial capital becomes negative in case of a large downward shock still remains. Next, we will show the welfare losses in three different model settings; a setting with only one risk factor that is stock market risk, a setting with stock market risk and interest rate risk as risk factors and exclusion of speculative demand, and the setting with again stock market risk and interest rate risk as risk factors, but including both hedging and speculative demand.

4.1.1 Costs of limitations on equity exposure and interest rate risk hedging

In Bovenberg et al. (2007), it is reported that in a setting where stock market risk is the only risk factor, the presence of borrowing constraints result in a welfare loss of approximately 2.8%. In their setting, the risk-free interest rate was set at a fixed level of 2%. This implies that there is one argument for the effect that borrowing restrictions result in a welfare loss, which is the fact that you cannot invest the desired fraction into stocks. In our model, there are two arguments where a welfare loss is based on: not being able to commit the desired fraction to risky assets, and not approaching the desired percentage of interest rate risk hedging. Furthermore, the fixed interest rate of 2% is also used to describe the dynamics of bond returns, implying a positive premium on investing in bonds. In our model, the term premium on investing in bonds equals 0. Besides that, we need to denote that in Bovenberg et al. (2007), the rate of time preference was set to 2%, while in our model we worked with $\delta = 3\%$.

In this thesis, we used the model of Brennan and Xia (2002) to set up an optimal investment portfolio, extended by the implementation of survival rates. In this model, stock market risk is not the only risk factor, but also the interest rate is time-varying, implying that an investor is exposed to interest rate risk. As already mentioned, this also imposes that limits on interest rate risk hedging will result in an additional welfare loss under borrowing constraints. Furthermore, we also discussed that in a world with a time-varying interest rate, we can separate the demand for bonds in a speculative and hedging demand. Next, we will explore to which extend including both types of demand results in additional welfare losses.

The welfare losses we mention will be expressed in terms of certainty equivalent consumption. The level of certainty equivalent consumption is the constant, certain consumption level that achieves the same utility level as the various stochastic consumption streams over the life cycle (Bovenberg et al. (2007)). In our setting, we can calculate utility over the life cycle using formula 1.1.17. Then, the certainty equivalent consumption level is defined as follows

$$ce = \left(\frac{U}{\int_0^{120} e^{-\delta t} \frac{1}{1-\gamma} dt} \right)^{\frac{1}{1-\gamma}} \tag{4.1.1}$$

And welfare losses will be expressed in the decrease in certainty equivalent consumption as a consequence of a certain restriction.

In fact, we consider the optimal investment mix as derived in figure 2.3, and compute the welfare loss in case we would impose the borrowing restriction on this asset allocation. In the financial setting for figure 2.3, stock market risk and interest rate risk are present, but speculative demand is excluded by setting the risk premium on investing in bonds equal to zero. In fact, while imposing the borrowing restriction on the asset allocation, we observe a welfare loss of approximately 5.9% in terms of certainty equivalent consumption. Furthermore, in case we implement a positive risk premium of 0.5% on investing in bonds, we are in the situation in which both stock market risk and interest rate risk are present, and where speculative demand is included. In this setting, the demand for bonds consists of both hedging and speculative demand, implying that the first part in formula 1.1.21 is non-zero. Considering the setting including speculative demand, we observe a welfare loss of approximately 6.7% due to borrowing constraints. The next figure displays the steps in the degree of welfare loss for these three financial settings

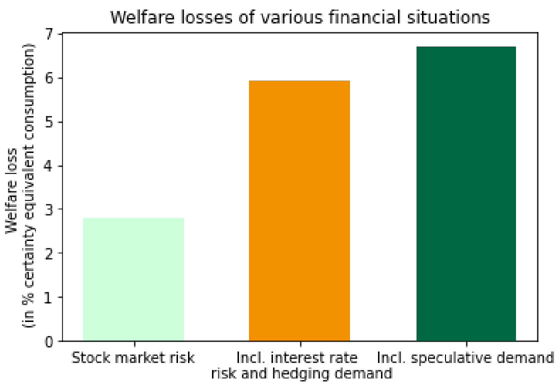


FIGURE 4.1: Welfare loss of borrowing constraints in three different financial settings: the setting has only stock market risk as risk factor, the second one includes stock market risk and interest rate risk as risk factors and excludes speculative demand for bonds, and the third setting has again stock market risk and interest rate risk as risk factors, but includes both hedging and speculative demand for bonds.

In summary, under borrowing constraints we observe the existence of additional welfare loss in case of implementing more financial effects. We learn from Bovenberg et al. (2007)

that the welfare loss of borrowing constraints equals 2.8% in terms of certainty equivalent consumption in a world where stock market risk is the only present risk factor. In the setting where we also impose a variable interest rate, and we exclude speculative demand for bonds, this welfare loss increases to 5.9% in terms of certainty equivalent consumption. This shows that the welfare loss arising from the fact that you cannot approach the desired fraction of interest rate risk hedging is around $5.9 - 2.8 = 3.1\%$, indicating that it is similar to the welfare loss of not being able to commit your desired fraction to risky assets. Additionally, by also implementing speculative demand for bonds in a world with stock market risk and a varying interest rate, the welfare loss increases again to 6.7% in terms of certainty equivalent consumption.

In the next part, we will discuss some short-cut investment strategies that are frequently offered by pension funds, because in practice a pension fund does not always have the time and flexibility to use a complex model for the optimal investment strategy for every single participant[1]. We assume that, in a world without borrowing restrictions, a fund would invest according to the optimal investment allocation we derived in section 2. As explained in 3.2, in the new Dutch pension contract it will be an option to calculate the optimal strategy for each age cohort and aggregate those strategies across the age cohorts to obtain the fund's collective optimal investment portfolio. Another option would be to provide your participants a few deterministic options for their investment strategy, from which they can then choose themselves. This is more ad-hoc, but saves a lot of time for a fund's investment strategists.

4.1.2 Practical investment policies and their welfare costs

In a framework where borrowing restrictions are represent, pension funds offer can multiple life-cycle investment strategies to their participants[2]. Within the offer, the strategies differ in risk profile, from a quite risk-averse investment profile (defensive investment strategy) to a more risk-seeking investment strategy (offensive investment strategy). In this subsection, we will explore some of the regularly provided choices in life-cycle pension schemes, investigate the differences with respect to the optimal investment allocations in section 2.3, and compute what the consequences of those differences are in terms of welfare losses. Welfare losses will be measured in terms of certainty equivalent consumption again.

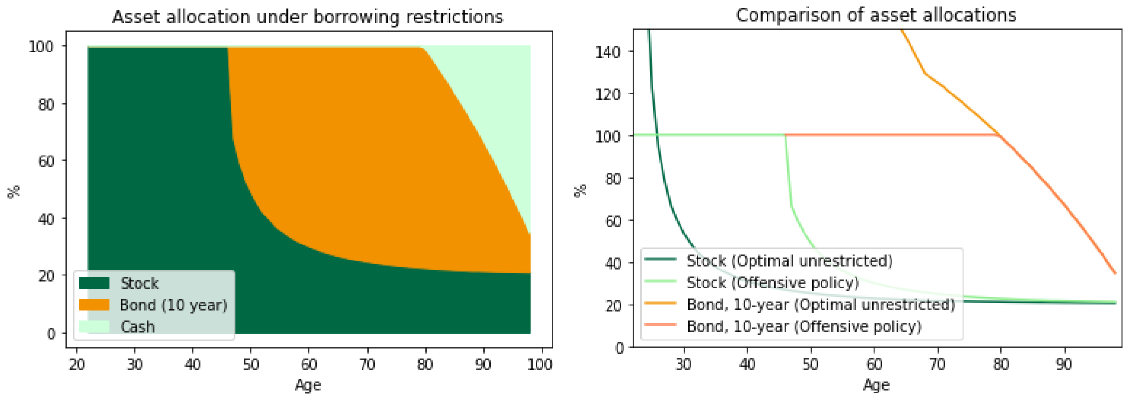
We start with a so-called offensive investment plan, in which a participant that enters the pension system at age 20 invests 100% in equities until age 45, and from that age the stock exposure declines proportionally to the decline as in the optimal, unrestricted setting in figure 2.3. As from age 45, the agent does not invest 100% in equities anymore, implying that part of financial capital can be invested in bonds. We will also set the exposure to bonds proportionally to the exposure in optimal unrestricted setting in 2.3. For the financial market parameters, we will assume they are equal to the values as defined in section 1.2. The only change that is made with respect to figure 2.3 is that we change time to maturity of the bond from 30 years to 10 years, which is allowed according to formula 3.1.2. Under borrowing restrictions, we start investing in the bond portfolio at a later age, and figure 3.1 shows us that at a later age a shorter time to maturity is desired. We

[1] An elaboration on the dilemmas between theory and practice can be found in the blog on https://www.cardano.nl/het-nieuwe-pensioencontract/leeftijdsafhankelijk-beleggingsbeleid-nutsfunctie-of-houtje-touwtje/

[2] A discussion of the investment options for some of the Dutch pension providers can be found on:https://www.sprenkelsenverschuren.nl/media/publicaties/2019_-_SV_Rapport_Onderzoek_DC-aanbieders_2018.pdf.

know that, around the retirement age, the duration of bonds in the average life-cycle of Dutch pension funds is around 12 years to maturity[3]. Furthermore, at the retirement age, the average percentage of interest rate risk hedging presented in Dutch life-cycles equals approximately 80%[4]. Therefore, we know that the average duration of the portfolio is around $12 \cdot 80\% \approx 10$ years.

By using this offensive investment strategy in a framework where we can only invest up to 100% of financial capital, we obtain the investment portfolio displayed in figure 4.2a



(A) Offensive life-cycle investment strategy in a situation where the fund is not allowed to invest more than 100% of the financial wealth of the participant.

(B) Comparison of the optimal unrestricted investment policy in figure 2.3 and the investment strategy in figure 4.2a under borrowing restrictions.

We see that this strategy substantially differs from the optimal unrestricted strategy as defined in section 2.3. We can consider this investment strategy as quite risky, since the position in risky assets is 100% of the financial capital along multiple years of the investment horizon. With this investment strategy, we can derive various scenarios of the development of financial wealth using formula 1.1.12. Then, by definition, the total wealth pattern can be derived by adding financial wealth to human capital. From this, the resulting consumption pattern can be extracted, as it equals total wealth divided by the optimal annuity factor in the unrestricted setting.

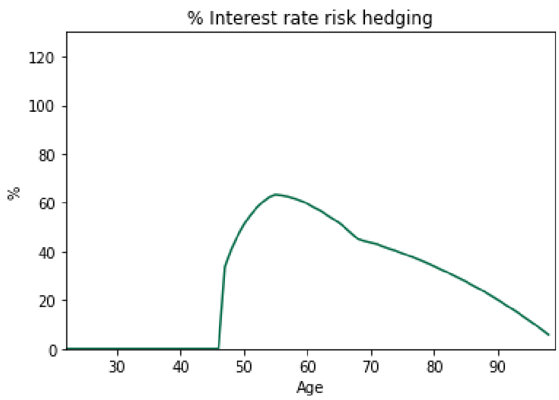
Then, we can calculate the certainty equivalent level for the expected consumption pattern resulting from the investment strategy as described in 4.2a, to conclude that the expected welfare loss of following this strategy compared to the optimal unrestricted asset allocation equals approximately 6.4% in terms of certainty equivalent consumption. This difference arises of course arises due to the limitations on total investments, but also because of a different implementation of risk-aversion by the fund. Notice that the welfare loss is evaluated on the average risk-aversion parameter of $\gamma = 5$ that we used throughout this thesis. The welfare loss of implementing the wrong risk-aversion parameter will be discussed in section 5.1.

In figures 4.3a and 4.3b, we compare the degree of interest rate risk hedging of this practical offensive policy to the desired percentage of interest rate risk hedging we saw in the

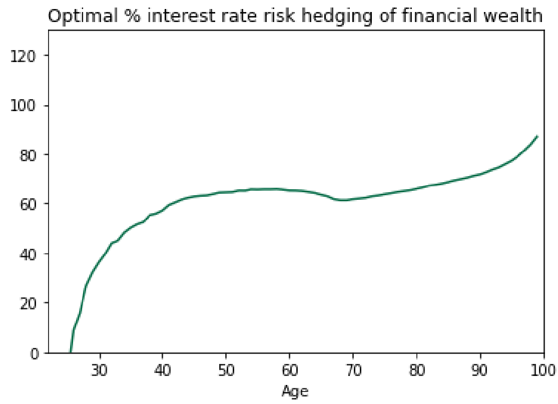
[3]Source not publicly available.

[4]`http://www.lcpnl.com/media/321809/20180606_lc_pensioen_2018.pdf` shows the average results of different life-cycle strategies based on a research on 12 Dutch pension funds. Notice that this research is based on the current Dutch DC market, which consists mainly of fixed instead of variable annuities, while in the new Dutch pension contract we will to a large extent shift to variable annuities.

optimal unrestricted setting. Having a look at the differences, we see that due to the unavailability of swaps and shrunk bond position, the degree of interest rate risk hedging lowers substantially. Especially for the first years, where financial wealth is fully dedicated to risky assets. Having a look at figure 4.3b, we see that in the first few years of the optimal setting, interest rate risk hedging is not desired. But, from approximately age 25, already a solid proportion of interest rate risk hedging is desired. At age 45, the optimal degree of interest rate risk hedging is already around 75% of financial capital, while in the strategy of 4.2a we still have not approached any level of interest rate risk hedging by then, as displayed in figure 4.3a.

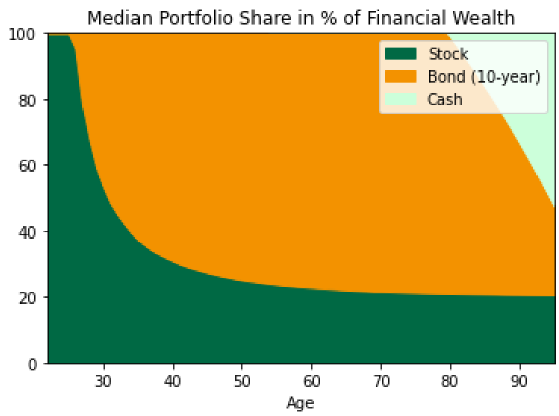


(A) % Actual interest rate risk hedging corresponding to the asset allocation displayed in figure 4.2a.

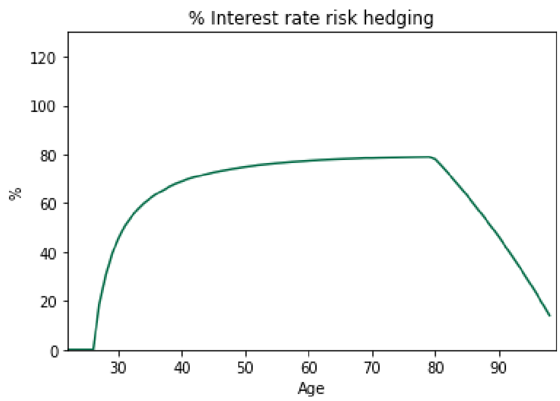


(B) Optimal % interest rate risk hedging in the unrestricted setting.

Instead of choosing for an offensive investment strategy, the participant is often offered a defensive investment option as well. We consider the following defensive life-cycle investment pattern: The participant invests 100% in risky assets until the age of 25, then the exposure to risky assets lowers proportionally to the reduction of stock exposure in the optimal setting of figure 2.3. This defensive investment strategy implies that we obtain an allocation of the investment portfolio over the life cycle as displayed in figure 4.4a, implying a level of interest rate risk hedging as displayed in figure 4.4b below.



(A) General pattern of a defensive life-cycle investment strategy in case you are not allowed to invest more than 100% of the financial wealth of a participant.



(B) % Actual interest rate risk hedging corresponding to the asset allocation displayed in figure 4.4a

The exposure to the 10-year real bond is held as close as possible equal to the bond exposure in the optimal unrestricted setting. Again, we can calculate the certainty equivalent level resulting from the expected consumption according to this strategy, and we find that the welfare loss of this defensive strategy under borrowing constraints is approximately equal to 6.8% in terms of certainty equivalent consumption. The welfare loss of this defensive strategy is above the loss of the offensive strategy, because larger part of financial wealth is invested in bonds and we assumed that the risk premium on bonds equals 0, while the risk premium on investing in stocks is positive. The deviation from the optimal level of interest rate risk hedging is less than in the offensive investment strategy, because in the defensive policy the participant invests a larger part of his wealth in the 10-year real bond, and from a younger age there is already wealth allocated to bonds.

To create a perspective for talking about welfare losses, we will shortly have a look at the welfare losses of the situations in which it is either not possible to invest in stocks or in bonds. Especially the situation in which stocks are not available is a quite extremely limited situation, because in our parameter setting, this is the only financial product that is associated with a positive risk premium, as the risk premium on the bond equals zero. In this situation, we see a welfare loss of approximately 13.5%. In the case in which bonds are not available, wealth is either dedicated to stocks or cash, as we also saw in figure 2.1, we observe a welfare loss of approximately 6.1%. Notice that the welfare loss of not investing in bonds results only from the fact that the hedging demand cannot be met, because we excluded the speculative demand by setting the risk premium of investing in bonds equal to zero.

In summary, we discussed that under borrowing constraints, the practical solution of many pension funds is to offer a bunch of strategies from which the participant can choose, varying from an offensive to a more defensive investment profile. We can conclude that both an offensive and defensive investment strategy differ substantially from the optimal investment pattern as we saw in figure 2.3, resulting in relevant welfare losses in terms of certainty equivalent consumption. In the next table, we summarized the results of the welfare losses of the different situations we considered in this paragraph, and the welfare losses of the unrestricted cases in which either bonds or stocks are not available.

Situation	**Welfare loss**
Optimum	-
Availability of products	
No stocks available	13.5%
No bonds available	6.1%
Borrowing restrictions: frequently offered strategies	
Offensive strategy	6.4%
Defensive strategy	6.8%

TABLE 4.1: Summary of computed welfare losses for different situations.

In the next part of this section, we will elaborate on another market limitation, and investigate the impact of illiquidity of long-term bonds in terms of welfare losses.

4.2 Illiquidity of long-term bonds

In section 3.3, we discussed that not all interest rate risk hedging instruments are fully available on the financial market, and that especially bonds with a maturity higher than 30 years and swap contracts does not always show the level of liquidity the Dutch pension section would need to fulfill its investment portfolios. Furthermore, in section 3.2.1, we discussed in which way to calculate the natural maturity for participants in a certain age cohort, and explained that for perfect interest rate risk hedging, the natural maturity of a participant would then be matched to the maturity of the bond portfolio to be invested in. In this part, we will show the difference in investment policy in case the bond portfolio with interest rate sensitivity equal to the natural maturity of the participant is not fully available on the financial market.

To demonstrate the effect of bond illiquidity onto the investment policy of a participant, we consider the differences to the optimal investment strategy for the simplified fund that consisted of one age cohort with age 25, for which the optimal strategy was displayed in figure 3.3. In this investment portfolio, the interest rate sensitivity of the bond portfolio equals the natural maturity corresponding to the age cohort of the participant. In line with the findings of section 3.3, we will now impose that in the financial market, the inflation linked bonds show an appropriate level of liquidity for maturities up to 30 years. Therefore, the bond with the longest maturity that is available on the financial market has a maturity of 30 years. In figure 3.1, we can see that the natural maturity of someone aged 25 is around 55 years, and that the natural maturity is below 30 years from approximately age 48 on. This suggests that the investment policy will probably change for at least the first 20 years, in case we take into account that only bonds with time to maturity up to 30 years are available.

To investigate the change in investment policy, we take into consideration only bonds up to 30 years. At a certain point, the participant reaches an age for which the natural maturity falls below this 30 years, and from that moment on we assume that the bond portfolio of the participant shows an interest rate sensitivity corresponding to the natural maturity again. This implies that the fraction of exposure to bonds of the age cohort with age 25 that was also considered in 3.3 changes to the bond allocation in the figure below, to approach the same interest rate risk hedging fraction as displayed by the blue line in figure 3.3.

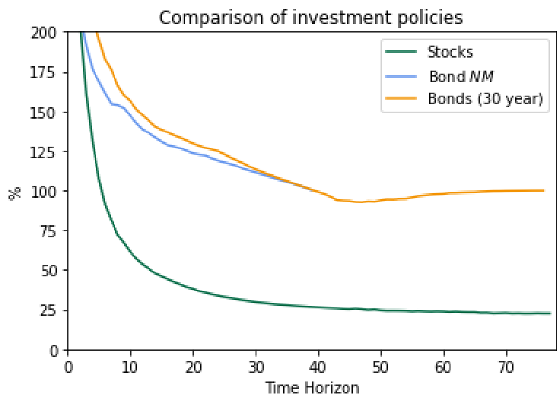


FIGURE 4.5: Investment policy of the age cohort with age 25, taken into account that only bonds with time to maturity up to 30 years are available.

In this figure, we see a comparison of the suboptimal investment policy to what we saw before in the situation where we could actually invest in the bond portfolio with natural maturity for a 25-year old. In the investment pattern, we observe an upward shift in the fraction of financial capital that is allocated to the bond portfolio for the time period during which the natural maturity for the participant is above 30 years. In fact, if we would impose the borrowing constraint as described in section 4.1 on this suboptimal investment policy, a welfare loss of approximately 6.5% in terms of certainty equivalent consumption arises.

On the other hand, we can have a look at the difference in investment policy in case we invest in a bond portfolio consisting of bonds with a longer maturity than the natural maturity. In case the natural maturity of a particular age cohort is lower than 30 years, and the fund still invests in an inflation-linked bond with 30 years to maturity, the share of cash lowers, implying that cash will be saved. To show this, we can have a look at the collective investment policy that was displayed in figure 3.4, for the pension fund that consisted of participants who all had age 60. In case this fund would invest in 30-year real bonds over the remaining life cycle of those participants, the fraction of financial wealth invested in bonds would change to the blue line in the figure below.

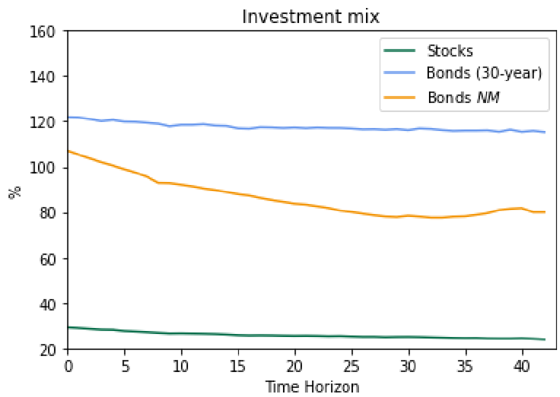


FIGURE 4.6: Optimal investment policy of the age cohort with age 60, compared to the difference in investment policy in case the fund would invest in 30-year real bonds.

We see that in case of investing in 30-year real bonds, the fraction of wealth invested into bonds shifts upwards. Because the fraction of wealth allocated to assets is now above 100% over the full remaining life cycle, cash is saved compared to the optimal policy. The part of cash that is saved over the remaining life cycle is displayed in the next figure.

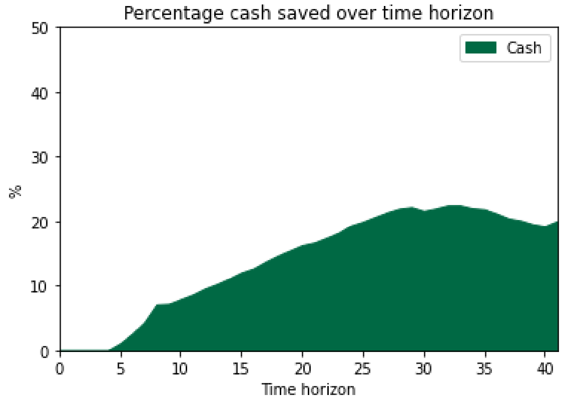


FIGURE 4.7: Percentage of cash that is saved for over the remaining life-cycle of 60 year old participants in case the fund invests in real bonds with a maturity of 30 years instead of the natural maturity.

We see that on average, we could save around 13.5% in cash. However, this would be the average if all our 60-year old participants would pass away at the end of the life cycle. In case we implement survival rates, implying that we take into account that not all of the participants probably will become this old, we observe that we can save 6.4% in cash on average along the remaining life cycle. On the other hand, by investing in 30-year real bonds instead of a bond portfolio with the natural maturity at each age, we observe a welfare loss of 1.3% in terms of certainty equivalent consumption, due to the difference in interest rate risk hedging policy.

In conclusion, in this section we saw some of the limitations that we can probably face while investing in the financial market, and calculated their welfare losses in terms of certainty equivalent consumption. Under borrowing constraints, we observed that implementing an additional risk factor in the model significantly increases the welfare loss. The welfare loss is then also based on the fact that you cannot reach the desired fraction interest rate risk hedging, besides the fact that you cannot invest your desired fraction into risky assets. Besides that, accounting for speculative demand also increases the welfare loss of borrowing constraints. Furthermore, we saw that the illiquidity of long-term bonds, as discussed in section 3.3, results in different asset allocations. On the other hand, investing in a bond with a maturity that is above your natural maturity can also save some amount of cash.

In the next section, we will perform a sensitivity analysis of the optimal investment mix with respect to specific parameters. Furthermore, we will discuss the fundamentals of stress-scenario testing, a strategy that is often implemented in practice to test for robustness of asset allocations under stress-scenarios. Lastly, we will discuss the implementation of a minimum required consumption level while deriving the optimal asset allocation and corresponding interest rate risk hedging policy.

Chapter 5

Robustness testing

In this chapter, we will discuss two approaches to check for robustness of an individual optimal asset allocation as defined in section 2. While performing a robustness test, an investor examines in which way the optimal asset allocation behaves in case an outstanding financial market scenario materializes, often resulting into a change financial market parameters. Therefore, in this section we discuss the response of the optimal asset allocation to a change in particular parameters, and consider the welfare losses of pursuing the optimal asset allocation on a specified parameter, while the actual parameter turns out to be different. Furthermore, we discuss the practical implication of stress-scenario testing. Regulators require financial institutions to carry out various stress-scenario tests and report on their internal procedures for managing capital and risk. To this end, we will discuss in which way an investor can implement relevant stress-scenarios, and a way to manage the effects of a stress-scenario in finding the optimal asset allocation, by controlling for a minimum required consumption level.

5.1 Sensitivity of optimal asset allocation to parameters

In section 2.3, we discussed the interest rate sensitivity of the optimal asset allocation under our assumed parameters in section 1.2. In this section, we will actually have a look at the optimal asset allocation in case we change one of those parameters. Besides its sensitivity to interest rates, an optimal investment mix as we saw in figures 2.3 to 2.7 can also depend highly on the assumptions on the parameters used to find the optimum. Therefore, an important step in interest rate risk modelling is to investigate the response of your optimal asset allocation and percentage interest rate risk hedging to a change in one of your assumed parameters. To this end, we will explore in this section the response of the asset allocation and optimal fraction of interest rate risk hedging to a change in ρ_{rS}, representing the correlation between stocks and interest rates, and a change in the risk-aversion parameter γ. We will calculate the welfare losses with respect to the individual optimal investment mix as defined in section 2. For clarification, this optimal asset allocation is displayed again below, including the optimal percentage of interest rate risk hedging, displayed by the blue line. From this line, we observe that the optimal percentage of interest rate risk hedging is around 80% over the biggest part of the life cycle. In the explanatory memorandum of Ministerie van Sociale Zaken en Werkgelegenheid (2020), it is stated that basically, older participants and retirees have to be protected against interest rate risk to a larger extent than younger participants. This is supported by the statement that elderly and retirees can bear less risk, as a result of relatively large accumulated pension capital combined with shorter or missing future accumulation of additional pension capital. However, our results suggest that from age 30, it is already optimal to hedge more than 50% of interest rate risk, which slightly increases along the rest of the life-cycle. The exact fraction of desired interest rate risk hedging depends, as discussed in section 3.2.4, also highly on investment beliefs of the investor.

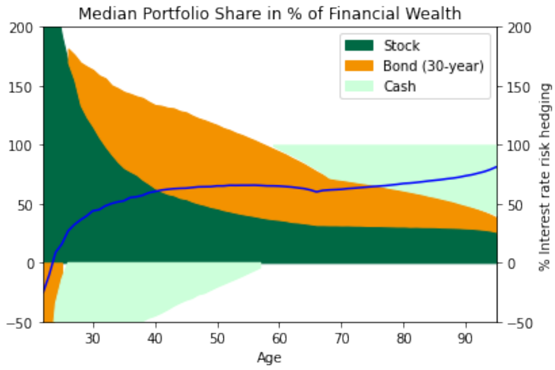
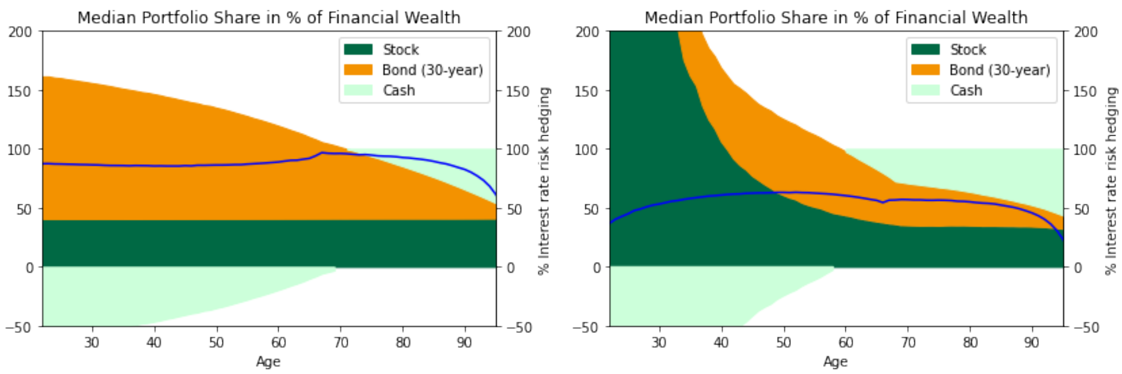


FIGURE 5.1: Optimal individual investment mix and corresponding optimal percentage of interest rate risk hedging, as derived in figures 2.3 and 2.10b.

5.1.1 Sensitivity with respect to correlation between stocks and interest rates

We start with the correlation between stocks and interest rates. Until now, we assumed that stocks and interest rates are not correlated, by setting $\rho_{rS} = 0$. For an investor, it is valuable to see in which way the optimal asset allocation reacts in case we would actually assume that, to a certain extend, stocks and interest rates are correlated to each other. For interest rate risk hedging, this is especially important in case of a negative correlation, because then equity prices go up in case interest rates fall, implying that equities would provide some sort of hedging qualities. On the other hand, in case of a strong positive correlation between stocks and interest rates, bonds loose part of their hedging qualities.

In the figures below, we display the optimal equity allocations and optimal percentage of interest rate risk hedging, displayed by the blue line, for the situations in which $\rho_{rS} = 0.2$ and $\rho_{rS} = -0.2$, while we keep the other parameters as described in section 1.2. According to formulas 1.1.4 and 1.1.5, the market price of interest rate risk and investing in stocks will also change due to a difference in ρ_{rS}.



(A) Optimal asset allocation in case $\rho_{rS} = 0.2$, while the other parameters remain the same as described in part 1.2.

(B) Optimal asset allocation in case $\rho_{rS} = -0.2$, while the other parameters remain the same as described in part 1.2.

In figure 5.3a, we observe that in case of a positive correlation between stocks and interest rates, the fraction of financial wealth allocated to stocks lowers, and the part of financial wealth that is dedicated to bonds increases. This effect arises because in case of a positive correlation between stocks and interest rates, a rising interest rate will imply that bonds become more attractive to invest in, as a rising interest rate increases the yield on bonds, and bonds are associated with a lower risk level than stocks. Simultaneously, an increase of interest rates results in a higher price for borrowing money, causing that less capital will be spent or invested which results into decreasing stock prices. In figure 5.3b, we see that in case of a negative correlation between stocks and interest rates, the fraction of wealth allocated to stocks increases, while the investment fraction to bonds decreases. This is caused by the fact that in this setting, stocks provide some sort of hedging qualities for interest rate risk, as equity prices go up in case of falling interest rates. Therefore, this results into a lower bond allocation and a higher equity allocation.

Moreover, we observe that the optimal percentage of interest rate risk hedging, as defined in formula 2.3.1, is slightly below 100% over the whole life cycle in case of a positive correlation between stocks and interest rates, while we saw in figure 2.10b that for $\rho_{rS} = 0$, the optimal percentage interest rate risk hedging increased along the life cycle. In figure 5.3b, we see that for a negative correlation between stocks and interest rate, the optimal percentage of interest rate risk hedging is lower, varying from approximately 30 till 60% along the life cycle. The desired interest rate risk hedging percentage lowers in this setting, because investing in equities also provide hedging qualities for interest rate risk.

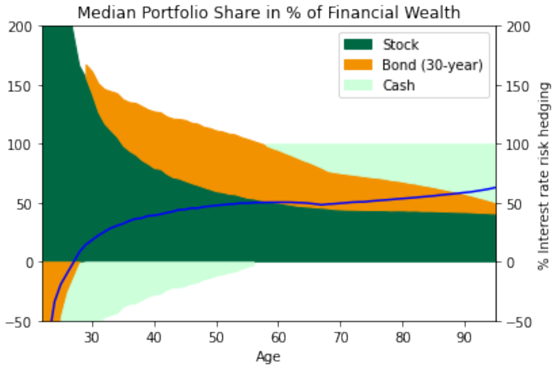
While keeping the other parameters as defined in section 1.2, we can investigate the welfare loss of using $\rho_{rS} = 0$ for the determination of the optimal asset allocation and interest rate risk hedging policy, while the actual parameter for the correlation between stocks and interest rates turns out to be unequal to zero. In fact, if the investor would assume that there is no correlation, while the actual parameter equals $\rho_{rS} = 0.2$, we observe a welfare loss of 1.6% in terms of certainty equivalent consumption. On the other hand, in case the correlation between stocks and interest rates turns out to move into the opposite direction to $\rho_{rS} = -0.2$, we see an existing welfare loss of 1.5% in terms of certainty equivalent consumption.

Hence, despite the fact that we observe such large differences in the optimal asset allocation for different values of ρ_{rS}, the welfare losses of using a different parameter for the correlation between stocks and interest rates than the actual one are relatively limited. However, welfare losses actually turn out to be substantial for a deviation in risk-aversion, which we will derive and discuss in the next part of this chapter.

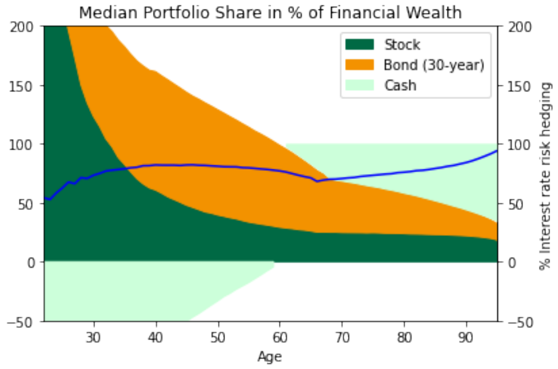
5.1.2 Sensitivity analysis to a change in risk-aversion

Additionally to the sensitivity of the optimal asset allocation to a change in the correlation between stocks and interest rates, we can also investigate the way in which the optimal asset allocation and optimal interest rate risk hedging policy respond in case of a change in the risk-aversion parameter γ. In Bovenberg et al. (2007), it was discussed that in case the optimal asset allocation is derived on the base of a risk-aversion parameter equal to 5, but the actual risk-aversion parameter turns out to be equal to 3, a welfare loss of 5% is observed. In the financial market setting for this result, the interest rate is not considered to be time-varying, implying that equities are the only risky instrument in the financial setting. In our setting, interest rates do also vary across time, implying that also interest rates are considered to be a risky instrument. Therefore, we can have a look whether a setting also including interest rate risk changes the welfare loss of using a different γ to determine your optimal investment allocation than the actual γ. We will consider the same difference in γ as in Bovenberg et al. (2007), and calculate the welfare loss of using $\gamma = 5$ while the actual risk-aversion parameter for the participant would be 3.

Besides $\gamma = 3$, representing an agent that is more risk-seeking than on average, we will also investigate the way in which the optimal asset allocation would respond in case γ is changed into the more risk-averse direction, to $\gamma = 8$. In the figures below, the optimal equity allocations and percentage interest rate risk hedging for respectively $\gamma = 3$ and $\gamma = 8$ are displayed



(A) Optimal asset allocation in case $\gamma = 3$, while the other parameters remain the same as described in part 1.2.



(B) Optimal asset allocation in case $\gamma = 8$, while the other parameters remain the same as described in part 1.2.

In these figures, we see that an agent who is more risk-seeking than average will invest a higher fraction in stocks over the whole life cycle, while an agent that is more risk-averse has lower equity exposure and invests larger part of his wealth into the 30-year real bond. Furthermore, we see that the level of desired interest rate risk hedging is lower for the risk-seeking agent than for the more risk-averse agent, especially at the start of the life-cycle.

In fact, in case an investor imposes $\gamma = 5$ to derive the optimal asset allocation, while the actual risk-aversion parameter turns to be equal to 8, the welfare loss equals approximately 2.2%. For the opposite direction, the welfare loss turns out to be more significant, which we already learned from Bovenberg et al. (2007). The consequences of using $\gamma = 5$ while the risk-aversion parameter should actually be equal to 3 are more extreme, because for a relatively more risk-seeking agent, the investor could have invested a higher fraction of wealth into equities, implying more benefit from the positive risk premium on investing in risky assets. Bovenberg et al. (2007) already reported a welfare loss of 5% in a setting without a variable interest rate. In our setting, a time-varying interest rate is actually implemented, suggesting that the use of a different risk-aversion parameter has even more consequences. In case the actual risk-aversion parameter is lower than the γ that was used for determining the optimum, we realize that next to an inaccurate allocation to equities, additional welfare loss arises because of an imprecise interest rate risk hedging policy.

In our financial market setting, we observe that considering the actual parameter for risk-aversion turns out to be 3, while $\gamma = 5$ was used for approaching the optimal asset allocation, the welfare loss is approximately 6.3% in terms of certainty equivalent consumption. This welfare loss arises due to two main reasons. On the one hand, the investor could have dedicated a larger part of capital to equities in case of a lower risk-aversion parameter, implying that the participant could have taken more advantage of the positive risk premium on stocks. On the other hand, the investor hedges larger part of the interest rate risk than desired by the participant, resulting in additional welfare loss, especially since the risk premium on investing in bond was set to zero. In case we incorporate a positive risk premium for bonds, speculative demand for bonds arises, and part of the existing welfare loss due to a higher interest rate risk hedging than desired by the participant will be offset by the fact that the participant will now obtain higher benefit from investing in bonds. By implementing a positive risk premium of 0.5% on investing in bonds, the welfare loss of the use of $\gamma = 5$ while the actual risk-aversion parameter equals $\gamma = 3$ reduces to 5.2% in terms of certainty equivalent consumption. Hence, the introduction of speculative demand causes a decrease in welfare loss for this situation, while in figure 4.1 we saw that the implementation of speculative demand increases the welfare loss of borrowing constraints.

We can conclude that the differences in investment policy due to a change in either the correlation between stocks and interest rates, or the risk-aversion parameter are substantial. This implies that the optimal asset allocation depend highly on the assumed parameters, and an important part of interest rate risk modelling is therefore to check in which way your optimal life-cycle changes due to different parameter settings. The welfare losses for the implementation of an inaccurate correlation between stocks and interest rates are relatively limited, while the losses for an imprecise risk-aversion parameter are substantial. Therefore, we can recommend pension funds to dedicate sufficient resources when researching the risk-attitude of their participants.

Besides knowing in which way your optimal investment allocation and hedging policy responds to changes in parameters, it is important to investigate in the situations in which parameters will actually change substantially. Therefore, in the next part of this chapter, we will discuss in which way an investor can implement stress-scenario testing in finding a robust optimal investment mix and a relevant interest rate risk hedging policy.

5.2 Stress-scenario testing

In multiple professions, it is straightforward to implement scenario thinking in everyday businesses: Policemen and firefighters are exposed to different cases during their training days to learn how to react in all possible scenarios. Also multinationals take into account various different scenarios while composing their multi-year plans. Pension funds build their investment strategies on the outcomes of an ALM-model. Based on a mathematical model, a range of scenarios are constructed around the average outcome. But, reality does often act differently compared to our the average outcome in our mathematical model, and therefore implementing scenario thinking in the pensions sector is definitely important, too. The aim of scenario thinking is to act less based on statistical probabilities, but act on consequences of specific scenarios instead.

Stress-scenario testing is important to measure investment risk and the adequacy of assets, as well as to evaluate internal processes and controls in a pension fund. Therefore, it is a useful tool to inform pension funds about what the consequences could be if one of the stress-scenarios materializes. Furthermore, stress-scenarios may be used by pension funds in their risk management framework as a tool to recognize parameter uncertainty and model uncertainty, as the real world might not act the same as their assumed economic model. Therefore, an important part of implementing scenario thinking is about using the available information to try to imagine what might happen in the future. By thinking about the various things that could occur, we prepare ourselves to deal with uncertainty in actual situations, which shortens the time to react in case a certain scenario materializes. It won't always be possible to adapt immediately in particular circumstances, but recognizing (part of) a stress scenario in advance will allow to improve the strategy in case the scenario would materialize. In the new pension contract, every age cohort is faced to a different investment horizon, and the stress scenarios for each age cohort can also be linked to their particular investment horizon. As a consequence, you have to take into account both long-term stress scenarios for younger participants, and short-term stress scenarios for the older age cohorts.

The better the understanding of the dynamics in the real world, the more realistic scenarios you can build and the more value scenario thinking can add. In case the investor manages to come up with a number of scenarios which together provide a representative range of possible future outcomes, he will have a powerful test region for the investment strategies. In this part, the use of scenario thinking for investments in a pension fund will be further exploited, and we will discuss in which way an alternative utility function can be implemented to increase the robustness of the optimal asset allocation in stress scenarios, provided by the introduction of a minimum required consumption level.

5.2.1 Derive relevant scenarios

One of the crucial things is to come up with a number of deterministic scenarios with which you are able to provide a representative range of possible future outcomes. Before you can set up multiple relevant scenarios, you will need to execute a few steps. In this part, we will discuss the thinking steps which you will have to consider before a particular scenario-set can be built.

On the one hand, you will need to determine the pension ambition you want to reach, and the corresponding objective to actually arrive at this ambition. The pension ambition is defined as the level of long-term pension that you pursue. Next to that, the trends and uncertainties on the path towards the pension ambition have to be identified. For this, a research on financial market trends and risk factors will be necessary. Furthermore, the drivers of change in future markets are important to identify, because some aspects can become of more importance in the future than they are at this point in time. Drivers of change might not be directly linked to the financial market at the first sight, but can possibly play an important role in the future. Examples of currently essential drivers of change are global warming and sustainability, but also employment aspects such as a transition in the labor market and working from home. You can rank different drivers of change with respect to uncertainty and potential impact to your pension ambition, such as displayed in the figure below. For high-impact drivers of change, you need to report the chances and risks they establish for your investment policy.

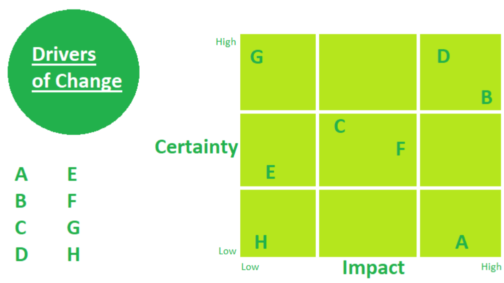


FIGURE 5.4: Rank drivers of change on uncertainty and potential impact

In case you have considered multiple drivers of change that could be important for your future policy, the next step is to combine them to create perspectives about the world in the future. In this step, main dimensions for the scenarios are formalized, and the scenarios distinguish in direction. For each dimension based on different drivers of change, the impact on the financial market and the corresponding consequences for the pension fund has to be concrete, which e.g. implies that you can express it in a certain amount of profit or loss relative to your ambition.

In the Scenariostudie 2020 of Cardano Risk Management, it is discussed how to use scenario thinking to come up with a robust investment policy. The first main lesson from this report is that it is just as important to take into account which investment outcomes you absolutely want to avoid as to specify the ambitions which you want to realize. Both the desired and undesired outcomes then have to be converted to measurable indicators. By using those measurable indicators, you can set a certain framework where you want your outcomes to be in. On the base of this, you can investigate which instruments you can adapt to change your investment policy such that it becomes in line the thresholds that you set. Once you know all the means you can use to calibrate your investment machinery, you have to determine your investment and policy decisions: Which products do you need to capture for sure and at which phase in recognizing scenario characteristics are you going to adapt your strategy? Decision making in this approach is supported by a utility function based on the Minimax method (Neumann (1928)), which is often used in game-theory.

The study represents four (global) extreme scenarios, which are depression, inflation, stagflation and prosperity, and one scenario that is the extended situation of developments in the past year, which is called Ice Age and in which leading factors are ageing, low interest rates and social inequality. These 5 scenarios provide possible dimensions in which the economy could develop in the upcoming years, aimed to let financial industries protect themselves against the negative consequences once one of the scenarios would materialize. The drivers of change are divided into five different categories: social, technological, ecological, financial and political. Then, their relation with financial markets is investigated and expressed in significance levels.

By using the impact on combinations of growth and inflation: two crucial factors that drive the financial market, almost all possible economic directions are included in the discussion of the five scenarios. With those economic directions, a translation to the investment policy can be made, since the value of an investment is determined on the base of its volume to economic growth and inflation. For financially relevant factors such as interest rates, spreads and stock volatility, this is translated into quantifiable impact or market shocks that are expressed in percent points change per year of the scenario horizon.

Having discussed the important ingredients to include while generating relevant financial scenarios, it is important to find the objective to arrive at the pension ambition. In the last part of this thesis, we will discuss the implementation of an alternative utility function in this objective that includes a minimum required consumption level, which will provide another step in improving robustness in the model.

5.2.2 Minimum required consumption level

To avoid that future consumption will drop below a certain threshold level in case a stress-scenario materializes, an investor can choose to insert a minimum required consumption level into the objective, such that the investment policy is determined while taking the minimum required consumption level into account. This implies that we implement alternative preferences, and for this we will deviate from the utility function as defined in formula 1.1.17.

For the previous derived expected utility, we already formulated the individual optimal consumption choice using formula 1.1.19 and the portfolio weights to approach the desired consumption pattern by formulas 1.1.24 and 1.1.25. In the last part of this thesis, we will formulate the individual optimization problem including a required minimum consumption level. For this, we first introduce variable $\bar{c}(t)$, representing the minimum required consumption level at time t. This $\bar{c}(t)$ can for example be set equal to the benefits of a guaranteed old age pension annuity. Then, we define

$$\hat{c}(t) = c(t) - \bar{c}(t) \tag{5.2.1}$$

With $c(t)$ as defined in formula 1.1.19. Then, if we consider T to be the moment the participant will pass away, the investor must solve the following defined maximization problem

$$\max_{\hat{c}} \mathbb{E}\left(\int_0^T e^{-\delta t} u(\hat{c}(t))dt\right) \tag{5.2.2}$$

$$\text{s.t. } V(0) \leq W(0) \tag{5.2.3}$$

In which the price of the consumption strategy $V(0)$ equals

$$V(0) = \mathbb{E}\left(\int_0^T M(t)\hat{c}(t)dt\right) + \mathbb{E}\left(\int_0^T M(t)\bar{c}dt\right) \tag{5.2.4}$$

Using the Lagrange Method (Everett III (1963)) and implementing that u represents CRRA utility, we can derive that the individual optimal portfolio decision must satisfy

$$\hat{c}^*(t) = (e^{\delta t} y M(t))^{-\frac{1}{\gamma}} \tag{5.2.5}$$

In which $M(t)$ represents the stochastic discount factor, with dynamics as specified in formula 1.1.3, and y is the Lagrange multiplier. Using formula 5.2.1, we know that $c^*(t)$ must satisfy

$$c^*(t) = (e^{\delta t} y M(t))^{-\frac{1}{\gamma}} + \bar{c}(t+h) \tag{5.2.6}$$

Besides this, we can derive, in line with van Bilsen et al. (2019), that the market value of optimal consumption equals

$$V^*(t) = \mathbb{E}_t\left(\int_0^{T-t} \frac{M(t+h)}{M(t)}\hat{c}^*(t+h)dh\right) + \int_0^{T-t} \mathbb{E}_t\left(\frac{M(t+h)}{M(t)}\bar{c}(t+h)\right)dh \tag{5.2.7}$$

In formula 5.2.6, we can insert the closed-form expression for $M(t)$ to arrive at a closed-form expression for the individual optimal consumption choice. Next to that, we could use formula 5.2.7 as ingredient for the application of the Martingale Method (Cox and fu Huang (1989)) to derive the optimal portfolio weights. Due to time conditions, the elaboration of the resulting individual optimal consumption choice, the optimal portfolio weights and a numerical application will not be provided in this thesis.

In summary, it is important not to rely blindly on your optimal investment solution depending on a specified set of parameters, but to check for robustness of the resulting investment policy with respect to changes in particular parameters. We have seen that even though changes in some of the specified parameters might result in a completely different investment policy, the welfare loss of implementing another parameter than the actual parameter can be acceptable. Furthermore, it is important to investigate whether your investment strategy is robust enough to avoid substantial decreases in expected pension benefits in case a stress-scenario materializes. To be certain that the expected pension benefits will not drop below a certain threshold, an investor can include a minimum required consumption level in the optimal investment problem. Such a minimum required consumption level could be supported by a risk-attitude research project with the pension fund's participants. Alternatively, it could be supported by a general research on expenditure levels of the retired population of the Netherlands, as e.g. provided in Nibud (2018).

Conclusions and Recommendations

In this thesis, we have developed a framework to derive a collective investment strategy in a bottom-up procedure. For this, we first explored the model of Brennan and Xia (2002) and additionally, we implemented individual survival rates and the bio-metric return into this model. Within this framework, we derived the optimal individual asset allocation and discussed the interest rate sensitivity of the pension assets resulting from this asset allocation. Considerable sensitivity with respect to interest rates was deduced, highlighting the importance of interest rate risk hedging. Furthermore, from the optimal asset allocation, we derived the optimal fraction of interest rate risk hedging over the life-cycle. Our results supports the statement in the explanatory memorandum of Ministerie van Sociale Zaken en Werkgelegenheid (2020) that for older participants, a larger fraction of interest rate risk hedging is desired, but the differences to younger participants are not as big as suggested by the ministry.

In section 3, we demonstrated a bottom-up approach to build towards a collective investment policy. In this method, we implemented an age-specific maturity for bond portfolios, as suggested by the documents of Centraal PlanBureau (2020) regarding the new Dutch pension contract. We derived that the age composition of the fund is relevant a fund's collective investment policy. Moreover, we discussed that especially for younger participants, there is no availability of inflation-linked bonds with maturity corresponding to their natural maturity. Besides that, we argued that the market for inflation-linked bonds is too limited to meet the demand of the Dutch pension sector. Part of this gap can be closed with swaps, but the total supply in the Eurozone definitely falls short of the demand of pension funds and insurers in Europe.

Throughout section 4, we discovered that expanding the financial setting with factors as time-varying interest rate and speculative demand for bonds results in additional welfare losses from borrowing constraints compared to a financial setting with a fixed interest rate. We concluded that the welfare loss from restrictions on interest rate risk hedging is similar to the welfare loss from restrictions on equity exposure. Additionally, we had a look at age cohorts that have no bonds available with a time to maturity corresponding to their natural maturity. We showed in which way the optimal asset allocation changes in case they still want to approach their desired fraction of interest rate risk hedging.

Section 5.1 reports the differences in investment policy for different financial parameters, and shows the welfare losses of implementing an inaccurate parameter in your model. The welfare losses for implementing an inaccurate correlation between stocks and interest rates are relatively limited, while the losses for an imprecise risk-aversion parameter are substantial. We can therefore recommend pension funds to dedicate sufficient resources when researching the risk-attitude of participants. Lastly, we discussed the relevance for a pension fund of considering the response of its asset allocation to particular stress-scenarios. We would recommend not to rely blindly on statistical probabilities, but instead act on consequences of specific stress-scenarios.

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