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“Fama-French 5-factor model: The performance in Developed markets excluding the U.S.”

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Abstract

There are several factors such as value, investment, size, profitability and momentum that provide premia for holding stocks with these features. The aim of this research is to describe the performance of a variety of multifactor models with an updated dataset of stock portfolio returns, from 1990 to 2019, in developed countries excluding the U.S. market. Also, this research compares the performance of a six-factor model, that includes the momentum factor, to the performance of other asset pricing models and shows how the model captures stock returns. Furthermore, heteroscedasticity and the GRS tests are used in order to evaluate the performance of the models. The results suggest that, though they are imperfect, Fama-French 5-factor model and the 6-factor model shrink intercepts to zero and solve some of the problems of Fama-French 3-factor model. Also, the value factor is non-redundant.

Key words: Asset pricing; Factors; Developed markets.

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1.Introduction

Modern world is consisted by markets that are becoming more integrated and an increasing number of potential investing companies and investors. Factor investing is continuously researched since it might be a way to explain abnormal returns. Factors or firm characteristics are considered variables that affect portfolio returns. Many tried to understand how these factors explain portfolio returns by using a variety of models such as CAPM, ICAPM, APT and the FF 3- and 5-factor models. There is a long history of research in the subject mostly in the American stock market, as it is the most developed one and probably with the longest history of performance. Therefore, it would be interesting to see how these models perform in other developed markets besides the U.S. one. Those markets are big and the amount of transactions and money is increasing in them but there still is much research that needs to be done. The European Union is a strong economic union but the most dominant models have been tested mostly in the U.S. market.

In addition, the rapid development of technology makes available a lot of data mining techniques. Machine learning, database systems and statistics alongside with programming languages give the opportunity to easily access data and extract information from those datasets. Another reason that motivated me to select asset pricing and more specifically Fama and French 5-factor model is the larger amount of data that is available now. The dataset that I am using is updated until December 2019. The period that I am investigating contains a big crisis and a lot of changes such as the establishment of the European Union. Moreover, Asian economies emerged as great economic powers. It is interesting to see how these models perform in this updated dataset, especially with the changes in economy and the world as a whole. Harvey, Liu and Zhu (2016) find that many discoveries of factors have been untrue and the idea of more factors is inconsistent with the principal component analysis that finds 5 factors. Thus, it is essential to always test in new regions with recent data since there might be the possibility that the robustness of factors is due to data mining.

The important Fama-French 5-factor model shows that market, size, value, operating profitability and investment adequately capture the returns of the U.S. stock market. Though there are many more factors that can affect the returns and one of them is momentum. Momentum can be a factor or an anomaly and momentum strategy is often chosen by investors and thus many researched it and confirmed that it produces significant returns. In this research project I take into account the momentum factor and I test the performance of the models in the Size-Mom portfolios. This research follows the Fama and French (2015) methodology for the most part. I use the two ratios of Fama and French (2017) in the pricing details part instead of the ones of Fama and French (2015). I am also presenting results for almost all models of all combinations of factors, although, in this research as well, it seems that the most dominant models are the traditional ones. I am also testing for heteroscedasticity in my sample and I implement a GJR model.

There are a few interesting results that can be seen in this research. First of all, when I do factor spanning tests using the five original Fama-French factors, I find that in this sample the value factor is not redundant and thus I am using it in my regressions and I am not using the orthogonal HML as Fama and French (2015) do. Furthermore, it seems that Fama-French 5-factor model, Carhart's 4-factor model and a 6-factor model that contains the momentum factor present the lowest pricing errors compared to other models but still are not perfect models. Fama and French argue that it is highly improbable to find a perfect model but even a not perfect model can be important for describing the variation in average returns. The aim of this research project is to test the performance of a variety of models in an updated sample of portfolio returns for different markets excluding the U.S. one and test parameters such as momentum.

2.Literature Review

The Efficient Market Hypothesis, introduced by Bachelier (1900), suggests that all the information about an asset is reflected on the asset's price making it impossible to gain abnormal returns. Consequently, positive alphas cannot be generated using any type of analysis, fundamental or technical. However, Grossman and Stiglitz (1980) argue that since obtaining information is costly and investors are compensated about their effort to gather information and find mispriced assets this information cannot be reflected to the price. The paradox is called the "Grossman Stiglitz Paradox".

The first important asset pricing model, that now serves as the foundation of financial economics, is the Capital Asset Pricing Model (CAPM) developed by William Sharpe (1964). CAPM is a single factor model, in which the only priced factor is the market. It suggests that there is a positive relationship between a stock's beta and the stock's expected returns. CAPM helps to calculate investment risk and the potential return on an investment. Empirically CAPM fails to explain the abnormal returns of stocks, but it is still used as a calculator of the cost of capital and as an evaluation technique for the performance of portfolios. Critique in CAPM usually refers to the simplicity of the model and the flat relationship of the systematic risk with the expected return of a stock. Stephen Ross (1976) proposes an alternative to the mean variance CAPM. The Arbitrage Pricing Theory (APT), through a multifactor asset pricing model, suggests that there is a linear relationship between a stock's expected return and a number of macroeconomic variables that capture systematic risk.

Fama and French (1993), put CAPM to the test and consequently introduce the three-factor model. The model includes two, additional to the market, factors that can explain the excess returns of the stocks, the size and the book-to-market (B/M) ratio of the firm. They find that the 3-factor model is a good model for the returns of portfolios that are formed not only on size and book-to-market, but also on Earnings/Price ratio, Cash Flow/Price ratio and sales growth.

Daniel and Titman (1997) criticize the research of Fama and French (1993) and suggest the characteristics model. Fama and French show that the cross-sectional variation in expected returns can be explained by only size and value factors. Daniel and Titman (1997) find that it is more characteristics rather than factor loadings that determine expected returns and also there are more than two characteristics that are important. Their results also indicate that value stocks comove because of their sensitivities to similar factors and not because of a unique factor. Davis, Fama and French (2000) find that the value premium in average stock returns is robust. The 3-factor model explains the value premium better than the characteristics model of Daniel and Titman (1997), in their 68-year period and there is no evidence against the fact that value loading determines expected returns. They believe that the evidence of Daniel and Titman (1997) in favor of the characteristics model is due to their short sample period. If they omit the period examined by Daniel and Titman (1997) the intercepts of their regressions could hardly be close to the zero-intercepts that the risk model gives.

Carhart (1997) extended Fama and French's (1993) three-factor model to a four-factor model including the momentum factor, alongside the size, value and market factors. It appears that Carhart's model explains more of the variation in average stock returns than the original Fama and French (1993) 3-factor model. Rouwenhorst (1998) exhibits his results about momentum strategies and finds that an internationally diversified portfolio with a long position in medium-term winners and a short position in medium-term losers generated a return of 1% monthly. This outperformance is present in all markets, it holds across size and lasts for about one year, but this relationship is negatively correlated with size. Blackburn and Cakici (2017) focus on momentum and study returns from a variety of developed markets. They interestingly find significant returns in a strategy that goes long in long-term losers and short in short-term winners, a result that holds over the entire sample period and the majority of markets.

Griffin, J. M. (2002) examines different versions of the Fama and French three-factor model in international datasets and individual securities. He finds that none of the models completely captures the variation in average returns but domestic versions of the model do a better job than international and global versions of the 3-factor model. Fama and French (2012) examine if empirical asset pricing models capture factor patterns in international average returns. In their dataset they have 23 international markets divided in four regions, North America, Europe, Japan, and Asia Pacific, and they are trying to examine if asset pricing is integrated across these four regions. They are trying to detail the size, value and momentum patterns in average returns for developed markets and to examine how well the 3-factor and Carhart's 4-factor model capture average returns for portfolios formed on combinations of size, value and momentum. They also use global factor models to explain global and regional returns. They extend their range of markets with cost the reduced size of the sample. Their results indicate that there are common patterns in developed markets. There are value premiums and there is a momentum premium in all regions except Japan. The global models do not sufficiently explain average returns on regional portfolios

Titman, Wei and Xie (2004) find that the level of a firm's investment has an effect on the firm's stock. More specifically, there is a negative relationship between abnormal capital investment and stock returns. Novy-Marx (2013) identifies the profitability factor as he finds that profitable stocks generate significantly higher returns than unprofitable stocks. He also finds that, controlling for profitability, value strategies perform better. In their research Watanabe, A., Xu, Y., Yao, T., & Yu, T. (2013) examine if the value effect in international stock markets is consistent with the results in the U.S. and evaluate the possible economic causes of the value factor. They find that the value effect exists in international equity markets and that there are large differences of this effect in the countries that they examine. The effect is stronger in markets that are more informationally efficient.

Subsequently, Fama and French (2015) add profitability and investment factors to their initial three-factor model, as they identify evidence that stock returns are related to these 5 factors and they introduce the five-factor model which is the base of this research. Based on the Dividend Discount Model they add CMA (=Conservative Minus Aggressive Investment) and RMW (=Robust Minus Weak Profitability). They also describe the momentum factor without including it to their model as it is likely that adding more factors will result in poor diversification of the portfolios used to construct the factors. Under the success of the 5-factor model, which explains the size, B/M, profitability and investment patterns, Fama and French (2017) try the model internationally and they find that average stock returns of three out of four regions they use (North America, Europe, and Asia Pacific) increase with B/M ratio and profitability. They also find the expected negative relationship between returns and investment. In Japan this investment-average returns relationship is weak but the relationship between average returns and B/M ratio is strong.

Kewei Hou, G. Andrew Karolyi, Bong-Chan Kho (2011) use a sample period from 1981 to 2003 with a large number of stocks. They are searching for the firm-level characteristics that have great explanatory power over the variation of stock returns. They find that the value factor has great explanatory power. This factor is based on C/P and not on B/M and this is consistent both for cross-section and time-series tests. To this power of C/P adds a medium-term stock-price momentum. They discover that local and international versions of multifactor models have low pricing errors and the lowest rejection rate. Additionally, they notice that C/P is linked to a global covariance risk factor. Finally, they identify that momentum and C/P matter more as global risk factors than as characteristics, both in a local and a foreign level. Bekaert, G., R. J. Hodrick, and X. Zhang (2009) use linear factor models in order to capture the international return comovements. They find that an APT model and a factor model with similar global and regional Fama-French factors perform well. They use country specific portfolios and find that global market integration is more important than the regional one. In addition, testing for within-country and within-industry returns they conclude that despite globalization there are still international diversification benefits.

De Moor and Sercu (2013) document the size effect for international stocks for the time period 1980-2009. They find that the unexplained returns can be linked to a dividend-yield factor. The two factors they use, one as in Fama and French for size and the second for small stocks, seem to be consistently correlated with this dividend yield factor. Karolyi and Wu (2018) propose a new multi-factor asset pricing model based on, among others, size, value and momentum characteristics and they test it for 46 developed and emerging markets. The main difference is in the way that they build their factor portfolios because they use a partial-segmentation approach that captures the variation in international stock returns and achieves low pricing errors and rejection rates compared to conventional methods.

3.Data

3.1 Dataset and Potential issues

All the data of this research is downloaded from the Kenneth R. French Data Library. The dataset contains portfolio returns from 22 developed countries excluding the U.S. with the countries included to be Australia, Austria, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, Great Britain, Greece, Hong Kong, Ireland, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Sweden and Singapore, and it is updated from November 1990 until December 2019. All returns are monthly, measured in U.S. dollars and include dividends and capital gain and they are not continuously compounded.

By using a dataset that contains a variety of countries, I assume market integration and I gain some diversification benefits in my Left-Hand-Side (LHS) portfolios that are going to be the dependent variables. One benefit is the more accurate regression fits that I am expecting to get. More specifically by having as many countries as possible I expect to get higher R-squared and smaller errors, since my factors are global factors and not country or region specific. Most European countries in the sample belong to the European Union and can be considered integrated since there is a free movement of goods, services and labor. Japan and Canada can be considered two different independent markets and only in South Pacific area (Hong Kong, Singapore, New Zealand and Australia) the assumption of market integration is debatable.

In the sample there is a large amount of countries with different currencies. As mentioned, all returns are monthly, measured in U.S. dollars and include dividends and capital gain and they are not continuously compounded. In their entire research Fama and French assume that there is no exchange risk. Investors would normally be interested in capital gains by taking advantage of potential currency fluctuations and to avoid that we would need a segmented market. This is what Solnik (1974a) observes as one of the limitations of the CAPM. He also finds that the risk premium of a security over its national risk-free rate is proportional to its international systematic risk. Solnik (1974b) in order to describe the international relations of stock prices takes into account both national and international factors. According to Purchasing Power Parity two currencies are in equilibrium when a set of goods is priced the same in two countries, taking into consideration the exchange rates. Adler and Dumas (1983) argue that investors will evaluate differently a security's returns because of derivations of the Purchasing Power Parity. They also state reasons that constrain financial transactions such as taxes and border controls. Perold and Schulman (1988) talk about currency hedging and they argue that it is preferable to follow a long-run investment policy in terms of hedged portfolios than unhedged portfolios. Zhang (2006), under the hypothesis of market integration, tests the performance of a variety of conditional and unconditional international asset pricing models. He finds that both conditional and unconditional models have priced exchange risk factors and that allowing for time variation (conditional models) and risk premiums improves the performance of the models and the hypothesis of integration is supported. Balvers and Klein (2014) arrive in the conclusion that currency risk should be priced but although it might be statistically significant, usually it is not economically significant. Therefore, I will not take into account an exchange risk because the market doesn't seem to price a risk like that.

Investors are looking to exploit currency fluctuations in order to gain an abnormal return. Sometimes though, changes in currencies can dramatically reduce the return of a portfolio that has high exposure to international markets. Some ways to hedge against exchange risk are currency forwards, currency futures and currency options or swaps. For instance, in currency options an investor would have the right to sell or buy a currency at a specified amount and thus hedges against the downward case but he doesn't lose the upward potential. However, options have fees to enter and that may cost a respectable amount to the investor. In a currency swap there are a few scenarios. Both parties pay a fixed of different currencies, both pay a floating rate of different currencies or one party pays a fixed on one currency and the other party pays a floating rate on a different currency. Swaps have transaction costs as well. Therefore, investors and institutions have a variety of practices to help them prevent problems associated with exchange risk.

3.2 Construction of factors

The factors that are going to be investigated in this research are: market, size, value, investment, operating profitability and momentum. In order to create them, the stocks have to be categorized based on their characteristics. Stocks in a region are sorted into two market cap categories and into three categories for each of the book-to-market equity, operating profitability, and investment at the end of each June. The B/M, OP and INV breakpoints for each region are the 30th and 70th percentiles of respective ratios for the big stocks of the specific region. Big stocks are those that are above the 90% of the market capitalization of the region and small stocks those at the lowest 10% of the market capitalization of the region. In order to construct the factors some accounting variables of the fiscal year ending in year t-1 are used. Value is measured by the book-to-market ratio at the end of fiscal year t-1. Operating profitability is the following ratio: $\frac{Operating\ Profits}{Book\ Value\ of\ Equity}$. $Operating\ Profits = Sales - Cost\ of\ goods\ sold - (Selling + General + Administrative\ Expenses) - Interest\ Expenses.$ Investment is the annual rate of total assets (growth of total assets for the fiscal year ending in t-1 divided by the total assets at the end of year t-2).

Market (MKT): The return of the regional value-weighted market portfolio minus the U.S. one-month T-bill rate.

SMB (SMB=Small Minus Big): The return of a portfolio with a long position on small stocks and a short position on big stocks. Therefore, below there are the nine small portfolios and nine big portfolios.

$$SMB_{B/M} = \frac{1}{3} * (SmallValue + SmallNeutral + SmallGrowth) - \frac{1}{3} * (BigValue + BigNeutral + BigGrowth)$$

$$SMB_{OP} = \frac{1}{3} * (SmallRobust + SmallNeutral + SmallWeak) - \frac{1}{3} * (BigRobust + BigNeutral + BigWeak)$$

$$SMB_{INV} = \frac{1}{3} * (SmallConservative + SmallNeutral + SmallAggresive) - \frac{1}{3} * (BigConservative + BigNeutral + BigAggresive)$$

$$SMB = \frac{1}{3} * (SMB_{\frac{B}{M}} + SMB_{OP} + SMB_{INV})$$

The rest of the factors are created with the same logic as the SMB factor.

HML: The average return of portfolio that goes long on value (high B/M) stocks and short on growth (low B/M) stocks.

$$HML = \frac{1}{2} * (SmallValue + BigValue) - \frac{1}{2} * (SmallGrowth + BigGrowth)$$

RMW: The average return of portfolio that goes long on stocks with robust operating profitability and short on stocks with weak operating profitability.

$$RMW = \frac{1}{2} * (SmallRobust + BigRobust) - \frac{1}{2} * (SmallWeak + BigWeak)$$

CMA: The average return of portfolio that goes long on stocks with conservative (low) investments and short on stocks with aggressive (high) investments.

$$CMA = \frac{1}{2} * (SmallConservative + BigConservative) - \frac{1}{2} * (SmallAggressive + BigAggressive)$$

WML is the momentum factor (Winners Minus Losers). Sorting is done by using the lagged momentum of stocks, which is the stock’s cumulative return for month t-12 to month t-2. It is the return of a portfolio that goes long on stocks that were Winners the last 12 months and short on stocks that were Losers the last 12 months.

$$WML = \frac{1}{2} * (SmallHigh + BigHigh) - \frac{1}{2} * (SmallLow + BigLow)$$

3.3 Construction of Portfolios

The 25 portfolios formed on size and B/M are as follows: For each region the size breakpoints are the 3rd, 7th, 13th and 25th percentiles of the region's aggregate market capitalization. For the other four factors the breakpoints are the 20th, 40th, 60th and 80th percentiles for the big stocks for the region. The 5x5 portfolios are the intersections of the 5x5 size sorts with the sorts of each of the other factors. For the 32 portfolios the logic is the same.

4.Summary statistics

4.1 Summary statistics for factor returns

Table 1, Panel A gives us the average returns of the factors. Market return is large at 0.37% per month. In disagreement to Fama and French (2015) where their size premium averages to 0.29% here it is only 0.10%. Average value return is high at 0.34%, profitability is at 0.35% and investment at 0.15%. The momentum premium (WML) is the largest of all at 0.66%.

Panel B gives us the small and big components of the 2x3 factors. It is evident that the momentum return is extremely high for small stocks (WML_S=0.93%) and the value premium is low for big stocks (HML_B=0.21%). The profitability premium is lower for bigger stocks, RMW_B=0.26%, compared to RMW_S=0.45% for small stocks. Last but not least investment average return for small stocks is more than double compared to that for big stocks (CMA_S=0.23%, CMA_B=0.08%).

Panel C shows the correlations between the 2x3 factors. There is an unexpected negative correlation between MKT and SMB with a considerable magnitude (-0.22). In Fama and French (2015) this relationship is positive (0.28). This result indicates that small stocks do not always have higher betas and consequently a size premium is not required in order to hold them. SMB has the lowest correlation with most of the other factors. Profitability and investment are negatively correlated, while I would expect the opposite. There is a normal, and high, positive correlation between CMA and HML as in Fama and French (2015). Finally, there is a fairly high positive correlation between profitability and momentum.

Fama and French (2015) believe that adding the momentum factor in their 5-factor model will result to poor diversification of the portfolios that are used to construct the factors. In Table 1, the correlations between the momentum factor and the other factors are not high, except from the one with profitability, as mentioned.

Table 1: Summary statistics for the 2x3 monthly factor percent returns. The sample period is November 1990-December 2019. In Panel A there are summary statistics such as mean, standard deviation and the t-statistic for the factors and in Panel C there is the correlation between the factors. In Panel B there are the Small and Big Components of Factors(2x3). HML_B is the average return on portfolios with big high B/M stocks minus the average return on portfolios with big low B/M stocks, HML_S is exactly the same logic for portfolios of small stocks, HML is the average of HML_B and HML_B. The rest of the components are defined in the same way. All means are in percentages.

Panel A: Mean, Standard Deviation and t-statistic for 2x3 Factors

	2x3 Factors					
	Rm-Rf	SMB	HML	RMW	CMA	WML
Mean	0.37	0.10	0.34	0.35	0.15	0.66
Std dev.	4.55	1.98	2.11	1.37	1.75	3.49
t-Statistic	1.54	0.97	3.05	4.83	1.63	3.54

Panel B: Small and Big Components of Factors (2x3)

	HML_S	HML_B	RMW_S	RMW_B	CMA_S	CMA_B	WML_S	WML_B
Mean	0.48	0.21	0.45	0.26	0.23	0.08	0.93	0.39
Std dev.	2.33	2.61	1.29	2.05	1.79	2.12	3.36	4.05
t-Statistic	3.86	1.49	6.49	2.38	2.35	0.70	5.20	1.78
Differences	HML_{S-B}		RMW_{S-B}		CMA_{S-B}		WML_{S-B}	
Mean	0.27		0.19		0.15		0.55	
Std dev.	2.58		2.05		1.77		2.58	
t-Statistic	1.97		1.70		1.54		3.98	

Panel C: Correlations between Factors (2x3)

	Rm-Rf	SMB	HML	RMW	CMA	WML
Rm-Rf	1.00					
SMB	-0.22	1.00				
HML	-0.05	0.08	1.00			
RMW	-0.36	-0.06	-0.34	1.00		
CMA	-0.29	0.04	0.59	-0.27	1.00	
WML	-0.27	0.11	-0.26	0.39	-0.03	1.00

4.2 Summary statistics for Portfolios

Table 2 presents the average monthly excess returns for 4 different sets of value-weight portfolios sorted on size with B/M, Inv, OP and Mom accordingly. In each Panel there are 25 sorted portfolios and their average excess returns.

In Panel A an inconsistent relationship between average returns and size is evident. For the first two categories of B/M, returns increase with size while for the next 3 categories of B/M average returns decrease with size, as expected. Both value and size patters are ambiguous in Panel A. Controlling for size, we see a value effect as higher B/M corresponds to higher average returns. The effect is stronger among small stocks. For example, the increase for small stocks is from 0.12% to 0.83% and for big stocks from 0.24% to 0.47%, as we move from the lowest B/M to the highest.

Table 2: Average monthly percent excess returns for the 25 portfolios formed on all combinations of Size, B/M, Inv, OP and Mom. The sample period is November 1990-December 2019. On the vertical axis we always have the Size from Small to Big and on the horizontal axis we have the rest of the characteristics. All numbers are in percentages. All Panels show the average excess returns of the portfolios, which are the intersections of Size with B/M, Inv, OP and Mom respectively.

5x5 Portfolios					
	Low	2	3	4	High
B/M ->			*Panel A: Size-B/M Portfolios*		
Small	0.12	0.33	0.48	0.61	0.83
2	0.03	0.31	0.40	0.56	0.60
3	0.22	0.36	0.36	0.43	0.60
4	0.27	0.40	0.42	0.49	0.45
Big	0.24	0.38	0.41	0.48	0.47
Inv->			*Panel B: Size-Inv Portfolios*		
Small	0.62	0.72	0.77	0.68	0.35
2	0.41	0.62	0.56	0.46	0.21
3	0.48	0.55	0.47	0.40	0.23
4	0.43	0.45	0.47	0.46	0.29
Big	0.42	0.40	0.34	0.33	0.30
OP->			*Panel C: Size-OP Portfolios*		
Small	0.32	0.66	0.78	0.89	0.92
2	0.13	0.48	0.50	0.59	0.70
3	0.22	0.39	0.48	0.54	0.60
4	0.15	0.42	0.51	0.52	0.50
Big	0.02	0.32	0.47	0.41	0.42
Mom->			*Panel D: Size-Mom Portfolios*		
Small	-0.19	0.46	0.68	0.95	1.30
2	-0.09	0.31	0.50	0.66	0.95
3	0.00	0.32	0.41	0.60	0.77
4	0.07	0.36	0.44	0.48	0.74
Big	0.10	0.24	0.46	0.48	0.46

In Panel B the sort is done using size and investment. For year portfolios the investment variable is the total assets growth for the fiscal year ending in t-1 divided by the total assets at the end of year t-2. In every size quantile the average return of the portfolio in the highest investment quantile is lower than the one in the lowest investment quantile, indicating that companies that invest aggressively have lower average returns. This result is clearly evident for small stocks in the 4^{th} and 5^{th} investment quantile where the average return drops from 0.68% to 0.35%. However, the relation is not consistent. For the first investment quintiles the average return increases and then drops in the 4^{th} and 5^{th} investment quintile, for every size category except the last one. The size effect is also evident in all investment quintiles but not strong and consistent in the highest Investment quintile as average returns fluctuate.

Panel C shows the average excess returns for the 25 value-weight portfolios formed on size and operating profitability. Controlling for profitability average returns decrease as size increases and thus the size effect is present. For small stocks the profitability effect is stronger as it is 0.32% in the lowest OP quintile and 0.92% in the highest OP quintile. For big stock portfolios the effect is also present but not strong.

Panel D presents the average excess returns for the 25 portfolios formed on size and momentum. It is seen that for the first momentum quantile as the stocks are bigger the average return is larger. However, in the next four momentum quantiles the average returns are negatively related to the size. As momentum increases average returns increase as well. The effect is stronger for small stocks (-0.19% compared to 1.30%).

Table 3: Averages of monthly excess returns for the 32 value-weighted portfolios formed on a) size, B/M, OP b) Size, B/M, Inv and c) Size, OP and Inv. The Size is split into 2 groups and not in 5 categories. In each Size group the rest of the characteristics are split into 4 categories and we get the 32 portfolios. The sample period is November 1990-December 2019. All numbers are in percentages.

2x4x4 Portfolios								
	Small				Big			
Panel A: Portfolios formed on Size, B/M and OP								
B/M ->	Low	2	3	High	Low	2	3	High
Low OP	-0.32	-0.09	0.31	0.45	-0.16	-0.09	0.18	0.36
2	-0.25	0.15	0.46	0.67	0.27	0.42	0.48	0.60
3	0.25	0.53	0.71	0.88	0.30	0.40	0.54	0.58
High OP	0.55	0.74	0.96	0.91	0.30	0.51	0.59	0.77
Panel B: Portfolios formed on Size, B/M and Inv								
B/M ->	Low	2	3	High	Low	2	3	High
Low Inv	0.22	0.43	0.66	0.69	0.27	0.47	0.44	0.52
2	0.36	0.53	0.63	0.81	0.19	0.51	0.52	0.62
3	0.43	0.50	0.68	0.68	0.19	0.40	0.44	0.55
High Inv	0.18	0.33	0.37	0.63	0.32	0.22	0.44	0.31
Panel C: Portfolios formed on Size, Inv and OP								
OP ->	Low	2	3	High	Low	2	3	High
Low Inv	0.18	0.43	0.71	0.74	0.14	0.47	0.54	0.53
2	0.23	0.46	0.66	0.81	0.17	0.47	0.53	0.49
3	0.09	0.39	0.56	0.77	0.16	0.40	0.39	0.39
High Inv	-0.35	0.07	0.38	0.54	-0.03	0.42	0.38	0.37

Table 3 shows average excess returns for the 2x4x4 portfolios. Sorting is done by using 2 categories of size and 4 quartiles for each of the other factors so there are 32 portfolio returns in each Panel. For small stocks there are strong value and profitability effects in average returns. The investment effect is present but the relation (Panel B) is not obvious as average returns vary a lot. For the big stock portfolios, the trends are the same, though weaker.

For small stock portfolios the summary statistics show that, controlling for OP or Inv, the average returns increase with B/M in every level of operating profitability. Controlling for B/M or OP, average returns decrease as investment increases. Last but not least, controlling for B/M or Inv, average returns increase with OP. For big stock portfolios the results are similar but weaker in magnitude. In general, anomalies are stronger in small size both in terms of magnitude and in terms of consistency.

An interesting result is the negative average excess return, -0.32%, of the small, low B/M and low OP stock portfolio in Panel A. In Fama and French (2015) this portfolio has an average excess return of 0.03% which is extremely low. In this sample the slopes of RMW and CMA when regressing the small, low B/M and low OP portfolio on the Fama-French 5-factor model factors, are negative, as in Fama and French (2015). These negative slopes are typical of firms that invest a low despite low profitability. Consistent to Fama and French (2015) there is a negative average monthly excess return for the small, low OP and high Inv portfolio, at -0.35%. The 5-factor slopes of RMW and CMA are also negative and that indicates again that in this portfolio firms invest a lot despite low profitability.

5.Methodology

In this section I am going to discuss some asset pricing methodologies and specify the reasons that made me use the Fama French methodology. Sharpe (1964) introduced the CAPM but empirically CAPM does not work and this might be the case because many of its assumptions in reality do not hold. Daniel and Titman (1997) use three models in order to investigate what determines expected returns. A model consistent with Fama and French (1993) views with a factor with positive risk premium, an alternative model where the structure is stable over time and where the returns are determined by a stock's loading on factors with time varying return premia and lastly a characteristics-based pricing model. Their results indicate that there is no distress factor but most of the co-movement of stocks is because stocks with comparable sensitivities tend to behave similarly at the same time. They directly test if the returns of high B/M and small size stocks can be associated to their factor loadings and they find that it is more characteristics than factor loadings that determine expected returns. Davis, Fama and French (2000) find that there is a strong market and a reliable value premium in returns. In their test for their 68-year period the risk model outperforms the characteristics model. They believe that the evidence in favor of the characteristics model is due to their short sample period since they find that if they omit the 20.5-year period examined by Daniel and Titman (1997) the intercept could hardly be close to the zero-value predicted by the risk model.

Another change in the methodology could be to use individual stock returns instead of portfolio returns. In fact, much of the literature on the U.S. stock market and international markets portfolio returns are used and this might be due to the high availability of portfolios created by Fama and French and other researchers. The importance of specific factors varies with different geographic regions and this is evident in Fama and French (2017) where their model performs differently from region to region. A large amount of literature argues that the creation of portfolios reduces the idiosyncratic volatility and allows factor loadings to be estimated more precisely. Many argue that better estimates of factor loadings consequently result to factor risk premia be estimated more precisely. Researchers tried to compare the performance of a variety of traditional asset pricing models using individual stock returns as the dependent variable. A large amount of the results show that the explanatory power of the factors drops when the dependent variable is stock returns and not portfolio returns.

In order for all this research to be done I need to make some assumptions. As Sharpe (1964) and Lintner (1965) I assume that all investors have access to funds at a common risk-free rate. Furthermore, all investors are free to invest any fraction of their wealth at any security with no constraints. All these securities are traded in a perfect competitive market with no transaction costs and taxes. Short sales are also allowed and I do not take into account the currency risk.

5.1 Proposed model

Fama French 5-factor model

Fama and French (2015), based on past research such as the ones from Titman, Wei and Xie (2004) and Novy-Marx (2013), incorporated the factors of investment and profitability in their original Fama-French 3-factor model and created the 5-factor model as below:

$$R_{i,t} - R_{f,t} = a_i + b_i * (R_{m,t} - R_{f,t}) + s_i * SMB_t + h_i * HML_t + r_i * RMW_t + c_i * CMA_t + \varepsilon_{i,t}$$

The purpose of the model is to capture the variability of the portfolio returns using these factors.

<u>6-factor model</u>

A large amount of past research, such as Carhart (1997), Rouwenhorst (1998) and more, indicates that momentum strategies generate significant returns and that models that include the momentum perform better and I incorporate this idea in my model. The model that this research tests is a 6-factor asset pricing model that includes the momentum factor in addition to the 5 factors used by Fama and French.

$$R_{i,t} - R_{f,t} = a_i + b_i * (R_{m,t} - R_{f,t}) + s_i * SMB_t + h_i * HML_t + r_i * RMW_t + c_i * CMA_t + w_i * WML_t + \varepsilon_{i,t}$$

Where: $R_{i,t}$ is the return of the diversified portfolio i at time t, $R_{f,t}$ is the 1-month T-bill which is the risk-free rate at time t, $R_{m,t}$ is the return of the developed markets' value-weighted market portfolios, SMB_t is the size factor, HML_t is the value factor, RMW_t is the profitability factor, CMA_t is the investment factor, WML_t is the momentum factor, b_i, s_i, h_i, r_i, c_i, w_i the corresponding factor coefficients, a_i is the intercept and finally $\varepsilon_{i,t}$ is the error term. The factors are defined in Section 3.

The plan is to create models using different combinations of the six common factors and test their performance. Thus, the model performance section will have 3-, 4-, 5- and 6- factor models with different combinations of factors.

5.2 Hypothesis

The initial (null) hypothesis is that the 6-factor asset pricing model effectively explains expected returns, thus that the intercepts a_i of the regressions are jointly indistinguishable from zero. In order to test this hypothesis, the use of GRS Test (Michael R. Gibbons, Stephen A. Ross, Jay Shanken (1989)) is necessary since this method can test the joint significance of the intercepts. The null hypothesis is:

$$H_0: a_\iota = 0, \qquad \forall\, i$$

Where i indicates the regression performed.

If the intercepts a_i are indistinguishable from zero that means that the model performs well. The hypothesis is tested for all models, since I am interested in comparing their performance. The value of a_i indicates the return of the left-hand side of the regression that is left unexplained. GRS Test is an F-test but it tests the joint significance of the intercepts and it is also used by Fama and French (2015).

I am interested in the improvements in descriptions of average returns provided by adding more factors to the original 3-factor model and especially by adding momentum in the 5-factor model. There are a) four 3-factor models that combine MKT, SMB with HML, CMA, RMW, WML. b) Six 4-factor models that combine MKT, SMB with pairs of HML, CMA, RMW and WML. c) Four 5-factor models that combine MKT, SMB with combinations of HML, CMA, RMW and WML and d) one 6-factor model with all the factors.

5.3 Controlling for Heteroscedasticity

Heteroscedasticity means that the variance of the error term is not constant but depends on the independent variables. Therefore, if I do not account for heteroscedasticity, the standard errors are going to be wrong and I might accept or reject hypotheses based on a wrong statistic. In this case I might accept or reject the significance of the coefficient and thus the significance of a factor of my model.

In this research an Autoregressive Conditional Heteroscedasticity model is used in order to incorporate heteroscedasticity by modelling the conditional variance of the error term as a function of its lag(s) and the squared return of the previous period(s). This is a GJR (1,1) model, where GJR is an asymmetric case of a GARCH model and it allows for the variance to react depending on the shocks it receives. The model would look like below:

$$\sigma_t^2 = \omega + \beta_1 * \sigma_{t-1}^2 + \alpha_1 * u_{t-1}^2 + \gamma_1 * u_{t-1}^2 * I_{t-1}$$

Where: $I_{t-1} = 1$ if $u_{t-1} < 0$ and 0 otherwise. When this term is lower than zero then it is a bad period, a recession and when it is higher, then the term is adjusted by γ_1.

By using a GJR model, I model volatility and therefore I capture not only heteroscedasticity but also the fact that volatility clusters, mean reverts, is highly autocorrelated and is higher when returns are negative.

6.Factor Spanning Tests

Fama and French (2015) show that other factors capture the return of HML factor and thus it becomes redundant. In this section I investigate the behavior of the 2x3 factors by regressing each one of them onto the others and by interpreting the magnitude of the intercept and the coefficients.

Table 4: Intercepts, coefficients and R-squared of regressing each factor on the other five. In the second row there is the corresponding t-statistic for each coefficient. The regressions are shown vertically. Using the p-value we see the significance: *** p<0.01, ** p<0.05, * p<0.1

Regression	(1)	(2)	(3)	(4)	(5)
Variables	MKT-RF	SMB	HML	RMW	CMA
SMB	-0.547***		0.066	-0.099***	-0.083**
	-5.45		1.41	-3.15	-2.29
HML	0.164	0.087		-0.107***	0.414***
	1.38	1.41		-2.95	11.75
RMW	-1.568***	-0.281***	-0.230***		-0.314***
	-10.17	-3.15	-2.95		-5.33
CMA	-1.173***	-0.181**	0.689***	-0.243***	
	-8.31	-2.29	11.75	-5.33	
MKT-RF		-0.145***	0.033	-0.147***	-0.142***
		-5.45	1.38	-10.17	-8.31
Intercept	1.107***	0.254**	0.301***	0.493***	0.183**
	5.26	2.27	3.1	7.99	2.42

Table 4 shows that market is an important factor for describing the returns with a really high intercept of 1.107% that is statistically significant at the 1% significance level. In this regression a remarkable result is the high negative coefficients of factors operating profitability and investment (both statistically significant).

Factors size and investment are important as their intercepts are significant at a 5% level though not large, only at 0.254% and 0.183% respectively. It seems that operating profitability factor is also really important in describing portfolio returns with a high (0.493%) and statistically significant intercept.

An important result concerns the value factor. It appears that in this sample the factor does not become redundant and this result is inconsistent with the result of Fama and

French (2015). In fact, the intercept is actually quite large, at 0.301% and it is statistically significant. This result is in line to the result of Cakici (2015) that includes all 5 factors and finds that HML is important in all of the geographic regions of his sample and on the contrary investment and profitability factors are not always significant. A possible explanation of this result can also be the sample period.

To summarize it seems that all five factors are important in capturing portfolio returns. It is also interesting that profitability has the second highest intercept, after market. Unlike Fama and French (2015), in this project it seems that HML does not become redundant and the returns of book-to-market ratio are not captured by other factors.

An interesting result in the factor spanning tests above is the huge intercept, 1.107% per month that I get when I regress the market factor on the other factors. This intercept is also statistically significant. That indicates that a time-series investing in market beta seems a profitable idea. We know that historically investing in high market-beta stocks is associated with low returns. As we know beta is a statistical measure of a portfolio against the market and cannot be diversified. CAPM uses as basis that everyone invests in portfolios with the highest Sharpe ratio (expected excess return per unit of risk). CAPM also assumes that all investors can leverage. But in reality, this is not the case as many individuals and funds have restrictions in leveraging and therefore, they overweight risky stocks in order to achieve high returns. This behavior, as Frazzini and Pedersen (2014) support, suggests that high beta stocks require lower returns than stocks with low beta that require leverage. Frazzini and Pedersen (2014) use a Betting-against-beta factor in their research. This factor is a portfolio that is long in low-beta assets, leveraged to a beta of 1 and short in high-beta stocks, de-leveraged to a beta of 1. The factor has positive average returns and those returns depend on the tightness of the leverage constraints. Tight liquidity constraints result in negative returns for the factor. Tight liquidity means overweighting risky stocks and therefore expected returns rise. They find that the return of their Betting-against-beta factor can compete against all the conventional factors in terms of significance and robustness. To summarize the betting against beta strategy is the strategy where you go long in underpriced (low-beta stocks) and short in overpriced (high-beta stocks). The prices of these stocks will eventually come back in a balance and therefore we exploit the mispricing as long as it is available. This seems feasible in a cross-sectional environment but the factor spanning tests above indicate that in a time-series, investing in the market is a good idea.

7.Model Performance Summary

Using the GRS test (Gibbons, Ross and Shanken (1989)), I am going to check if the intercepts α are indistinguishable from zero. In Table 5 below there are the GRS test results (GRS statistic and p-value) and also the following statistics: the average absolute value of intercept A$|\alpha_i|$ (A indicates an average value) is the average distance of intercept α_i from zero, in absolute value and it shows the unexplained part of the portfolios' return in absolute value. Therefore, a low value of A$|\alpha_i|$ indicates that the model describes well the LHS portfolios. Another statistic is $\frac{Aa_i^2}{A\bar{r}_i^2}$, where $\overline{r_i}$ is the difference between the time-series average excess return on the LHS portfolio i, $\overline{R_i}$ and the average excess return of the value-weight market portfolio, $\overline{R_M}$. In Fama and French (2015) instead of the average excess return of the market portfolio they use the cross-section average of $\overline{R_i}$ as a benchmark point, but in Fama and French (2017) they argue that the average excess return of the market portfolio is better. The ratio itself is the ratio of dispersion of the unexplained average returns of the LHS portfolios relative to the dispersion of the LHS average portfolio returns. Therefore, a high ratio signifies that a greater proportion of the dispersion of the LHS portfolio average returns comes from the dispersion of the unexplained average returns. Consequently, the higher the ratio, the worse the model. In the second ratio, $\frac{As^2(\alpha_i)}{Aa_i^2}$, the denominator as explained before is the dispersion of the unexplained average returns of the LHS portfolios. The numerator is the average of the squared sample standard errors of α_i. Thus, the ratio gives us the proportion of unexplained dispersion in average returns that is caused by sampling errors. A small value of the ratio corresponds to a bad-performing model because it means that little of the dispersion of the intercepts is sampling error and most is due to the dispersion of the true returns. The last statistic in Table 5 is the Adjusted R^2 (AR2).

There are 7 sets of LHS portfolios and 15 models for which I show the statistics. The GRS test rejects all models for most of the LHS portfolios and all p-values of getting a GRS statistic larger than the one presented in the table are zero. These results confirm that all models are incomplete descriptions of the LHS portfolios and that the intercepts are statistically significant and different from zero. Therefore, I am interested in showing the improvements that some of the models offer compared to the other models in these 7 sets of LHS portfolios.

In Panel A for the 25 portfolios formed on size and B/M, the best 3-factor model is the Fama and French one and has the lowest average absolute alpha. The model that performs the best in terms of average absolute intercept is the 6-factor model with an A$|\alpha_i|$ of 0.065%. Carhart's 4-factor model and Fama-French 5-factor model give similar results (at 0.071% and 0.072% respectively) and they show improvement over the 3-factor model. Interestingly, a 5-factor model with HML, CMA and WML performs quite well with an A$|\alpha_i|$ of 0.065%. The two ratios confirm the improvements that the models offer in my sample. Fama-French 5-factor model has a ratio of 0.34, thus only 34% of the dispersion of the LHS portfolio average returns results from the dispersion of the unexplained average returns, when the proportion for the Fama-French 3-factor model is 53%. Also, a smaller proportion of the dispersion of the unexplained part is due to sampling errors for the Fama-French 5-factor model compared to the 3-factor one. This proportion is even smaller for the 6-factor model.

In Panel B the results are more consistent to the Fama and French (2015) ones. The Fama-French 5-factor model performs well with around half the average absolute intercept of the Fama-French 3-factor model (at 0.092% and 0.183% respectively). Carhart's 4 factor model gives a small improvement over the 3-factor model (A$|\alpha_i|$=0.165%). The 6-factor model performs quite similar to the Fama and French 5-factor model, GRS Stat: 3.07, A$|\alpha_i|$=0.094 and GRS Stat=3.09, A$|\alpha_i|$=0.092 respectively. Weirdly the 4-factor model with the lowest average absolute alpha (0.107) is one with RMW and CMA, maybe because the portfolios are formed on size and OP. The two ratios confirm the results. The Fama-French 5-factor model and the 6-factor model have the same ratios. 27% of the dispersion of the LHS portfolio average returns result from the dispersion of the unexplained average returns (while 95% for the Fama-French 3-factor model). In addition, a small proportion of the dispersion of the unexplained part is due to sampling errors for the 6-factor model (and the Fama-French 5-factor model) compared to the proportion for the Fama-French 3-factor model which is larger.

Table 5: The table presents statistics that help evaluate the ability of the models to capture the LHS portfolio returns. Panel A shows the results for 25 Size-B/M portfolios, Panel B for 25 Size-OP portfolios, Panel C for 25 Size-Inv portfolios and Panel D for 25 Size-Mom portfolios. The GRS statistic tests if the intercepts of the models are indistinguishable of zero. The p-value, the average absolute intercept and two ratios. The first ratio is the ratio of dispersion of the unexplained average returns of the LHS portfolios relative to the dispersion of the LHS portfolio average returns. The second ratio gives us what proportion of the dispersion of the intercepts results from sampling errors. Market and Size are always the first 2 factors of each model. The table is split in two parts due to the size of it. A|a| in percentages.

Table 5A	**GRS**	**p-value**	**A|αi|**	**AR2**	$\frac{A\alpha_i^2}{A\bar{r}_i^2}$	$\frac{As^2(\alpha_i)}{A\alpha_i^2}$
Panel A: 25 Size-B/M Portfolios						
HML	2.60	0.00	0.093	0.95	0.53	0.25
RMW	3.00	0.00	0.210	0.92	2.39	0.09
CMA	2.79	0.00	0.100	0.93	0.71	0.24
WML	2.67	0.00	0.170	0.92	1.74	0.12
HML RMW	1.78	0.01	0.075	0.95	0.32	0.48
HML CMA	2.59	0.00	0.094	0.95	0.53	0.24
HML WML	1.90	0.01	0.071	0.95	0.35	0.40
RMW CMA	2.19	0.00	0.110	0.94	0.70	0.27
RMW WML	3.00	0.00	0.222	0.92	2.66	0.08
CMA WML	2.36	0.00	0.112	0.94	0.85	0.20
HML RMW CMA	1.79	0.01	0.072	0.95	0.34	0.43
HML RMW WML	1.59	0.04	0.065	0.95	0.25	0.61
RMW CMA WML	2.19	0.00	0.116	0.94	0.81	0.23
HML CMA WML	1.90	0.01	0.072	0.95	0.35	0.38
HML RMW CMA WML	1.65	0.03	0.065	0.95	0.28	0.52
Panel B: 25 Size-OP Portfolios						
HML	5.90	0.00	0.183	0.95	0.95	0.07
RMW	4.41	0.00	0.126	0.95	0.42	0.14
CMA	6.24	0.00	0.188	0.95	1.01	0.06
WML	5.72	0.00	0.172	0.94	0.78	0.09
HML RMW	3.22	0.00	0.081	0.95	0.23	0.27
HML CMA	6.04	0.00	0.187	0.95	0.98	0.06
HML WML	4.99	0.00	0.165	0.95	0.76	0.09
RMW CMA	3.49	0.00	0.107	0.95	0.37	0.17
RMW WML	4.54	0.00	0.139	0.95	0.47	0.13
CMA WML	5.59	0.00	0.179	0.95	0.87	0.07
HML RMW CMA	3.09	0.00	0.092	0.96	0.27	0.23
HML RMW WML	3.15	0.00	0.086	0.95	0.24	0.27
RMW CMA WML	3.61	0.00	0.115	0.95	0.39	0.16
HML CMA WML	5.08	0.00	0.166	0.95	0.76	0.08
HML RMW CMA WML	3.07	0.00	0.094	0.96	0.27	0.23

Panel C shows the statistics for LHS portfolios formed on Size and Inv. Between the 4 important models the 6-factor one has the lowest average absolute intercept at 0.070% and as we add factors A|α_i| decreases. It is also evident that Fama-French 5-factor has similar performance to the 6-factor model in this portfolio set (A|α_i|=0.076%, GRS statistic=1.90). In these three sets of LHS portfolio it is evident that in terms of average absolute intercept, the Fama-French 5-factor model and the 6-factor model offer improvements in the description of the LHS portfolios compared to other models. The ratios of dispersion confirm these results.

Table 5B	**GRS**	**p-value**	**A|αi|**	**AR2**	$\frac{A a_i^2}{A \bar{r}_i^2}$	$\frac{A s^2(\alpha_i)}{A a_i^2}$
Panel C: 25 Size-INV Portfolios						
HML	3.12	0.00	0.089	0.95	0.53	0.25
RMW	3.41	0.00	0.192	0.94	1.80	0.09
CMA	3.32	0.00	0.099	0.94	1.80	0.09
WML	3.23	0.00	0.148	0.94	1.18	0.14
HML RMW	2.15	0.00	0.085	0.95	0.44	0.35
HML CMA	3.11	0.00	0.089	0.96	0.54	0.20
HML WML	2.42	0.00	0.080	0.95	0.41	0.34
RMW CMA	2.30	0.00	0.089	0.95	0.47	0.27
RMW WML	3.30	0.00	0.196	0.94	1.85	0.09
CMA WML	2.91	0.00	0.104	0.95	0.65	0.17
HML RMW CMA	1.90	0.01	0.076	0.96	0.33	0.38
HML RMW WML	1.87	0.01	0.076	0.95	0.34	0.45
RMW CMA WML	2.24	0.00	0.087	0.96	0.47	0.27
HML CMA WML	2.41	0.00	0.080	0.96	0.41	0.27
HML RMW CMA WML	1.71	0.02	0.070	0.96	0.26	0.47
Panel D: 25 Size-Mom Portfolios						
HML	5.50	0.00	0.323	0.89	1.41	0.05
RMW	5.39	0.00	0.092	0.09	0.60	0.13
CMA	5.57	0.00	0.287	0.89	1.16	0.06
WML	5.00	0.00	0.160	0.94	0.31	0.10
HML RMW	4.61	0.00	0.220	0.89	0.78	0.11
HML CMA	5.49	0.00	0.321	0.89	1.39	0.89
HML WML	4.24	0.00	0.126	0.94	0.28	0.13
RMW CMA	4.54	0.00	0.170	0.89	0.53	0.15
RMW WML	5.17	0.00	0.174	0.94	0.41	0.09
CMA WML	4.64	0.00	0.134	0.94	0.31	0.11
HML RMW CMA	4.45	0.00	0.208	0.90	0.69	0.12
HML RMW WML	4.16	0.00	0.133	0.94	0.32	0.13
RMW CMA WML	4.36	0.00	0.138	0.94	0.34	0.11
HML CMA WML	4.24	0.00	0.126	0.95	0.28	0.12
HML RMW CMA WML	4.10	0.00	0.135	0.95	0.33	0.12

Fama-French 3-factor model performs quite poorly in explaining the returns of the Size-Mom portfolios with an A$|\alpha_i|$ of 0.32%. Interestingly in Panel D the 3-factor model with the lowest average absolute intercept is the one with MKT, SMB and RMW. Carhart’s 4-factor model has an average absolute alpha of 0.126% per month and is the best 4-factor model in this set. Surprisingly Fama-French 5-factor model produces the highest A$|\alpha_i|$ which makes it a bad model for explaining the Size-Mom portfolio returns. The rest of the 5-factor models have similar performance to each other. The 6-factor model has a similar average absolute intercept of 0.135%. Regarding the 25 portfolios formed on size and momentum as I add more factors to the models the average absolute intercept shrinks to zero.

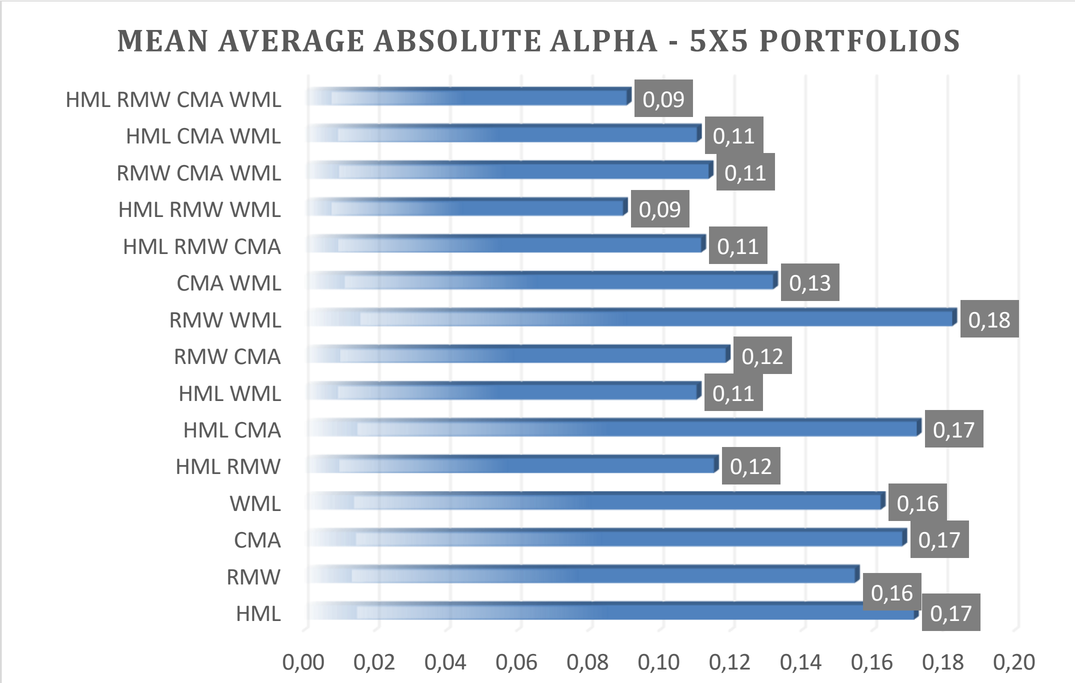
The two ratios presented in Panel D confirm the results discussed above. Carhart's 4-factor model performs well and according to the first ratio, a small percentage (28%) of the dispersion of the LHS portfolio average returns results from the dispersion of the unexplained average returns while for the Fama-French 5-factor model is 69% which again confirms the poor performance of this model. 33% of the dispersion of the LHS Size-Mom portfolio average returns results from the dispersion of the unexplained average returns for the 6-factor model. Unfortunately, the second ratio does not offer much in terms of interpretation in this set since there are not big differences between the models that can confirm a better performance.

I get similar results when the LHS portfolios are the 32 portfolios formed on combinations of size with value, profitability and investment factors. In this case the Fama-French 5-Factor model is often the best performer when I compare the average absolute intercept of the models. The 6-factor model performs equally well. The results are shown in Table A2 in Appendix.

The following graph shows the mean A$|\alpha_i|$ of the models in all 4 sets of 5x5 portfolio returns and it is an intuitive way to conclude what models produce the lowest average absolute intercepts. The results are close but looking in more detail it is evident that Carhart's 4-factor model, Fama-French 5-factor model and the 6-factor model perform better than other models in terms of A$|\alpha_i|$.

The model that describes least well the average returns is a 4-factor model with MKT, SMB, HML and RMW with a mean average absolute intercept of 0.18% per month. It is evident that as the number of factors increases, A$|\alpha_i|$ decreases. The 3-factor model performs almost equally well with a difference of only 1 basis point. Carhart's 4-factor model is the best among all 4-factor models with an average absolute intercept of 0.11%, a considerable improvement compared to the Fama-French 3-factor (0.17%).

Graph 1: The graph shows the performance of all models. Each bar represents the mean of the average absolute intercepts that a model produces for each of the 4 sets of 5x5 portfolios.



Comparing the overall (average) performance of all 5-factor models it seems that a model with MKT, SMB, HML, RMW and WML best explains the average returns. This model performs exactly as the Fama-French 5-factor model in explaining the returns of the 3 first portfolio sets but since it includes the momentum factor it does a far better job than the Fama-French 5-factor model in explaining the returns of portfolios formed on size and momentum. Therefore, the mean A$|\alpha_i|$ of this model is lower but that does not mean that this model performs better in general. As it is evident from Table A1 in Appendix the momentum factor seems to be the second most important after the market since it is not redundant and produces a high and statistically significant intercept of 0.479%. The rest of the 5-factor models seem to perform equally well. Fama-French 5-factor model performs well in the first 3 sets of 25 portfolios but poorly in explaining the 25 Size-Mom portfolios and that increases its mean A$|\alpha_i|$. The 6-factor model seems to explain the average returns better than any other model. Therefore, overall, the 6-factor model performs better in explaining average returns. The FF 5-factor model performs great in explaining the returns of 25 Size-BM, Size-OP and Size-Inv portfolios but performs poorly with 25 Size-Mom portfolios. Carhart's 4-factor model is overall the best 4-factor performer.

8.Regression Details

In this section I present tables with regression details about the Fama-French 3-factor and 5-factor models. More specifically the intercepts and coefficients and their t-statistics are going to be discussed alongside the pricing details of section 7. Fama and French (2015) find that HML factor is redundant and therefore they could discard their 5-factor model. However, they use the orthogonal HML, HMLO, which is defined as the sum of the intercept and residual from the regression of HML on Rm-Rf, SMB, RMW and CMA, because they believe there is a value premium that the model should capture. In section 6, the results show that none of the factors is redundant for my sample and therefore I am going to discuss the 5-factor model, using the HML factor. The 5-factor model therefore would be:

$$R_{i,t} - R_{f,t} = a_i + b_i * (R_{m,t} - R_{f,t}) + s_i * SMB_t + h_i * HML_t + r_i * RMW_t + c_i * CMA_t + \varepsilon_{i,t}$$

As Fama and French (2015) also state the intercept and the residuals of this regression are going to be the same either HML or HMLO is used. However, the slopes of the rest of the factors are going to change.

8.1 25 and 32 Portfolios

25 Size-B/M Portfolios

Panel A of Table 6 shows the regression intercepts of Fama-French 3-factor model when dependent variables are the 5x5 Size-B/M portfolios returns. The 3-factor model produces only 3 statistically significant intercepts at 1% significance level and 8 more at the 5% and 10% levels. I fail to reject the null for the rest of the intercepts because of their low t-statistics. Consistent to Fama and French (2015), I observe a negative intercept for portfolios of small and low B/M stocks and a positive intercept for big and low B/M stocks. The first intercept is sufficient in order to reject the 3-factor model. In Panel A there is only one intercept that is positive and significantly large, at 0.27%, for small and high B/M stocks.

Panel B shows the 5-factor intercept and the slopes of HML, RMW and CMA. Market slopes are always close to 1 and the ones for SMB are strongly positive for small stocks and then gradually become smaller and finally negative for big stocks. There are now 4 statistically significant intercepts at 1% significance level and 3 in the other levels. The rest are now indistinguishable from zero. The intercepts of extreme growth stocks converge to zero. Focusing on microcap stocks, HML's slope in the lowest

B/M quantile is -0.57 (t=-11.33) and CMA's slope is -0.37 (t=-5.72) and equal to the one of RMW (-0.37 and t=-5.02). There are 9 portfolios that have negative RMW and CMA slopes but only for a few of them both the intercepts and the slopes are statistically significant. HML slopes are strongly negative for low B/M portfolios and strongly positive for high B/M portfolios. Negative RMW and CMA slopes imply that a portfolio contains stocks with returns that have similar behavior to the returns of unprofitable firms that grow rapidly.

Table 6: Regressions for 25 value-weighted Size-B/M portfolios. The sample period is November 1990 to December 2019. The LHS variables are the monthly excess returns on the 25 Size-B/M portfolios. On the RHS there are Rm-Rf, SMB, HML, RMW and CMA, all constructed using the 2x3 sorts. Panel A shows the intercepts that the 3-factor model produces while Panel B shows the 5-factor intercepts and the coefficients of HML, RMW and CMA (h, r and c respectively). On the right side of both Panels there are the t-statistics of the coefficients.

B/M ->	Low	2	3	4	High		Low	2	3	4	High
					Panel A: 3-factor: Rm-Rf, SMB, HML						
			a						***t(a)***		
Small	-0.20	-0.04	0.05	0.12	0.27	Small	-2.22	-0.54	0.73	2.13	5.01
2	-0.28	-0.10	-0.04	0.03	-0.02	2	-4.09	-1.92	-0.99	0.65	-0.39
3	-0.09	-0.06	-0.14	-0.13	-0.03	3	-1.21	-1.09	-2.30	-2.27	-0.51
4	0.01	-0.01	-0.04	-0.03	-0.20	4	0.21	-0.11	-0.63	-0.56	-3.31
Big	0.13	0.09	0.00	0.02	-0.20	Big	2.23	1.71	-0.03	0.31	-2.36
					Panel B: 5-factor: Rm-Rf, SMB, HML, RMW and CMA						
			a						***t(a)***		
Small	-0.01	0.02	0.08	0.09	0.25	Small	-0.09	0.26	1.23	1.5	4.36
2	-0.13	-0.05	-0.03	0.02	0.03	2	-1.82	-0.87	-0.51	0.39	0.51
3	0.05	-0.03	-0.18	-0.17	0.01	3	0.7	-0.62	-2.82	-2.7	0.17
4	0.11	-0.02	-0.08	-0.06	-0.17	4	1.68	-0.39	-1.24	-0.94	-2.72
Big	0.15	-0.01	-0.03	0.00	0.01	Big	2.39	-0.1	-0.63	0.04	0.1
			h						***t(h)***		
Small	-0.57	-0.20	0.03	0.20	0.32	Small	-11.33	-4.45	0.89	6	10.21
2	-0.60	-0.15	-0.02	0.21	0.35	2	-15.4	-4.86	-0.8	7.91	11.86
3	-0.56	-0.14	0.10	0.25	0.35	3	-13.72	-4.43	2.82	7.51	10.72
4	-0.50	-0.09	0.08	0.25	0.49	4	-13.3	-2.79	2.38	7.3	14.31
Big	-0.66	-0.22	0.21	0.40	0.94	Big	-19.38	-7.63	7.37	11.87	20.79
			r						***t(r)***		
Small	-0.37	-0.11	-0.05	0.08	0.03	Small	-5.02	-1.64	-0.9	1.73	0.6
2	-0.29	-0.10	-0.04	0.02	-0.11	2	-5.19	-2.17	-0.99	0.41	-2.62
3	-0.28	-0.04	0.08	0.05	-0.11	3	-4.61	-0.97	1.64	1.11	-2.23
4	-0.20	0.03	0.07	0.04	-0.07	4	-3.59	0.58	1.49	0.72	-1.35
Big	-0.05	0.18	0.07	0.04	-0.40	Big	-0.94	4.22	1.59	0.85	-6.03
			c						***t(c)***		
Small	-0.37	-0.41	-0.36	-0.23	0.00	Small	-5.72	-7.05	-7.44	-5.34	-0.08
2	-0.19	-0.19	-0.02	0.08	0.23	2	-3.72	-4.7	-0.71	2.44	6.11
3	-0.13	0.00	0.12	0.22	0.33	3	-2.39	-0.08	2.6	5.15	7.93
4	-0.18	0.12	0.16	0.24	0.19	4	-3.66	2.78	3.7	5.44	4.34
Big	0.05	0.30	-0.03	-0.15	-0.53	Big	1.13	8.08	-0.9	-3.47	-9.05

Fama and French (2017) show that for small stock portfolios in the lowest B/M quintile neither the 3-factor nor the 5-factor model works. In their US results this is evident from their strong negative intercepts. Although the 5-factor model seems to perform well for the NA and Europe portfolios their GRS statistics show the opposite. In my research, the results are consistent with Fama and French (2017), as it is evident that the 3-factor model fails to explain the average returns of the 5x5 Size-

B/M set as there are strong negative intercepts for the microcap extreme growth portfolios. Although the 5-factor model improves the results and shrinks the intercepts to zero only a few of these intercepts are statistically significant. Hou, Karolyi and Kho (2011) evaluate a variety of characteristics and confirm that the value factor has a strong explanatory power in global stock returns but in their research this factor is based on the Cash flow to Price ratio and not on the B/M ratio.

<u>25 Size-OP Portfolios</u>

In Table 7 the LHS portfolios are formed on size and operating profitability. According to the pricing details in Table 5, Fama-French 5-factor model performs quite well in terms of average absolute intercept and this is confirmed in Table 7. In Panel A there are 11 intercepts that are statistically significant at the 1% level and 4 more at the other levels. The intercepts of low-profitability stock portfolios are negative and those of high-profitability stocks are positive. In Panel B, the 15 statistically significant intercepts become 9 (at all levels) and consistent to Fama and French (2015), for those intercepts that are close to zero I fail to reject the null. An interesting result is the large intercept (0.31%) of microcap stocks with high profitability. In fact, for microcap stocks as operating profitability increases the intercept increases. Only 3 intercepts jointly have negative exposures to RMW and CMA and only 2 of these slopes are statistically significant (at 1%), but the corresponding intercepts are not.

Fama and French (2017) find that the 3-factor model does not capture the profitability effect as the 3-factor intercepts increase from low to high profitability in the NA and Europe portfolios. Their 5-factor model absorbs the profitability pattern. The intercepts that the 5-factor model produces do not follow any pattern. The results for the developed countries portfolios are consistent to the ones of Fama and French (2017). In the summary statistics there is a pattern in profitability. Average monthly returns increase with profitability and especially in the small stock portfolios the difference between the low profitability portfolio and the high profitability portfolio is 0.60% per month. Consistent to Fama and French (2017), the 3-factor model intercepts show the same profitability pattern as the summary statistics and the 5-factor model solves the issue and there is no strong pattern in the 5-factor intercepts. However, there are some negative intercepts in the lowest profitability quintile that are statistically significant, especially the one in the second size quintile that remains significant in both Panels and is the strongest negative in Panel B at -0.13% (t=-2.73).

Table 7: Regressions for 25 value-weighted Size-OP portfolios. The sample period is November 1990 to December 2019. The LHS variables are the monthly excess returns on the 25 Size-OP portfolios. On the RHS there are Rm-Rf, SMB, HML, RMW and CMA which are all constructed using the 2x3 sorts. Panel A shows the intercepts that the 3-factor model produces while Panel B shows the 5-factor intercepts and the coefficients of HML, RMW and CMA (h, r and c respectively). On the right side of both Panels there are the t-statistics of the coefficients.

OP->	Low	2	3	4	High		Low	2	3	4	High
			Panel A: 3-factor: Rm-Rf, SMB, HML								
		a							***t(a)***		
Small	-0.16	0.17	0.31	0.41	0.48	Small	-2.32	3.34	6.08	7.95	7.28
2	-0.39	-0.03	-0.05	0.11	0.27	2	-6.93	-0.51	-0.92	2.45	4.62
3	-0.31	-0.13	-0.02	0.03	0.17	3	-4.48	-2.12	-0.4	0.59	2.91
4	-0.33	-0.06	0.04	0.08	0.09	4	-5.15	-1.02	0.65	1.3	1.5
Big	-0.47	-0.11	0.10	0.07	0.21	Big	-4.57	-1.63	1.99	1.31	3.57
			Panel B: 5-factor: Rm-Rf, SMB, HML, RMW and CMA								
		a							***t(a)***		
Small	0.01	0.17	0.24	0.29	0.31	Small	0.15	3.17	4.55	5.63	4.67
2	-0.13	0.06	-0.05	0.01	0.11	2	-2.73	1.07	-0.98	0.22	1.83
3	-0.09	-0.07	-0.06	-0.05	0.03	3	-1.47	-1.12	-1.04	-0.81	0.46
4	-0.12	-0.08	0.01	0.01	0.01	4	-1.96	-1.39	0.12	0.14	0.15
Big	0.12	0.12	0.02	-0.10	-0.03	Big	1.58	1.97	0.47	-1.97	-0.49
		h							***t(h)***		
Small	-0.02	0.20	0.17	0.20	0.05	Small	-0.65	6.62	5.72	7.08	1.29
2	-0.11	0.06	0.22	0.12	0.01	2	-4.24	2.06	7.31	4.46	0.23
3	-0.09	0.05	0.12	0.21	0.02	3	-2.75	1.4	3.52	6.24	0.58
4	-0.10	0.08	0.12	0.10	0.03	4	-3.02	2.37	3.58	2.81	1.02
Big	0.03	0.05	0.14	0.13	-0.29	Big	0.68	1.52	4.62	4.77	-9.27
		r							***t(r)***		
Small	-0.33	-0.01	0.14	0.24	0.37	Small	-5.91	-0.19	3.33	5.8	7.03
2	-0.53	-0.18	0.01	0.21	0.34	2	-13.71	-4.28	0.24	5.48	7.31
3	-0.46	-0.14	0.07	0.17	0.31	3	-9.13	-2.94	1.49	3.52	6.33
4	-0.45	0.03	0.05	0.13	0.16	4	-9.15	0.67	1.04	2.62	3.37
Big	-1.22	-0.47	0.16	0.36	0.48	Big	-19.33	-9.46	3.71	8.86	10.75
		c							***t(c)***		
Small	-0.28	-0.06	-0.11	-0.10	-0.13	Small	-5.65	-1.5	-2.97	-2.8	-2.78
2	0.03	0.16	0.06	-0.01	-0.07	2	0.78	4.23	1.57	-0.15	-1.75
3	0.24	0.28	0.17	-0.01	-0.09	3	5.3	6.52	3.91	-0.31	-2.19
4	0.13	0.27	0.15	0.09	-0.09	4	3.01	6.44	3.53	2.12	-2.06
Big	-0.27	0.03	-0.06	-0.13	0.15	Big	-4.83	0.75	-1.52	-3.7	3.75

25 Size-Inv Portfolios

According to Table 5 Fama-French 5-factor model and the 6-factor model improve the description of the portfolio returns over the Fama-French 3-factor model, as they produce a lower A$|\alpha_i|$. In Panel A of Table 8 there are 6 significant intercepts and in Panel B there are 7 but most of them not at 1% level. Consistent to Fama and French (2015), the 3-factor model produces negative intercepts for high-investment stock portfolios. In addition, there are some negative intercepts for medium size stocks with low investment. All of these negative intercepts tend to shrink to zero in Panel B but they are not statistically significant. The strongest negative intercept, -0.21 (t=-3.39), in Panel A for high investment at the second size quintile, becomes -0.11 with t=-2.06 with the Fama-French 5-factor model and this is probably due to the statistically significant slopes of RMW and CMA factors (-0.17, t=-4.04 and -0.54, t=-14.53 accordingly).

Table 8: Regressions for 25 value-weighted Size-Inv portfolios. The sample period is November 1990 to December 2019. The LHS variables are the monthly excess returns on the 25 Size-INV portfolios. On the RHS there are Rm-Rf, SMB, HML, RMW and CMA which are all constructed using the 2x3 sorts. Panel A shows the intercepts that the 3-factor model produces while Panel B shows the 5-factor intercepts and the coefficients of HML, RMW and CMA (h, r and c respectively). On the right side of both Panels there are the t-statistics of the coefficients.

Inv->	Low	2	3	4	High		Low	2	3	4	High
Panel A: 3-factor: Rm-Rf, SMB, HML											
			a						***t(a)***		
Small	0.09	0.24	0.30	0.20	-0.05	Small	1.47	5.29	6.05	3.31	-0.64
2	-0.17	0.12	0.07	-0.02	-0.21	2	-2.92	2.3	1.46	-0.31	-3.39
3	-0.08	0.03	-0.05	-0.10	-0.18	3	-1.25	0.46	-0.9	-1.62	-3
4	-0.10	-0.03	0.00	0.01	-0.08	4	-1.48	-0.53	-0.03	0.16	-1.23
Big	0.03	0.00	-0.03	0.01	0.05	Big	0.35	-0.03	-0.57	0.22	0.63
Panel B: 5-factor: Rm-Rf, SMB, HML, RMW and CMA											
			a						***t(a)***		
Small	0.12	0.17	0.23	0.14	0.05	Small	1.85	3.54	4.38	2.26	0.64
2	-0.11	0.13	0.04	0.00	-0.11	2	-2.21	2.76	0.92	0.05	-2.06
3	-0.06	0.02	-0.08	-0.11	-0.07	3	-0.94	0.42	-1.36	-1.67	-1.24
4	-0.08	-0.06	0.00	0.00	-0.02	4	-1.26	-1.07	-0.08	0.07	-0.27
Big	0.05	-0.07	-0.07	0.03	0.07	Big	0.8	-1.46	-1.26	0.67	1.13
			h						***t(h)***		
Small	0.11	0.16	0.20	0.21	-0.12	Small	3.18	6.12	6.96	6.31	-2.65
2	0.04	0.09	0.15	0.08	-0.10	2	1.48	3.26	5.79	2.51	-3.36
3	0.04	0.13	0.17	0.15	-0.19	3	1.35	4.24	5.41	4.37	-6.15
4	0.12	0.07	0.14	0.12	-0.21	4	3.22	2.24	4.36	3.39	-6.18
Big	-0.20	0.03	0.06	0.10	-0.02	Big	-6.32	1.16	2.03	3.66	-0.57
			r						***t(r)***		
Small	-0.07	0.14	0.15	0.14	-0.18	Small	-1.32	3.51	3.51	2.8	-2.78
2	-0.15	-0.05	0.04	-0.03	-0.17	2	-3.69	-1.39	0.92	-0.72	-4.04
3	-0.08	-0.02	0.04	0.03	-0.19	3	-1.61	-0.4	0.93	0.55	-4.17
4	-0.05	0.03	-0.01	0.02	-0.10	4	-1	0.72	-0.12	0.3	-2.01
Big	-0.09	0.13	0.08	-0.01	-0.01	Big	-1.86	3.04	1.87	-0.31	-0.13
			c						***t(c)***		
Small	0.03	0.12	-0.07	-0.23	-0.64	Small	0.73	3.52	-1.99	-5.31	-11.21
2	0.44	0.34	0.16	-0.08	-0.54	2	12.25	9.9	4.64	-2.2	-14.53
3	0.44	0.39	0.25	-0.06	-0.37	3	10.41	9.52	6.01	-1.37	-9.05
4	0.35	0.42	0.19	-0.07	-0.35	4	7.64	10.74	4.59	-1.64	-8.01
Big	0.73	0.41	0.02	-0.48	-0.66	Big	17.85	11.21	0.42	-13.65	-15.1

The intercept of the second size category stocks with high investment is enough in order to reject the 5-factor model as a description of returns on the Size-Inv portfolios. This is a similar problem to the one observed in Table 6 about the microcap portfolio in the lowest B/M category. There is a negative exposure to RMW and CMA like the one of firms with high investment despite their low profitability. In Panel B there are some slope patterns. All portfolios in the highest investment quintile have negative exposures to the HML and the RMW factor. The slopes of HML for most low investment portfolios are positive and close to zero.

Fama and French (2017) find that the 5-factor intercepts for Europe are close to zero. For NA both models produce strong positive intercepts for microcap portfolios in the lowest investment quintiles. They argue that the improvements that the 5-factor model offers are due to negative slopes of RMW and CMA. In my sample, Fama-French 5-factor model offers improvements over the 3-factor model, as intercepts are indeed closer to zero, but I fail to reject the null for most of them, and the improvements seem to be due to the negative RMW and CMA slopes, as in Fama and French (2017).

25 Size-Mom Portfolios

Table 9, shows the Fama-French 3-factor and 5-factor intercepts and coefficients for portfolios formed on size and momentum. According to Table 5, Fama-French 3-factor model produces an average absolute intercept of 0.32% and the 5-factor model of 0.21%. The rest of the statistics also confirm that the Fama-French 5-factor model performs better than the Fama-French 3-factor model as a description of expected returns of portfolios formed on size and momentum. In Panel A there are 19 significant intercepts, 15 of them at 1% significance level, and in Panel B they become 13 and only 8 at 1% significance level. In Panel B many intercepts shrink to 0 but for most of them I fail to reject the null hypothesis because of their low t-statistics.

Both models produce negative intercepts for low-momentum stocks. A similar problem is evident in Tables 6 and 7 where the models produce strong negative intercepts for microcap and low profitability stocks, accordingly. The intercepts become positive for the respective portfolios with high momentum. The strongest negative intercept that the Fama-French 5-factor model produces is the one for extremely small stock portfolios with low momentum at -0.47% (t=-3.76). This portfolio has a positive slope of HML (0.24) and negative slopes of RMW and CMA (-0.65 and -0.53 respectively). Similar negative slopes of RMW and CMA are evident in the 25 Size-B/M and Size-OP portfolios. The large number of significant negative intercepts though, makes the 5-factor model inappropriate to describe expected returns on portfolios formed on size and momentum, in this sample of developed countries.

Table 9: Regressions for 25 value-weighted Size-Mom portfolios. The sample period is November 1990 to December 2019. The LHS variables are the monthly excess returns on the 25 Size-Mom portfolios. On the RHS there are Rm-Rf, SMB, HML, RMW and CMA which are all constructed using the 2x3 sorts. Panel A shows the intercepts that the 3-factor model produces while Panel B shows the intercepts and the coefficients of HML, RMW and CMA (h, r and c respectively). On the right side of both Panels there are the t-statistics of the coefficients.

Mom->	Low	2	3	4	High		Low	2	3	4	High
					Panel A: 3-factor: Rm-Rf, SMB, HML						
		a						***t(a)***			
Small	-0.80	-0.04	0.21	0.52	0.89	Small	-6.46	-0.70	3.96	8.50	7.68
2	-0.75	-0.23	0.02	0.23	0.56	2	-5.55	-3.41	0.38	3.85	5.59
3	-0.65	-0.24	-0.08	0.17	0.39	3	-4.86	-3.35	-1.31	2.65	3.87
4	-0.54	-0.16	-0.04	0.08	0.41	4	-4.12	-2.06	-0.61	1.14	3.84
Big	-0.40	-0.19	0.05	0.17	0.27	Big	-2.61	-2.57	0.89	2.31	2.05
					Panel B: 5-factor: Rm-Rf, SMB, HML, RMW and CMA						
		a						***t(a)***			
Small	-0.47	-0.01	0.15	0.46	0.84	Small	-3.76	-0.18	2.63	7.04	6.71
2	-0.38	-0.15	-0.02	0.15	0.53	2	-2.74	-2.14	-0.26	2.51	4.93
3	-0.32	-0.19	-0.12	0.08	0.30	3	-2.31	-2.46	-1.84	1.30	2.72
4	-0.22	-0.09	-0.07	-0.04	0.31	4	-1.61	-1.13	-1.15	-0.61	2.69
Big	-0.05	-0.09	-0.04	-0.04	0.06	Big	-0.34	-1.16	-0.66	-0.48	0.41
		h						***t(h)***			
Small	0.24	0.21	0.16	0.01	-0.13	Small	3.43	5.77	5.19	0.31	-1.95
2	0.27	0.17	0.11	-0.05	-0.28	2	3.59	4.31	3.48	-1.51	-4.71
3	0.29	0.22	0.13	-0.06	-0.32	3	3.78	5.22	3.53	-1.72	-5.46
4	0.38	0.20	0.09	-0.04	-0.28	4	5.13	4.25	2.62	-1.15	-4.40
Big	0.22	0.23	0.18	-0.01	-0.41	Big	2.52	5.15	5.91	-0.19	-5.42
		r						***t(r)***			
Small	-0.65	-0.06	0.12	0.11	0.11	Small	-6.52	-1.15	2.54	2.04	1.12
2	-0.74	-0.16	0.06	0.14	0.04	2	-6.70	-2.77	1.32	2.80	0.47
3	-0.66	-0.12	0.06	0.15	0.18	3	-5.92	-1.92	1.12	2.99	2.08
4	-0.64	-0.15	0.05	0.23	0.20	4	-5.89	-2.18	0.99	4.08	2.13
Big	-0.68	-0.20	0.17	0.41	0.43	Big	-5.36	-3.05	3.66	6.85	3.80
		c						***t(c)***			
Small	-0.53	-0.04	0.11	0.14	-0.17	Small	-6.05	-0.79	2.78	3.06	-1.98
2	-0.36	0.09	0.22	0.28	0.05	2	-3.67	1.76	5.39	6.48	0.59
3	-0.30	0.16	0.29	0.34	0.20	3	-3.08	2.95	6.27	7.63	2.59
4	-0.31	0.14	0.34	0.30	0.08	4	-3.26	2.37	7.41	5.87	0.94
Big	-0.34	-0.10	0.12	0.17	0.13	Big	-3.02	-1.84	3.08	3.32	1.28

Hou, Karolyi and Kho (2011) make a large number of experiments. They use different ratios and types of characteristics and they find that there is a medium-term stock-price momentum in international markets. The strong momentum effect is consistent through their entire sample. A factor constructed based on this momentum can add to the explanatory power of their global value factor that is based on the Cash-flow to Price ratio. They test local, global and international multifactor models and they find that an international version of a multifactor model that contains the momentum factor provides the lowest pricing errors and rejection rates compared to other versions that they test. The three-factor model includes Cash-flow to price ratio, momentum and the global market factor and also captures strong variation in global stock returns.

32 Size-OP-Inv Portfolios

Table 10 shows the intercepts and coefficients that the Fama-French models produce in the Size-OP-Inv portfolio set. For small stocks, only 3 out of the 7 strongly significant intercepts of Panel A remain significant at 1% level in Panel B. Fama-French 3-factor model presents a variety of problems in Panel A. Some of the problems are absorbed by the slopes of the factors in the 5-factor model.

Table 10: Regressions for 32 value-weighted Size-OP-Inv portfolios. The sample period is November 1990 to December 2019. The LHS variables are the monthly excess returns on the 32 Size-OP-Inv portfolios. On the RHS we there are Rm-Rf, SMB, HML, RMW and CMA which are all constructed using the 2x3 sorts. Panel A shows the intercepts that the 3-factor model produces while Panel B shows the 5-factor intercepts and the coefficients of HML, RMW and CMA (h, r and c respectively). On the right side of both Panels there are the t-statistics of the coefficients.

	Small								Big							
OP->	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
	Panel A: Three-factor intercepts: RM-RF, SMB, and HML															
	α				***t(α)***				***α***				***t(α)***			
Low Inv	-0.36	-0.14	0.15	0.19	-3.15	-1.84	2.55	2.74	-0.42	-0.01	0.12	0.28	-3.53	-0.15	1.44	2.65
2	-0.29	-0.05	0.13	0.31	-2.86	-0.94	3.12	6.01	-0.35	0.04	0.08	0.18	-3.39	0.59	1.15	2.27
3	-0.40	-0.15	0.07	0.29	-3.65	-2.53	1.65	5.01	-0.34	0.02	0.05	0.09	-3.27	0.22	0.70	1.02
High Inv	-0.79	-0.40	-0.09	0.13	-6.84	-4.76	-1.66	2.02	-0.40	0.06	0.08	0.18	-3.21	0.75	0.80	1.83
	Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW, and CMA															
	α				***t(α)***				***α***				***t(α)***			
Low Inv	-0.13	-0.09	0.14	0.07	-1.16	-1.29	2.27	0.95	0.00	0.11	-0.01	-0.04	0.00	1.17	-0.10	-0.40
2	-0.13	0.01	0.08	0.16	-1.29	0.13	1.84	3.05	0.01	0.03	-0.09	-0.07	0.06	0.33	-1.22	-0.95
3	-0.21	-0.05	0.01	0.19	-1.78	-0.75	0.30	3.32	0.09	0.05	-0.01	0.00	0.93	0.58	-0.09	0.02
High Inv	-0.44	-0.16	-0.03	0.01	-3.88	-2.03	-0.59	0.21	0.12	0.13	-0.02	0.03	1.13	1.57	-0.21	0.38
	h				***t(h)***				***h***				***t(h)***			
Low Inv	-0.41	0.04	0.18	0.17	-6.93	1.09	5.25	4.18	-0.12	-0.06	0.09	-0.29	-2.25	-1.11	1.97	-5.48
2	-0.12	0.12	0.21	0.24	-2.16	4.42	8.79	8.41	0.10	0.20	0.24	-0.07	1.83	4.89	5.92	-1.77
3	-0.11	0.19	0.26	0.24	-1.79	5.84	10.73	7.60	0.30	0.14	0.07	-0.08	5.73	2.91	1.74	-1.49
High Inv	-0.39	-0.03	0.08	-0.08	-6.24	-0.68	2.59	-2.53	-0.04	0.14	0.14	-0.21	-0.79	3.14	2.46	-4.29
	r				***t(r)***				***r***				***t(r)***			
Low Inv	-0.53	-0.14	0.01	0.23	-6.06	-2.47	0.17	4.01	-0.91	-0.29	0.24	0.61	-11.91	-3.98	3.50	7.72
2	-0.37	-0.14	0.10	0.31	-4.68	-3.50	2.72	7.20	-0.75	0.03	0.35	0.49	-9.52	0.43	5.75	8.05
3	-0.40	-0.21	0.12	0.21	-4.27	-4.36	3.41	4.53	-0.86	-0.05	0.13	0.19	-11.10	-0.76	2.18	2.44
High Inv	-0.69	-0.46	-0.10	0.26	-7.60	-7.35	-2.31	5.29	-1.00	-0.11	0.25	0.33	-12.16	-1.62	3.08	4.62
	c				***t(c)***				***c***				***t(c)***			
Low Inv	0.55	0.53	0.28	0.23	7.21	10.79	6.51	4.59	0.61	0.65	0.39	0.65	8.99	10.16	6.43	9.36
2	0.48	0.35	0.21	0.11	6.91	9.90	6.71	2.95	0.16	0.10	0.26	0.34	2.29	1.83	4.91	6.44
3	0.03	0.07	-0.09	-0.20	0.35	1.74	-2.94	-4.94	-0.44	-0.20	-0.19	-0.07	-6.43	-3.28	-3.86	-1.01
High Inv	-0.57	-0.58	-0.34	-0.30	-7.11	-10.46	-8.74	-7.05	-0.98	-0.51	-0.55	-0.57	-13.55	-8.70	-7.66	-8.96

Both for small and big stocks RMW slopes are negative for low profitability and positive for high profitability as in Fama and French (2015). In addition, for both small and big stock portfolios CMA slopes are strongly negative for the highest investment quartiles and positive for the lowest. Fama-French 5-factor model offers some improvements (consistent with the results in Table 5) as the intercepts shrink towards zero but I fail to reject the null hypothesis for the majority of them. In Panel B Fama-French 5-factor model has some problems in some portfolios. The strongest negative intercept is the one for small stock portfolios with low operating profitability and high investment at -0.44% (t=-3.88). This problem is also evident in Fama and French (2015). This extreme significant negative intercept rejects Fama-French 5-factor model as a model that can describe the expected returns on the 32 Size-OP-Inv portfolios. For small stocks in the lowest profitability quartile all intercepts are negative but those for high Inv are stronger therefore, it seems that 5-factor problems are associated mostly with small stocks with high investment. The biggest intercept for big stock portfolios is the one for portfolios with low profitability and high investment at 0.12% (t=1.13). Therefore, the problems are not carried over to big stock portfolios, as the intercepts in the lowest profitability quartile are positive, though the null is not rejected.

Overall, it is obvious that the Fama-French 3-factor model fails to explain all sets of portfolios as it has many problems, especially in the microcap stock portfolios. It gives strongly negative intercepts for portfolios of small stocks in all 4 sets of the 5x5 portfolios. Fama-French 5-factor model and the 6-factor model offer some improvements regarding the intercepts. The intercept of low B/M stocks at the second size quintile goes from -0.28% (t=-4.09) in the 3-factor model to -0.13% (t=-1.82) in the 5-factor model. Similar improvements are seen in the other sets of portfolios. For high Inv stocks at the second size quintile (Table 8) the intercept shrinks from -0.21% (t=-3.39) to -0.11% (t=-2.06). In the big and low operating profitability stock portfolio (Table 7) the change in the intercept from -0.47% (t=-4.57) in the 3-factor model to 0.12% (t=1.58) in the 5-factor model is quite large. Finally, although 5-factor model's performance in the 25 Size-Mom portfolios is poor there still is a big improvement over the 3-factor model (from -0.80% to -0.47%) for the extreme microcap low momentum portfolios. Therefore, it seems that Fama-French 5-factor model, although it has many problems, offers improvements in capturing the variation in average returns related to B/M, profitability, investment and momentum. Most of the improvements trace to the negative slopes of factors RMW and CMA.

8.2 6-factor model

Table 11 shows the 6-factor intercepts and their t-statistics. All the statistically significant intercepts are far from zero and for the ones that are closer to zero I fail to reject the null hypothesis. Consistent to Table 5 statistics and the low $A|\alpha_i|$ (0.065%), the 6-factor model shrinks most intercepts to zero in the Size-B/M portfolio set. In

Panel B there is a profitability pattern for small stock portfolios, where 4 out of 5 intercepts are significant at 1% significance level, but patterns are not present for big stock portfolios. In Panel C there are no clear patterns. The two statistically significant intercepts at 1% level are far from zero and that makes the model incapable of explaining the Size-Inv expected returns. In Panel D the difference in returns of small high momentum stocks with that of small low momentum stocks is extremely large. The momentum trend is evident for small, but not for big stocks. According to the results in Panel D of Table 5 the 6-factor model is one of the best models in explaining the average returns of portfolios formed on size and momentum but the results cannot be confirmed based on the t-statistics.

Table 11: Regressions for all 25 value-weighted portfolios. The sample period is November 1990 to December 2019. The LHS variables are the monthly excess returns on the 25 portfolios. On the RHS there are Rm-Rf, SMB, HML, RMW, CMA and WML which are all constructed using the 2x3 sorts. Panels A, B, C and D show the intercepts that the 6-factor model produces with different sets of 5x5 portfolios. On the right side of both Panels there are the t-statistics of the coefficients.

6-factor: Rm-Rf, SMB, HML, RMW, CMA, WML											
Panel A: 25 Size-B/M Portfolios											
B/M->	Low	2	3	4	High		Low	2	3	4	High
			a						***t(a)***		
Small	-0.04	0.01	0.08	0.09	0.23	Small	-0.43	0.16	1.18	1.45	3.95
2	-0.12	-0.01	-0.01	0.03	0.02	2	-1.62	-0.17	-0.26	0.54	0.45
3	0.09	-0.02	-0.13	-0.13	0.01	3	1.17	-0.31	-2.13	-2.16	0.21
4	0.13	0.00	-0.06	-0.04	-0.16	4	1.85	-0.08	-0.92	-0.63	-2.52
Big	0.15	-0.01	-0.01	0.00	0.04	Big	2.4	-0.27	-0.25	0.05	0.51
Panel B: 25 Size-OP Portfolios											
OP->	Low	2	3	4	High		Low	2	3	4	High
			a						***t(a)***		
Small	-0.03	0.16	0.24	0.29	0.29	Small	-0.38	2.87	4.46	5.46	4.43
2	-0.14	0.05	-0.02	0.04	0.12	2	-2.87	0.9	-0.47	0.83	2.07
3	-0.08	-0.05	-0.03	-0.02	0.04	3	-1.33	-0.78	-0.49	-0.27	0.74
4	-0.13	-0.06	0.03	0.02	0.03	4	-2.12	-1.08	0.54	0.32	0.5
Big	0.15	0.14	0.04	-0.10	-0.05	Big	1.96	2.2	0.69	-1.89	-0.89
Panel C: 25 Size-INV Portfolios											
INV->	Low	2	3	4	High		Low	2	3	4	High
			a						***t(a)***		
Small	0.09	0.15	0.22	0.14	0.03	Small	1.42	3.16	4.13	2.27	0.36
2	-0.11	0.12	0.06	0.04	-0.10	2	-2.06	2.45	1.25	0.8	-1.93
3	-0.06	0.05	-0.05	-0.05	-0.05	3	-0.95	0.85	-0.9	-0.85	-0.95
4	-0.09	-0.05	0.03	0.03	0.00	4	-1.29	-0.82	0.5	0.51	-0.04
Big	0.04	-0.07	-0.03	0.04	0.04	Big	0.71	-1.39	-0.53	0.83	0.64
Panel D: 25 Size-MOM Portfolios											
Mom->	Low	2	3	4	High		Low	2	3	4	High
			a						***t(a)***		
Small	-0.22	0.07	0.15	0.38	0.63	Small	-2.69	1.10	2.53	6.50	6.45
2	-0.06	-0.04	0.00	0.09	0.33	2	-0.91	-0.68	0.04	1.54	4.39
3	-0.01	-0.07	-0.09	0.01	0.07	3	-0.08	-1.12	-1.36	0.21	1.05
4	0.07	0.05	-0.05	-0.12	0.08	4	0.87	0.79	-0.74	-1.82	1.05
Big	0.30	0.04	-0.03	-0.18	-0.25	Big	3.40	0.71	-0.46	-3.60	-3.26

8.3 GJR Analysis

In addition to the previous analysis I decided to test for heteroscedasticity and try to correct it using a GJR(1,1) model. The goal of the GJR model is to give more accurate standard errors, which might change the significance of the intercepts and the coefficients. By creating a variety of twoway scatterplots of residuals against date for a variety of portfolios and models I see that there is heteroscedasticity in my data. An heteroscedasticity test and the rejection of the constant variance confirms the scatterplots. The graphs of autocorrelation and partial autocorrelation confirm that no more lags are necessary and a GJR(1,1) model is sufficient.

However, when running the loop of the regressions with the GJR, Stata fails to converge to a unique log likelihood for many models and this issue doesn't allow me to show complete regression tables as in part 8. Therefore, I am showing the intercepts and standard errors alongside with AIC and BIC statistics for a few randomly chosen portfolios and models and I compare them to the OLS results.

Table 12: OLS and GJR results for random models and portfolios. And their AIC (Akaike) and BIC statistics. Below each estimate, the respective t or z statistics are displayed.

	Alpha	Mktrf	SMB	HML	RMW	CMA	WML	AIC	BIC
Big-High B/M-High Inv	FF 3-Factor								
OLS	-0.30	1.15	-0.09	0.56				1530.75	1546.18
	-2.59	44.58	-1.53	10.29					
GJR	-0.22	1.14	0.00	0.52				1447.81	1478.67
	-2.44	90.36	-0.09	10.99					
Big-Low B/M-Low Inv	3-factor								
OLS	-0.08	0.90	-0.28				0.07	1584.00	1599.43
	-0.65	31.24	-4.33				1.86		
GJR	0.02	0.84	-0.29				0.13	1516.49	1547.35
	0.21	41.33	-5.67				4.23		
Small High B/M	FF 5-Factor								
OLS	0.25	0.92	1.06	0.32	0.03	0.00		981.81	1004.96
	4.36	64.75	38.51	10.21	0.60	-0.08			
GJR	0.25	0.93	1.08	0.31	0.03	0.01		970.45	1009.03
	4.30	66.84	41.28	9.55	0.65	0.31			
Small Low Mom	6-Factor								
OLS	-0.22	1.04	1.16	-0.01	-0.24	-0.34	-0.52	1228.02	1255.03
	-2.69	51.25	29.23	-0.16	-3.43	-5.74	-21.30		
GJR	-0.20	1.04	1.16	0.00	-0.24	-0.36	-0.52	1234.27	1276.71
	-2.22	49.50	30.73	-0.06	-3.42	-6.28	-24.10		

It is evident that for each combination of model and portfolio, AIC and BIC for OLS and GJR are close. Therefore, an important conclusion is that the GJR model doesn't really improve the results. Sometimes the BIC is even smaller for the OLS regression. Comparing just the BIC, I see that bad models (of Table 5) become slightly better with GJR while for the best models the opposite is true. To summarize it seems that implementing a GJR(1,1) model in my sample is not so useful since the improvements that I get are minor. As can be seen, the coefficients of OLS differ from those of GJR, which might be due to the presence of endogeneity bias. This is consistent with the findings from Allen and McAleer (2019), which show evidence that a key issue on Fama and French regressions is the endogeneity of the factors. This endogeneity in the Fama-French factors leads to biased and inconsistent estimates in their multiple regressions.

9.Conclusions and Discussion

The perfect factor model would efficiently explain the expected returns on the portfolios and therefore there would not be any intercept to exploit. A variety of factors and characteristics can explain the returns. In my sample there are strong patterns for size, value, profitability, investment and momentum in both the summary statistics and the regression details. Average returns increase when B/M, profitability and momentum increase and there is a negative investment-returns relationship. The patterns are stronger for small stocks. An interesting and inconsistent with Fama and French (2015) result is that the HML factor is not redundant in this sample and therefore its returns are not captured by other factors, like RMW and CMA.

According to Table 5 the model that seems to capture more efficiently the variation in average portfolio returns, although an imperfect model, is a 6-factor model that is an extension of Fama-French 5-factor model and includes momentum. This model produces the lowest average absolute intercepts and the best dispersion ratios for most sets of LHS portfolios. Fama-French 5-factor model has a comparable good performance, although it explains poorly the average returns of portfolios formed on size and momentum. Carhart’s 4-factor model always explains the returns more sufficiently than the Fama-French 3-factor model and it is overall a model with relatively low average absolute intercepts. Interestingly a 5-factor model that includes momentum instead of investment produces low average absolute intercepts and this might have to do with the high importance of momentum in this sample, seen by the factor spanning tests in Appendix 1. Although there is heteroscedasticity in the data a GJR(1,1) model does not offer great improvements, as someone would expect. Consistent with Fama and French (2015), the biggest asset pricing problems are connected with portfolios of small stocks, that have significant negative intercepts, and especially for portfolios that have significant negative exposures to RMW and CMA, as they include firms that behave like unprofitable firms that invest a lot. The problem is present also in the 6-factor model and thus neither the updated dataset nor the addition of a new factor solves the problems of these misbehaving portfolios.

Fama-French 5-factor model and the 6-factor model offer improvements in all sets of portfolios by shrinking the intercepts to zero and the 2 models solve some of the problems that are present with the Fama-French 3-factor model. The solutions to some of the problems could possibly be due to the negative exposures to some of the factors. However, they are still imperfect models. A model that completely captures the expected returns, the perfect model, is the one that the intercept of the regression of a portfolio on the factor returns of the model is indistinguishable from zero. The GRS tests this hypothesis and easily rejects it for all models in most of the LHS portfolios. Therefore, it seems that there is no perfect model. An imperfect model could still be useful in the subject of asset pricing. An extension to this research project could be a Fama-MacBeth cross-sectional analysis of this updated sample of portfolio and factor returns. Finally, a GARCH model could possibly solve some of the computational issues that the GJR model presents.

10.Appendix

10.1 Factor spanning tests including Momentum factor

Table A1: Intercepts, coefficients and R-squared of regressing each factor on the other five. In the second row there is the corresponding t-statistic for each coefficient. Also using the p-value we see the significance: *** p<0.01, ** p<0.05, * p<0.1

Regression	(1)	(2)	(3)	(4)	(5)	(6)
Variables	MKT-RF	SMB	HML	RMW	CMA	WML
SMB	-0.522***		0.090*	-0.113***	-0.095***	0.216**
	-5.15		1.96	-3.72	-2.63	2.47
HML	0.117	0.123*		-0.051	0.435***	-0.470***
	0.95	1.96		-1.41	12.22	-4.69
RMW	-1.478***	-0.342***	-0.112		-0.359***	0.811***
	-8.97	-3.72	-1.41		-5.97	5.52
CMA	-1.127***	-0.208***	0.697***	-0.261***		0.382***
	-7.83	-2.63	12.22	-5.97		2.96
WML	-0.098	0.081**	-0.128***	0.100***	0.065***	
	-1.52	2.47	-4.69	5.52	2.96	
MKT-RF		-0.137***	0.022	-0.128***	-0.134***	-0.068
		-5.15	0.95	-8.97	-7.83	-1.52
Intercept	1.147***	0.211*	0.344***	0.404***	0.147*	0.479***
	5.41	1.88	3.63	6.6	1.94	2.62

When momentum factor is added, there are changes in the coefficients and the intercepts. It is important to notice that when momentum is added SMB and CMA are statistically significant at a 10% level instead of a 5% level as in Part 6. In addition, momentum factor seems quite important as its intercept is statistically significant at a 1% significance level and it is the second larger after the market one, at 0.479%. In the first regression the intercept of the market (1.147%) is even larger when momentum is added and it is statistically significant at a 1% significance level. Another interesting result is that the intercept of HML increases (0.344%) and the one of RMW (0.404%) decreases when momentum is taken into consideration.

10.2 Pricing Details for the 2x4x4 Portfolios

Table A2: Summary statistics for the 4 dominant factor models. The table shows the ability of the models to capture the LHS portfolio returns. Panel A shows the results for 32 Size-B/M-Inv portfolios, Panel B for 32 Size-B/M-OP portfolios, Panel C for 32 Size-OP-Inv portfolios. The statistics are explained in Table 5.

	GRS	p-value	A|αi|	AR2	$\frac{A\alpha_i^2}{A\bar{r}_i^2}$	$\frac{As^2(\alpha_i)}{A\alpha_i^2}$
Panel A: 32 Size-B/M-INV Portfolios						
HML	2.77	0.00	0.092	0.91	0.38	0.43
HML WML	2.28	0.00	0.084	0.92	0.29	0.61
HML RMW CMA	1.35	0.11	0.069	0.93	0.20	0.82
HML RMW CMA WML	1.35	0.11	0.066	0.93	0.18	0.89
Panel B: 32 Size-B/M-OP Portfolios						
HML	4.00	0.00	0.207	0.89	0.59	0.13
HML WML	3.66	0.00	0.193	0.89	0.52	0.16
HML RMW CMA	2.08	0.00	0.138	0.90	0.26	0.30
HML RMW CMA WML	2.24	0.00	0.142	0.90	0.27	0.29
Panel C: 32 Size-OP-INV Portfolios						
HML	3.99	0.00	0.208	0.90	0.95	0.10
HML WML	3.71	0.00	0.191	0.90	0.77	0.14
HML RMW CMA	1.84	0.00	0.084	0.92	0.20	0.47
HML RMW CMA WML	2.23	0.00	0.095	0.92	0.23	0.41

10.3 Gibbson Ross Shanken test

The GRS test and its statistic are used in order to evaluate the ex-ante efficiency of a portfolio. In fact, Gibbons, Ross and Shanken (1989) use multivariate statistical methods in order to check if any portfolio is ex-ante mean-variance efficient. If a portfolio is mean-variance efficient then the next two conditions must be satisfied: E($\widetilde{r_{it}}$)=β_{ip}*E($\widetilde{r_{pt}}$) and H_0: $\alpha_{ip} = 0$, where $\widetilde{r_{it}}$ is the excess return on asset i , $\widetilde{r_{pt}}$ the excess return on the portfolio, β_{ip} its beta and α_{ip} the intercept. In this research, the GRS test examines the efficiency of a model and how well it explains the average returns of the portfolios, by checking the intercept α that the model produces, and its behavior. Instead of using an F-test for each alpha, GRS jointly tests if the intercepts are statistically different from zero.

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