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An Empirical Portfolio Perspective on Option Pricing Anomalies*

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Abstract. We empirically study the economic benefits of giving investors access to index options in the standard portfolio problem, analyzing both expected-utility and nonexpected-utility investors in order to understand who optimally buys and sells options. Using data on S&P 500 index options, CRRA investors find it always optimal to short out-of-the-money puts and at-the-money straddles. The option positions are economically and statistically significant and robust to corrections for transaction costs, margin requirements, and Peso problems. Loss-averse and disappointment-averse investors also optimally hold short option positions. Only with highly distorted probability assessments can we obtain positive portfolio weights for puts (cumulative prospect theory and anticipated utility) and straddles (anticipated utility).

JEL Classification: G11, G12

1. Introduction

The portfolio choice literature has grown tremendously over the past decade and has considered a variety of extensions of existing asset allocation models, such as the analysis of alternative preferences, different asset classes, frictions, stochastic labor income, return predictability, learning, etc. (see e.g., Campbell and Viceira (2002) for a survey). Surprisingly, very few papers have considered

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the role of equity (index) derivatives in portfolio choice.[1] This is surprising for the following three reasons.

First, it has been shown that hedge funds feature option-like risk-return characteristics (Fung and Hsieh (1997), Mitchell and Pulvino (2001) and Agarwal and Naik (2004)). Similarly, so-called "structured products" that embed a capital guarantee to provide portfolio insurance (typically consisting of long positions in an equity index and an index put) have gained popularity in recent years (Ang et al. (2005, p. 500)). Therefore, analyzing the role of hedge funds and alternative investments in asset allocation necessitates an understanding of portfolio choice with options.

Second, a common finding in empirical work on equity derivatives is that index options embed large risk premia for jump and/or volatility risk.[2] The source and nature of these risk premia is not well understood and it has been argued that the prices of these index options are anomalous and excessively high (Jones (2006) and Bondarenko (2003a,b)). The empirical evidence of market incompleteness and option risk premia suggests that options are needed to complete the market (and can therefore not be treated as redundant assets), and that they may improve an investor's risk-return trade-off. It is therefore of interest to study optimal portfolio demand in the presence of equity options.

Most importantly, gaining insight about the type of equilibrium model that could rationalize observed option prices requires an understanding of who would optimally buy index options at these (high) prices, since options are in zero net supply. A first fundamental question we study is which investor optimally holds long positions in index options. Interestingly, Garleanu et al. (2005) develop a model in which risk-averse market makers cannot perfectly hedge a book of options, so that demand pressure increases the equilibrium price of options. The authors document empirically that end users are net long index options, which can explain their high prices, but the model is completely agnostic about the source of the exogenous demand by end users. The goal of

[1] Some notable exceptions are Leland (1980), Brennan and Solanki (1981), and Liu and Pan (2003), as discussed at the end of the introduction.

[2] It is now well accepted that the underlying index value is subject to stochastic volatility and/or jumps, generating market incompleteness (see Ait-Sahalia (2002), Andersen et al. (2002), and Eraker et al. (2003) for recent contributions). Moreover, the incompleteness seems to be priced (Buraschi and Jackwerth (2001)). Among others, Coval and Shumway (2001) and Bakshi and Kapadia (2003) show the presence of a negative volatility risk premium and Bates (2002) and Pan (2002) estimate a positive jump risk premium. Ait-Sahalia et al. (2001) use option-based trading strategies to provide evidence for jump risk premia. Jones (2006) argues that multiple risk factors, besides the return on the underlying index, are priced. See Bates (2003) for a survey of the recent literature documenting a variety of intriguing stylized facts about the prices of equity index options.

this paper, instead, is to attempt to explain the demand of end users within a flexible and tractable portfolio choice framework.

A third motivation for including index options in the menu of available assets stems from the recent attention given to portfolio choice with nonexpected utility specifications, in particular loss aversion and disappointment aversion. Besides being supported by experimental evidence, these preferences have been applied successfully in the equity-only portfolio choice literature, explaining for instance nonparticipation in equity markets. These same preferences could also help to explain observed portfolio behavior when options are available to investors, like the demand for portfolio insurance. In fact, the asymmetric nature of the payoffs of certain derivatives (e.g., out-of-the-money (OTM) puts) has led to conjectures in the literature that some nonexpected utility preferences (e.g., loss aversion) are necessary to explain the demand for options. This is another important question we address. Studying portfolio choice with options constitutes a strong test of nonexpected utility preferences that has not been conducted in the literature.

To answer these questions, we consider both standard and behavioral preferences in a portfolio choice setting in which investors have access to option-based strategies (puts and straddles[3]) and in which we correct for realistic market frictions. Our analysis uses a simple and flexible framework for empirical portfolio choice, due to Brandt (1999) and Ait-Sahalia and Brandt (2001). The only inputs required are time series of returns on equity index options and on the index itself. We focus on the S&P 500 index and options on S&P 500 index futures from 1987 to 2001, thereby including the 1987 crash.

We find first of all that constant relative risk-aversion (CRRA) investors always take economically and statistically significant short positions in OTM puts and at-the-money (ATM) straddles, and that portfolio insurance is never optimal. For instance, a CRRA investor with risk aversion coefficient of 2, is willing to pay 1.23% of her wealth *per month* to be able to *short* the OTM put and 1.93% to have access to the *short* straddle position. Surprisingly, the optimality of large negative derivatives positions also holds for loss-averse and disappointment-averse investors. Even though aversion to losses or disappointment makes these "behavioral" investors avoid stock market risk entirely in the absence of derivatives, they hold large negative derivatives positions when OTM puts and ATM straddles are available. In fact, their positions are often more extreme than the ones held by CRRA investors.

[3] A long straddle position involves the simultaneous purchase of a call and put option on the same underlying and with the same strike price and maturity, and benefits from unexpected increases in volatility. Coval and Shumway (2001) demonstrate that zero-beta straddles earn negative excess returns, which they interpret as evidence that volatility risk is a priced factor.

In addition to their contribution to the portfolio choice literature, these findings also have fundamental implications for attempts to develop heterogeneous-agent equilibrium models in which options play a nontrivial role: if an equilibrium model is to produce option prices and risk premia that are in line with the challenging historical data, at least some investors must have a positive demand for these assets, since they are in zero net supply.[4] In fact, we show that standard expected-utility investors and commonly studied behavioral investors *never* have a positive demand for straddles and OTM puts given observed prices. Therefore, generating similar prices in equilibrium (with some positive demand by at least some investors) will require the inclusion of rather different and nonstandard preferences.

As examples of these nonstandard preferences, we show as a second contribution that cumulative prospect theory, which combines loss aversion with distorted probabilities, and anticipated utility (preferences with rank-dependency) can potentially generate positive put and straddle holdings.[5] In the case of cumulative prospect theory, the positive put weights coexist with highly levered equity positions. For anticipated utility we show that positive derivatives holdings requires not only an upward distortion of the probability of poor portfolio outcomes, but especially of favorable outcomes. The latter induces a preference for positive skewness, which makes portfolio insurance and especially long straddle positions attractive. In particular, we find that distorting only the left tail of the portfolio return probability distribution does not result in strictly positive portfolio weights for puts or straddles.

Our results do not require (costly) continuous trading and are remarkably robust to a variety of extensions like transaction costs, margin requirements, and crash-neutral derivatives strategies, as well as to the choice of sample period and return frequency. Our findings provide strong evidence that the jump and volatility risk premia documented in the option pricing literature are economically substantial. It is worth emphasizing that although our empirical framework is cast in discrete time, it is meaningful to talk about (the effect of) jump risk and jump risk premia. This is because the option returns faced by the investor reflect key properties of the underlying continuous-time price process, like the presence of priced jump risk. To substantiate this claim we show that an investor facing discrete-time option returns generated from a complete-market Black-Scholes model (rather than empirical option

[4] We should emphasize that this paper is not a search for a representative agent that would price derivatives correctly. Recent contributions to the representative-agent option pricing literature include Ait-Sahalia and Lo (2000), Brown and Jackwerth (2001), Liu et al. (2005), Rosenberg and Engle (2002), and Bliss and Panigirtzoglou (2004).

[5] A first exploration of heterogeneous-agent equilibrium option pricing with nonstandard preferences can be found in Bates (2002).

returns) effectively ignores derivatives. This implies that our results can only be explained by economically important deviations from the complete-market paradigm where only one factor (market risk) is priced.

Few papers have included options when studying portfolio choice. The seminal work of Leland (1980) and Brennan and Solanki (1981) studies the demand for derivatives for portfolio insurance purposes in a complete-market setting. Liu and Pan (2003) are the first to add nonredundant options to a dynamic asset allocation problem, using continuous-time dynamic programming. Our paper differs from Liu and Pan in several ways. First, our modeling approach is different. While their approach generates intuitive closed-form solutions for the optimal derivatives demand, it requires specified price dynamics and risk premia. Also, their quantitative examples specialize to either a pure jump risk or a pure volatility risk setting, so that a single derivative completes the market. Using instead the approach of Brandt (1999) and Ait-Sahalia and Brandt (2001), we need not impose specific price dynamics or a pricing kernel, or take a stand on prices of risk or on the number of nonspanned factors. Secondly, our approach is empirical in nature, while they analyze the quantitative implications of a theoretical model for different parameter settings. The portfolio weights they report depend crucially on the choice of parameters. For instance, they consider 24 different parameter sets that all imply a positive jump risk premium and obtain 13 positive put weights and 11 negative ones. Instead we directly estimate the optimal portfolio weights for different preferences and obtain unambiguous conclusions. We incorporate realistic frictions like transaction costs and margin requirements, and account for Peso problems. A final important difference is that we study nonexpected-utility investors in addition to standard preferences.

The organization of the paper is as follows. Section 2 introduces the model that is used to obtain optimal derivative portfolios, and Section 3 describes the data. The benchmark results for expected utility are given in Section 4. Section 5 presents the results for a variety of nonexpected-utility preferences. Robustness checks and sensitivity analysis are reported in Section 6. In Section 7 we estimate the economic value of having access to derivatives in terms of wealth certainty equivalents. The analysis is extended by allowing for multiple nonspanned factors in Section 8, before concluding in Section 9.

2. Model

We consider an investor with utility from end-of-period wealth and access to the riskfree asset, an equity index (which may be implemented using an index futures contract to enable easy short-selling), and a derivative on the index futures contract. We study optimal portfolios for a variety of preferences and derivative contracts. As emphasized before, market incompleteness is fundamental to the analysis, but we want to remain agnostic about the precise nature of the incompleteness and in particular about the risk premia associated with any nonspanned factor(s). Essentially, options are treated like any other asset and we "let the data speak" about the importance of options in completing markets and in improving the risk-return trade-off for investors.

Denoting the fraction of wealth invested in equity by α_E and the fraction of wealth invested in the derivative by α_D, the investor solves:

$$\max_{\alpha_E,\alpha_D} E\left[U\left(W_T\right)\right] \tag{1}$$

Given initial wealth W_0 and denoting the return on asset i by R_i (where R_f is the gross return on the riskless asset), we have

$$W_T = \left[R_f + \alpha_E\left(R_E - R_f\right) + \alpha_D\left(R_D - R_f\right)\right]W_0. \tag{2}$$

In the absence of market frictions and for a differentiable utility function $U(.)$, the first-order conditions for $i \in \{E, D\}$ are:

$$E\left[U'\left(\left[R_f + \alpha_E\left(R_E - R_f\right) + \alpha_D\left(R_D - R_f\right)\right]W_0\right)\left(R_i - R_f\right)W_0\right] = 0. \tag{3}$$

This asset allocation problem can be solved without imposing any parametric structure on the return dynamics and risk premia by using the methodology developed in Brandt (1999) and Ait-Sahalia and Brandt (2001). When returns are stationary, the conditional expectations operator in the Euler equations associated with the portfolio problem can be replaced by the sample moments and the optimal portfolio shares are estimated from the first-order condition in GMM fashion. We analyze unconditional portfolios, assuming returns are i.i.d.[6] The number of parameters (unconditional portfolio weights) and Euler restrictions coincide and exact identification obtains. In the case of market frictions or a nondifferentiable utility function, we directly replace the expectation in condition (1) by its sample counterpart and maximize this expression over the portfolio weights, given possible constraints due to market frictions.

This approach presents the following major advantages. First, the nonparametric nature of the method is particularly appealing when including

[6] It is straightforward to allow for conditioning information and time-varying portfolio weights. Unreported results (available upon request) show that this has no impact on the findings, since the slope coefficients in portfolio rules that are affine functions of an instrument (based on option prices) are not statistically significant.

derivatives in the investment opportunity set given the difficulties in identifying risk premia reflected in option prices. Second, the approach is sufficiently general to allow for numerous extensions. Subsequent to the benchmark analysis, we will consider different types of nonexpected-utility preferences and introduce realistic transaction costs. Third, in addition to point estimates, the methodology also produces standard errors of the portfolio weights since the portfolio weights are parameters that are estimated using a standard GMM setup. Formal tests can then be conducted to determine whether the demand for options is significantly different from zero and whether the inclusion of derivatives in the asset space leads to welfare gains as measured by certainty equivalents. Finally, the approach can accommodate situations where markets remain incomplete even after the introduction of nonredundant derivatives.

While our framework is cast in discrete time, it is clear that, given an underlying continuous-time model, both stochastic volatility and jumps have an important impact on discrete-time equity and option returns. In particular, jumps and stochastic volatility generate higher-order dependence between the discrete-time equity and option returns. In addition, the risk premia for both sources of risk obviously modify the risk-return trade-off. Both effects are present in our analysis and turn out to play a major role. In Section 6.3 we explicitly demonstrate that without these effects the introduction of derivatives is quantitatively irrelevant. More precisely, with Black-Scholes generated option returns, the option would be redundant in continuous time and only matters in discrete time to the extent that it improves spanning. The latter effect is shown to be insignificant.

For the benchmark results and for most of the subsequent analysis, we implement the model using monthly returns for an investor with a one-month horizon, thus focusing on the static portfolio problem. While this ignores the intertemporal Merton-style hedging demands, it provides a very useful benchmark.[7] Furthermore, the theoretical examples in Liu and Pan (2003) show that the direct intertemporal hedging demands for derivatives are small. A more subtle aspect of intertemporal hedging demands for options concerns the fact that options represent dynamic trading strategies, so that an investor who would like to rebalance (e.g., because returns are not i.i.d.), but is not allowed to do so, may have a hedging demand for options. We show that this effect is quantitatively small in our set-up (Section 6.3) and unlikely to explain our results.

[7] Clearly, the multiperiod dynamic setting in which intertemporal hedging demands play a role is an interesting extension for future work.

3. Data Description

The empirical analysis is based on time series of returns on a riskfree asset, an equity index and associated index options. For the riskfree asset we use 1-month LIBOR rates, obtained through Datastream. Datastream is also used for S&P 500 index returns, which include dividends. We construct both weekly and monthly returns for these assets.

The option data consist of S&P 500 futures options, which are traded on the Chicago Mercantile Exchange. Although futures options are American-style options, they are used in many recent studies because of data availability and a number of other advantages over index options (issues related to liquidity, dividends and nonsynchronicity for index options, as discussed in, e.g., Bondarenko (2003b, page 5)). The dataset contains daily settlement prices for call and put options with various strike prices and maturities, as well as the associated futures price and other variables such as volume and open interest. The sample runs from January 1987, thus including the 1987 crash, until June 2001. We apply the following data filters to eliminate possible data errors. First, we exclude all option prices that are lower than the direct early exercise value. Second, we check the put-call parity relation, which consists of two inequalities for American futures options. Using a bid-ask spread of 1% of the option price and the riskfree rate data, we eliminate all options that do not satisfy this relation. In total, this eliminates less than 1% of the observations.

Since these options are American with the futures as underlying, we apply the following procedure to correct the prices for the early exercise premium. We use a standard binomial tree with 200 time steps to calculate the implied volatility of each call and put option in the dataset. Given this implied volatility, the same binomial tree is then used to compute the early exercise premium for each option and to deduct this premium from the option price. By having a separate volatility parameter for each option at each trading day, we automatically incorporate the volatility skew and changes in volatility over time. On the basis of this procedure, the early exercise premia turn out to be small (about 0.2% of the option price for the short-maturity options we analyze). Compared to options that have the index itself as underlying, these early exercise premia are small because the underlying futures price does not necessarily change at a dividend date. Therefore, even if the model used to calculate the early exercise premia is misspecified, we do not expect that this will lead to important errors in the option returns that are constructed below.

To convert the option price data into monthly option returns we follow a similar procedure as in Buraschi and Jackwerth (2001) and Coval and Shumway (2001). First, we fix several targets for the strike-to-spot ratio: 92, 96 and 100%. At the first day of each month, we select the option with strike-to-spot ratio closest to the target ratio. We exclude options that mature in the same month (on the third Friday of that month). Next, we calculate the monthly return on the selected options up to the first day of the subsequent month. In this paper, we focus on the short-maturity options that have about 7 weeks to maturity at the moment of buying and at least 2 weeks to maturity when the options are sold. These options typically have the largest trading volume and we exclude in this way automatically options with very short maturities, which may suffer from illiquidity (Bondarenko (2003b)).[8] In the end, this gives us time series of option returns for several strike-to-spot ratios.

The procedure discussed above implies that we do not hold options to maturity. The advantage of our procedure is that it yields equally spaced return series. In addition, the constructed option returns are more sensitive to changes in volatility and jump probabilities than returns on options that are held to maturity. This is crucial here, since the analysis focuses exactly on the role of options as vehicles for trading volatility and jump risk.

We do not allow the investor to choose from all available options simultaneously, since our investors may then exploit small in-sample differences between highly correlated option returns, leading to extreme portfolio weights (see, e.g., Jorion (2000) for a discussion of this issue). Instead, we focus on a number of economically intuitive derivative strategies that are often used in practice. In particular, we focus on two benchmark strategies:

- A (short-maturity) OTM put with 96% moneyness (strike-to-spot ratio)
- A (short-maturity) ATM straddle.

Both strategies have remaining maturities between 8 and 2 weeks, as described above. OTM puts and ATM straddles with these characteristics in terms of moneyness and maturity are known to be very liquidly traded (see e.g., Figure 1 in Bondarenko (2003b)) and have been analyzed extensively in the recent option pricing literature, making both obvious choices as benchmark strategies. In addition, we consider a number of alternative strategies:

- A "crash-neutral" OTM put, consisting of a long position in the 96%-OTM put option and a short position in the 92%-OTM put option
- A "crash-neutral" ATM straddle, consisting of a long position in the ATM straddle and a short position in the 92%-OTM put option.

The crash-neutral put and straddle have also been studied by Jackwerth (2000) and Coval and Shumway (2001), respectively. By adding an opposite position

[8] We construct weekly option returns in a similar way, each week selecting the appropriate strike prices and switching to the next delivery month at the beginning of each month.

Table I. Summary statistics

This table reports mean, standard deviation, Sharpe ratio, skewness and the correlation with S&P 500 index returns for monthly returns on several S&P 500 futures option strategies over our January 1987–June 2001 sample. The benchmark option strategies are an OTM put with 0.96 strike-to-spot ratio and an ATM straddle. The "crash-neutral" (CN) OTM put (and ATM straddle) consists of a long position in the 0.96 OTM put (ATM straddle) and a short position in the 0.92 OTM put. The options are short-maturity and have about 7 weeks to maturity at the moment of buying and at least 2 weeks to maturity when the options are sold. We use 1-month LIBOR for the riskfree rate.

Strategy	Mean	Std. dev.	Sharpe	Skewness	Corr. index
Equity	0.013	0.044	0.176	−0.826	1.000
0.96 OTM put	−0.406	1.110	−0.370	5.452	−0.759
ATM straddle	−0.130	0.360	−0.375	2.074	−0.071
CN OTM put	−0.314	1.080	−0.295	2.440	−0.515
CN ATM straddle	−0.074	0.370	−0.213	1.070	0.386
0.92 OTM put	−0.480	1.760	−0.275	10.458	−0.610

in a deep OTM put option, a short position in the straddle (or the 96%-OTM put option) is protected against large crashes.[9] These strategies are studied in Section 6.

Table I provides summary statistics of the data. Most striking are the negative average returns on long positions in all option strategies. In terms of Sharpe ratios, a short position in each option strategy outperforms the equity index. Especially, a short position in the OTM put and ATM straddle perform extremely well with a monthly Sharpe ratio of about 0.37, implying an annual Sharpe ratio of around $\sqrt{12} \times 0.37 = 1.28$. It should immediately be pointed out that Sharpe ratios can be highly misleading when analyzing derivatives (Goetzmann et al. (2002)). For example, it is clear that the skewness of the return on the option strategies is also much larger than for the equity index. These summary statistics are comparable to the ones reported in Coval and Shumway (2001) and Bondarenko (2003b).

4. Benchmark Results: Expected Utility

As a benchmark, we consider an investor with CRRA and a one-month horizon, facing frictionless markets. Initial wealth W_0 is normalized to one

[9] Note that the crash-protection is only approximate since the positions are not held till maturity and because the size of a crash or downward jump may be stochastic.

without loss of generality. The values for the coefficient of relative risk aversion γ are $\frac{1}{2}$, 1, 2, 5, 10 and 20.

4.1 NO DERIVATIVES

It will prove useful to analyze the demand for equities (α_E) in the absence of derivatives ($\alpha_D \equiv 0$). The portfolio weights in Table II are significantly different from zero and roughly proportional to risk-tolerance. Only for low risk aversion does the investor choose levered positions in equity. For $\gamma = 5$, the equity weight is more moderate and drops below 75%. This highlights the fact that our analysis is partial equilibrium: extreme levered equity portfolios, often viewed as the portfolio or partial-equilibrium consequence of the equity premium puzzle, only show up for risk-tolerant investors. Simultaneously however, even very risk-averse investors hold equity positions, so that CRRA preferences fail to explain the participation puzzle. It will be useful to keep these results in mind when studying the demand for derivatives with nonexpected-utility preferences.

4.2 OTM PUTS

When considering OTM puts (with 0.96 moneyness), the investor is better able to trade jump and (to a lesser extent) volatility risk than with equities only. The optimal put weights give insight into the extent to which these risks are spanned by derivatives but not (optimally) by equity markets and especially into the attractiveness of the risk premia associated with these risks.

The main result from the middle panel of Table II is that all portfolio weights, both for equity and for the OTM put, are negative. The negative put weights reflect the high market price of the risk factors present in option returns as documented in the empirical option pricing literature. Liu and Pan (2003) demonstrate that the optimal portfolio weight in puts is positive whenever jump risk is not priced. The negative weights obtained here are therefore strong evidence for a nontrivial jump risk premium. These put weights are also strongly statistically significant. This may perhaps be surprising given that the sample includes both the 1987 and the 1990 crash (invasion of Kuwait).

Turning to the effect of the introduction of the derivative on the demand for equity, the positive correlation between the return on the short put position and the index return plays an important role. A short put position can be hedged partially by a negative equity weight. In other words, a short put position has a positive delta, so that delta-hedging requires a short equity position. The equity premium obviously makes this hedge expensive.

Table II. Portfolio weights for CRRA preferences

This table reports the optimal portfolio weights α_E (equity) and α_D (derivative strategy) and their standard errors for a CRRA investor with risk aversion γ obtained by estimating (3) with GMM over our January 1987–June 2001 sample. The derivative strategies are a (short-maturity) OTM put with 0.96 strike-to-spot ratio and a (short-maturity) ATM straddle, using S&P 500 futures options. The implied equity weight corresponding to each α_D is calculated by multiplying the optimal put (straddle) weight with the empirical beta of the put (straddle).

γ	0.5	1	2	5	10	20
			No derivatives			
α_E	4.2805	3.0458	1.7145	0.7208	0.3653	0.1838
SE	0.5489	1.1541	0.8035	0.3587	0.1841	0.0931
			OTM put			
α_E	−4.5063	−2.6977	−1.5418	−0.6790	−0.3512	−0.1787
SE	4.1646	2.3900	1.2708	0.5313	0.2702	0.1363
α_D	−0.2086	−0.1535	−0.1001	−0.0477	−0.0253	−0.0130
SE	0.0969	0.0581	0.0370	0.0192	0.0106	0.0056
Implied equity	3.9762	2.9259	1.9080	0.9092	0.4822	0.2478
			ATM straddle			
α_E	0.4765	0.5189	0.4924	0.2731	0.1503	0.0787
SE	1.0803	1.0539	0.7360	0.3511	0.1852	0.0950
α_D	−0.5756	−0.4709	−0.2932	−0.1318	−0.0683	−0.0348
SE	0.0646	0.1112	0.0859	0.0421	0.0223	0.0115
Implied equity	0.3302	0.2702	0.1682	0.0756	0.0392	0.0200

In Table II, α_E is nonetheless negative for all coefficients of risk aversion, although not statistically significant. Note that the term hedging is used in a static sense, since the static portfolio problem we focus on excludes Merton-style intertemporal hedging, as discussed at the end of Section 2. The notion of hedging is closely related to delta-hedging. However, since perfect delta-hedging requires continuous trading and complete markets, and is in practice infeasible, the term hedging (or risk-diversification) seems more appropriate.

To shed more light on the economic significance of the portfolio weights in derivatives, Table II also reports the implied equity weights corresponding to each α_D. Keeping in mind that options are not redundant and cannot be perfectly replicated, the implied equity weights are calculated by multiplying the optimal put weights with the empirical beta of the put (−19.0612). This reveals the empirical equity exposure that investors optimally hold in the form of derivatives. The implied equity weights are very large and range from 398%

for $\gamma = \frac{1}{2}$ to 25% for $\gamma = 20$. If derivatives were redundant and only reflected stock market risk, the introduction of puts would not affect the total amount of equity exposure that investors optimally take. For instance, the $\gamma = 2$ investor would still hold a total implicit equity weight of 171%. Instead we find that this investor chooses a total implicit equity weight of only 37% (191% through the short put and −154% through the short equity). This finding illustrates once more that options are not redundant, but reflect economically important risk premia, which allow the investor to achieve a substantially superior risk-return trade-off than can be achieved in the equity market. The investor is better off by giving up exposure to stock market risk in exchange for exposure to the jump and volatility risk factors in put options. The certainty equivalent wealth gains associated with this are quantified in Section 7.

4.3 ATM STRADDLE

While the OTM put can be thought of as mainly giving exposure to jump risk, an ATM straddle allows the investor to trade volatility risk. The empirical option pricing literature has documented a negative volatility risk premium. In our portfolio setting, this manifests itself in the form of large negative optimal straddle positions in Table II.

The portfolio weights are much larger than for the OTM put and grow to almost −50% for $\gamma = 1$ (−58% for $\gamma = \frac{1}{2}$). The weights become more reasonable as risk aversion grows, but remain very statistically significant. Even though an ATM straddle is close to delta-neutral (more precisely, the correlation between straddle returns and equity returns is only −0.071) and the (static) hedging demand for equity is therefore expected to be small, the equity weight is substantially affected by the introduction of the straddle. Investors hold long equity positions due to the positive equity risk premium, but since the risk-return trade-off presented by the straddle is superior, the equity position is much smaller than when derivatives are not available. In fact, the equity position is no longer significant.

Even though the straddle portfolio weights are roughly three times the put weights, the implied equity positions for straddles (obtained by multiplying the optimal straddle weights with the empirical beta of the straddle (−0.5737)) are substantially smaller. This is not surprising, since the straddle combines a call and put with equity exposures of opposite signs. However, as for puts, it is still the case that the optimal portfolio weights with the straddle represent an important deviation from the amount of equity exposure taken when derivatives are absent. For example, the $\gamma = 2$ investor has a total implicit equity exposure of only 66% (49% through equity directly and 17% through the straddle), compared to 171% in the top row of Table II. The investor is willing to sacrifice exposure to stock market risk in order to take on volatility risk instead, attracted by the large volatility risk premium.

4.4 EQUILIBRIUM IMPLICATIONS

The results for standard expected-utility CRRA investors have important equilibrium implications. We find that all investors optimally hold short positions in puts and straddles that are economically and statistically significant. Because equity derivatives are in zero net supply, the important question is which investors would optimally hold the other side of these contracts. For option markets to clear at historically observed prices (i.e., consistent with the empirically observed option returns), the large negative demands of CRRA investors must be offset by positive demands of other market participants. This is one of the motivations of the analysis of nonexpected utility preferences in the next section.

When markets are complete and derivatives are redundant, the negative demands of some investors can easily be offset by positive demands of other investors, who would simply "undo" their position in derivatives by shorting the replicating portfolio, that is by appropriately adjusting their holdings of the underlying (equity index) and riskfree asset. However, our findings provide strong evidence against the ability of investors to undo any option holdings through their equity and bond portfolio. This can be understood by recalling the large impact of the introduction of derivatives on the total implicit equity exposure chosen by CRRA investors. In a complete market where only stock market risk is priced, the demand for options is not identified and the investor only cares about the total implicit equity exposure. For example, whether derivatives are available or not, the $\gamma = 2$ investor would always optimally hold the equivalent of 171% total equity exposure in a complete market setting. Instead, we find that this investor shifts to 37% total exposure when puts are available and to 66% with straddles. Put differently, Section 7 will demonstrate that the optimal short positions we obtain represent very large welfare gains to investors relative to when $\alpha_D = 0$. Correspondingly, when being forced to hold positive amounts of puts or straddles, the same investor would suffer a major welfare loss, even when he is able to optimally adjust his equity and riskfree asset holdings. The economic reason is market incompleteness and the fact that jump and volatility risk constitute additional priced risk factors beyond stock market risk, which investors cannot fully trade through the equity market alone.

5. Nonexpected Utility

In this section, we examine whether the large short positions in derivatives chosen by expected-utility investors also obtain for different specifications of nonexpected utility.[10] This is important since some of these (behaviorally motivated) preferences have been suggested in the literature as explanations for the equity premium puzzle and the participation puzzle.

5.1 PROSPECT THEORY

Prospect Theory, as introduced by Kahneman and Tversky (1979), is based on experimental evidence against expected utility and has allowed numerous researchers to explain a variety of empirical regularities and phenomena that are puzzling from the point of view of expected utility.[11] Loss aversion is the feature of (Cumulative) Prospect Theory (Tversky and Kahneman (1992)) that has received most attention in the finance literature and that is crucial in explaining well-documented behavior. Three deviations from expected-utility decision-making lie at the heart of Prospect Theory. First, individuals derive utility from losses and gains X (relative to a reference level) rather than from a level of wealth W. Second, marginal utility is larger for infinitesimal losses than for tiny gains so that investors are loss averse. Note that loss aversion generates first-order risk aversion (Segal and Spivak (1990)). Third, the value function exhibits risk aversion in the domain of gains, but is convex in the domain of losses. A typical specification for the value function $V(X)$ of a loss-averse investor is:

$$V(X) = \begin{cases} \frac{X^{\widehat{\gamma}}}{\widehat{\gamma}} & \text{for } X \geq 0 \\ -\lambda \frac{(-X)^{\widehat{\gamma}}}{\widehat{\gamma}} & \text{for } X \leq 0 \end{cases} \tag{4}$$

The parameter λ controls the degree of first-order risk aversion and makes the value function kinked at zero. Tversky and Kahneman (1992) suggest $\lambda = 2.25$. In the portfolio choice problem solved below, we also use $\lambda = 1.25$ and $\lambda = 1.75$ to allow for smaller first-order risk aversion. The curvature parameter $\widehat{\gamma}$ is constrained to belong to the interval [0, 1] and is estimated at 0.88 by Tversky and Kahneman (1992).[12] Barberis et al. (2001) use $\widehat{\gamma} = 1$,

[10] We also analyzed expected-utility mean-variance preferences, which differ from CRRA in discrete time. Since mean-variance preferences do not "punish" negative skewness, we find even more negative option weights in general.

[11] For an excellent survey, see Barberis and Thaler (2003).

[12] The curvature parameter $\widehat{\gamma}$ in Prospect Theory should not be confused with the coefficient of relative risk aversion γ in the expected-utility analysis.

which proves very tractable in their equilibrium setting. We also include this specification in our analysis and consider $\widehat{\gamma} \in \{0.8, 0.9, 1.0\}$.

It is important to point out that Kahneman and Tversky first formulated their theory in an atemporal setting and focused on experiments where subjects faced gambles with two possible nonzero outcomes (Barberis and Thaler (2003)). Bringing this theory to a temporal setting with gambles characterized by a richer support—a typical setting in financial economics—requires therefore that one imposes more structure on the dynamics of the reference point. Issues related to narrow framing or mental accounting and the updating of the reference point ("intertemporal framing") become crucial elements of the analysis (see e.g., Benartzi and Thaler (1995) and Barberis et al. (2006)). The evolution of the reference point may prove particularly important when considering put options. A reasonable assumption seems to be to have the reference level equal to initial wealth grown at the riskless rate: $X \equiv W_T - R_f W_0$.

A second implementation issue that arises in a portfolio setting relates to the convexity of the value function over losses. Risk-seeking behavior when facing losses is a robust finding in experiments when the losses are small. However, there seems to be far less consensus among decision scientists for large losses as some evidence suggests concavity (Laughhunn et al. (1980)). In the finance literature, Gomes (2005) argues that having marginal utility decrease as wealth approaches zero is unappealing. This is especially relevant in our setting where investors have access to derivative-based returns with unusually asymmetric distributions. Risk-seeking behavior becomes extreme and investors mainly take on positions for which the nonnegativity constraint on wealth becomes binding. Rather than imposing default penalties to avoid these extreme positions, we follow Gomes and have the value function become concave again for substantial losses, consistent with Laughhunn et al. (1980). We set the inflection point at 50% of initial wealth and use logarithmic utility from there onwards. Ait-Sahalia and Brandt (2001) impose portfolio constraints to rule out extreme positions due to the convexity of the value function. These constraints are often binding. In our setting, however, leverage constraints are less meaningful since derivative strategies per definition allow for leverage.

Finally, a last ingredient of (Cumulative) Prospect Theory as formulated in Tversky and Kahneman (1992) makes decision-makers transform probabilities in a nonlinear way when taking expectations of the value function. In particular, the probabilities of extreme outcomes are distorted upwards by taking probability mass away from outcomes with moderate losses or gains. For ease of exposition, we first present results without the nonlinear probability transformation and thus focus on the part of prospect theory that is most commonly studied in the finance literature, namely loss aversion. Subsequently (Section 5.1.2) we additionally introduce probability distortions. This will prove of great importance in the context of derivative portfolios.

The empirical methodology is similar to the expected utility case, but with three differences. First, we replace the utility function in (1) by the value function $V(.)$. Second, we directly optimize the expression in (1) (after replacing the expectation by the sample counterpart), because the value function is not differentiable at the "kink". Finally, standard errors for the portfolio weights in this subsection are not computed for the following reason. If the optimal portfolio weights are equal to zero, the value function (evaluated at the observed portfolio returns) is not differentiable for all observations, so that smoothing will give essentially arbitrary results. If the optimal portfolio weights differ from zero, it may still be the case that some of the observed portfolio returns are close to the "kink" in the value function, so that even in this case the calculated standard errors would be sensitive to the smoothing method chosen.

Loss aversion

First, when derivatives are not available, loss aversion produces nonparticipation for $\lambda = 1.75$ and $\lambda = 2.25$, that is for sufficient first-order risk aversion. When $\lambda = 1.25$, however, the positions are highly levered and more extreme than for the logarithmic expected-utility investor. The convexity of the value function in the domain of losses is not innocuous and in fact makes the positions more extreme relative to the linear case $\widehat{\gamma} = 1$.

Adding a put option with 0.96 moneyness has dramatic effects in Table III. All preference parameters result in large negative equity and put positions. These results are quite strong and surprising in light of the nonparticipation obtained when derivatives are absent. For $\lambda > 1.25$, loss-averse investors completely ignore the equity premium and invest nothing in equities. When puts are available however, these same loss-averse investors find it optimal to short the options and to simultaneously short the equity index. In fact the short put position is almost as large as the one chosen by a relatively risk-tolerant logarithmic investor (Table II). The risk premia priced in derivatives are too substantial to be ignored, unlike the equity premium, which is ignored by most loss-averse investors. These results are striking if one thinks of loss-averse investors as potentially having an obvious demand for portfolio insurance and hence for protective put positions. The same insight can be gained from the implied equity positions: for example, the linear $\lambda = 2.25$ investor holds a total implicit equity position of 98% with puts, while not participating in stock market risk when derivatives are not accessible.

Table III. Portfolio weights for loss aversion

This table reports the optimal portfolio weights α_E (equity) and α_D (derivative strategy) for a loss-averse investor with value function (4) over our January 1987–June 2001 sample, obtained by replacing the expectation in (1) by its sample counterpart and maximizing over the portfolio weights. The derivative strategies are a (short-maturity) OTM put with 0.96 strike-to-spot ratio and a (short-maturity) ATM straddle, using S&P 500 futures options. The implied equity weight corresponding to each α_D is calculated by multiplying the optimal put (straddle) weight with the empirical beta of the put (straddle).

λ	1.25			1.75			2.25		
$\widehat{\gamma}$	0.8	0.9	1	0.8	0.9	1	0.8	0.9	1
				No derivatives					
α_E	3.706	3.657	3.595	0	0	0	0	0	0
				OTM put					
α_E	−1.218	−1.694	−3.087	−1.164	−1.306	−1.620	−1.148	−1.169	−1.247
α_D	−0.125	−0.135	−0.164	−0.123	−0.124	−0.129	−0.121	−0.119	−0.117
Impl. eq.	2.383	2.573	3.126	2.345	2.364	2.459	2.306	2.268	2.230
				ATM straddle					
α_E	0.495	0.363	0.322	0.867	0.942	0.901	1.544	1.500	1.399
α_D	−0.517	−0.524	−0.528	−0.471	−0.459	−0.448	−0.369	−0.361	−0.355
Impl. eq.	0.297	0.301	0.301	0.270	0.263	0.257	0.212	0.207	0.204

It is worth pointing out that the suboptimality of protective put strategies (long equity combined with a long put position) does not result from some particular features of our setup, such as the assumed evolution of the reference point or the fact that puts are not held until maturity. While both features do play a role in determining whether or not losses can be avoided with certainty, loss-averse investors also take the risk-return trade-off into account. Before demonstrating this in more detail, it is useful to explain why these features may play a role. First, recall that options are not held till maturity when we construct option returns. Even a deep OTM put does then not necessarily provide a guaranteed floor. Second, whether protective puts allow the investor to avoid losses actually depends on the evolution of the reference point (and on the strike price chosen for the put). We follow the literature here and let the reference point grow at the riskfree rate. In that case, nontrivial portfolios ($\alpha_i \neq 0$) cannot avoid losses with certainty (returns bounded from below by R_f) unless arbitrage opportunities exist. Only if the investor has a sufficiently low reference point (e.g., at 0.95 of initial wealth W_0) or “positive surplus wealth” could losses be avoided by an investment strategy based on a put option with a specific strike price (struck sufficiently above the reference point since the put premium needs to be paid).[13] To demonstrate now that the suboptimality of portfolio insurance is not driven by these properties of the setup, we simply remove them. In particular, we consider the asset allocation problem of a loss-averse investor who can invest in put-protected equity (equity plus put), and in the put. The puts are ATM and held until maturity, and the reference level is chosen to equal the minimum wealth level guaranteed by a put-protected equity position (a reference level of around $0.97 \times W_0$). For all parameter values, the loss-averse investor actually shorts both assets. Even though put-protected equity can now literally guarantee that no losses are incurred, it is still suboptimal in terms of risk-return trade-off, highlighting once more the very negative average return on puts in our sample.

As a further robustness check, we also consider an alternative reference level, namely initial wealth grown at the optimal equity portfolio return without derivatives, rather than initial wealth grown at the riskless rate. The idea is that the equity-only portfolio can be viewed as a benchmark for investors who gain access to derivatives. Note that this reference level endogenously coincides with the previous specification (initial wealth grown at R_f) in cases of nonparticipation ($\lambda > 1.25$ in Table III). Unreported results show that this alternative specification does not alter the conclusion of the optimality of short derivatives positions and has in fact a very small impact on the size of the optimal portfolio weights.

Finally, considering straddles in Table III, even stronger results are obtained than for put options. Again nonparticipation disappears for all parameter values and the optimal portfolio always involves extremely large negative straddle positions, worth at least one third of initial wealth. The option portfolio weights become more extreme as the first-order risk-aversion parameter λ decreases. When λ equals 1.25, the optimal short straddle position is even larger than for logarithmic expected utility. As for expected utility, the optimal equity weights are positive.

To summarize, nonparticipation results typically obtained with loss aversion disappear as soon as derivatives are introduced. In fact, even loss-averse investors find it optimal to not only participate, but, more strikingly, to hold short positions in either puts or straddles.

[13] Assuming prices generated by a complete-market model, Siegmann and Lucas (2002) demonstrate theoretically that loss-averse investors may optimally invest in nonlinear (option-like) securities, depending on their surplus wealth.

Cumulative prospect theory

Cumulative Prospect Theory adds probability distortions according to a nonlinear transformation to the loss-averse preferences of the previous subsection. In particular, the expectations in (1) are based on transformed probabilities ("decision weights"), which overweight both extremely positive and extremely negative outcomes of the optimal portfolio.

The distorted probabilities or decision weights are obtained from the objective probabilities as follows. States or outcomes are ordered from worst to best according to the endogenously chosen portfolio by the investors and are labeled accordingly: $R_1 \leq \cdots \leq R_k \leq R_f \leq R_{k+1} \leq \cdots \leq R_N$. Denoting the objective probability of portfolio outcome n by p_n, its subjectively distorted probability π_n is obtained as follows:

$$\pi_i = w\left(p_1 + \cdots + p_i\right) - w\left(p_1 + \cdots + p_{i-1}\right) \text{ for } 2 \leq i \leq k \tag{5}$$
$$\pi_i = w\left(p_i + \cdots + p_N\right) - w\left(p_{i+1} + \cdots + p_N\right) \text{ for } k+1 \leq i \leq N-1$$

where

$$w\left(p\right) = \frac{p^c}{\left[p^c + (1-p)^c\right]^{1/c}} \tag{6}$$

and $\pi_1 = w\left(p_1\right)$, $\pi_N = w\left(p_N\right)$. The nonlinear transformation function $w(.)$ was proposed in Tversky and Kahneman (1992), who suggest $c = 0.65$ on the basis of experimental evidence. Note that $c = 1$ brings us back to the previous subsection without distortions. We also consider an intermediate parameter value for c of 0.8. Figure 1 illustrates the probability distortion for both parameter values.

Even though the probability distortion is considered by decision scientists to be a fundamental ingredient of prospect theory (see e.g., Abdellaoui (2000)), it has been ignored by financial economists. In fact the only other application in finance of cumulative prospect theory that we know of is Polkovnichenko (2005), who studies diversification issues. We will show that this feature of prospect theory is absolutely essential to our analysis. Bondarenko (2003a,b) demonstrates that historical put prices cannot be rationalized by any model within the broad class of models with a path-independent pricing kernel and rational updating of beliefs. Cumulative prospect theory falls outside this class, since the beliefs are not only biased, but furthermore not updated (rationally): Bondarenko's results go through with biased beliefs as long as investors learn rationally.[14] A final motivation for distorting the probabilities of extreme

[14] While the issue does not arise here directly, it is in fact not obvious how the decision weights or distorted probabilities are to be updated. Even when updating rationally, convergence may

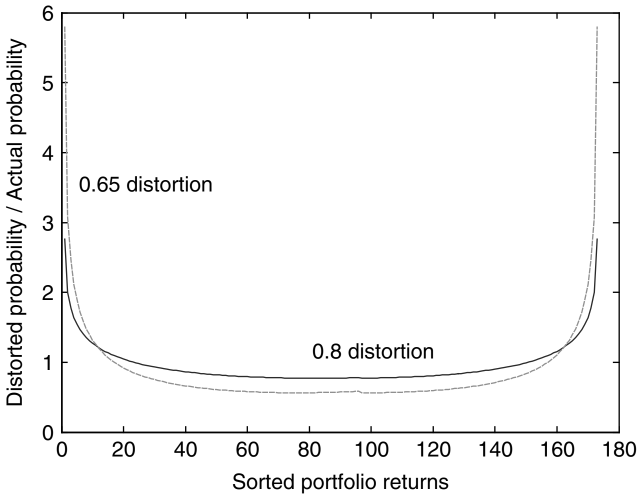

Figure 1. Ratio of distorted to actual probabilities in cumulative prospect theory. This figure shows the ratio of distorted to actual probability in cumulative prospect theory with weighting function $w(p) = \frac{p^c}{[p^c+(1-p)^c]^{1/c}}$ (Section 5.1.2) for $c = 0.65$ and $c = 0.8$.

portfolio outcomes for an investor who is long the market, is that it can be seen as a (partial) justification for the crash-aversion preferences in Bates (2002). Bates solves a heterogeneous-investor equilibrium model and generates a number of challenging stylized facts in options markets, but needs to impose that some investors are crash-averse.[15]

When derivatives are not available (top panel of Table IV), the effect of $c < 1$ is to push the portfolio demands for equity towards zero. For $\lambda = 2.25$, we already obtained nonparticipation without the probability distortion. For $\lambda = 1.25$, nonparticipation results if the probability distortion is sufficiently severe ($c = 0.65$ as suggested by Tversky and Kahneman), but not for moderately nonlinear probability transformations ($c = 0.8$). Therefore the distortion acts as a substitute for a high degree of first-order risk aversion. Even when first-order risk aversion is moderate, the fact that extreme portfolio outcomes are overweighted makes the investor sufficiently worried about

be slow since the events concerned are extreme and may occur infrequently. An alternative interpretation is that the decision-maker is well aware of the actual probabilities, but nonetheless distorts them for the purpose of utility evaluation and decision-making, in which case the issue of learning becomes moot.

[15] Liu et al. (2005) study the equilibrium option pricing implications of robustness for investors that are averse to model uncertainty concerning rare events. This can be viewed as an alternative approach to generating effective crash aversion.

Table IV. Portfolio weights for cumulative prospect theory

This table reports the optimal portfolio weights α_E (equity) and α_D (derivative strategy) for cumulative prospect theory (value function (4) and distorted probability assessments according to (5) and (6)), over our January 1987–June 2001 sample, obtained by replacing the expectation in (1) by its (distorted) sample counterpart and maximizing over the portfolio weights. The derivative strategies are a (short-maturity) OTM put with 0.96 strike-to-spot ratio and a (short-maturity) ATM straddle, using S&P 500 futures options. The implied equity weight corresponding to each α_D is calculated by multiplying the optimal put (straddle) weight with the empirical beta of the put (straddle).

λ	1.25				2.25			
$\widehat{\gamma}$	0.8		1		0.8		1	
c	0.65	0.8	0.65	0.8	0.65	0.8	0.65	0.8
				No derivatives				
α_E	0	2.279	0	2.279	0	0	0	0
				OTM put				
α_E	0	−1.037	5.575	0.043	0	−0.676	0	0
α_D	0	−0.100	0.078	−0.052	0	−0.071	0	0
Implied equity	0	1.906	−1.487	0.991	0	1.353	0	0
				ATM straddle				
α_E	0.445	0.557	0	0.299	0	0	0	0
α_D	−0.282	−0.389	0	−0.360	0	0	0	0
Implied equity	0.162	0.223	0	0.207	0	0	0	0

stock market risk to ignore the equity premium: both positive and negative returns are overweighted, but the left-hand tail of the distribution matters more because of (even moderate) loss aversion and the negative skewness of the equity return distribution.

Introducing OTM puts, the probability distortion has a large impact on portfolio choice. For $\lambda = 2.25$, we find nonparticipation for all parameter values, except for $\widehat{\gamma} = 0.8$ and $c = 0.8$ (moderate distortion), where the short put position remains optimal. The combination of loss aversion and the large skewness of the put option return explains the effect of the probability distortion. Interestingly, protective-put strategies are not optimal. With less first-order risk aversion ($\lambda = 1.25$), short put positions are still optimal if the probability distortion is moderate ($c = 0.8$). The weights are remarkably smaller than in Table III though. For $c = 0.65$, we finally obtain a positive put weight. However, the puts protect a levered equity position that can be considered unreasonable. Also, the result is unlikely to be robust in light of the nonparticipation for $\lambda = 2.25$ or for $\widehat{\gamma} = 0.8$. Positive put weights require large probability distortions and moderate loss aversion. When loss aversion becomes more pronounced, the same investor simply stops investing.

The short straddle positions we previously found would also be expected to change and to become unattractive when extreme portfolio outcomes are considered more likely. Table IV's bottom panel shows a similar pattern as its middle panel for puts. When first-order risk aversion is substantial ($\lambda = 2.25$), investors never participate, but they continue to short straddles when both the distortion and first-order risk aversion are moderate ($c = 0.8$ and $\lambda = 1.25$). However, unlike for puts, we do not find strictly positive straddle weights: when the distortion is pronounced ($c = 0.65$) and the investor is not too loss-averse, a (smaller) short straddle position remains optimal for $\widehat{\gamma} = 0.8$, while the investor chooses not to participate for $\widehat{\gamma} = 1$. The intuition for the optimality of the long put (in contrast to a long straddle position) is that a long put exhibits substantially more positive skewness than a straddle, as is clear from Table I. The distortion in Figure 1 overweighs the left and right tail of the portfolio return distribution in a symmetric fashion, which favors return distributions with positive skewness (like the long equity plus long put).

As in the previous subsection without distorted decision weights, we also analyze the alternative specification where the reference level equals initial wealth grown at the optimal equity portfolio return when derivatives are not available. This robustness check is relevant for prospect theory with rank-dependency, since the reference level and the difference between losses and gains not only matters for the value function in (4), but also for the distorted decision weights in (5). As before, we obtain qualitatively and quantitatively virtually identical results, whether the investor participates in equity markets or not in the absence of derivatives.

The results above substantiate the claim made earlier that distorted probabilities are an essential ingredient of prospect theory if one wants to explain nonparticipation with loss aversion, since we found large (and negative) portfolio weights with unbiased beliefs in the previous subsection. Also, positive weights in derivatives can only be obtained for puts and for particular parameter values, namely moderate loss aversion and sufficiently large probability distortions, and are accompanied by an equity allocation that can be considered unreasonable.

5.2 DISAPPOINTMENT AVERSION

Disappointment aversion (Gul (1991)) has recently been advocated as an interesting alternative to prospect theory for portfolio choice problems. It shares with prospect theory the intuitively appealing notion of loss aversion or first-order risk aversion (Segal and Spivak (1990)), but it is axiomatically founded. Ang et al. (2005) first analyzed the portfolio implications of disappointment aversion, and show how it generates reasonable equity holdings, including nonparticipation, while having a number of modeling advantages. In particular, the reference point is endogenous and the preferences satisfy global concavity (this in fact by virtue of the endogenous reference point) thus avoiding the excessive risk-taking problem for loss aversion in the domain of losses as argued by Gomes (2005). Finally, a single parameter controls the degree of disappointment aversion, and standard CRRA is nested as a special case. Disappointment aversion is therefore an interesting alternative to loss aversion. An additional reason for including it in the current analysis is the conjectures of Pan (2002, p. 34) that disappointment aversion (and the aversion to negative skewness it implies) may explain the magnitude of estimated jump risk premia, and of Ang, Bekaert and Liu (p. 500) that it may provide a rationale for the recent popularity of put-protected products. Finally, as shown by Backus et al. (2004), disappointment aversion can also be viewed as a class of preferences that distorts probabilities.

A disappointment-averse investor solves (1) where $U(W_T)$ is given by

$$U(W_T) = \begin{cases} \frac{W_T^{1-\gamma}}{1-\gamma} & \text{for } W_T > \mu_W \\ \frac{W_T^{1-\gamma}}{1-\gamma} - \left(\frac{1}{A} - 1\right)\left[\frac{\mu_W^{1-\gamma}}{1-\gamma} - \frac{W_T^{1-\gamma}}{1-\gamma}\right] & \text{for } W_T \leq \mu_W \end{cases} \tag{7}$$

where $A \leq 1$ is the coefficient of disappointment aversion, γ is the coefficient of relative risk aversion and μ_W is the implicitly defined certainty equivalent wealth, which acts as the reference point and which depends on the endogenously chosen portfolio. $A = 1$ corresponds to standard expected utility. Ang, Bekaert and Liu show that $A = 0.6$ generates nonparticipation for all levels of risk aversion, while $A = 0.85$ leads to a reasonable 60% equity allocation (in the i.i.d. case) for an investor with $\gamma = 2$. We consider $A = 0.6$ and $A = 0.8$. To further interpret these parameter values, it may be useful to compare with loss aversion: the degree of first-order risk aversion is given by A^{-1}, so that these parameter values correspond to $\lambda = 1.67$ and $\lambda = 1.25$.[16]

In Table V, when the investor can invest only in the riskfree asset or the equity index, $A = 0.6$ always leads to nonparticipation, in line with the results of Ang, Bekaert and Liu. Less disappointment aversion ($A = 0.8$) makes the

[16] We also considered $A^{-1} = 2.25$, as in Section 5.1, and obtained similar results.

Table V. Portfolio weights for disappointment aversion

This table reports the optimal portfolio weights α_E (equity) and α_D (derivative strategy) for a disappointment-averse investor with utility function (7) over our January 1987–June 2001 sample, obtained by replacing the expectation in (1) by its sample counterpart and maximizing over the portfolio weights. The derivative strategies are a (short-maturity) OTM put with 0.96 strike-to-spot ratio and a (short-maturity) ATM straddle, using S&P 500 futures options. The implied equity weight corresponding to each α_D is calculated by multiplying the optimal put (straddle) weight with the empirical beta of the put (straddle).

γ	1		2		5		10	
A	0.6	0.8	0.6	0.8	0.6	0.8	0.6	0.8
				No derivatives				
α_E	0	1.855	0	0.965	0	0.393	0	0.198
				OTM put				
α_E	−1.090	−1.510	−0.689	−0.991	−0.313	−0.458	−0.164	−0.241
α_D	−0.100	−0.121	−0.063	−0.081	−0.028	−0.038	−0.015	−0.020
Implied equity	1.906	2.306	1.201	1.544	0.534	0.724	0.286	0.381
				ATM straddle				
α_E	0.957	0.815	0.577	0.593	0.250	0.287	0.129	0.153
α_D	−0.278	−0.393	−0.156	−0.234	−0.067	−0.103	−0.034	−0.053
Implied equity	0.159	0.225	0.089	0.134	0.038	0.059	0.020	0.030

portfolio weights substantially smaller than for expected utility, but seems not sufficient to generate nonparticipation.

When the investor can also allocate wealth to OTM puts, nonparticipation always disappears. In fact, the investor always shorts OTM puts and equity. This is true even for the high coefficient of disappointment aversion, for which nonparticipation is optimal for all risk aversion coefficients when puts are absent. Similar results are obtained in Table V for straddles: all disappointment-averse investors short straddles. Interestingly, the equity weight actually increases in many cases relative to the expected utility results ($A = 1$, Table II). This can be understood by noting that disappointment aversion makes the investor more averse to negative skewness. For a given risk exposure, skewness can be reduced by investing less in the straddle and more in equity.

In conclusion, disappointment aversion leads to nonparticipation without derivatives, but always results in participation when derivatives are included in the menu of assets. Most importantly, the investor chooses short positions that are economically significant. The results are in fact quite similar to what happens for expected utility, although the derivatives positions are naturally somewhat smaller. This suggests that disappointment aversion is not sufficient to explain option pricing puzzles even though it generates stock-market nonparticipation (Ang et al. (2005)).

5.3 ANTICIPATED UTILITY

The analysis so far has shown that highly distorted decision weights combined with moderate loss aversion are needed to obtain positive put weights. However, this specification leads to unreasonably levered equity positions (when the investor chooses a long put position) and fails to produce positive straddle weights. Moreover, cumulative prospect theory requires assumptions about the evolution of the reference point, which restricts its applicability in dynamic settings. For these reasons we now consider Anticipated Utility (Quiggin (1982) and Yaari (1987)), which also features distorted decision weights or rank dependency, but which nests expected utility and does not require assumptions about the reference point.[17] Epstein and Zin (1990) integrate the model of Quiggin and Yaari in a dynamic context and show that it can resolve the equity premium puzzle.

For the static portfolio problem we study, Anticipated Utility weights the utilities of outcomes with decision weights π_n, which are based on distorted cumulative probabilities of ranked portfolio outcomes. Denoting the (objective) cumulative probability of outcome n by $P_n \equiv \sum_{j=1}^{n} p_j$, the subjectively distorted probability (or decision weight) π_n is obtained as follows:

$$\pi_i = w\left(P_i\right) - w\left(P_{i-1}\right) \tag{8}$$

for a weighting function $w(.)$. Unlike for cumulative prospect theory in (5), the transformation function is applied uniformly for gains and losses and the weighting does not involve "mirroring" around the reference level.

For the utility of outcomes we take the power specification $\frac{W_T^{1-\gamma}}{1-\gamma}$ so that the model nests CRRA expected utility for $w(P) = P$. We use the functional form suggested by Epstein and Zin (1990):

$$w(P) = P^{\alpha} \text{ for } 0 < \alpha \leq 1 \tag{9}$$

and consider $\alpha \in \{0.65, 0.8\}$. Figure 2 presents the corresponding decision weights for our portfolio problem. Having $w(P) = P^{\alpha}$ clearly penalizes

[17] We thank an anonymous referee for suggesting the analysis of Anticipated Utility.

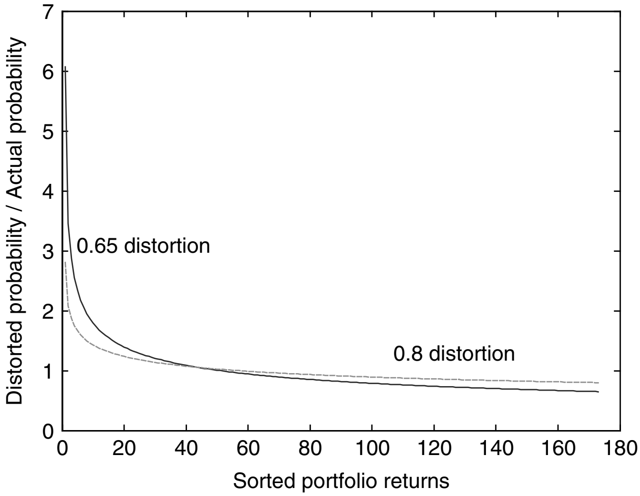

Figure 2. Ratio of distorted to actual probabilities in Anticipated Utility, $w(P) = P^{\alpha}$. This figure shows the ratio of distorted to actual probability in Anticipated Utility (Section 5.3) with $w(P) = P^{\alpha}$ for $\alpha = 0.65$ and $\alpha = 0.8$.

extremely negative outcomes and would be expected to instill conservative behavior.

Table VI shows that the overweighting of the left tail of the portfolio return distribution generates nonparticipation without derivatives, reflecting the first-order risk aversion exhibited by Anticipated Utility. When adding puts or straddles, we still obtain nonparticipation if the distortion is sufficiently severe ($\alpha = 0.65$). The more moderate probabilitity distortion ($\alpha = 0.8$) produces negative portfolio weights for both derivatives strategies. In this case, the distortion only acts to substantially decrease the size of the portfolio weights. For both puts and straddles, the weights are roughly one third of the expected-utility results in Table II.

It is clear from these results that "paranoia" alone (increasing the probability of unfavorable portfolio outcomes) is not sufficient to generate positive demand for puts or straddles. We therefore now analyze Anticipated Utility with the transformation function of Kahneman and Tversky (Equation (6)). Applying this function to cumulative probabilities of outcomes, without loss aversion or mirroring around R_f as in (5), poor portfolio outcomes are overweighted (as in the case of $w(P) = P^{\alpha}$), but extremely positive portfolio outcomes are also overweighted and in fact more so (see Figure 3). This probability distortion therefore induces a stronger preference for positively skewed portfolio return distributions.

Table VI. Portfolio weights for anticipated utility, $w(P) = P^{\alpha}$

This table reports the optimal portfolio weights α_E (equity) and α_D (derivative strategy) for anticipated utility (power specification for the utility of outcomes and decision weights according to (8) and (9)), over our January 1987–June 2001 sample, obtained by replacing the expectation in (1) by its (distorted) sample counterpart and maximizing over the portfolio weights. The derivative strategies are a (short-maturity) OTM put with 0.96 strike-to-spot ratio and a (short-maturity) ATM straddle, using S&P 500 futures options. The implied equity weight corresponding to each α_D is calculated by multiplying the optimal put (straddle) weight with the empirical beta of the put (straddle).

γ	1		2		5		10	
α	0.65	0.8	0.65	0.8	0.65	0.8	0.65	0.8
				No derivatives				
α_E	0	0	0	0	0	0	0	0
				OTM put				
α_E	0	−0.672	0	−0.373	0	−0.158	0	−0.081
α_D	0	−0.062	0	−0.035	0	−0.015	0	−0.008
Implied equity	0	1.182	0	0.667	0	0.286	0	0.152
				ATM straddle				
α_E	0	0.447	0	0.247	0	0.105	0	0.053
α_D	0	−0.192	0	−0.103	0	−0.043	0	−0.022
Implied equity	0	0.110	0	0.059	0	0.025	0	0.013

The results in Table VII are interesting. Without derivatives, the distortion of the probability of favorable portfolio outcomes breaks the nonparticipation we found in Table VI. Not surprisingly, the weights are smaller than for CRRA expected utility. For both puts and straddles, we still obtain negative portfolio weights when the distortion parameter is modest. However, for $c = 0.65$, we now find positive weights for puts and for straddles. For puts, the weights are quite small, and certainly more reasonable than for cumulative prospect theory. For instance, the logarithmic investor holds 8% of her wealth in equity and 11% in puts. The investor buys put options attracted by their substantial positive skewness. The straddle weights are substantially larger and accompany levered equity positions for $\gamma = 1$ and $\gamma = 2$. The positive portfolio weight for the straddle is also driven by the positive skewness of its return distribution. Owing to the larger overweighting of favorable portfolio outcomes than of unfavorable outcomes (Figure 3), the investor exhibits a strong preference for positive skewness, and a long straddle position becomes optimal. Note that cumulative prospect theory failed to produce positive straddle weights since the probability distortion is essentially symmetric in that case (by virtue of the

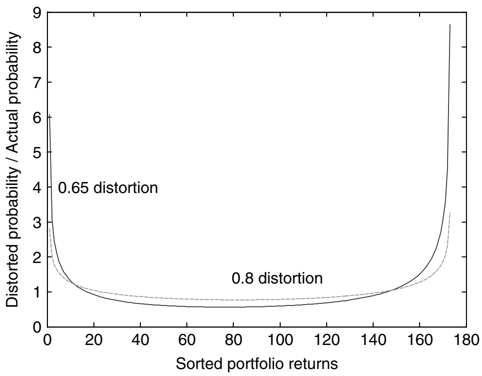

Figure 3. Ratio of distorted to actual probabilities in Anticipated Utility, $w(P) = \frac{P^c}{\left[P^c+(1-P)^c\right]^{1/c}}$.
This figure shows the ratio of distorted to actual probability in Anticipated Utility (Section 5.3) with $w(P) = \frac{P^c}{\left[P^c+(1-P)^c\right]^{1/c}}$ for $c = 0.65$ and $c = 0.8$.

Table VII. Portfolio weights for anticipated utility, $w(P) = \frac{P^c}{\left[P^c+(1-P)^c\right]^{1/c}}$

This table reports the optimal portfolio weights α_E (equity) and α_D (derivative strategy) for anticipated utility (power specification for the utility of outcomes and decision weights according to (8) and (6)), over our January 1987–June 2001 sample, obtained by replacing the expectation in (1) by its (distorted) sample counterpart and maximizing over the portfolio weights. The derivative strategies are a (short-maturity) OTM put with 0.96 strike-to-spot ratio and a (short-maturity) ATM straddle, using S&P 500 futures options. The implied equity weight corresponding to each α_D is calculated by multiplying the optimal put (straddle) weight with the empirical beta of the put (straddle).

γ	1		2		5		10	
c	0.65	0.8	0.65	0.8	0.65	0.8	0.65	0.8
				No derivatives				
α_E	1.811	2.074	0.961	1.106	0.396	0.456	0.200	0.230
				OTM put				
α_E	0.082	1.980	0.036	0.969	0.013	0.378	0.006	0.187
α_D	0.107	−0.003	0.048	−0.005	0.017	−0.003	0.009	−0.002
Implied equity	−2.040	0.057	−0.915	0.095	−0.324	0.057	−0.172	0.038
				ATM straddle				
α_E	3.168	0.291	1.558	0.217	0.611	0.094	0.303	0.048
α_D	0.248	−0.322	0.122	−0.179	0.048	−0.076	0.024	−0.039
Implied equity	−0.142	0.185	−0.070	0.103	−0.028	0.044	−0.014	0.022

mirroring around R_f in (5)) and does not induce a strong enough preference for positive skewness.

6. Sensitivity Analysis

As a next step in the analysis, we consider a number of extensions and sensitivity checks to study the robustness of the results. For brevity, we focus on expected utility.

6.1 TRANSACTION COSTS AND MARGIN REQUIREMENTS

To analyze whether the results are robust to the presence of transaction costs, we now introduce the following market frictions. A first trading cost stems from the bid-ask spread. For equity, we follow Fleming et al. (2001) and impose a relatively small transaction cost of two basis points round trip since the equity position can be implemented with index futures. For the index options, we use the information on bid-ask spreads in Bakshi et al. (1997). Importantly, the spreads depend on the moneyness of the options and are allowed to change for a given option as moneyness evolves from the start of the period over which we compute the return to the end of the month. The spreads imply average round trip costs of about 6% for OTM puts and about 4% for ATM options. These estimates are conservative and may represent upper bounds to the extent that investors are able to trade within the quoted bid-ask spreads, as shown for individual options in Mayhew (2002). A second important friction comes in the form of margin requirements on short equity and option positions. Margin requirements affect the analysis only if the investor does not invest sufficiently in the riskfree asset, since otherwise the riskfree asset holdings serve as margin, and if in addition the borrowing rate exceeds the lending rate. On the basis of Hull (2003) and of the CBOE and CME websites, the margin for short options positions is set at 15% (minus the percentage by which the option is OTM) of the value of the underlying plus the premium. For short equity positions, we use the margin for CME equity index futures of almost 8% of the equity value. Initial wealth of the investor is taken to be $100,000 and the borrowing spread is chosen to be 300 basis points per year.

The effect of these frictions for the equity-only case in Table VIII is intuitive. Highly risk-averse investors ($\gamma \geq 5$) do not hold levered positions and are therefore only affected by the bid-ask spread. Since the spread is small for equity index futures, the portfolio weights in Table VIII are only marginally smaller than the ones for frictionless markets in Table II. Risk-tolerant investors,

Table VIII. Portfolio weights with market frictions

This table reports the optimal portfolio weights α_E (equity) and α_D (derivative strategy) and their standard errors for a CRRA investor with risk aversion γ over our January 1987–June 2001 sample, incorporating bid-ask spreads and margin requirements as explained in Section 6.1. The portfolio weights are obtained by replacing the expectation in (1) by its sample counterpart and maximizing over the portfolio weights, given the constraints due to market frictions. The derivative strategies are a (short-maturity) OTM put with 0.96 strike-to-spot ratio and a (short-maturity) ATM straddle. The implied equity weight corresponding to each α_D is calculated by multiplying the optimal derivative weight with the empirical beta of the derivative.

γ	0.5	1	2	5	10	20
			No derivatives			
α_E	3.1872	2.2592	1.1974	0.7046	0.3569	0.1795
SE	0.7602	1.4377	0.8270	0.3579	0.1835	0.0928
			OTM put			
α_E	−1.7397	−1.3178	−0.8250	−0.3835	−0.2018	−0.1035
SE	2.3401	2.5339	1.2973	0.5330	0.2697	0.1358
α_D	−0.1499	−0.1138	−0.0767	−0.0370	−0.0197	−0.0102
SE	0.0612	0.0503	0.0328	0.0166	0.0102	0.0053
Implied equity	2.8121	2.1349	1.4389	0.6941	0.3696	0.1914
			ATM straddle			
α_E	0.9801	0.7562	0.5398	0.3896	0.2062	0.1060
SE	1.1204	1.2506	0.7683	0.3487	0.1819	0.0928
α_D	−0.4591	−0.3426	−0.1989	−0.0924	−0.0478	−0.0243
SE	0.1456	0.1377	0.0874	0.0399	0.0209	0.0107
Implied equity	0.2077	0.1550	0.0900	0.0418	0.0216	0.0110

however, hold levered positions and these become substantially more expensive with the introduction of margin requirements. The equity portfolio weights drop substantially for $\gamma \leq 2$ and become statistically insignificant (except for $\gamma = \frac{1}{2}$).

Despite the introduction of bid-ask spreads and margin requirements, the optimal put weights are still negative and statistically significant. Comparing Table VIII with Table II, it can be seen that market frictions actually mainly affect the equity portfolio weights. Relative to Table II, (long) equity has become more attractive given the lower trading cost on equity than on options. This makes it relatively more costly to short equity to hedge negative option positions. The decrease in the absolute value of the put weights reflects both the direct effect of the transaction cost and an indirect effect due to the fact that hedging short puts with short equity is now more expensive.

Since straddles do not lead to large hedging demands for equity, only the direct effect is at work in Table VIII's bottom panel. The positive equity positions increase substantially relative to Table II. The straddle weights become smaller in absolute value but remain strongly statistically significant. Notice that the shrinking of the straddle weights is more pronounced than it is for puts even though straddles are only directly affected by the presence of the market frictions. However, the expected return on short straddles is substantially smaller than the expected return on short puts, so that a given transaction cost affects the straddle more. Hence the larger effect on straddle positions.

6.2 CRASH-NEUTRAL PUTS AND STRADDLES

Our analysis may suffer from a Peso problem: perhaps returns on short puts and straddles turned out substantially higher *ex post* than expected *ex ante* by market participants, simply because fewer stock market "crashes" occurred than expected. Indeed, our sample includes one of the most impressive bull markets of recent history. Even though the sample does contain the 1987 and 1990 stock market crashes, our analysis may so far still be vulnerable to this criticism. In order to make our results robust to the ex post absence of major crashes, we now consider crash-protected straddles following Coval and Shumway (2001) and crash-neutral puts as in Jackwerth (2000). We crash-neutralize short OTM puts with 0.96 moneyness by simultaneously going long 0.92 moneyness "deep" OTM puts, creating what is often referred to as a vertical bull spread. Short ATM straddles are crash-neutralized in the same way (a "ratio-call spread"). It is important to realize that these strategies may be substantially less attractive given the positive jump risk premium (and to a lesser extent negative volatility risk premium) present in the 0.92 OTM put. Crash-neutralizing the short positions in the put option and straddle will lower their expected return. Simultaneously, it lowers the risk of the strategies and in particular the likelihood of extremely negative returns.

Crash-neutralizing the OTM puts in Table IX makes the portfolio weights for puts somewhat smaller in absolute value. This illustrates that crash-protection, although expected to be useful given the 1987 and 1990 crashes in the sample, does not come free and lowers the expected return of the position. Importantly though, even with crash insurance, the optimal put weights remain statistically significantly negative. This may be surprising if, based on the smirk-like pattern of Black-Scholes implied volatilities for OTM

Table IX. Portfolio weights with crash-neutral strategies

This table reports the optimal portfolio weights α_E (equity) and α_D (derivative strategy) and their standard errors for a CRRA investor with risk aversion γ obtained by estimating (3) with GMM over our January 1987–June 2001 sample. The derivative strategies are a (short-maturity) crash-neutral OTM put and a (short-maturity) crash-neutral ATM straddle, using S&P 500 futures options. The crash-neutral OTM put consists of a long position in the 0.96 OTM put and a short position in the 0.92 OTM put. The crash-neutral ATM straddle consists of a long position in the ATM straddle and a short position in the 0.92 OTM put. The implied equity weight corresponding to each α_D is calculated by multiplying the optimal crash-neutral put (straddle) weight with the empirical beta of the put (straddle).

γ	0.5	1	2	5	10	20
			Crash-neutral OTM put			
α_E	−1.5250	−1.3022	0.5050	0.1683	0.0792	0.0384
SE	1.8912	2.0562	1.0993	0.4501	0.2258	0.1130
α_D	−0.1366	−0.1045	−0.0718	−0.0337	−0.0177	−0.0090
SE	0.0579	0.0425	0.0348	0.0177	0.0095	0.0049
Implied equity	1.7157	1.3125	0.9018	0.4233	0.2223	0.1130
			Crash-neutral ATM straddle			
α_E	4.4064	3.3609	2.1047	0.9484	0.4913	0.2498
SE	1.2802	1.2793	0.7965	0.3747	0.1968	0.1007
α_D	−0.5032	−0.3952	−0.2639	−0.1211	−0.0630	−0.0321
SE	0.1317	0.1093	0.0912	0.0459	0.0244	0.0125
Implied equity	−1.6203	−1.2725	−0.8498	−0.3899	−0.2029	−0.1034

puts, one were to think of the deep OTM puts as being more "overpriced" than the 0.96 OTM puts, so that short crash-neutral put positions would seem unattractive. Table IX shows that this intuition is misguided, since a much smaller percentage of wealth is invested in the 0.92 put than in the 0.96 put. The crash neutrality of the puts also affects the optimal equity position. First of all, the correlation with equity of the crash-neutral put is smaller than for the unprotected put, making the negative equity position needed for hedging purposes smaller in Table IX than in Table II. Secondly, the optimal put position itself changes. Both effects substantially reduce the need for hedging with a short equity position. Except for $\gamma \leq 1$, the optimal equity weight actually becomes positive (but remains insignificant).

In Table IX we find negative and very significant weights for the crash neutral straddles. The weights are slightly smaller than before. As for puts, crash-protection involves two opposing effects. First, the correlation of a short straddle with equity returns changes and becomes negative, so that long equity can be used to hedge a short position. This makes the crash-protected straddle more attractive than the uninsured alternative. At the same time, crash protection leads to a reduction in expected return (for a short position). The latter effect dominates here, making, on net, the short straddle less attractive with the crash protection and resulting in a less negative weight.

Our results are therefore robust to the Peso problem: crash-neutralizing the puts and straddles does not qualitatively change the optimality of negative put and straddle positions.

6.3 THE IMPACT OF DISCRETE TIME AND LACK OF REBALANCING

Our analysis studies the importance of index options for portfolio choice in a discrete-time setting. One of the advantages of our approach is that the attractiveness of investing in index options does not rely on the ability to trade continuously. However, the concern may arise that the discreteness of the time period in the analysis makes options attractive, not because of their superior risk-return trade-off, but simply since it allows the investor to achieve some dynamic trading strategy that could not be implemented with equities only.[18] To demonstrate that this is not driving the results, we now simulate monthly returns from the Black-Scholes model where the risk-return trade-off in options is, by construction, not superior, and where options would indeed be redundant if continuous trading were allowed. This allows us to isolate the effect of the discreteness of the trading period. Comparing the optimal portfolios based on Black-Scholes-generated return series with the portfolios presented before will then shed light on the validity of our claim that options are indeed attractive investments purely because of the jump and volatility risk premia they incorporate.

Given estimates of the riskfree rate and index volatility over our sample period, the Black-Scholes model is used to simulate 10,000 time series of equity and option returns, each of the same length as the empirical sample. For each time series of returns, the optimal portfolios (α_E and α_D) and associated standard errors are estimated as before. Table X presents averages of the portfolio weights and of the associated t-ratios across these 10,000 simulations.

Interestingly, we find that the optimal put and straddle weights in Table X are very close to zero. Not surprisingly, the optimal equity weights are therefore very similar to the weights obtained in Table II without derivatives, except

[18] Haugh and Lo (2001) analyze to what extent buy-and-hold portfolios of options allow investors to achieve certain dynamic investment policies in an environment where markets are otherwise complete.

Table X. Portfolio weights with simulated Black-Scholes returns

This table reports optimal portfolio weights α_E (equity) and α_D (derivative strategy) for a CRRA investor with risk aversion γ using simulated monthly returns from the Black-Scholes model. The derivative strategies are a (short-maturity) OTM put with 0.96 strike-to-spot ratio and a (short-maturity) ATM straddle. Given estimates of the riskfree rate and S&P 500 volatility over our sample period (January 1987–June 2001), the Black-Scholes model is used to simulate 10,000 time-series of equity and option returns, each of the same length as the empirical sample. For each time-series of returns, the optimal portfolios (α_E and α_D) and associated standard errors are obtained by estimating (3) with GMM. The table presents averages of the portfolio weights and of the associated t-ratios across these 10,000 simulations.

γ	0.5	1	2	5	10	20
			OTM put			
α_E	8.5412	4.1738	1.9296	0.7270	0.3555	0.1757
av. t	1.6561	1.4986	1.4285	1.3946	1.3842	1.3791
α_D	0.0134	0.0037	−0.0052	−0.0040	−0.0023	−0.0013
av. t	0.0238	0.0052	−0.0939	−0.1363	−0.1494	−0.1559
			ATM straddle			
α_E	8.0352	3.9668	2.0724	0.8497	0.4278	0.2150
av. t	2.6353	2.4054	2.2063	2.0886	2.0506	2.0348
α_D	0.0542	0.0276	−0.0039	−0.0059	−0.0037	−0.0020
av. t	0.2592	0.1093	−0.0263	−0.0993	−0.1217	−0.1344

for $\gamma \leq 1$. In line with Liu and Pan (2003), log investors hold small long positions in the put or straddle, while more risk-averse investors have very small negative portfolio weights. Most importantly though, the derivatives weights are completely statistically insignificant. These results vindicate our claim that options matter to investors because of the generous risk premia embedded in their prices. The fact that the trading period is discrete is not driving our strong results.

We also conduct an additional experiment with stochastic volatility, which induces time variation in investment opportunities and intertemporal hedging demands for equity (Chacko and Viceira (2005)) or for options (Liu and Pan (2003)). Facing stochastic volatility, an investor with a multiperiod horizon would want to rebalance his equity position. If rebalancing the equity position is not possible, a static position in derivatives may represent a useful substitute, as it allows the investor to indirectly implement a dynamic trading strategy. This role of derivatives is emphasized by Haugh and Lo (2001) and ignored in our analysis above. To quantify the importance of this effect and the extent to which it may alter our conclusions, we simulate a stochastic volatility model using the parameter values of Liu and Pan (2003). The investor has a one-month horizon and is not allowed to rebalance his equity position, but returns are sampled at a higher frequency. Importantly, the volatility risk premium is set to zero, so that derivatives do not carry a different risk-return trade-off. The demand for options stems only from their ability to satisfy the investor's rebalancing needs. Unreported results show small positive weights for puts and straddles, which are always statistically insignificant. The hedging demands for options are therefore both statistically and economically small, in line with Liu and Pan, who find that the myopic demand dominates the hedging demand for reasonable values of the volatility risk premium. Haugh and Lo (2001) conduct a similar experiment and find a large demand for options despite the absence of volatility or jump risk premia. The difference in findings can be understood from the fact that their investor has a 20-year horizon and is not allowed to rebalance his equity position at all over this 20-year period.

We also consider additional robustness checks in the context of the benchmark model, namely a change in the frequency of the return time-series and the horizon of the investor from monthly to weekly, as well as a sample split. All the results (unreported for space reasons, but available upon request) survive and some become in fact stronger.

7. The Economic Value of Investing in Derivatives

To further quantify the economic value of including derivatives optimally in a portfolio, we now report the certainty equivalent wealth that the investor demands as compensation for not being able to invest in derivatives. This summary metric can be interpreted as the maximum fixed cost that the investor is willing to pay to gain access to derivatives. Since the investor's preferences are homothetic, the certainty equivalent is computed in percentage terms. Also, this is a percentage of initial wealth over a period that corresponds to the investor's horizon (one month). We focus on expected utility and also consider transaction costs and crash-neutral strategies.

We see in Table XI that the economic value of investing optimally in puts and especially in straddles is substantial. The certainty equivalent declines as risk aversion increases, reflecting the smaller positions in derivatives chosen by more risk-averse investors. It is important to keep in mind that these are certainty equivalents for investors with a one-month horizon. An investor with \$100,000 of investable wealth and risk aversion coefficient of 10, is therefore willing to pay \$330 per month to be able to short puts and \$450 per month to

Table XI. Monthly certainty equivalent wealth of investing in derivatives

This table shows the monthly certainty equivalent wealth, which is the percentage of initial wealth needed to compensate the CRRA investor with a one-month horizon and risk aversion γ when derivatives are not available, and which reflects the optimal portfolios of Table II (put and straddle), Table VIII (put and straddle with transaction costs) and Table IX (crash-neutral put and straddle).

γ	0.5	1	2	5	10	20
Put	0.0186	0.0172	0.0123	0.0061	0.0033	0.0017
Straddle	0.0413	0.0314	0.0193	0.0086	0.0045	0.0023
Put (tr. cost)	0.0178	0.0149	0.0091	0.0040	0.0022	0.0011
Straddle (tr. cost)	0.0181	0.0140	0.0081	0.0042	0.0022	0.0011
Crash-neutral put	0.0173	0.0141	0.0087	0.0039	0.0020	0.0010
Crash-neutral straddle	0.0341	0.0244	0.0150	0.0067	0.0034	0.0017

access straddles. For a lower risk aversion of two, these numbers grow to \$1230 and \$1930, respectively. When transaction costs and margin requirements are added, the certainty equivalents become smaller, but remain very large, again keeping in mind that these are monthly numbers. While the certainty equivalent is higher for straddles than for puts without transaction costs, the put has slightly more economic value than the straddle when transaction costs are taken into account. With crash insurance, puts and straddles become somewhat less valuable, reflecting of course the change in optimal weights due to crash protection discussed in Section 6.2. The economic value of being able to invest in derivatives remains substantial however.

8. Multiple Nonspanned Factors

The analysis throughout the paper indicates that jump risk and volatility risk are priced rather generously and that this has significant implications for portfolio choice. In particular, most preferences lead to substantial short positions in straddles and in puts. While it is intuitive that OTM puts load mainly on jump risk and ATM straddles mainly on volatility risk, it remains to be seen whether there is portfolio evidence for the existence of multiple nonspanned factors.[19] Including the ATM straddle and OTM put that we have been considering so far simultaneously in the portfolio problem may be problematic given the high correlation between the returns on these assets (0.587). Therefore in an attempt to disentangle exposure to volatility risk and

[19] Jones (2006) estimates a general nonlinear latent factor model for put returns. He shows that allowing for a second priced factor reduces mispricing, but adding a third factor seems to make mispricing worse.

Table XII. Equity, OTM put and CN-ATM straddle weights

This table reports the optimal portfolio weights α_E (equity), α_{Put} ((short-maturity) OTM put with 0.96 strike-to-spot ratio), $\alpha_{Straddle}$ ((short-maturity) crash-neutral ATM straddle consisting of a long position in the ATM straddle and a short position in the 0.92 OTM put), and their standard errors for a CRRA investor with risk aversion γ, obtained by estimating (3) with GMM over our January 1987–June 2001 sample. The implied equity weight is calculated by multiplying the optimal derivative weight with the empirical beta of the derivative.

γ	0.5	1	2	5	10	20
α_E	−1.2401	−0.6397	−0.3171	−0.1318	−0.0678	−0.0345
SE	2.4521	2.5981	1.4415	0.6373	0.3311	0.1689
α_{Put}	−0.1474	−0.1271	−0.0831	−0.0390	−0.0205	−0.0105
SE	0.0690	0.0658	0.0428	0.0213	0.0115	0.0060
Implied equity	2.8096	2.4227	1.5840	0.7434	0.3908	0.2001
$\alpha_{Straddle}$	−0.2893	−0.2390	−0.1706	−0.0803	−0.0419	−0.0213
SE	0.1035	0.0983	0.0927	0.0507	0.0276	0.0144
Implied equity	−0.9315	−0.7696	−0.5493	−0.2586	−0.1349	−0.0686

to jump risk, we consider the crash-neutral straddle (ATM straddle insured by a 0.92 put) along with the 0.96 OTM put from the benchmark analysis. The idea is that these assets are economically meaningful factor-mimicking portfolios that load mainly on volatility risk and jump risk, respectively. The correlation between the returns on these assets is indeed much lower and close to zero (−0.025).

There is fairly strong evidence for the existence of at least two nonspanned factors in Table XII. The optimal investment strategy consists of short positions in both puts and crash-neutral straddles. Both weights are statistically significant, except for high risk aversion. The equity weight is negative, but insignificant.

Table XIII considers the same portfolio problem, but now incorporating the bid-ask spreads and margin requirements of Section 6.1. Adding transaction costs naturally reduces the derivatives portfolio weights. Even with transaction costs and costly margin requirements, all investors hold short positions in both derivatives, although the statistical evidence of multiple nonspanned factors weakens.

Finally, we report in Table XIV the certainty equivalent wealth levels in order to shed light on the economic importance of accessing both derivatives simultaneously, without and with frictions. Without frictions, these certainty equivalents can be compared with the results for the put only or for the crash-neutral straddle only, i.e., Table XI. Adding a short put to a portfolio

Table XIII. Equity, OTM put, and CN-ATM straddle weights, with transaction costs

This table reports the optimal portfolio weights α_E (equity), α_{Put} ((short-maturity) OTM put with 0.96 strike-to-spot ratio), $\alpha_{Straddle}$ ((short-maturity) crash-neutral ATM straddle consisting of a long position in the ATM straddle and a short position in the 0.92 OTM put), and their standard errors for a CRRA investor with risk aversion γ (as in Table XII), but now incorporating the bid-ask spreads and margin requirements of Section 6.1. The implied equity weight is calculated by multiplying the optimal derivative weight with the empirical beta of the derivative.

γ	0.5	1	2	5	10	20
α_E	−0.0000	−0.0000	−0.0349	−0.0471	−0.0307	−0.0173
SE	–	–	1.4419	0.6326	0.3282	0.1674
α_{Put}	−0.1141	−0.1050	−0.0685	−0.0324	−0.0172	−0.0088
SE	0.0698	0.0615	0.0402	0.0202	0.0110	0.0057
Implied equity	2.1405	1.9698	1.2851	0.6078	0.3227	0.1651
$\alpha_{Straddle}$	−0.1764	−0.1892	−0.1172	−0.0504	−0.0255	−0.0128
SE	0.1103	0.1014	0.0912	0.0477	0.0258	0.0134
Implied equity	−0.5063	−0.5430	−0.3364	−0.1446	−0.0732	−0.0367

Table XIV. Monthly certainty equivalent wealth of investing in both derivatives strategies

This table shows the monthly certainty equivalent wealth, which is the percentage of initial wealth needed to compensate the CRRA investor with a one-month horizon and risk aversion γ when derivatives are not available, and which reflects the optimal portfolios of Table XII (benchmark, i.e., put and crash-neutral straddle) and Table XIII (put and crash-neutral straddle with transaction costs).

γ	0.5	1	2	5	10	20
Benchmark	0.0442	0.0329	0.0212	0.0097	0.0050	0.0026
Transaction costs	0.0303	0.0229	0.0129	0.0054	0.0028	0.0014

that already includes a short straddle is still very valuable and increases the certainty equivalent by about 50%. This strongly suggests both nonspanned factors are important economically and that at least two derivatives are needed to complete the market. With frictions, adding an optimal short crash-neutral straddle position to a short put position is also economically valuable, but mainly for relatively risk-tolerant investors.

9. Conclusion

Adding OTM index put options and ATM index straddles to the standard portfolio problem has dramatic effects. Expected-utility investors hold statistically and economically significant short positions in these derivatives in order to exploit the sizeable premia for jump risk and volatility risk priced in these assets. This result is robust to a number of extensions and sensitivity checks like trading costs, margin requirements and Peso problems. Negative optimal derivatives positions also obtain for the nonexpected-utility specifications that have previously been proposed to explain zero equity holdings: loss-averse and disappointment-averse investors who ignore the equity premium and do not participate in equity markets, hold short positions in puts and straddles when these assets become available. Remarkably, positive put holdings that would implement portfolio insurance are never optimal given historical option prices, even when investors are extremely loss-averse or disappointment-averse.

Only for investors using sufficiently distorted probabilities do we find positive derivatives weights, in some specific cases. Cumulative prospect theory results in positive put weights (but never in positive straddle weights) when investors have moderate loss aversion. However, these puts protect highly levered equity positions. Anticipated utility is able to generate a strictly positive demand for puts and straddles, but only when the distorted decision weights induce a sufficiently strong preference for positive skewness, namely by distorting the probability of extremely favorable portfolio outcomes even more than the probability of unfavorable portfolio outcomes.

This makes it challenging to explain the popularity of put options and of put-protected strategies: long positions seem anomalously suboptimal in our portfolio choice problem. Simultaneously however, certain institutional investors are often described as buying index puts for portfolio insurance purposes (see e.g., Bates (2003, p. 400) and Bollen and Whaley (2004, p. 713)). Agency problems in portfolio delegation may be responsible for this. Analyzing the agency problem further and studying optimal contract design in this context are interesting topics for future research.

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