Reassessing the Equity Premium Puzzle: A Banking Sector Perspective

by
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Management Summary

This thesis investigates the equity premium puzzle in the Netherlands by applying both standard and extended versions of the consumption-based capital asset pricing model (CCAPM). The equity premium puzzle refers to the discrepancy between returns on equities and risk-free assets, which standard models cannot account for without assuming unrealistically high risk aversion.

The first part of the thesis confirms that the standard CCAPM fails to generate plausible risk-free rate or equity premium using recent Dutch data, consistent with Mehra and Prescott (1985). The second part extends the model by incorporating financial intermediaries following Dzhumashev (2025). In this extension, banks hold risk-free assets and issue loans under regulatory constraints.

Although this extended model is more realistic, its empirical application was challenging and revealed inconsistencies between predicted and observed risk-free rates and withdrawal rates. Overall, while the inclusion of financial intermediaries helps to address the equity premium puzzle. Further research is recommended to fully estimate the model's equilibrium and to test its applicability across countries.

1 Introduction

Traditionally, market indexes have been considered a more favorable long-term investment than bonds due to their consistently higher average returns. The discrepancy between the returns of the market index and the risk-free assets is known as the equity premium. The equity premium has been the subject of significant research in financial economics. However, attempts to explain this premium using standard consumption-based models have encountered substantial difficulty. The joint work of Mehra and Prescott (1985) brought this discrepancy into focus by showing that the observed equity premiums in the U.S. could not be explained with realistic levels of risk aversion in standard intertemporal utility models. This shaped significant academic interest towards the equity premium puzzle (EPP).

So far, a variety of theoretical extensions and preference structures have been proposed to find a solution to the equity premium puzzle, such as habit formation, ambiguity aversion, and recursive utility. Despite these innovations, a key dimension remains overlooked: the role of financial intermediaries. In particular, standard consumption-based asset pricing models assume that households are the primary holders of risk-free assets such as government bonds. However, empirical data show that these assets are mainly held by banks and other financial institutions. The results of Baklanova, Kuznits, and Tatum (2022) support this finding, as it is reported that households hold merely 3% of US government securities.

Recent work by Dzhumashev (2025) extends the capital consumption asset pricing model to incorporate the banking sector as an additional agent. In this stylized model, banks play a central role in determining the risk-free rate by demanding risk-free assets, issuing loans, and facing capital and reserve requirements. This extension offers a more realistic representation of asset demand.

This thesis aims to apply the extension to the CCAPM model as proposed by Dzhumashev (2025) to the Dutch economy, using recent macroeconomic and financial data. Two core research questions are addressed:

- Research question 1: Does the traditional equity premium puzzle persist in the Netherlands when using conventional asset pricing models?
- Research question 2: Can the extended CCAPM, accounting for financial institutions, better explain the observed equity premium for Dutch data?

The thesis will first provide a short overview of the contributions that different researchers have proposed to explain the equity premium puzzle. Then information regarding the data will be provided as well as a short summary of the statistics. To answer the first research question, these statistics will be implemented in the log-normal approximation of the consumption-based asset pricing model as proposed by Campbell, Lo, and MacKinlay (1997) to obtain a relation between risk aversion and equity premium. However, this leads to unrealistic values of risk aversion or implausible returns of the risk-free rate. These results imply that the consumption capital asset pricing model with standard power utility is indecisive when it comes to linking risk aversion to asset returns. Thus, the second Research question intends to seek whether the inclusion of financial intermediaries can account for the observed equity premium puzzle. Dzhumashev (2025) proposes in his working paper an extension that will be thoroughly described. Furthermore, the equilibrium that Dzhumashev (2025) briefly described in his working paper will be more explicitly investigated. One of the results of the equilibrium is to obtain a model that yields risk-free rates similar to that of the observed equity premium. However, calibration of the risk-free rate as suggested by Dzhumashev (2025) leads to inconclusive results. Using parameters that are consistent with recent financial data in the Netherlands led to withdrawal rates that were not supported by Dzhumashev (2025). Due to the fact that the results of the second research question are inconclusive, it is recommended that further research is done on this topic with data from multiple countries.

The remainder of this thesis is structured as follows. Section 2 provides a comprehensive review of the existing literature on the equity premium puzzle and its theoretical extensions. Section 3 details the data sources and the methodology used to calibrate the models. Section 4 addresses the first research question by applying the lognormal approximation to the CCAPM for Dutch data. Section 5 investigates the second research question by extending the model to include financial intermediaries as proposed by Dzhumashev (2025). Section 6 provides the conclusion of the paper.

2 Literature

Mehra and Prescott (1985) attempted to explain the equity premium through the use of a consumption-based capital asset pricing model with standard intertemporal power utility. This benchmark model at the time was unable to generate the observed equity premium for realistic levels of risk aversion. This revealed that the consumption-based capital asset pricing model had some fundamental shortcomings. Thus, the equity premium puzzle became a standard for testing the capabilities of asset pricing models.

The framework used by Mehra and Prescott (1985) is based on time preferences and consumption smoothing. In particular, consumption smoothing implies that households prefer stable consumption over volatile consumption. By modeling preferences through intertemporal utility, households now balance the trade-off between immediate satisfaction and the potential gains from future consumption. One of its economic intuitions is that current consumption and future consumption are valued differently. The intertemporal utility model is provided in equation (1). Households obtain utility $\mathrm{u}(\cdot)$ over different consumption levels c_t discounted by the subjective discount factor $\beta \in (0, 1)$ given their current expectation $\mathbb{E}_0(\cdot)$. If β decreases, the intensity at which households are impatient and prefer to consume now increases.

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}, \quad 0 < \beta < 1 \tag{1}$$

In addition to the benchmark model at the time, Mehra and Prescott (1985) assumed that the representative agent has a utility function with constant relative risk aversion (CRRA). CRRA utility functions possess isoelastic properties, meaning that an agent's investment strategy remains unchanged regardless of their wealth level. This isoelasticity is particularly relevant in financial models, as it ensures consistent decision-making across varying levels of wealth. Equation (2) shows the utility of the agent as a function of consumption c_t and risk aversion γ.

$$u(c_t) = \begin{cases} \frac{c_t^{1-\gamma}}{1-\gamma}, & \text{if } 0 < \gamma < \infty \\ \ln(c_t), & \text{if } \gamma = 1 \end{cases} \tag{2}$$

Campbell, Lo, and MacKinlay (1997) co-authored a textbook on the application of econometric methods to financial markets. One of the core contributions is highlighted in Chapter 8. This chapter proposes the application of a log-linear approximation to derive a tractable formula for the equity premium. This approximation simplifies the relationship underlying the equity premium puzzle, making it easier to test empirically.

Mehra and Prescott (2003) took a revised look at their findings in 1985. Hereby, they re-estimated the risk aversion parameter with their data.[1] A plausible reason for the observed equity premium could be investors demanding a risk premium for bearing non-diversifiable risk associated with stocks. Modern asset pricing theory, as explained by Berk and DeMarzo (2009), suggests that the yield of an asset could be due to the risk-return trade-off. Generally, stocks are considerably riskier than bonds which investors account for by demanding a larger risk premium. However, the results of Mehra and Prescott (2003) did not seem to find results that support the risk-return trade-off.

Several researchers have proposed different models that use alternative preferences structures besides the classical standard power utility. Abel (1990) used an alternative preference structure that is known as "catching up with the Joneses", where utility depends on the consumption of the agent and how it compares to others. Campbell and Cochrane (1999) proposed the use of external habits that evolved over time depending on the agent's past consumption. This preference structure causes the agent to evaluate current consumption compared to the habit level. Maenhout (2004) implemented robustness to account for model misspecificiation, this changes the agent's utility function to be more concerned about worst-case scenarios. Hansen

[1]Some of their data has been updated.

and Sargent (2002) investigated how ambiguity aversion changed the utility function by penalizing uncertainty. Finally, Epstein and Zin (1991) developed recursive utility functions that detached risk aversion from the elasticity of intertemporal substitution.

Dzhumashev (2025) offers a fresh view of the equity premium puzzle as discovered by Mehra and Prescott (1985). In particular, the author proposes the incorporation of financial entities to explain the equity premium puzzle. In his work, he presents that the implementation of additional economic agents could help explain the observed equity premium puzzle. The report of Dzhumashev (2025) first describes the disparity between the consumption capital asset pricing model and U.S. market data. The equity premium described by the CCAPM is shown in equation (3). The $\mathbb{E}_t[r_{t+1}]$ is the expected return of a risky asset, $r_{f,t+1}$ is the risk-free return and $u'(c_{t+1})$ is the marginal utility of consumption.

$$\mathbb{E}_t[r_{t+1}] = r_{f,t+1} + cov_t[\frac{-u'(c_{t+1}), r_{t+1}}{\mathbb{E}_t[u'(c_{t+1})]}] \tag{3}$$

To add on, he motivates the inclusion of the banking sector within the framework of the consumption capital asset pricing model. Dzhumashev (2025) incorporates institutional demand into the model, acknowledging that banks play a dominant role in investing in risk-free assets. This implies that risk aversion from household investors alone does not fully determine the equity premium. Recent findings of Baklanova, Kuznits, and Tatum (2022) encourage this motivation since they found that only a small amount of the risk-free assets are included in the investment portfolio of households.

The motivation of the author to include banks in the CCAPM is developed from different sources in the field of quantitative finance. Corporate banks in the model of Dzhumashev (2025) are in the manner of Heuvel (2008). Heuvel (2008) describes the core functions of the bank in his paper. In particular, the bank provides long-term loans to firms which are financed by deposits from households. In particular, Chari, Christiano, and Eichenbaum (1995) explain that firms take on loans from banks to start production processes. Firms in the model of Dzhumashev (2025) require intermediate goods, financed by working capital to produce consumables for the goods market. Households are subject to a Cash-in-Advance (CIA) constraint as explained by Svensson (1985). This constraint forces households to make consumption decisions that disrupt their ability to smooth consumption.

Although this remains an ongoing area of research, Dzhumashev (2025) demonstrates that the equity premium cannot be fully explained by the risk aversion of households. His extension of the CCAPM, which incorporates banks as key financial intermediaries, is to help address the equity premium puzzle.

3 Data sources and scope

Annual and monthly data on the consumer price index (CPI) and household consumption were obtained from Statistics Netherlands (Centraal Bureau voor de Statistiek 2025a). The weekly prices of the AEX and the AEX GR were collected from Google Finance (Google Finance 2025). Monthly data for Dutch 1-year government bond yields were recovered from Market Watch (MarketWatch 2025). Annual data on the household deposit rate and the corporate lending rate were collected from DNB (De Nederlandsche Bank 2025). Information regarding the bank's Return on Equity (ROE) has also been gathered from DNB (De Nederlandsche Bank 2016). Information on regulatory constraints such as the minimum equity ratio and the required reserve ratio was obtained from the ECB (European Central Bank 2023).

The data used in this thesis uses several unique time series that span the period from 2006 to 2024. In contrast to the data used by Mehra and Prescott (1985), the dataset for this analysis is significantly smaller in scope.[2] This is due to one of the time series being very scarce in data. The Dutch 1-YR bond yields were only publicly available after 2006 (MarketWatch 2025). Since the data in this thesis covered Dutch financial and macroeconomic data that spanned 19 years compared to 90 years from Grossman and Shiller (1981). The data used for this thesis were significantly more susceptible to economic crises that disrupt the overall flow of the economy.

Another key distinction between the two datasets lies in their geographic location and economic strength. While Mehra and Prescott (1985) examined the U.S. economy, this thesis focuses on the Dutch economy. In addition, the data underwent two remarkable regime changes, such as the global financial crisis of 2008 and the corona pandemic of 2019. Furthermore, the methodology for calculating the real returns also differs. Real returns represent the earnings of an investment after accounting for taxes and inflation (Bajaj Finserv 2024). As explained by Investopedia (2023), these adjustments often result in real returns that are significantly lower than nominal returns.

In this thesis, the real return of the market index is calculated as equation (4). This equation highlights that the logarithmic real return of the market index r_{t+1}^{real} is difference between the logarithmic nominal return of the market index $r_{t+1}^{nominal}$ subtracted by the logarithmic inflation rate π_{t+1}. The logarithmic inflation rate has been calculated in equation (6). The nominal logarithmic return has been calculated in equation (7).

$$r_{t+1}^{real} = r_{t+1}^{nominal} - \pi_{t+1} \tag{4}$$

[2]Mehra and Prescott (1985) had data available covering the period 1889–1978, originally obtained from Grossman and Shiller (1981).

Equation (5) displays the method to get real risk-free returns. $r_{f,t+1}^{real}$ represents the logarithmic real risk-free rate derived from the difference between the logarithmic nominal risk-free rate $r_{f,t+1}^{nominal}$ and the logarithmic inflation rate. The logarithmic nominal risk-free rate has been calculated in equation (9).

$$r_{f,t+1}^{real} = r_{f,t+1}^{nominal} - \pi_{t+1} \tag{5}$$

3.1 Consumer price index

The consumer price index (CPI) reflects the price of a basket of goods and services consumed by benchmark Dutch household (Centraal Bureau voor de Statistiek 2025b). Figure 1 shows the consumer price index for the period 2006 to 2024. The year 2015 has been used to normalize the price. From figure 1 it is clear that the price of these goods has been steadily increasing after 2015 and rapidly increasing when Corona broke out. The logarithmic inflation rate π_{t+1} is calculated from equation (6). The consumer price index at year t is represented as P_t and the consumer price index at year t+1 subsequently defined as P_{t+1}. Inflation is measured as the logarithmic year-on-year change in the consumer price index. Hence, inflation reflects the percentage change in prices from a representative basket of goods and services purchased by the average Dutch household (Centraal Bureau voor de Statistiek 2025c).

$$\pi_{t+1} = \log(\frac{P_{t+1}}{P_t}) \tag{6}$$

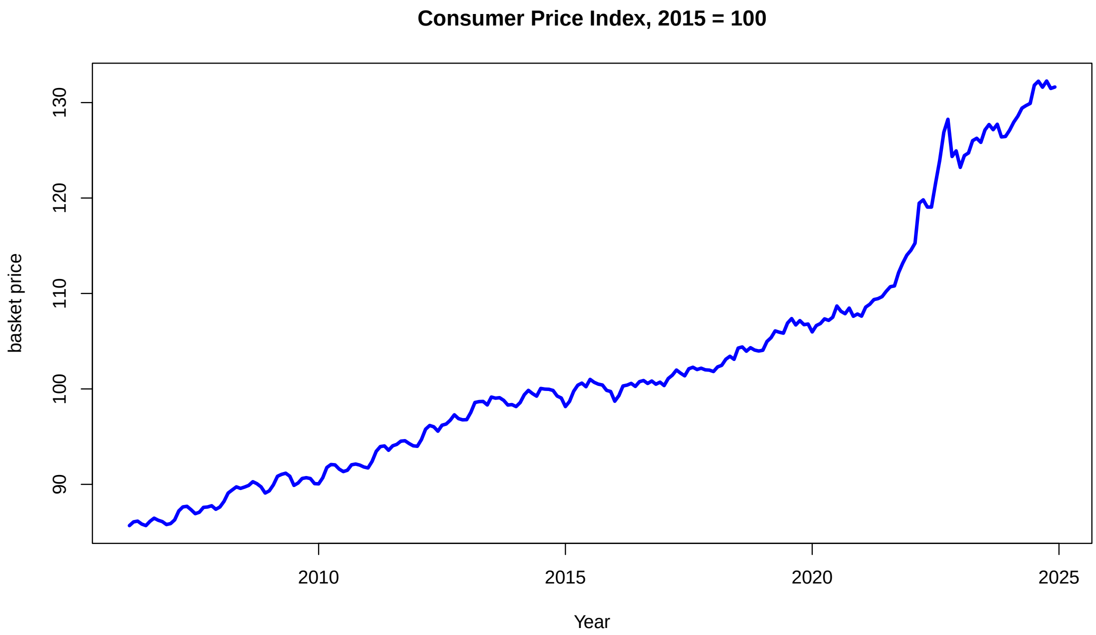

Figure 1: Monthly price development for a basket that reflects the Dutch consumer price index, 2006-2024.

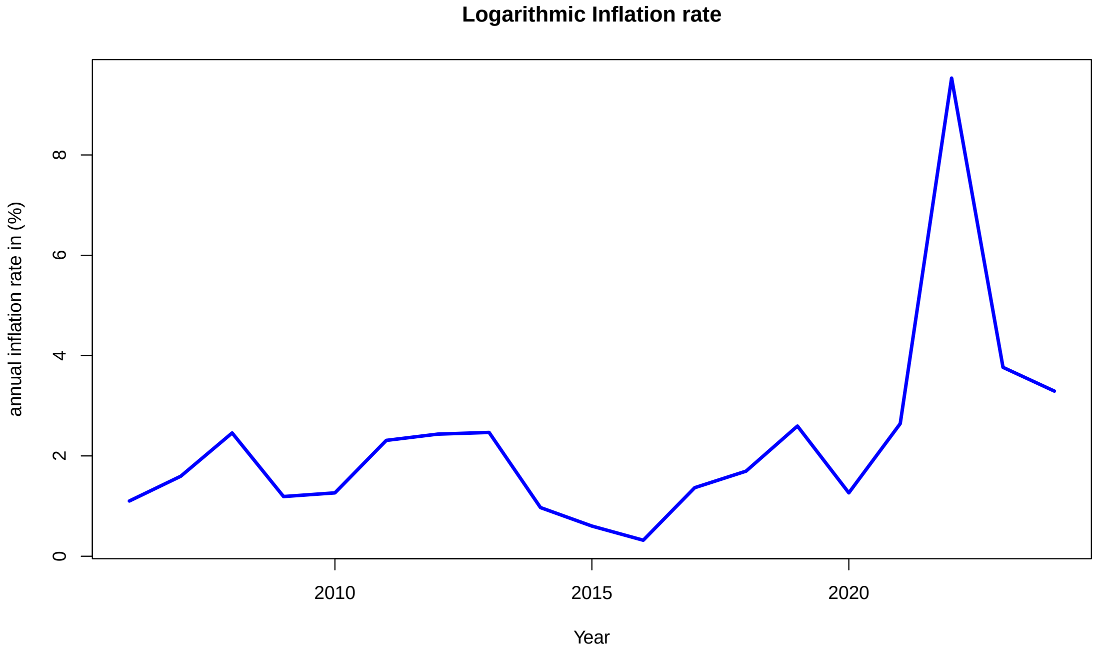

Figure 2: Annual logarithmic inflation in the Dutch economy, 2006-2024.

Figure 2 illustrates the annual logarithmic inflation rate derived from the CPI in the Dutch economy for the period 2006–2024. Figure 2 illustrates a remarkable increase in inflation around the period 2021-2022. Two recent economic events were responsible for this surge. The Statistics Netherlands points out that the COVID-19 recession as well as the invasion of Ukraine caused a major inflation spike (Centraal Bureau voor de Statistiek 2025c).

In addition to the figures, table 1 presents the summary statistics regarding the logarithmic inflation rate during the observed period. The average inflation rate in the Netherlands was 2.26% and had a standard deviation of 1.98%. In this time frame, the inflation rate in the Netherlands has been on average considered favorable by the ECB as the targeted inflation rate is around 2% per year.

Measure	**Time Period**	**Annual mean (%)**	**Std. Deviation (%)**
log inflation	2006 - 2024	2.26	1.98

Table 1: Summary statistics of the logarithmic inflation rate regarding the Dutch economy for the period 2006-2024.

3.2 The market index

A market index is constructed as a large composition of different stocks (Campbell, Lo, and MacKinlay 1997). Mehra and Prescott (1985) used the S&P 500 as a benchmark portfolio to examine the equity premium. Because the focus lies in the economy of the Netherlands, a different market index is used. The Amsterdam Exchange Index (AEX) is a market index that includes 25 of the most actively traded

Dutch stocks (Euronext 2025). The full representation of its components is shown in table 11 (Appendix). Notable companies include multi-nationals such as Shell, ASML and Unilever. The AEX is characterized by its relatively small and concentrated composition compared to larger indices such as the S&P 500.

Figure 3 shows the nominal prices for both the AEX and AEX GR for the period 2006 to 2024. This figure illustrates that there is a clear difference between the nominal price development between the AEX and the AEX GR. Because the constituent companies that are part of the AEX price index pay dividends to shareholders, the nominal price of the AEX index drops on the ex-dividend date (Kenton 2023). In contrast, the AEX GR assumes that the dividends are reinvested back into the price index. This is important because Mehra and Prescott (1985) analyzed real equity returns that included dividends. Thus, the AEX GR better represents the total return of this market index.

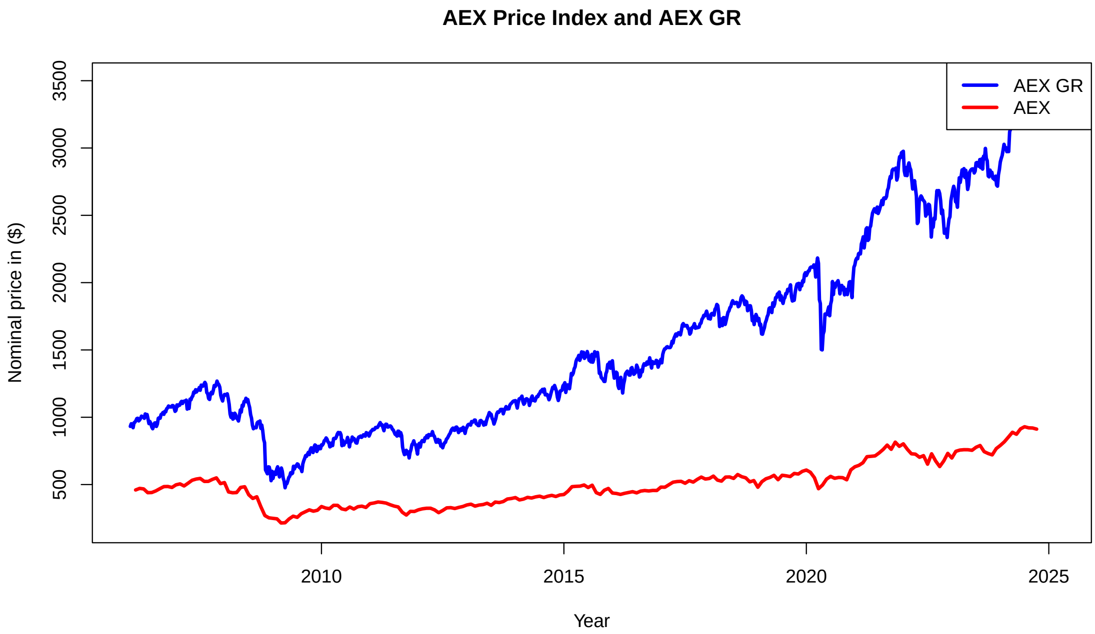

Figure 3: Monthly nominal prices in U.S. dollars ($) regarding the AEX and AEX GR, 2006-2024.

3.2.1 Returns on equity

In finance, Chen (2023) explains that investors are more interested in the return of equity, rather than the price of the equity itself. Modern portfolio theory by Markowitz (1952) strengthens this belief as investors should care solely about the returns and risk profile of their assets. The nominal logarithmic return of the AEX GR is provided in equation (7). S_t^n represents in this data the nominal closing price of the last available trading day in the year t and S_{t+1}^n is defined as the nominal closing price on the last available trading day in year t+1. To obtain the real return rate of the AEX GR, equation (4) is used.

$$r_{t+1}^{nominal} = \log(\frac{S_{t+1}^n}{S_t^n}) \tag{7}$$

Figure 4 displays the annual real and nominal logarithmic return of the AEX GR. Clearly, the effect of the GFC is clearly visible as a massive decrease in the year 2008. Moreover, the effect of the COV-19 pandemic is also visible in the graph, but it did not have such a detrimental effect on the return of the AEX GR as the financial crisis.

Figure 4: Real and nominal annual log-returns on the AEX GR, 2006-2024.

Finally, the summary statistics of the AEX GR is detailed in table 2. Clearly, the real logarithmic return of the AEX GR is considerably smaller than that of the S&P 500. This could be due to the observed time period containing two economic crises. This could explain why the real return of the AEX GR is rather small for a market index and the volatility is quite large.

Index Fund	**Time Period**	**Real Return (%)**	**Std. Deviation (%)**
AEX GR	2006 - 2024	4.63	23.07
S&P 500	1889 - 1978	6.98	16.54

Table 2: Summary statistics of the AEX GR using Dutch data and the U.S. Data used by Mehra and Prescott (1985) in their research.

3.2.2 The lognormal distribution

The lognormal model is often used in the literature as a useful distribution to model asset prices (Campbell, Lo, and MacKinlay 1997). This is because the lognormal assumption has a benefit over the normal model. In particular, the lognormal assumption implies that asset prices cannot be negative. If the gross returns $\frac{S_{t+1}^n}{S_t^n}$ are assumed to be normally distributed: $\frac{S_{t+1}^n}{S_t^n} \sim \mathcal{N}(\mu, \sigma^2)$, then these returns could be lower than -1. This means that the loss of the investor would also be more than a total loss. Thus, under the assumption that the log-returns are normally distributed: $\log(\frac{S_{t+1}^n}{S_t^n}) \sim \mathcal{N}(\mu, \sigma^2)$. This means that in the worst case prices can be zero, which means that the losses of the investor have a lower bound of -1.

Figure 5 illustrates for a simulated set of asset prices different outcomes depending on whether the distribution is being modeled as normal or lognormal. The gross returns simulated under the normal model will provide losses exceeding -1. In contrast, the losses of the lognormal model are bounded below by -1.

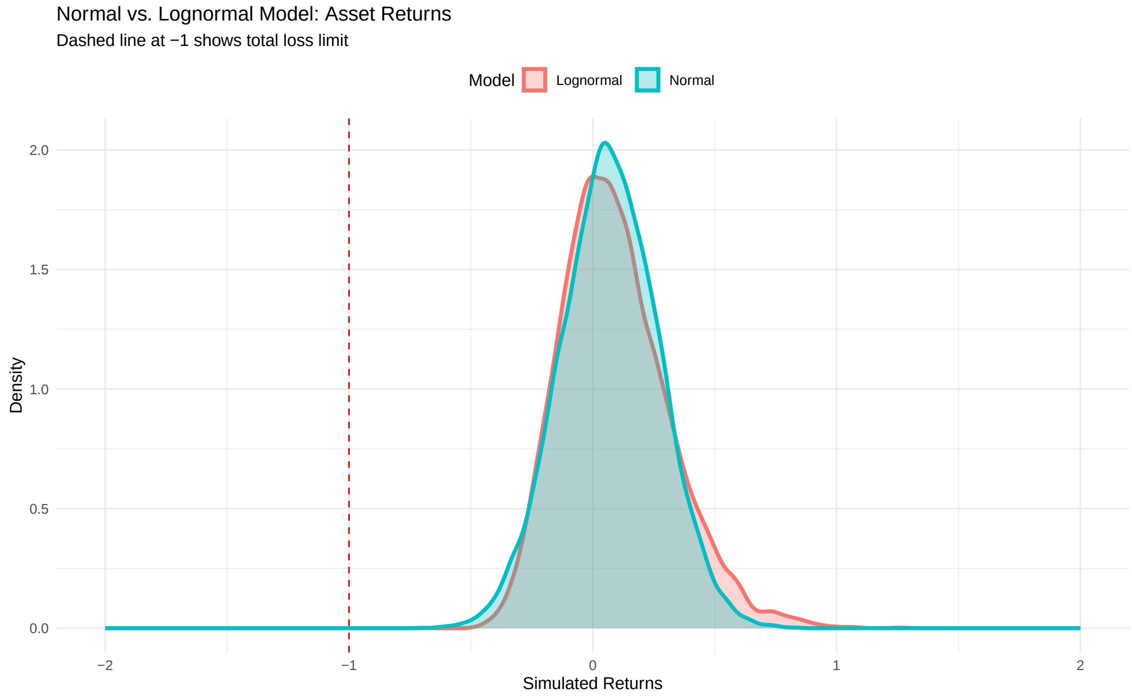

Figure 5: Simulated returns under the normal and lognormal model.

3.3 Consumption Growth

In their analysis of the EPP, Mehra and Prescott (1985) used aggregate per capita consumption data to test the consumption capital asset pricing model on the equity premium. Dzhumashev (2025) explains that this model assumes that the return of equity is related to the intertemporal marginal utility of consumption. The marginal utility of consumption can be valued differently depending on the state of the economy. If the investor prefers consumption smoothing, marginal utility will be high during a recession and low during an economic boom (Mehra and Prescott 1985). Additionally, if the investor is risk averse, it aims to minimize consumption drops as much as possible. This implies that there must be a substantial equity premium for risky assets that are positively correlated with consumption. However, the consumption-based asset pricing model with standard power utility, as used by Mehra and Prescott (2003), was unable to match the observed equity premium with reasonable levels of risk aversion.

In this thesis, consumption data was used to gain inference on the equity premium. This measure of consumption, as defined by Statistics Netherlands, includes the final consumption expenditure on goods and services for the average Dutch household (Statistics Netherlands 2024). Specifically, the total actual individual consumption by households and NPISHs, while adjusted for inflation (Statistics Netherlands (CBS) 2025).[3] The inclusion of NPISH aims to better mimic the total consumption used by Dutch households. Moreover, using data adjusted for inflation gives real consumption growth. Equation (8) reflects the logarithmic growth of consumption

[3]NPISH is an abbreviation for non-profit institutions serving households.

as the logarithmic ratio between the annual aggregate consumption c_{t+1} this year compared to that of the previous year c_t.

$$\Delta c_{t+1} = \log(\frac{c_{t+1}}{c_t}) \tag{8}$$

Figure 6 shows the annual growth rate of real consumption per capita in the Nether-

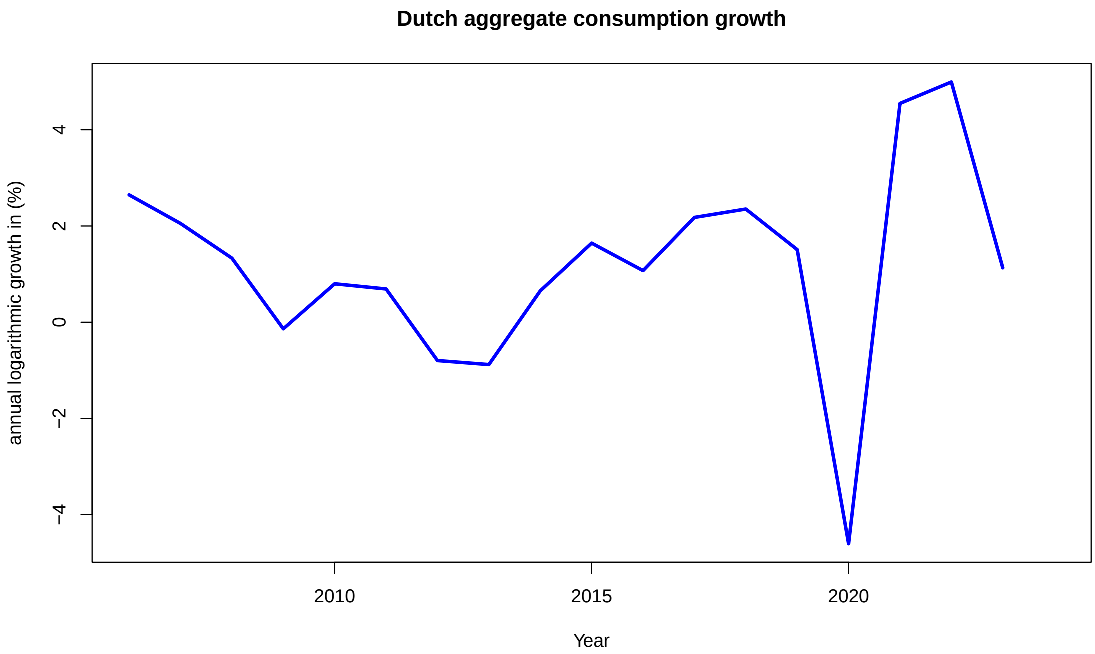

Figure 6: Annual logarithmic consumption growth in the Netherlands, 2006-2024.

lands. This figure shows that the Corona recession substantially impacted consumption growth in the Netherlands. Otherwise, consumption growth remained fairly stable in the Netherlands.

Table 3 displays the low volatility of consumption, as the standard deviation of consumption growth in the Netherlands is 2.18% for the entire observed period. Compared to the U.S. growth rate of consumption, the Netherlands has a smaller growth rate on average.[4]

3.4 The Risk-free asset

In this thesis, Dutch Treasury Certificates (DTC) are used as a proxy for risk-free securities. DTCs are government-backed bonds with a 1-year term of maturity, issued by the Dutch State Treasury Agency. The Dutch State Treasury Agency (2025) describes these certificates as risk-free because they carry virtually zero credit risk.

[4]The summary statistics regarding consumption growth by Mehra and Prescott (1985) was not logarithmic.

Consump. growth	Time Period	Real Return (%)	Std. Deviation (%)
the Netherlands	2006 - 2024	1.18	2.18
U.S.	1889 - 1978	1.83	3.57

Table 3: Summary statistics of the growth rate of consumption regarding recently used Dutch data and the U.S. Data used by Mehra and Prescott (1985) in their research..

This is because the government distributes them, ensuring repayment. These assets are generally without exposure to credit default risk or market fluctuations typical of other securities. Corporate finance theory points out that investors expect risk-free assets to compensate only for the fundamental risk associated with lending capital to the government (Berk and DeMarzo 2009). The bond yields related to this government treasury bill have been obtained from MarketWatch (2025).

Equation (9) shows how the nominal risk-free rate $r_{f,t+1}^{nominal}$ was approximated by the logarithmic returns. The data in this thesis consists of the 1-year forward bond yields denoted as $y_{m,t}$ for each month m in year t. The nominal return was obtained as the average of the logarithmic returns of that year. This approximation is used to obtain an annualized nominal risk-free return, rather than holding the bond for a specific month for a year (Campbell, Lo, and MacKinlay 1997).

$$r_{f,t+1}^{\text{nominal}} = \frac{1}{12} \sum_{m=1}^{12} \log(1 + y_{m,t}) \tag{9}$$

Figure 7 highlights the returns of the nominal and real risk-free asset. The difference between real returns and nominal returns is particularly noticeable during the corona period. This is due to the rapid increase in inflation. Furthermore, the year 2015 highlights the moment when the ECB used monetary policies that significantly lowered the yields of government bonds (Corsetti, Dedola, and Leduc 2019). This caused bond yields to drop below the zero lower bound, lowering the returns of the risk-free asset.

Table 4 displays the real returns of the risk-free asset. The real return of the risk-free asset (-1.42%) during the observed period has been notably lower than the real return of the risk-free asset approximated by Mehra and Prescott (1985) (0.8%). Furthermore, the real returns of the U.S. risk-free asset (5.67%) have been more volatile than the Dutch risk-free asset (2.43%).

Asset	Time Period	Real Return (%)	Std. Deviation (%)
Dutch risk-free	2006 - 2024	-1.42	2.43
U.S. risk-free	1889 - 1978	0.80	5.67

Table 4: Summary statistics of the risk-free assets regarding recently used Dutch Data and the U.S. Data used by Mehra and Prescott (1985) in their research.

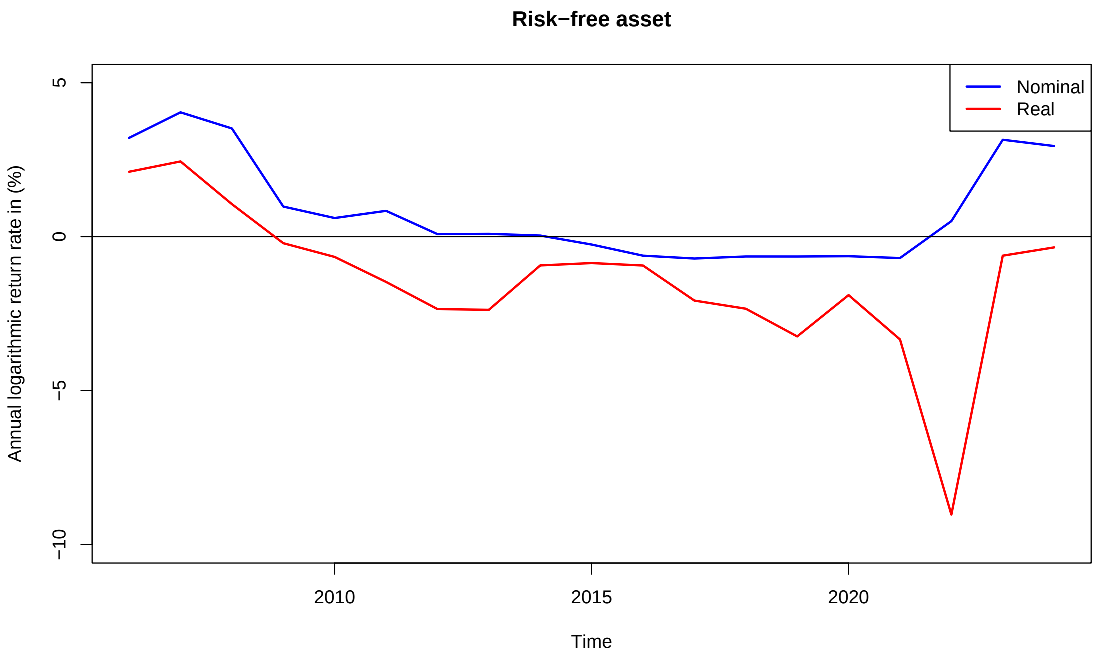

Figure 7: Annual real and nominal return on a risk-free asset, 2006-2024.

3.5 Bank related parameters

An essential parameter in the model of Dzhumashev (2025) is the minimum equity requirement (κ), which is a regulatory standard that limits how loans can be financed with the equity of banks. The minimum equity requirement is frequently reported in financial stability reports from central and corporate banks. This regulatory measure has been progressively strengthened in the Basel III framework, especially after the 2008 financial crisis (European Central Bank 2024). The Common Equity Tier 1 (CET1) ratio is a representation of the minimum equity requirement. The CET1 ratio compares the banks' core equity to its risk-weighted assets. According to De Nederlandsche Bank (2023), their Financial Stability Report estimates that the Dutch banking sector currently maintains a CET1 ratio of 16.3%.

The required reserve ratio (ρ) for European banks is currently set at 1% as mandated by the European Central Bank. The European Central Bank (2023) explains that this parameter is a fraction of bank liabilities, including but not limited to customer deposits and short-term debt securities with maturities of up to two years. The required reserve ratio plays a role in the lending operations of corporate banks in the model of Dzhumashev (2025). Specifically, the minimum reserve ratio influences the balance sheet of the bank in regards to how many household deposits have to be kept as reserves.

The Return on Equity (ROE) of Dutch banks has been described in a technical report by (De Nederlandsche Bank 2016). This measure reflects the profitability of the bank and has changed over time due to stricter regulation. Before the great financial crisis, most western banks often had ROE exceeding 15%. However, after policy adjustments in the BASEL III accord the banks' capital buffer had to be more resilient to economic shocks. This lowered the profitability of banks. The technical report of De Nederlandsche Bank (2016) discusses that in a stable economic scenario, Dutch banks have a ROE of 7% ± 2% depending on the economic situation.

3.6 Interest rate

To accurately model the method of Dzhumashev (2025), the corporate loan rate and the household deposit rate were used. The household deposit rate reflects the interest i_t^s in a deposit account held by a corporate bank, while firms borrow money at the interest i_t from the bank to purchase some intermediate goods. Data for these interest rates are obtained from the DNB (De Nederlandsche Bank 2025). Figure 8 shows the interest rate (redeemable at notice) for Dutch households between the period 2006 to 2024. This reflects the interest rate for deposits that can be withdrawn at short notice. From figure 8 it is clear that the household deposit rate has been steadily decreasing after monetary policy changes from the ECB. Moreover, during the pandemic, the household deposit rate briefly crossed the zero lower bound. After this period, the household deposit rate has been steadily recovering.

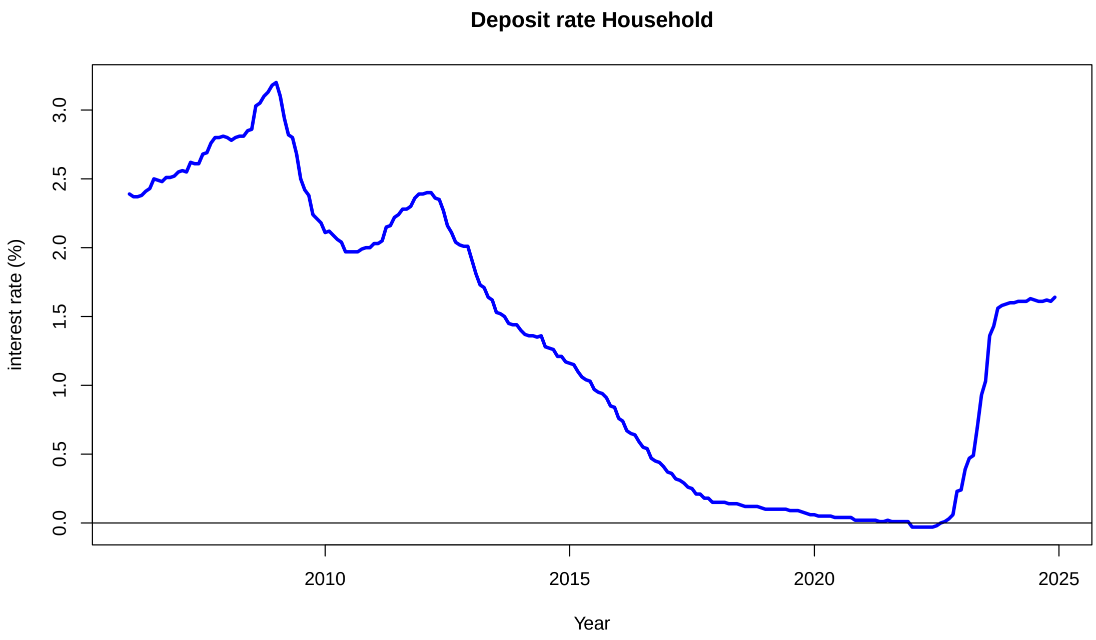

Figure 8: Interest rate for Dutch households where their savings are redeemable at notice, 2006-2024.

Figure 9 shows the interest rate offered to non-financial corporate firms for outstanding bank loans. The trend observed in Figure 8 is similar here, as the interest rates of corporate loans were steadily decreasing. During the height of the COVID-19 pandemic, corporate loan rates reached their lowest point as banks sought to provide liquidity to businesses impacted by the lockdown. During this period, firms faced reduced consumer demand and disrupted supply chains. Similarly to the household deposit rate, the corporate loan rate has been steadily improving after the pandemic.

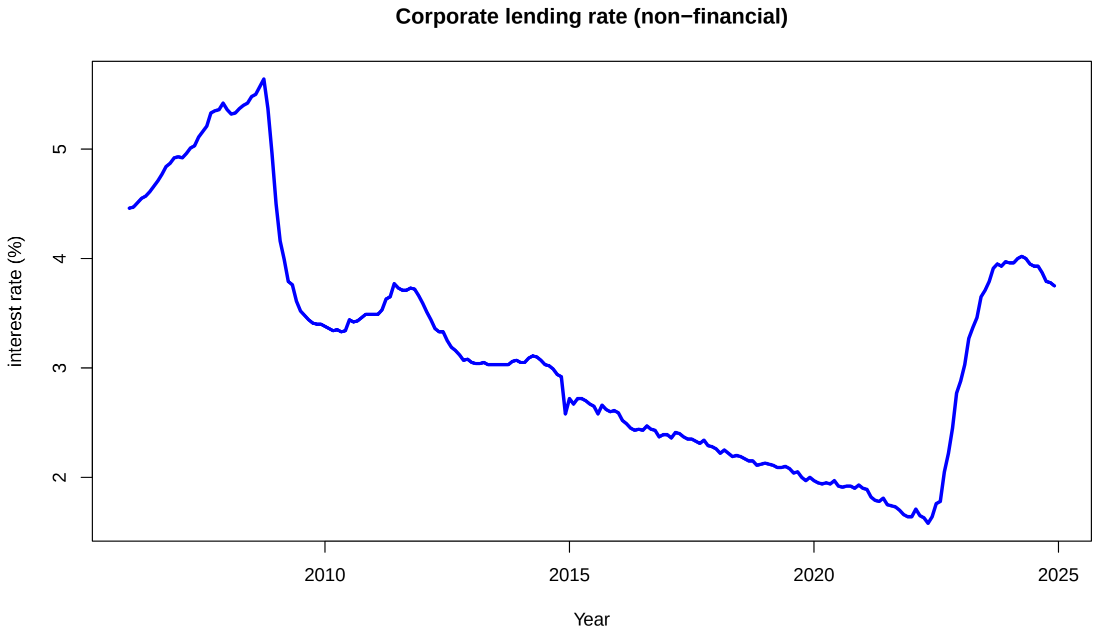

Figure 9: Interest rate for loans to Dutch non-corporate firms, 2006-2024.

Similar trends between household savings rates (Figure 8) and corporate loan rates (Figure 9) highlight the broad impact of both monetary policy and crises in different interest rates of the Dutch economy. Table 5 displays the average interest rates for both household deposits and outstanding corporate loans. On average, the household deposit rate (1.30%) has been considerably lower than the corporate loan rate (3.16%). This is because the bank earns interest on corporate loans and pays interest to households. As seen from the standard deviation, both rates were generally stable throughout the observed period.

Interest rate	**Time Period**	**Mean (%)**	**Std. Deviation (%)**
deposit rate	2006 - 2024	1.30	1.04
corp. loan rate	2006 - 2024	3.16	1.08

Table 5: Summary statistics of household deposit rate and corporate loan rate for the period 2006-2024.

4 Research Question 1

So far the literature has not been able to explain the observed equity premium for reasonable levels of risk aversion.[5] Weil (1989), Mehra and Prescott (2003) have fitted data to examine the equity premium puzzle with the conclusion that the investor must be abnormally risk averse to obtain the observed equity premium. To better understand the equity premium puzzle, the first research question will analyze the methods used by Mehra and Prescott (1985) and Campbell, Lo, and MacKinlay (1997) to better understand the equity premium puzzle at hand. The models obtained will be tested on recent observed Dutch market data.

First, basic financial notions and theory that are essential for the EPP will be discussed and described. Second, a connection will be made between the First Fundamental Theorem of Asset Pricing (FFTAP) and the equity premium puzzle. Third, the log-normal approximation of Campbell, Lo, and MacKinlay (1997) will be used to obtain a formula to calculate the equity premium. Finally, recent Dutch data will be applied to investigate the relations between the variables that cause the equity premium puzzle.

4.1 Notions & theory of Finance

The definition of arbitrage can be found in Harrison and Kreps (1979). Suppose a trader has access to both the U.S. and Japanese stock market. Assuming that both markets are extremely liquid and that transaction costs are negligible, an arbitrage opportunity arises if there is a price difference between identical assets traded in each market. In such a case, the trader can construct a portfolio that requires no upfront investment, has no risk, and yields a guaranteed positive return. Then in finance, this concept is known as an arbitrage opportunity. In essence, it is a risk-less profit with zero net investment.

Many asset pricing models assume that there are no arbitrage opportunities. The First Fundamental Theorem of Asset Pricing establishes an equivalence between market conditions and asset pricing. As explained in Campbell, Lo, and MacKinlay (1997), the absence of arbitrage opportunities in a market is equivalent to the existence of a positive stochastic discount factor (SDF) that prices all assets. Theorem 1 is a stylized excerpt from Campbell, Lo, and MacKinlay (1997).

Theorem 1 (First Fundamental Theorem of Asset pricing)**.** The following is equivalent:
1. The market is free from arbitrage
2. There is a positive stochastic discount factor which prices all assets.

Equation (10) represents the notion of the First Fundamental Theorem of Asset Pricing, where the current value of the stock price S_t is the current expectation of the stochastic discount factor m_{t+1} and the future payoff X_{t+1}. The stochastic

[5] Normal risk aversion is according to the literature $1 \leq \gamma \leq 5$ (Mehra and Prescott 1985).

discount factor m_{t+1} adjusts for risk and time preferences, linking the current price of an asset to its expected future return.

$$S_t = \mathbb{E}_t[m_{t+1}X_{t+1}] \tag{10}$$

From equation (10) it is shown that when dividing both sides by S_t and setting the gross return as $R_{t+1} = \frac{X_{t+1}}{S_t}$, the First Fundamental Theorem of Asset Pricing can be given as:

$$\mathbb{E}_t[m_{t+1}R_{t+1}] = 1 \tag{11}$$

This formulation highlights that the SDF acts as a weighting factor, ensuring that the expected discounted returns across all assets in an arbitrage-free market are one. The presence of the SDF is central to ensuring market consistency and pricing equilibrium, serving as a foundation for a variety of asset pricing models. The main takeaway is that the SDF captures the risk-adjusted value of future cash flows, adjusting them for uncertainty and preference for consumption across time (Campbell, Lo, and MacKinlay 1997).

The agent chooses an investment strategy that aims to maximize profit while maintaining low risk. Sharpe (1964) strengthens this belief, as any rational investor would choose a portfolio with the highest risk-return trade-off that the investor accepts. Hereby, under the inclusion of a consumption-based model, the stochastic discount factor can be mathematically stylized. Suppose that there is an intertemporal framework, where an investor gets utility only from consumption c_t. The investor gets a certain exogenous level of income y_t. The investor values the future less than the current period by discounting the future with a subjective discount factor $0 < \beta < 1$. If the investor considers consumption smoothing by consuming some amount in both periods, the investor may want to optimize consumption for now and in the future. Mathematically this framework can be defined in a two-period setting as the set of equations (12):

$$\begin{aligned} \max_{h_t} \quad & \mathbb{E}_t[u(c_t) + \beta u(c_{t+1})] \\ \text{s.t.} \quad & c_t = y_t - s_t^T h_t, \\ & c_{t+1} = y_{t+1} + x_{t+1}^T h_t \end{aligned} \tag{12}$$

The vector $h_t \in \mathbb{R}^N$ describes the holdings of the agent for N different stocks. Likewise, the vectors $s_t \in \mathbb{R}^N$ and $x_{t+1} \in \mathbb{R}^N$ represent the purchased stock price and stock payoff respectively. Here, the agent has some current disposable income y_t and purchases a portfolio of assets with price vector s_t and holdings h_t. Therefore, his current consumption c_t is his current capital minus the investment made in period t. In period t+1, the agent enjoys the profits obtained from the payoff $x_{t+1}^T h_t$. This can be used for additional consumption c_{t+1}.

When the constraints are substituted in the objective function and first-order conditions are calculated, then equation (13) is obtained.

$$s_t = \mathbb{E}_t[\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1}] \tag{13}$$

Equation (13) shows that the First Fundamental Theorem of Asset Pricing is derivable from the consumption-conscious investor in a two-period setting when optimizing utility from consumption. Equation (13) explains that the current value of the stock s_t can be broken down as the current expectation of the investor that values the future payout x_{t+1}, based on the difference between the marginal utility of consumption in period t and t+1 $\frac{u'(c_{t+1})}{u(c_t)}$ and the subjective discount factor of the investor β. Additionally, it shows that equation (13) is a specific form of the FTAP if the stochastic discount factor is stylized as shown in equation (14).

$$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)} \tag{14}$$

4.2 The lognormal approximation for the equity premium

To derive the equity premium puzzle identified by Mehra and Prescott (1985), a log-normal approximation is implemented to establish the relationship between the equity premium and the underlying risk aversion parameter. By starting with the First Fundamental Theorem of Asset Pricing and applying the consumption-based model to derive a stochastic discount factor (SDF), we connect asset returns with consumption growth. Under the assumption that the agent has constant relative risk aversion preferences, this framework forms the basis for analyzing the equity premium. Campbell, Lo, and MacKinlay (1997) demonstrated that under the assumption of log-normality for returns, the equity premium can be expressed as a function of the risk aversion coefficient, the variance of returns, and the covariance between asset returns and consumption growth.

From the First Fundamental Theorem of Asset Pricing the relation between the stochastic discount factor and the asset price was provided. Let the First Fundamental Theorem of Asset Pricing be restated as:

$$1 = \mathbb{E}_t[R_{t+1} m_{t+1}]. \tag{15}$$

The stochastic discount factor is defined in equation (16) and c_t is the aggregate consumption:

$$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)} \tag{16}$$

If we assume that the representative agent who is trying to maximize a time-separable utility function has CRRA preferences so that his/her utility is described by:

$$u(c_t) = \begin{cases} \frac{c_t^{1-\gamma}}{1-\gamma}, & \text{if } 0 < \gamma < \infty \\ \ln(c_t), & \text{if } \gamma = 1. \end{cases} \tag{17}$$

Then substitution of the CRRA function in the stochastic discount factor becomes possible. A visual representation of the CRRA function is provided in Figure 10. The curvature of the function illustrates how much more or less risk-averse an individual is. Moreover, when $\gamma = 1$, the CRRA function transforms to the common logarithmic function.

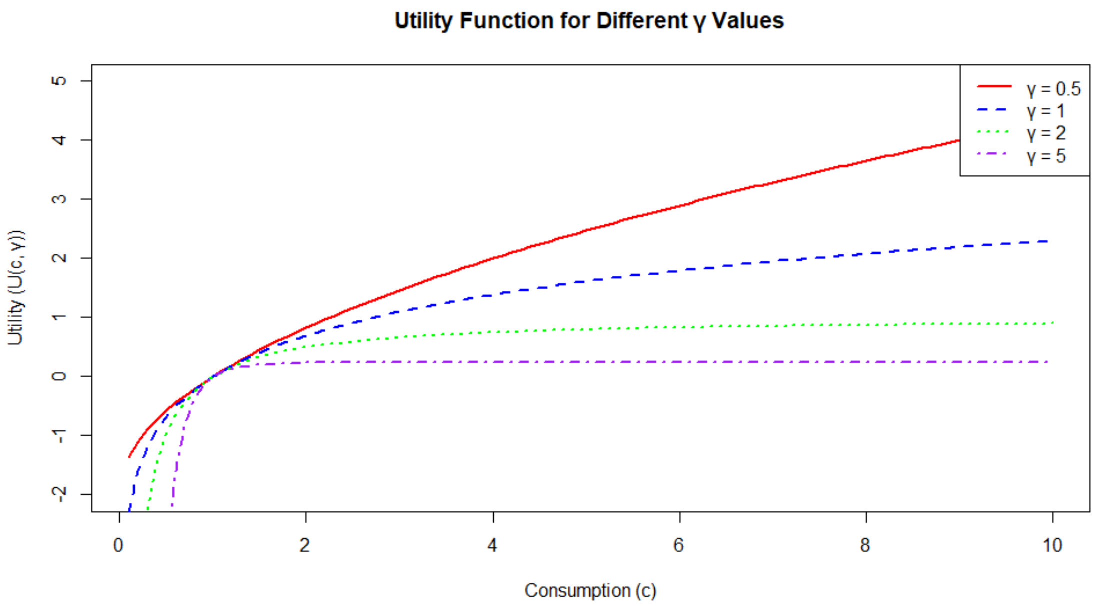

Figure 10: Constant Relative Risk Aversion model for different values regarding risk aversion γ.

When the derivative of equation (17) is taken, the marginal utility becomes $u'(c_t) = c_t^{-\gamma}$. Which will be substituted into the stochastic discount factor to obtain:

$$1 = \mathbb{E}_t[\beta(\frac{c_{t+1}}{c_t})^{-\gamma} R_{t+1}] \tag{18}$$

Campbell, Lo, and MacKinlay (1997) point out that if a random variable X is conditionally lognormally distributed $\log(X) \sim N(\mu, \sigma^2)$, then the conditional expectation is given as in equation (19).

$$\log \mathbb{E}_t[X] = \mathbb{E}_t[\log X] + \frac{1}{2} Var_t[X] \tag{19}$$

The essence of equation (19) is to serve as an approximation to connect asset pricing theory to empirical data. The log-normal approximation is used to gain inference on the equity premium puzzle as researched by Mehra and Prescott (1985). Hence, if the identity of a random variable X is set to the First Fundamental Theorem of Asset Pricing for agents with CRRA preferences, then equation (20) displays this relation.

$$X = \mathbb{E}_t[R_{t+1}\beta(\frac{c_{t+1}}{c_t})^{-\gamma}] \tag{20}$$

Assuming joint conditional lognormality and homoskedasticity for the asset returns and consumption growth. The logs of equation (20) can be taken, which results in equation (21).

$$\log(X) = \log(\beta) - \gamma \log(\frac{c_{t+1}}{c_t}) + \log(R_{t+1}). \tag{21}$$

Setting the log consumption growth as $\log(\frac{c_{t+1}}{c_t}) = \Delta c_{t+1}$ and writing the net return as $r_{t+1} = \log(R_{t+1})$. Then from these new notations, equation (21) can be rewritten as:

$$\log(X) = \log(\beta) - \gamma \Delta c_{t+1} + r_{t+1} \tag{22}$$

When the expectation of equation (22) is taken, then this results in equation (23).

$$\mathbb{E}_t[\log(X)] = \log(\beta) - \gamma \mathbb{E}_t[\Delta c_{t+1}] + \mathbb{E}_t[r_{t+1}] \tag{23}$$

Equation (24) describes the variance of the conditionally lognormally random variable X under the identity of equation (20). Under the assumption of homoskedasticity for consumption growth and asset returns, the variance of the risky asset return is σ_r^2, the variance of consumption growth $\gamma^2\sigma_c^2$ and the covariance between the consumption growth and the risky asset return which is $-2\gamma\sigma_{rc}$ yields:

$$Var_t[\log X] = \sigma_r^2 - 2\gamma\sigma_{rc} + \gamma^2\sigma_c^2. \tag{24}$$

Additionally, from equation (18) and equation (20) if follows that equation (25) holds.

$$\log(\mathbb{E}_t[X]) = 0 \tag{25}$$

Thus, from the set of equations (23), (24) and (25) a relation has been established between the net returns of risky assets as a function of the risk aversion, subjective discount factor, growth of consumption and the variances and covariances of the consumption and risky asset returns. This is displayed in equation (26).

$$0 = \log(\beta) - \gamma \mathbb{E}_t[\Delta c_{t+1}] + \mathbb{E}_t[r_{t+1}] + \frac{1}{2}(\sigma_r^2 - 2\gamma\sigma_{rc} + \gamma^2\sigma_c^2) \tag{26}$$

If the risk-free asset is substituted in equation (26), a relation between consumption growth, subjective discount factor, risk aversion and risk-free asset returns can be obtained. Since, the future payoff of a risk-free asset is known $\mathbb{E}_t[r_{t+1}^f] = r_{t+1}^f$. Since the return for a risk-free asset assumes certainty, there is no risk and therefore no variance (Campbell, Lo, and MacKinlay 1997). In this case, the following holds: $\sigma_r^2 = 0, \sigma_{rc} = 0$. With these implementations, equation (27) displays this relation.

$$0 = \log(\beta) - \gamma \mathbb{E}_t[\Delta c_{t+1}] + r_{f,t+1} + \frac{1}{2}\gamma^2\sigma_c^2 \tag{27}$$

The risk-free rate is linear in expected consumption growth with its slope coefficient being equal to the coefficient of relative risk aversion.

Hence, the equity premium as shown in equation (28) can be found from equation (27) and equation (26):

$$\mathbb{E}_t[r_{t+1}] - r_{f,t+1} = \gamma\sigma_{rc} - \frac{1}{2}\sigma_r^2 \tag{28}$$

4.3 Equity premium puzzle in the Netherlands

Table 6 shows the sample statistics found in the Dutch economy during the observed period of 2006 to 2024 and the sample statistics of Mehra and Prescott (1985) obtained from Grossman and Shiller (1981).[6] These sample statistics are essential to calibrate the equity premium puzzle obtained from the log-normal approximation from Campbell, Lo, and MacKinlay (1997).

Statistic	**Dutch data**	**U.S. data**
$r_{f,t+1}$	-1.42%	0.80%
$\mathbb{E}_t[r_{t+1}]$	4.63%	6.98%
σ_r	23.07%	16.54%
$\mathbb{E}_t[\Delta c_{t+1}]$	1.18%	1.72%
σ_c	2.18%	3.52%
σ_{rc}	0.29%	n/a
$\mathbb{E}_t[r_{t+1}] - r_{f,t+1}$	6.05%	6.18%

Table 6: Sample statistics for the Dutch Data for the period, 2006-2024 and U.S. data from Grossman and Shiller (1981) and Mehra and Prescott (2003).

The first sample statistic r^f_{t+1} displays the average real logarithmic risk-free return rate. For the Netherlands, investing in this approximation of these risk-free assets would on average yield a net return r^f_{t+1} of -1.42%. The low return can be explained for a few reasons. First, the bond yields on Dutch 1-YR government bonds for a long period below the zero lower bound. Moreover, high inflation during the observed period pushed logarithmic real returns far below the zero lower bound. Hence, DTCs were not a favorable investment during this period. The return of the risk-free asset of Mehra and Prescott (1985) was clearly higher (0.80%).

The second sample statistic $\mathbb{E}_t[r_{t+1}]$ denotes the mean logarithmic real return of the market index 4.63%. Figure 4 illustrates the effect of the 2008 financial crisis. This economic shock drastically reduced the returns of assets. Moreover, the high inflation during the corona recession could also have accounted for less than favorable real returns of the AEX GR. In contrast, there could be a couple of reasons why the real return of the S&P 500 was so large (6.98%). First, the post-period of the second World War strengthened the U.S. economy through the Marshall plan, which provided American made goods a new European market. Second, U.S. firms had a very favorable moment during this period, due to the global market expansion, which enhanced the performance of U.S. stocks. The third sample statistic represents the standard deviation (σ_r) or volatility which for the S&P 500 (16.54%) has been smaller than that of the AEX GR (23.07%). Ultimately, the AEX GR was observed in a short period with two economic crises.

The fourth and fifth sample statistics denote the mean growth rate of consumption $\mathbb{E}_t[\Delta c_{t+1}]$ and the standard deviation of the growth rate of consumption σ_c

[6] logarithmic consumption growth from Mehra and Prescott (2003).

respectively. Similarly, to the real return of the risk-free asset and the real return of the market index, U.S. consumption out-grew Dutch consumption by quite a large margin. A macroeconomic analysis performed by Maddison (2007), could provide an explanation. The period in which the U.S. economy was observed (1889-1978) was during strong economic development through rapid industrialization. In contrast, the Netherlands has seen lower consumption growth because the economy has matured and therefore been more stable.

The sixth sample statistic displays the covariance σ_{rc} between logarithmic consumption growth and logarithmic asset returns. For the data used in this thesis, the covariance is 0.29%. The literature supports similar findings, Campbell (2003) explains that empirical data often reflects that the covariance between stocks and consumption is typically positive but modest. Frequently, the covariance between equity returns and consumption growth is too small to explain the equity premium puzzle.

Lastly, the final sample statistic denotes the equity premium. This is ever so slightly smaller in the Netherlands.

4.3.1 Estimating the equity premium

To gain inference on the equity premium puzzle, equation (27) will be rewritten into:

$$r_{f,t+1} = -\log(\beta) + \gamma \mathbb{E}_t[\Delta c_{t+1}] - \frac{1}{2}\gamma^2\sigma_c^2 \tag{29}$$

to have the net return of the risk free rate as a function of the subjective discount factor β, the risk-aversion parameter γ, the mean of growth rate $\mathbb{E}_t[\Delta c_{t+1}]$ and the variance of consumption σ_c^2.

Following the calibration method used by Mehra and Prescott (2003), the risk-aversion parameter is fixed to 10 ($\gamma = 10$) and the subjective discount factor is fixed to 0.99 ($\beta = 0.99$). Then the risk-free rate and the return of the market index can be calculated.

Using the sample statistics from the data analysis in equation (29), gives:

$$r_{f,t+1} = 0.1043$$

From the calibration, the logarithmic real rate of return of the risk-free asset would be 10.43%. As shown in Figure 7, this is unexplainably higher than what has been observed in recent years.
Similarly, the equity return can be calculated with equation (30) using the sample statistics from table 6.

$$\mathbb{E}_t[r_{t+1}] = r_{f,t+1} + \gamma\sigma_{rc} - \frac{1}{2}\sigma_r^2 \tag{30}$$

$$\mathbb{E}_t[r_{t+1}] = 0.1067$$

This would mean that the logarithmic real return of a risky asset would be 10.67%. Compared to the logarithmic real return of 4.63% of the AEX GR this is still a massive overestimation. Likewise, the risk-free rate calibrated under these fixed parameters is massively overvalued. The equity premium, as calibrated from this example is 0.24%. The calibration calculated an equity premium that is too small compared to the observations from the data.

Table 7 provides supported values for the risk aversion parameter from the literature. Under these calibrations of the risk aversion parameter, the risk-free rate becomes extremely overvalued for each iteration of γ. For $\gamma = 1$ both the interest rate and the equity premium are negative. For any value of $\gamma \geq 2$, the return of the market index becomes positive and the equity premium increases.

Statistic for $\beta = 0.99$	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 4$	$\gamma = 5$
$r_{f,t+1}$	2.16%	3.27%	4.33%	5.34%	6.31%
$\mathbb{E}_t[r_{t+1}]$	-0.21%	1.19%	2.54%	3.85%	5.10%
$\mathbb{E}_t[r_{t+1}] - r_{f,t+1}$	-2.37%	-2.08%	-1.79%	-1.49%	-1.21 %

Table 7: Sample statistics for the Dutch Data for the period, 2006-2024 and U.S. data from Grossman and Shiller (1981)

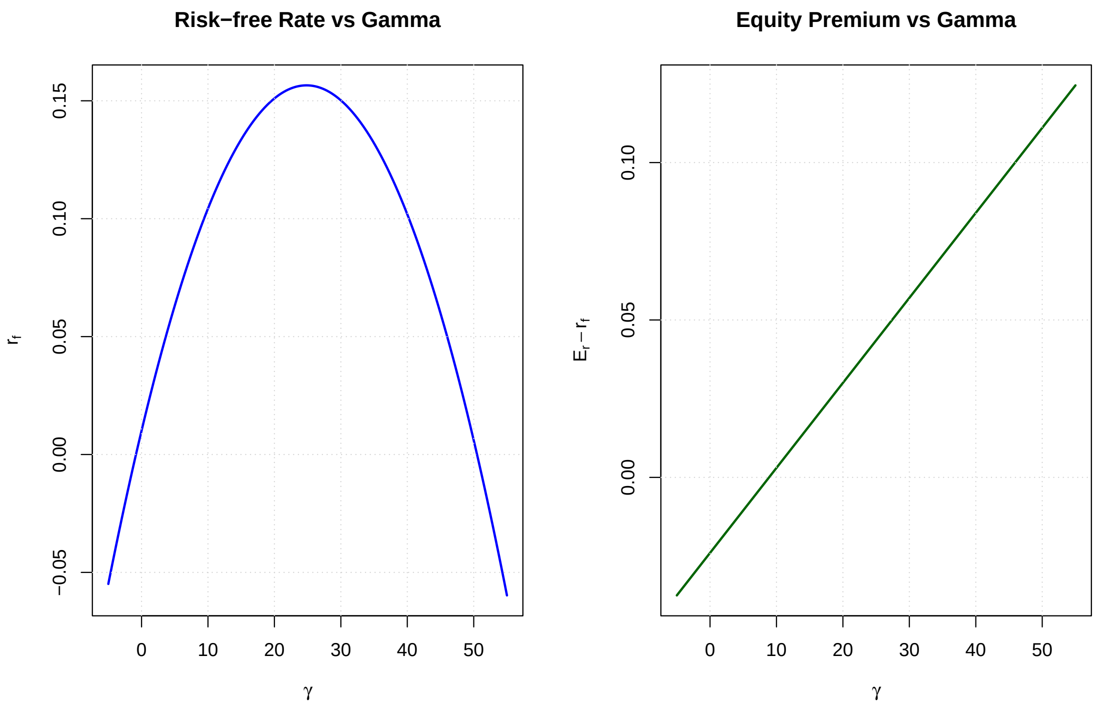

Figure 11: The risk-free rate and equity premium as a function of γ using Dutch sample statistics with $\beta = 0.99$

Figure 11 aims to show the effect of increasing the risk aversion parameter on the return of the risk-free rate and the equity premium. Suppose γ is fitted to get the observed equity premium of $\pm 6\%$, then that would result in a fitted $\gamma \approx 30$. Using this γ to obtain the risk-free rate would give an unrealistically high risk-free return rate of around $\pm 15\%$. Similarly, taking γ to fit the observed risk-free rate -1.45% would result in a $\gamma \approx -2$ or 52. If this fitted γ was used to determine the equity premium, the equity premium would be around $\pm - 3.5\%$ or $\pm 13\%$. If $\gamma = -2$, then this suggests that investors are either risk-seeking which produces an unrealistically negative equity premium or investors are extremely risk-averse which causes an unrealistically high equity premium.

4.3.2 Estimating the risk aversion parameter

A different approach is to calibrate the level of risk aversion and the subjective discount factor that is fitted to the observed data, as provided from table 6. Rewriting equation (29) provides the gamma to be a function of the risk premium, the variance of the risky asset and the covariance between the risky asset and consumption.

$$\gamma = \frac{\mathbb{E}_t[r_{t+1}] - r_{f,t+1} + \frac{1}{2}\sigma_r^2}{\sigma_{rc}} \iff \gamma = 30.03$$

The calculated value of $\gamma = 30.03$ suggests a unrealistic level of risk aversion, which is a far outlier to the empirically supported range of 1 to 5. Mehra and Prescott (2003) found a comparable observation since their calibrated risk aversion parameter was around 47.6.

Similarly, for β we use equation (29):

$$\log(\beta) = \gamma \mathbb{E}_t[\Delta c_{t+1}] - r_{f,t+1} - \frac{1}{2}\gamma^2\sigma_c^2 \iff \beta = 0.35$$

The result $\beta = 0.35$ implies that investors severely undervalue future consumption relative to the present. Because β is close to 0, this suggests an extremely strong preference for consumption in the present. Evidently, this suggests that agents are discounting future consumption extremely heavily. Similarly, Mehra and Prescott (2003) also calibrated a lower than expected subjective discount factor of $\beta = 0.55$. Considering that these values are vastly different than what the literature points out as ideal values for β and γ, a sensitivity analysis could be suitable.

We start with the given equation for β:

$$\beta = \exp\left(-r_{f,t+1} + \gamma \cdot \mathbb{E}_t[\Delta c_{t+1}] - 0.5 \cdot \gamma^2 \cdot \sigma_c^2\right)$$

Taking the natural logarithm on both sides:

$$\log(\beta) = -r_{f,t+1} + \gamma \cdot \mathbb{E}_t[\Delta c_{t+1}] - 0.5 \cdot \gamma^2 \cdot \sigma_c^2$$

Rearrange into standard quadratic form:

$$0.5 \cdot \sigma_c^2 \cdot \gamma^2 - \mathbb{E}_t[\Delta c_{t+1}] \cdot \gamma + (\log(\beta) + r_{f,t+1}) = 0$$

This is a quadratic equation of the form:

$$A\gamma^2 + B\gamma + C = 0$$

where:

$$A = 0.5 \cdot \sigma_c^2$$

$$B = -\mathbb{E}_t[\Delta c_{t+1}]$$
$$C = \log(\beta) + r_{f,t+1}$$

Using the quadratic formula:

$$\gamma = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Substituting our values:

$$\gamma = \frac{\mathbb{E}_t[\Delta c_{t+1}] \pm \sqrt{\mathbb{E}_t[\Delta c_{t+1}]^2 - 2 \cdot \sigma_c^2 \cdot (\log(\beta) + r_{f,t+1})}}{\sigma_c^2}$$

This equation provides two possible solutions for γ. If $\beta = 0.99$ as is standard in the literature, then the calibrated $\gamma =$ (-1.98 or 51.64). This means that if β is fixed for an ideal value, investors are either extremely risk-averse or somewhat risk-seeking.

Assuming a fixed level of relative risk aversion γ, it is possible to obtain the implied subjective discount factor β as a function of the different variables.

$$\log(\beta) = -r_{t+1}^f + \gamma \cdot \mathbb{E}[\Delta c_{t+1}] - \frac{1}{2}\gamma^2 \cdot \sigma_c^2 \tag{31}$$

Solving for β directly:

$$\beta = \exp\left(-r_{t+1}^f + \gamma \cdot \mathbb{E}[\Delta c_{t+1}] - \frac{1}{2}\gamma^2 \cdot \sigma_c^2\right) \tag{32}$$

Using equation (32) provides a function for β from the other variables. The literature keeps any plausible risk aversion level in the range of 1 to 5. So, fixing γ to any of these values would lead to a range of values for the calibrated beta with a lower bound of (1.02) and an upper bound of (1.07). This implies that $\beta > 1$ which is not feasible, since the supremum for the subjective discount factor is 1. This implies that investors overvalue future consumption so much more than current consumption that investors do not consume anything in the present and keep saving for consumption in the future, which is unrealistic. All in all, the equity premium seen in the Netherlands could not be reconciled by the standard consumption-based model with the standard power utility obtained from a log-normal approximation.

5 Research Question 2

The second Research question aims to find out if the implications of extended consumption capital asset pricing model proposed by Dzhumashev (2025) can account for the equity premium puzzle. This research question aims to see whether the inclusion of the banking sector can better explain the observed equity premium. First, the capital asset pricing model and the consumption capital asset pricing model will be examined. This is because Mehra and Prescott (1985) used the consumption capital asset pricing model with standard power utility to explain the observed equity premium. Second, the extension that Dzhumashev (2025) provides will be discussed and explained. In short, Dzhumashev (2025) includes the banking sector and firms in the consumption capital asset pricing model. Third, the framework of Dzhumashev (2025) will be explained. The model considers three entities that interact with each other. Fourth, the economic equilibrium will be explained. Finally, there will be a calibration exercise to test the model of Dzhumashev (2025).

5.1 CAPM and CCAPM

In finance, the capital asset pricing model is a well-known model that relates the return of an asset to the sensitivity of the market portfolio. The CAPM has been independently developed by Sharpe (1964), Lintner (1965) and Mossin (1966) as an asset pricing model. The CAPM assumes that the investor can borrow at the risk-free rate, the investor is only concerned with the mean-variance value of the portfolio, the market is frictionless and investors have homogeneous expectations Sharpe (1964). Equation (33) provides the linear relationship between the returns of an individual stock $\mathbb{E}_t[r_{i,t+1}]$, the risk-free rate $r_{f,t+1}$, the equity premium $\mathbb{E}_t[r_{t+1}]-r_{f,t+1}$ and the market sensitivity parameter β.

$$\mathbb{E}_t[r_{i,t+1}] = r_{f,t+1} + \beta_i(\mathbb{E}_t[r_{t+1}] - r_{f,t+1}) \tag{33}$$

Equation (33) explicitly holds if β_i captures the systematic risk between the individual asset and the market portfolio. As defined by Berk and DeMarzo (2009), systematic risk describes that risk persists in the market, unlike idiosyncratic risk which can be diversified away. Under the capital asset pricing model, the β is defined as in equation (34):

$$\beta_i = \frac{Cov(r_{i,t+1}, r_{t+1})}{Var(r_{t+1})} \tag{34}$$

The capital asset pricing model has some very strong assumptions which do not hold in the real financial market. For example, borrowing and lending at the same risk-free rate. An extension has been imposed by Black (1972) known as the Black CAPM, which has been developed to ease some of these CAPM assumptions.

A different extension of the capital asset pricing model has been developed by Lucas (1978) known as the consumption capital asset pricing model. A main component of this model is that asset returns are linked to consumption risk rather than equity returns. An essential assumption is that the utility function is a standard power utility function. Equation (35) is obtained from Dzhumashev (2025) and describes that the pricing of assets depends on how their returns co-vary with the marginal utility of consumption.

$$\mathbb{E}_t[r_{t+1}] = r_{f,t+1} + cov_t[\frac{-u'(c_{t+1}), r_{t+1}}{\mathbb{E}_t[u'(c_{t+1})]}] \tag{35}$$

5.2 The extended CCAPM model

Mehra and Prescott (2003) announced in their research paper that the consumption capital asset pricing model is unable to explain a high equity premium and a low risk-free rate at the same time for acceptable levels of risk aversion. Similarly, the first research question in this thesis was unable to describe reasonable parameters of risk aversion and the subjective discount factor to the observed equity premium puzzle with recent Dutch data.

Dzhumashev (2025) acknowledges that the consumption capital asset pricing model may lack variables or undefined processes that could explain why there is such a discrepancy between the observed equity returns and the risk-free rate. Dzhumashev (2025) builds upon the existing consumption capital asset pricing model by implementing banks and firms. The reason for including banks to the consumption capital asset pricing model is that banks are the main entity that demand bonds. Due to banks shaping the demand for bonds, they also shape the risk-free rate. Baklanova, Kuznits, and Tatum (2022) clarifies that only a small amount of risk-free assets are contained in the portfolios of households.

Banks are stylized in the manner of Heuvel (2008). Households earn a short-term interest rate for holding deposits in the bank. Firms require intermediate goods and capital to produce goods in the economy. This allows banks to offer long-term loans to initiate production for firms. The bank maintains their balance of deposit and loans on their balance sheet. Additionally, the bank's balance sheet is also mandated to structure their loans based on regulatory constraints. Additionally, these regulatory conditions also determine the bank's balance sheet how many risk-free assets they must hold.

The basic consumption capital asset pricing model assumes one type of agent that can invest in risk-free assets. This assumption may not be representative of the actual asset demand in the economy. Adding banks to the extended version of the consumption capital asset pricing model, allows the model to account for an additional agent who can invest in risk-free assets. Through the inclusion of banks, the risk-free assets will be primarily held by the bank due to regulatory conditions and the bank's balance sheet composition. Dzhumashev (2025) proposes that this extended model can provide two new insights into the equity premium puzzle. First, because banks are required to hold a significant amount of risk-free assets, bonds become exogenous to the portfolio decisions of households. Second, the equity premium becomes independent of the risk aversion of households. In this model, the first-order condition of the bank generates a formula for the risk-free rate that is calibrated to match the observed equity premium (Dzhumashev 2025).

5.2.1 The general framework

The model of Dzhumashev (2025) is set in discrete time for $t \in \{0, 1, ...\}$. There are effectively four unique players in the economy; households, firms, corporate banks and the central bank. Households can save money on the bank or purchase goods with them. In addition, households can invest in bonds or assets. Firms start with some capital and take on loans from banks to purchase intermediate goods to initiate production. This is to let households consume in the goods market, supplied by firms. Banks maintain a balance sheet based on loans, deposits and regulatory constraints set by the monetary authority. The model assumes that the bonds available at the bond market are provided by the government. Besides the bond market, there is also an asset market that allows investors to become shareholders. This allows firms and banks to raise capital. In this model, the role of the central bank is to support the health of the financial system, as well as to set the regulatory constraints on corporate banks.

5.3 Households in a CIA model

Households modeled by Dzhumashev (2025) are subject to intertemporal utility derived from consumption and do not have to consider choices between labor or leisure. The household's utility function is assumed to be that with constant relative risk aversion preferences, as shown in equation (17) with $\gamma = 1$. This utility function is consistent with the literature and implies that households have a modest level of risk aversion. Households may consider consuming c_t, save its money on bank deposits d_t d_t, it can invest its money in the assets of firms a_t or it can purchase risk-free assets b_t. Over the course of one singular discrete time period these investments have a return of i_t^s, i_t^e and i_t^b respectively.[7] Dzhumashev (2025) points out that households cannot directly use their wealth for consumption. If households want to enjoy a certain consumption level of c_t with cash price p_t, then they can do so by withdrawing money from the bank that is equivalent to the monetary value of those consumption goods m_t. To maintain the framework of the CIA model, all consumption decisions must be made at the start of the period.

5.3.1 Household optimization problem

For the discrete time, households optimize their utility function as represented by equation (36) subject to two constraints. The first constraint represented by equation (37) shows the Cash-in-Advance constraint. This constraint implies that consumption must be done at the start of the period. The budget constraint in equation (38)

[7]The notation of Dzhumashev (2025) is maintained here, thereby, the start of the period is t and the end of that same period is t+1.

shows the saving and investment decisions after the consumption allocation on the right-hand side of the equation. The holdings or returns at the end of the period are on the left-hand side. At the end of each period, households can retrieve their returns from savings in deposits d_{t+1}, stock investments a_{t+1} and bond returns b_{t+1}.

$$\max_{m_{t+1},b_{t+1},a_{t+1},c_t} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \tag{36}$$

subject to:

$$p_t c_t = m_t \leq d_t \tag{37}$$

$$d_{t+1} + b_{t+1} + a_{t+1} \leq (1 + i_t^s) d_t + (1 + i_t^b) b_t + (1 + \mathbb{E}[i_t^e]) a_t - p_t c_t \tag{38}$$

The optimization problem displayed as equation (36), equation (37) and equation (38) is solved in the paper of Dzhumashev (2025). Furthermore, Dzhumashev (2025) changes from nominal terms to real terms to solve the problem. One of the introduced variables is the gross rate of price change as $\pi_t = \frac{p_t}{p_{t-1}}$, which generally can be understood as inflation.

The first pricing equation that has been obtained by Dzhumashev (2025) is represented in equation (39). It displays how households price the return of assets in a cash-in-advance model. This equation resembles the no-arbitrage condition for a specific stochastic discount factor. The current price of equity is a function of the subjective discount factor, the difference between the marginal utility in time $t + 2$ and time $t + 1$ and the gross return of assets adjusted for inflation $R^e_{t+1} = (1 + i^e_{t+1}) \frac{\pi_{t+1}}{\pi_{t+2}}$. The Cash-in-Advance constraint forces households to make consumption decisions at the start of the period. This causes a friction in the consumption smoothing of households across time. Hence, households have to time their consumption differently.

$$1 = \beta \mathbb{E}_t [\frac{u'(c_{t+2})}{u'(c_{t+1})} R^e_{t+1}] \tag{39}$$

Bonds are priced in a similar fashion as assets. Equation (40) displays how the current price of bonds is priced similarly as the current price of assets. However, it regards the gross return of the risk-free asset adjusted for inflation $R^b_{t+1} = (1 + i^b_{t+1}) \frac{\pi_{t+1}}{\pi_{t+2}}$.

$$1 = \beta \mathbb{E}_t [\frac{u'(c_{t+2})}{u'(c_{t+1})} R^b_{t+1}] \tag{40}$$

Dzhumashev (2025) is able to decompose equation (39) and equation (40) into a relation for the equity premium as shown in equation (41). This equation is the consumption capital asset pricing model under the constraint of a Cash-in-Advance model. However, the implications remain the same as those seen from equation (35). Those assets with high returns during recessions or, if future marginal utility is low $u'(c_{t+2})$, are considered more valuable. Thereby, assets that only have a high return during economic booms or, if future marginal utility is high $u'(c_{t+2})$, must have a considerable equity premium.

$$\mathbb{E}_t[R^e_{t+1}] = R^b_{t+1} + cov_t[\frac{-u'(c_{t+2}), R^e_{t+1}}{\mathbb{E}_t[u'(c_{t+2})]}] \tag{41}$$

Equation (41) displays the relation between the expected return on equity, the risk-free rate and the risk-premium. Empirical data reflects that the observed risk-free rate is too low compared to the equity premium when predicted by consumption-based models with standard power utility (Dzhumashev 2025). Figure 11 of the first research question supports this, the risk aversion γ fitted to the observed risk-free rate yields a low equity premium. Dzhumashev (2025) proposes a lemma for the no-arbitrage condition represented as equation (41) not being satisfied empirically. If the no-arbitrage condition does not hold, the return of the risk-free rate is too low to be explained by household risk aversion. In this situation, households would reallocate their portfolio of assets by selling bonds and purchasing equity until the no-arbitrage condition holds. However, households are unable to short-sell government bonds. Thus the optimal holdings for households is infeasible and therefore has to be a boundary solution ($b_t = 0$).

Dzhumashev (2025) highlights that this result shows a shortcoming in the consumption-based pricing model. The consumption-based model assumes that there is only one agent who demands risk-free assets, while this is not true in the real economy. Because the observed low risk-free rate cannot be explained by household demand. To address this, Dzhumashev (2025) implements financial intermediaries as an extension of the consumption capital asset pricing model that are required to hold risk-free assets.

5.4 The banking sector

Dzhumashev (2025) implements the banking sector in the model to account for the demand of bonds issued from the government. As clarified by Dzhumashev (2025), the "vanilla" consumption capital asset pricing model cannot perform well if households do not demand risk-free assets. Thus, adding the banking sector can mitigate this problem. The banking sector in the model of Dzhumashev (2025) consists of the monetary authority and the corporate banks.

5.4.1 Central Banks

The monetary authority is introduced in a stylized manner. The core functions of the monetary authority in this model is to maintain the health of the financial system and to endorse the regulatory constraints on the corporate bank. The first regulatory condition is the minimum equity requirement given by the parameter $\kappa \in (0, 1)$. As explained by Dzhumashev (2025), the minimum equity requirement determines that for any long-term loan provided to a firm, a part of the loan must be from the bank's own equity e_t. Equation (42) aims to highlight that a certain amount of the loan l_t given by the bank must be leveraged by the bank's equity.

$$e_t \geq \kappa l_t \tag{42}$$

The second regulatory constraint is given by the minimum reserve requirement. The minimum reserve requirement is given by the parameter $\rho \in (0, 1)$ and determines how much total reserves r_t the corporate bank must maintain as a fraction of the deposits. Thereby, it is an additional parameter to limit the lending capabilities of the bank, to safeguard depositors. Equation (43) shows this constraint, for a certain level of total reserves given by the bank as r_t must be at least the required reserve ratio of the deposits d_t.

$$r_t \geq \rho d_t \tag{43}$$

Besides the regulatory constraints, Dzhumashev (2025) acknowledges the Open Market Operations of the central bank. Two Open Market Operations often discussed in Macro-economics is Quantitative Easing and Quantitative Tightening. Berk and DeMarzo (2009) points out that Quantitative Easing is an operation from the central bank to purchase government bonds to reduce the interest rate and increase the money supply. Essentially, Quantitative tightening is the reverse of Quantitative Easing to limit the growth of the economy. In the paper of Dzhumashev (2025) these Open Market Operations are used to effectively increase the reserves available to banks. In this case, the Open Market Operations considered here, is that the Central bank purchases government bonds at a high price and sells them to the Corporate bank at a low price. This new liquidity is defined as additional reserves x_t, where the model simply assumes that $x_t = gr_t$ for, g > 0. Equation (44) displays the Open Market Operations performed by central banks to stimulate growth in the economy and promotes the ability for a bank to be a credit provider (Dzhumashev 2025).

$$r_{t+1} = r_t + x_t \tag{44}$$

To conclude, central banks in the model of Dzhumashev (2025) is stylized to control, regulate and promote the corporate banks.

5.4.2 Corporate banks

Dzhumashev (2025) builds the functions of corporate banks with the same intuitions of Heuvel (2008) and Mierau and Mink (2018). Banks play a fundamental role in the economy by acting as a financial intermediary (Diamond and Dybvig 1983). Corporate banks provide liquidity in the asset market, lend out loans to accumulate capital and a whole other plethora of financially related activities. Regardless, the scope of this model limits the banking sector to a stylized form. It is explicitly assumed that the banking sector is based on a smaller set of activities rather than the total core of activities that realistically approximate the banking industry to absolute detail (Dzhumashev 2025).

In this model, banks with their own equity capital e_t hold deposits from households d_t. Because the bank operates as a financial intermediary, the bank is able to extend some of these short-term deposits from households into long-term loans l_t that firms use for investment and operational expenses. Furthermore, banks rely on a combination of equity and short-term liabilities to promote loans. The lending capabilities from the corporate bank are restricted by the regulatory conditions set by the central bank to ensure financial stability (Heuvel 2008). This allows firms to

access financing while ensuring liquidity for depositors who may need to withdraw funds at any time.
Banks earn revenue based on the interests paid on these loans extended to firms. The interest rate of these extended loans to firms is given by i_t. However, in the economy, it can occur that firms are unable to repay a part of their loan or default entirely. Because banks have an incentive to generate profits while also managing risk, they can account for this by charging a risk premium i_t^σ. Dzhumashev (2025) explains that this risk premium is considered an additional premium to the lending rate, as a compensation for the chance of default. Additionally, equation (45) describes how the bank's return is determined by its equity e_t and outstanding loans l_t. The total return is generated through the equity return which is the return of a market index, the leverage ratio $\frac{l_t}{e_t}$ and risk premium. The leverage ratio describes the fraction of outstanding loans relative to the amount of equity. This could generate high returns or higher losses depending on the risk-profile of the bank.

$$\text{BankROE} = i_t^e + \frac{l_t}{e_t} i_t^\sigma \tag{45}$$

For investing in government bonds, banks earn at the rate i_t^b. Because banks also hold deposits, they have to pay an interest of i_t^d to the households. However, households may withdraw some money m_t for consumption purposes. Since this can create some uncertainty for the bank, it should hold more reserves r_t than necessary as seen from equation (43).

5.4.3 Corporate Bank optimization

Table 8 aims to visualize the assets, liabilities and equity the bank has in the model of Dzhumashev (2025). It describes how the total amount of assets is the amount of extended loans l_t, purchased government bonds b_t and the total amount of reserves $r_t + x_t$. The liabilities of the bank are compromised of only the deposits from households, while the equity part of the balance sheet is the bank's own capital.

Assets		**Liabilities & Equity**	
Loans	l_t	Deposits	d_t
Bonds	b_t	Equity	e_t
Reserves	r_t		
Additional reserves	x_t		
Total	$l_t + b_t + r_t + x_t$	Total	$d_t + e_t$

Table 8: Stylized balance sheet of the bank showing assets, liabilities, and equity.

Equation (46) shows the profit maximization function of the bank. Deposits are a liability because the bank has to repay households at a rate i_t^s. Loans serve a dual purpose, because, on the one hand, they generate a return of i_t for the bank regarding whether the loan is repaid. On the other hand, high-risk loans may get defaulted upon by firms. Hence, banks have to take account for this, by assuming a loss at the premium i_t^σ. Equity e_t represents the cost of equity that shareholders demand from banks. Equation (47) and equation (49) represent the regulatory constraints as set by the monetary authority. Equation (48) describes the balance sheet identity of the bank: assets = liabilities + equity.

$$\max_{l_t, b_t, d_t} \mathbb{E}_t[\Pi] = i_t l_t + i_t^b b_t - i_t^s d_t - \mathbb{E}[i_t^e] e_t - i_t^\sigma l_t \tag{46}$$

subject to:

$$e_t \geq \kappa l_t \tag{47}$$

$$l_t + b_t + r_t + x_t = d_t + e_t \tag{48}$$

$$r_{t+1} = r_t + x_t \geq \rho d_{t+1} \tag{49}$$

Dzhumashev (2025) solves the optimization problem for the banking sector by proposing a set of simplifications. The first simplification is derived from Heuvel (2008) and Mierau and Mink (2018) where the regulatory condition for the minimum equity requirement as described in equation (47) is binding. As assumed by Dzhumashev (2025), the bank only holds government bonds to hedge against the uncertainty of money withdrawals from households in the next period. This implies that the required reserve condition as highlighted in equation (49) also is binding. However, the total amount of cash withdrawn by households could be larger than the total amount of available reserves. Equation (50) displays ω which is the fraction of cash withdrawn at the end of the period compared to the deposits at the begin of the period.

$$\omega = \frac{m_{t+1}}{d_t} \tag{50}$$

Since the amount of cash withdrawn ω could be larger than the required cash ratio ρ, banks must keep at least the amount of bonds as shown in equation (51) to hedge against risk. The risk-free bond condition may be assumed to be binding if there is little risk or uncertainty (Dzhumashev 2025).

$$b_t \geq (\omega - \rho) d_t \tag{51}$$

Through rewriting, adjusting and taking F.O.C's Dzhumashev (2025) is able to obtain an expression for the risk-free rate obtained from the profit maximization of the bank under the regulatory constraints. Equation (52) displays the expression of the risk-free rate.

$$i^b = \frac{1-\omega}{(\omega-\rho)(1-\kappa)}(i^s \frac{1-\kappa}{1-\omega} + \mathbb{E}[i^e]\kappa + i^\sigma - i) \tag{52}$$

The essence of equation (52) is that the banks are now the sole determinant for the risk-free rate unlike the implication of the baseline consumption capital asset pricing model. Moreover, this representation of the risk-free rate excludes the risk-aversion parameter that is known for generating large risk-free rates.

5.5 Firms

Dzhumashev (2025) adds firms to the model for a couple of reasons. First, firms are an additional economic player that requires loans to start production, thereby, justifying the bank's inclusion in this extension of the consumption capital asset pricing model. Second, the addition of firms allows for the implementation of economic markets such as the goods market and equity market. Here, households can purchase products in the goods market and trade in stocks in the asset market. Moreover, Dzhumashev (2025) announces the following assumptions to incorporate firms. First, firms do not hold any deposits at the bank. Second, all borrowed money l_t is invested in intermediate goods z_t for the price p_t to start production with no remains. Third, firms have some starting physical capital k_t to produce goods.

Equation (53) shows a production function with the inputs of physical capital and that is often seen in the literature of macroeconomics (Chari, Christiano, and Eichenbaum 1995). Physical capital is usually defined by machines, buildings and tools to transform intermediate goods into the gross output of goods and services q_t. In the paper of Dzhumashev (2025), Φ is referred to as the stochastic productivity coefficient. In the literature, this coefficient also is known as the productivity coefficient, reflecting the efficiency of production Jones (2011). To add on, α and γ are Cobb-Douglous parameters relating to the share of capital and intermediate goods respectively.[8]

$$q_t = \Phi(k_t^\alpha)^{1-\gamma} z_t^\gamma \tag{53}$$

Equation (54) displays the net output of production as y_t as only a fraction $(1 - \bar{z})$ of the gross output produced by firms is available for the economy.

$$y_t = (1 - \bar{z})q_t \tag{54}$$

Dzhumashev (2025) proposes the final output of the production from the firm in equation (55). Thereby, providing the final output function of goods as a function of intermediate output and physical capital. Here, $\mu \equiv (1-\bar{z})^{1-\gamma}\bar{z}^\gamma$ and $A \equiv (\Phi\mu)^{\frac{1}{1-\gamma}}$

$$y_t = (\Phi\mu)^{\frac{1}{1-\gamma}} k_t^\alpha = A k_t^\alpha \tag{55}$$

Finally, Dzhumashev (2025) describes the profit maximization function of the firm as described in the maximization problem (56). The revenue gained by the firm for selling goods or services is $p_t y_t$, while, the firm has costs for the depreciation of physical capital $\delta p_t k_t$, the cost of equity $i_t^e p_t k_t$ demanded by shareholders and its outstanding loans with interest $i_t l_t$.

[8]The notation is maintained to adhere to the working paper of Dzhumashev (2025) and γ is unrelated to risk-aversion in this context.

$$\max_{k_t, l_t} \Pi_t = p_t y_t - (i_t^e + \delta) p_t k_t - i_t l_t \tag{56}$$

5.6 Equilibrium

An equilibrium is obtained when the economic agents in this model optimize their objective function given the constraints, prices and the markets clear. In the model of Dzhumashev (2025), the agents are households, firms and banks. As explained by Mas-Colell, Whinston, and Green (1995), some assumptions must be satisfied to maintain economic equilibrium. First, it is assumed that each economic agent is rational. This means that agents are choosing their decision variables to maximize their own utility function. In the model Dzhumashev (2025), households optimize their utility, while firms and banks maximize profits. Second, the goods, bonds, loan and asset markets are considered complete. When a market is considered complete, an agent can hedge itself against all states of the world. Third, all economic agents are price takers and do not set prices to influence the equilibrium. Finally, the market clears through the match of supply equals demand. This condition will evidently set the prices of each economic agent. Because an equilibrium follows from the clearing condition of the market, it remains essential to highlight with which side of demand or supply each agent interacts.

5.6.1 Dynamic equilibrium

The equilibrium as shown in the working paper of Dzhumashev (2025) demonstrates that households gain utility through consumption. Considering that households want to consume goods and services produced by firms, consider the consumption of households c_t on the demand side. Households that want to become shareholders demand equity to invest in a_t, perhaps also risk-free assets b_t. Because households save their money in their bank account. This means that deposits are supplied d_t. Due to the empirical violation of equation (41) an additional non-negativity constraint is set of $b_t \geq 0$ relating to the holding of bonds as households not being able to short sell bonds. This non-negative constraint can be rewritten as $-b_t \leq 0$.

For the household, let the Lagrangian be defined as (58):

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \Bigg[& \sum_{t=0}^{\infty} \beta^t \log(c_t) \\ & + \sum_{t=0}^{\infty} \lambda_t \left(d_t - p_t c_t\right) \\ & + \sum_{t=0}^{\infty} \phi_t \Big((1 + i_t^s) d_t + (1 + \mathbb{E}[i_t^e]) a_t + (1 + i_t^b) b_t \\ & \qquad - p_t c_t - d_{t+1} - b_{t+1} - a_{t+1} \Big) \qquad (57) \\ & - \sum_{t=0}^{\infty} \eta_t b_t \Bigg] \qquad (58)\end{aligned}$$

Taking first-order conditions from the household Lagrangian gives a set of dynamic equations as shown in equation (59), (60), (61), (62), (63) and (64). This puts forth a dynamic equilibrium since the household considers multiple-time periods.

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \rightarrow \beta^t \frac{1}{c_t} - p_t \lambda_t - p_t \phi_t = 0 \tag{59}$$

$$\frac{\partial \mathcal{L}}{\partial d_{t+1}} = 0 \rightarrow \lambda_{t+1} + \phi_{t+1}(1 + i_{t+1}^s) - \phi_t = 0 \tag{60}$$

$$\frac{\partial \mathcal{L}}{\partial a_{t+1}} = 0 \rightarrow \phi_{t+1} \mathbb{E}[(1 + i_{t+1}^e)] - \phi_t = 0 \tag{61}$$

$$\frac{\partial \mathcal{L}}{\partial b_{t+1}} = 0 \rightarrow \phi_{t+1}(1 + i_{t+1}^b) - \phi_t - \eta_t = 0 \tag{62}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 \rightarrow \frac{d_t}{p_t} - c_t = 0 \tag{63}$$

$$\frac{\partial \mathcal{L}}{\partial \phi_t} = 0 \rightarrow (1 + i_t^s) d_t + (1 + \mathbb{E}[i_t^e]) a_t + (1 + i_t^b) b_t - p_t c_t - d_{t+1} - b_{t+1} - a_{t+1} = 0 \tag{64}$$

From equation (59) a dynamic function for the demanded consumption can be obtained as for the time t, as shown in equation (65).

$$\beta^t \frac{1}{c_t} = p_t(\lambda_t + \phi_t). \tag{65}$$

Thereby, for the next time period $t + 1$, consumption can be explained by equation (66).

$$\beta^{t+1} \frac{1}{c_{t+1}} = p_{t+1}(\lambda_{t+1} + \phi_{t+1}). \tag{66}$$

Equation (61) provides a recursion for ϕ in period $t+1$, that can be obtained as shown in equation (67).

$$\phi_{t+1} = \frac{\phi_t}{\mathbb{E}[(1 + i^e_{t+1})]} \tag{67}$$

Equation (68) provides a recursion for ϕ in period $t+1$ and Lagrange multiplier η_t, that can be obtained as shown in equation (68). The multiplier may be non-zero $\eta_t \neq 0$ if $b_t = 0$.

$$\phi_{t+1} = \frac{\phi_t}{(1 + i^b_{t+1})} + \frac{\eta_t}{(1 + i^b_{t+1})} \tag{68}$$

Similarly, equation (60) provides a function to isolate λ_{t+1} as shown in equation (69).

$$\lambda_{t+1} = \phi_t - \phi_{t+1}(1 + i^s_{t+1}) \tag{69}$$

Moreover, equation (69) allows λ to be defined as a function of ϕ and the interest rates. To gain a recursive formula for consumption, equation (66) will be rewritten. First, equation (69) will be substituted into (66) which gives equation (70).

$$\beta^{t+1}\frac{1}{c_{t+1}} = p_{t+1}(\phi_t - \phi_{t+1}(1 + i^s_{t+1}) + \phi_{t+1}) \tag{70}$$

To add on, the dynamic relation between ϕ_t and ϕ_{t+1} as highlighted in equation (67). This will be used to obtain an isolation of ϕ_t in equation (70). For this retraction, equation (67) will be used. This gives equation (71) with ϕ_t isolated.

$$\beta^{t+1}\frac{1}{c_{t+1}} = p_{t+1}\phi_t(1 - \frac{i^s_{t+1}}{1 + \mathbb{E}[i^e_{t+1}]}) \tag{71}$$

Rewriting equation (69) with the ϕ_t substituted by equation (67) allows λ_{t+1} to be a function of ϕ_{t+1}. Thereby, if the period is set back to time t equation (72) can be obtained.

$$\lambda_t = \phi_t(\mathbb{E}[i^e_t] - i^s_t) \tag{72}$$

Equation (65) together with equation (72) allows ϕ_t to be a function of the interest rates, consumption and subjective discount factor as shown in equation (73).

$$\phi_t = \frac{\beta^t \frac{1}{c_t}}{p_t(1 + \mathbb{E}[i^e_t] - i^s_t)} \tag{73}$$

Equation (73) can be substituted in equation (71) to gain a recursive formula for consumption in the household as shown in equation (74).

$$\beta^{t+1}\frac{1}{c_{t+1}} = p_{t+1}[\frac{\beta^t}{p_t c_t(1 + \mathbb{E}[i^e_t] - i^s_t)}][1 - \frac{i^s_{t+1}}{1 + \mathbb{E}[i^e_{t+1}]}] \tag{74}$$

Rewriting and adjusting the terms in equation (74) gives an isolated function of future consumption as a function of the subjective discount factor, the interest rates, the prices and previous consumption as provided in equation (75).

$$c_{t+1} = \beta c_t(1 + \mathbb{E}[i^e_t] - i^s_t)\frac{p_t}{p_{t+1}}(\frac{1 + \mathbb{E}[i^e_{t+1}]}{1 + \mathbb{E}[i^e_{t+1}] - i^s_{t+1}}) \tag{75}$$

Equation (75) provides a tractable formula between the consumption in previous and current period. A possible method to solve this recursion is under the life cycle hypothesis (LCH). The life cycle hypothesis describes that households prefer consumption smoothing over their expected lifetime. This implies that the consumption and investment horizon of the household becomes finite at time T, where T is the terminal period of the household. This describes the problem of the household as equation (76).

$$\max_{\{c_t\}_{t=0}^{T}} \left[\sum_{t=0}^{T} \beta^t u(c_t) \right] \tag{76}$$

If the household has its life terminated at time T, then the deposits in the bank-account at termination are depleted $d_{T+1} = 0$, the household does not own any bonds $b_{T+1} = 0$ and has no active investments $a_{T+1} = 0$. Furthermore, the implication of the no-arbitrage condition not satisfying empirical data gives $b_T = 0$. This allows the budget constraint for the final period to be rewritten so that the terminal consumption is known. Equation (77) highlights the terminal consumption c_T.

$$c_T = \frac{(1 + i_T^s)d_T + (1 + \mathbb{E}[i_t^e])a_T}{p_T} \tag{77}$$

Since the terminal consumption is known, the recursive consumption equation can be used to calculate the previous consumption levels $c_{T-1}, c_{T-2}, ..., c_0$. Backwards dynamic programming could be implemented to determine the optimal consumption levels and other variables one step backwards in time. Because recursive consumption formula rapidly increases in size, it is difficult to gain a tractable closed order solution for the optimal levels of the household.

5.6.2 Static equilibrium

The profit function of the firm is defined in equation (78). Since firms need loans to begin production, it is clear that the loans l_t are on the demand-side of the firm. The loans are used to purchase intermediate goods. Physical capital k_t is something the firm should have or bring up by itself. Since the final output of the company is expressed as $y_t = Ak_t^\alpha \equiv g_t$, this can be understood as the products and services that make it to the goods market that households can consume. In addition, for shareholders, it is possible to invest in firms a_t through the asset market, which allows investors to raise physical capital for the firms.

$$\max_{k_t, l_t} \Pi_t = p_t * Ak_t^\alpha - (i_t^e + \delta)p_t k_t - i_t l_t \tag{78}$$

The first-order condition for the physical capital for the firm is provided in equation (79).

$$\frac{\partial \Pi_t}{\partial k_t} = 0 \rightarrow p_t \cdot \alpha \cdot Ak_t^{\alpha-1} - (i_t^e + \delta)p_t = 0 \tag{79}$$

Rewriting equation (79) can provide an isolation for the optimal demanded level of physical capital as can be seen from equation (80).

$$k_t = (\frac{i_t^e + \delta}{\alpha A})^{\frac{1}{\alpha - 1}} \tag{80}$$

The first-order condition for the loans demanded by the firm is illustrated in equation (81).

$$\frac{\partial \Pi_t}{\partial l_t} = 0 \rightarrow \Phi((k_t)^\alpha)^{1-\gamma}(\frac{l_t}{p_t})^\gamma - i_t \frac{l_t}{p_t} = 0. \tag{81}$$

Rewriting equation (81) into the optimal loans demanded is provided in equation (82).

$$l_t = [\frac{i_t}{\gamma \Phi((k_t)^\alpha)^{1-\gamma}}]^{\frac{1}{\gamma - 1}} \cdot \frac{1}{p_t} \tag{82}$$

However, as seen from equation (82) the optimal level of loans demanded by the firm also considers the optimal level of physical capital by the firm. Hence, the gain an isolation for the loans, substituting equation (80) in equation (82), gives equation (83).

$$l_t = [\frac{i_t}{\gamma \Phi((\frac{i_t^e + \delta}{\alpha A})^{\frac{1}{\alpha - 1}})^\alpha)^{1-\gamma}}]^{\frac{1}{\gamma - 1}} \cdot \frac{1}{p_t} \tag{83}$$

As seen from equation (80) and equation (83), the optimal level of loans and physical capital demanded by the firm can be obtained as a function from the interest rates and the production variables of the firm.

The bank's profit function is derived from the balance sheet and the regulatory constraints. To extend loans to firms the bank uses a part of its deposits as well as its own capital equity to fund these advances. Additionally, the bank must require some risk-free assets to hedge any potential risks. Thereby, deposits d_t and bonds b_t are demanded while the loans l_t and equity e_t are supplied. This is demonstrated in the Lagrangian of the bank as in equation (84).

Let the Lagrangian for the bank be described as:

$$\begin{aligned} \mathcal{L} = {} & i_t l_t + i_t^b b_t - i_t^s d_t - \mathbb{E}[i_t^e]\, e_t - i_t^\sigma l_t \\ & + \lambda_1 \left(\frac{1 - \omega}{1 - \kappa} l_t - d_t \right) \\ & + \lambda_2 \left(\kappa l_t - e_t \right) \\ & + \lambda_3 \left((\omega - \rho) d_t - b_t \right) \end{aligned} \tag{84}$$

The first-order condition for the loans supplied by the bank is:

$$\frac{\partial \mathcal{L}}{\partial l_t} = 0 \rightarrow i_t - i_t^\sigma + \lambda_1 \frac{1-\omega}{1-\kappa} + \lambda_2 \kappa = 0 \tag{85}$$

The first-order condition for the deposits demanded by the bank is:

$$\frac{\partial \mathcal{L}}{\partial d_t} = 0 \rightarrow -i_t^s - \lambda_1 + \lambda_3(\omega - \rho) = 0 \tag{86}$$

The first-order condition for bonds demanded by the bank is:

$$\frac{\partial \mathcal{L}}{\partial b_t} = 0 \rightarrow i_t^b - \lambda_3 = 0 \tag{87}$$

The first-order condition for equity supplied by the bank is:

$$\frac{\partial \mathcal{L}}{\partial e_t} = 0 \rightarrow -\mathbb{E}[i_t^e] - \lambda_2 = 0 \tag{88}$$

The first-order conditions for the constraints within the Lagrangian are:

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = 0 \rightarrow \frac{1-\omega}{1-\kappa} l_t - d_t = 0 \tag{89}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = 0 \rightarrow \kappa l_t - e_t = 0 \tag{90}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_3} = 0 \rightarrow (\omega - \rho) d_t - b_t = 0 \tag{91}$$

The bank will optimize its holdings based on the given interest rates. The Lagrangian parameters can be isolated from the of set of equations (85), (86), (87) and (88).

$$\lambda_2 = -\mathbb{E}[i_t^e]$$

$$\lambda_1 = \frac{1-\kappa}{1-\omega} \mathbb{E}[i_t^e]\kappa + i_t^\sigma - i_t$$

$$\lambda_3 = \frac{1}{\omega - \rho}\left(\frac{1-\kappa}{1-\omega}(\mathbb{E}[i_t^e]\kappa + i_t^\sigma - i_t) + i_t^s\right)$$

Since $i_t^b = \lambda_3$ according to equation (87) the exact result of Dzhumashev (2025) can be obtained. This follows from equation (52):

$$i_t^b = \frac{1}{\omega - \rho} \frac{1-\kappa}{1-\omega}\left(\mathbb{E}[i_t^e]\kappa + i_t^\sigma - i_t + \frac{1-\omega}{1-\kappa} i_t^s\right) \tag{92}$$

The optimal values for the bank can be retrieved from the F.O.C. from the Lagrangian as well as the market clearing conditions and the choices made by the firms. The set of equations (89), (90) and (91) are derived from the regulatory constraints set by the central bank and the bank's balance sheet identity.

The market clearing condition for loans between firms and banks is given by equation (106). Thereby, under the market clearing condition, if the optimal level of physical capital and loans are determined. The market clearing condition can suggest the optimal values for equity and deposits. As seen from the market clearing condition (106) the optimal level of deposits can be determined, this is shown in equation (93).

$$d_t = \frac{1-\omega}{1-\rho} l_t \tag{93}$$

Since, the firm optimizes its loans taken based on the amount of physical capital needed. From equation (52) the ratio of $\frac{1-\kappa}{1-\omega}$ can be determined. Thereby, the optimal level of deposits demanded can be reformulated in equation (94) as a function of the different interest rates and regulatory requirements.

$$d_t = \frac{\frac{1}{\omega-\rho}(\mathbb{E}[i_t^e]\kappa + i_t^\sigma - i^t)}{i_t^b - \frac{1}{\omega-\rho} i_t^s} * [\frac{i_t}{\gamma\Phi((\frac{i_t^e+\delta}{\alpha A})^{\frac{1}{\alpha-1}})^\alpha)^{1-\gamma}}]^{\frac{1}{\gamma-1}} \cdot \frac{1}{p_t} \tag{94}$$

Evidently, the market clearing condition for loans also determines the optimal level for equity. The regulatory constraint forced by the central bank and the market clearing condition imply that the optimal level of equity is given in equation (95).

$$e_t = \kappa l_t \tag{95}$$

In similar fashion to the deposits, the optimal level of equity as function of the price, interest rates and regulatory variables is given in equation (96).

$$e_t = \kappa * [\frac{i_t}{\gamma\Phi((\frac{i_t^e+\delta}{\alpha A})^{\frac{1}{\alpha-1}})^\alpha)^{1-\gamma}}]^{\frac{1}{\gamma-1}} \cdot \frac{1}{p_t} \tag{96}$$

The market clearing conditions and regulatory constraints mandate that the bank must hold a certain amount of bonds as shown in equation (99). Since the optimal level of deposits was determined through the market clearing conditions of deposits on the demand side of the bank and the loans on the demand side of the firms. From this procedure the optimal level of bonds is obtained from equation (97).

$$b_t = \frac{\mathbb{E}[i_t^e]\kappa + i_t^\sigma - i^t}{i_t^b - \frac{1}{\omega-\rho} i_t^s} * [\frac{i_t}{\gamma\Phi((\frac{i_t^e+\delta}{\alpha A})^{\frac{1}{\alpha-1}})^\alpha)^{1-\gamma}}]^{\frac{1}{\gamma-1}} \cdot \frac{1}{p_t} \tag{97}$$

5.6.3 Market clearing conditions

The market clears if supply and demand matches between the agents. For the bond market it is implicitly stated that the government is issuing these risk-free assets to the market. The implication for households not partitioning in the bond market holds. Thus, only banks are effectively holding risk-free assets. Moreover, banks are the only agent who can significantly invest into risk-free assets as mandated by the central bank. Let the market clearing condition for bonds be described as in equation (98).

$$Bonds^{Demanded} = Bonds^{Supplied} \tag{98}$$

In the format of equation (98), it is clear that the demand-side of bonds is determined only by the banks. Evidently, the government is the exogenous supply-pool of bonds. Thereby, the bank purchases the amount of risk-free bonds from the government to satisfy the regulatory constraint shown in equation (99).

$$(\omega - \rho)d_t = b_t \tag{99}$$

The market clearing condition for goods is provided in equation (100).

$$Goods^{Demanded} = Goods^{Supplied} \tag{100}$$

Hereby the total supply-side of goods is entirely determined by the final output of the firm by $Ak_t^{\alpha} \equiv g_t$. The total demand-side for goods is decided by consumption of households c_t and the reinvestment of the firm $I_t \equiv k_{t+1} - (1-\delta)k_t$. This states that the market clearing condition for goods is given by equation (101).

$$c_t + I_t = g_t \tag{101}$$

The market clearing condition for equity is given in equation (102).

$$Equity^{Demanded} = Equity^{Supplied} \tag{102}$$

The demand-side for equity is obtained from shareholders who willing to invest a_t^D in either banks or firms to gain some return. The supply-side of the equity market is given by those companies who are willing to raise capital from the money of shareholders. The monetary value of the equity market is given by the total price of physical capital $p_t k_t$ from the firms and the bank's equity e_t. This provides the market clearing condition as stated in equation (103).

$$a_t = p_t k_t + e_t \tag{103}$$

From the market clearing condition as stated in equation (103). The optimal level of asset invested can be determined. Since the optimal level of physical capital is known from equation (80) and optimal level of equity is know from equation (96). The optimal level of assets is given in equation (104).

$$a_t = p_t((\frac{i_t^e + \delta}{\alpha A})^{\frac{1}{\alpha-1}}) + \kappa * [\frac{i_t}{\gamma\Phi((\frac{i_t^e+\delta}{\alpha A})^{\frac{1}{\alpha-1}})^{\alpha})^{1-\gamma}}]^{\frac{1}{\gamma-1}} \cdot \frac{1}{p_t} \tag{104}$$

The market clearing condition for loans is determined between the banks and firms. Firms need loans to initiate production and thereby are on the demand-side for loans. Banks extend loans to firms to gain profit. However, the amount of loans banks can extend is limited due to the regulatory constraints set by the monetary authority. Let the market clearing condition for loans be defined as equation (105).

$$Loans^{demanded} = Loans^{supplied} \tag{105}$$

Firms demand loans as l_t and banks have their own equity e_t which can consists only a fraction κ of the loans due to the minimum equity requirement. Additionally, the required reserve ratio and the uncertainty of withdrawals determine that the bank can only give (1 - ω) of their deposits as loans since the withdrawal ratio ω could be larger than the required reserve ratio ρ. Hence, the market clearing condition is provided in equation (106).

$$l_t = (1 - \omega)d_t + e_t \tag{106}$$

5.7 Model parameter calibration

The risk-free interest rate i^b equation (52) displayed how the risk-free rate that is observed from the equity premium, can be calibrated through the minimum equity ratio κ, the required reserve ratio ρ, the market index return i^e, deposit rate i^s, loan rate i and the bank's risk premium i^σ. The risk premium of the bank i^σ is calculated from equation (107). The data for the regulatory conditions have been obtained from the European Central Bank. The interest rates regarding loans, household deposits and the bank's return on equity have been gathered from the DNB. The Dutch market index, AEX GR, has been used as the market proxy and the Dutch Treasury Certificates are used for the risk-free rate. All of the data aims to reflect the recent economy from 2006 - 2024. Moreover, the bank's return of equity has been considered for the post-period of the Great Financial Crises. Table 9 illustrates the parameters that are used to calibrate the model of Dzhumashev (2025).

$$i^\sigma = (\text{BankROE} - i^e) * \kappa \tag{107}$$

Parameters	Values	Notes
κ	16.3%	Equity-to-loan ratio
ρ	1%	Required reserve ratio
i^s	1.30%	Interest rate paid on deposits
i^e	4.63%	Real Equity return
i	3.16%	Business loan interest rate
$BankROE$	7%	Average bank equity premium
i^σ	0.386%	Bank risk-premium
ω	29.4%	Deposit Withdrawal Rate
i^b	-1.42%	Real Risk-free rate

Table 9: Summary of the parameters that are essential to calibrate the model of Dzhumashev (2025).

An initial calibration of ω to fit the observed real risk-free rate observed in the Netherlands, results in a ω of 29.4%. Dzhumashev (2025) explains that ω in this scenario, infers to the liquidity to purchase goods and services by households. This implies that households keep 29.4% of their money in advance for consumption. It is difficult to critique this observation since there is not a lot of literature to compare this result to. Dzhumashev (2025) supports a withdrawal rate of 90% or higher. Moreover, the calibration executed by Dzhumashev (2025) resulted in a withdrawal rate of 99% which is significantly larger than the withdrawal rate found in this calibration. Finally, the period and economy the author considers is also notably different.

5.8 Sensitivity analysis

Considering the initial result is considerably different than what was expected, a sensitivity analysis will be implemented. It shall be assumed that the interest rate paid on deposits, business loan interest rates and bank equity return can deviate by one standard deviation without loss of generality. This analysis is highlighted in Table 10.

Parameters	$-\sigma_{i^s}$	$+\sigma_{i^s}$	$-\sigma_i$	$+\sigma_i$	$-\sigma_{BankROE}$	$+\sigma_{BankROE}$
κ	16.3%	16.3%	16.3%	16.3%	16.3%	16.3%
ρ	1%	1%	1%	1%	1%	1%
i^s	0.26%	2.34%	1.30%	1.30%	1.30%	1.30%
i^e	4.63%	4.63%	4.63%	4.63%	4.63%	4.63%
i	3.16%	3.16%	2.08%	4.24%	3.16%	3.16%
$BankROE$	7.00%	7.00%	7.00%	7.00%	5.00%	9.00%
i^σ	0.386%	0.386%	0.386%	0.386%	0.060%	0.712%
ω	56.5%	2.25%	$>$ 100%	50.2%	35.9%	21.4%
i^b	-1.42%	-1.42%	-1.42%	-1.42%	-1.42%	-1.42%

Table 10: Model Calibration Parameters for Sensitivity analysis

Table 10 shows the sensitivity analysis in a ceteris paribus setting. If the deposit rate i^σ decreases by one standard deviation. The ω increases to 56.5%. This could mean that households are taking their wealth out of their bank account for consumption. The corresponding i^s is 0.26%. This means that in this model, prefer to consume considerably because the interest rate on their deposits is small. In contrast, if the deposit rate increases by one standard deviation, then the model could suggest that the households keep all of their wealth on their bank account as the withdrawal rate is 2.25%. Thereby, households could consider this return of 2.34% so valuable that they nearly keep all of their money in the bank to earn interest. If the corporate lending rate i_t decreases by one standard deviation, the withdrawal rate gets larger than 100%. This has no economic interpretation as the withdrawal rate $0 \leq \omega \leq 1$. Thus this result is infeasible. If the lending rate increases by one standard deviation, the withdrawal rate is 50.2%. If the bank's return of equity increases or decreases by one standard deviation. The withdrawal rate is somewhat larger 35.9% compared to the initial result. The increase, leads to a considerably lower withdrawal rate 21.4%. Although, the most favorable outcome is presented in column 2, it is not close to the suggested ideal range of the author $\omega \geq 90\%$. Thus, the findings of this sensitivity analysis remain inconclusive.

6 Conclusion

This thesis contributed to the literature by applying both the conventional and extended version of the CCAPM to recent Dutch data. Furthermore, the thesis provided a more comprehensive overview of the equilibrium as proposed by Dzhumashev (2025).

The first research question confirmed that the consumption-based capital asset pricing model with standard power utility fails to account for the observed equity premium in the Netherlands without resorting to either implausibly high levels of risk aversion or unrealistically low subjective discount factors. Evidently, the observed equity premium and observed risk-free rate could not be calibrated from the standard consumption based asset pricing model for fixed levels of the risk aversion and subjective discount factor. The calibrated returns on the risk-free assets were significantly higher than observed. Meanwhile, the calibrated equity premium was significantly lower than observed. These findings reflect the same structural inconsistencies originally identified by Mehra and Prescott (1985). In short, the application of the conventional CCAPM on recent Dutch data reinforces the conclusion that the model lacks the ability to explain the observed equity premium for reasonable levels of risk aversion.

The second research question explored whether the extended CCAPM proposed by Dzhumashev (2025), which incorporates financial institutions into the framework, could offer a more realistic explanation for the equity premium. In this model, banks act as intermediaries that demand risk-free assets, extend credit to firms, while under the constraints imposed by the monetary authority. The model was described in detail, and both static and dynamic equilibrium conditions were derived. Several key variables, including deposit interest rates, corporate lending rates, and bank return on equity (ROE), were calibrated using Dutch data to obtain the withdrawal rate.

However, the equilibrium was not fully estimated using empirical data due to the complexity of the system. Although the observed risk-free rate could be calibrated, the resulting values diverged notably from those reported in the working paper of Dzhumashev (2025). This discrepancy may be attributed to economic differences between the U.S. and Dutch as well as the observed time frame. As such, it remains inconclusive whether the extended model provides a quantitative solution to the equity premium in the Netherlands. Future research should focus on empirically estimating the full equilibrium system using structural or numerical methods and extending the analysis to panel data across different European economies.

Ultimately, addressing the equity premium puzzle may not be defined from consumer preferences alone, but by better capturing the banks and other financial intermediaries into the model.

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8 Appendix

8.1 AEX components

Table 11: 30 March 2024 Holdings in the AEX Index

Company	Ticker	Industry	Index Holding (%)
Shell PLC	SHELL	Energy	15.16
ASML Holding	ASML	Technology	14.85
Unilever	UNA	Consumer Staples	13.79
RELX	REN	Consumer Discretionary	8.51
ING Groep N.V.	INGA	Financials	5.70
Adyen	ADYEN	Industrials	5.20
Prosus	PRX	Technology	4.79
Wolters Kluwer	WKL	Consumer Discretionary	4.06
DSM Firmenich AG	DSFIR	Consumer Staples	2.99
ASM International	ASM	Technology	2.99
Ahold Delhaize	AD	Consumer Staples	2.97
Heineken	HEIA	Consumer Staples	2.90
UMG	UMG	Consumer Discretionary	2.29
Philips KON	PHIA	Health Care	1.63
KPN KON	KPN	Telecommunications	1.54
ArcelorMittal SA	MT	Basic Materials	1.47
Akzo Nobel	AKZA	Basic Materials	1.33
NN Group	NN	Financials	1.30
BE Semiconductor	BESI	Technology	1.23
EXOR NV	EXO	Financials	1.22
IMCD	IMCD	Basic Materials	1.05
Aegon	AGN	Financials	0.87
ABN AMRO Bank N.V.	ABN	Financials	0.82
ASR Nederland	ASRNL	Financials	0.76
Randstad NV	RAND	Industrials	0.60