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Optimal CO_2 price threshold followed by a Poisson process under investment option approach for a carbon capture utilization and storage plant

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Abstract

Carbon capture utilization and storage (CCUS) is an opportunity to reduce emissions in the electricity supply sector based on coal fired power plants and transform these emissions into added value products. Given the challenge scenario and tighter emission objectives set by the authorities, the implementation of this type of technologies is necessary to achieve these new net zero objectives in the medium term. The objective of this paper is to obtain the price threshold when an electric producer has the option to invest in a CCUS plant and the subsidy price in CO_2 prices is provided with uncertainty.

First, the analytical results for the Real Option Value price thresholds for price and investment subsidy policies were obtained analytically. Then, the same model was considered when the subsidy price is provided under uncertainty. For the first model, the price thresholds were compared with respect the results by the Net Present Value Approach. the investment subsidy provision for CCUS utilization rate about 20% was the lowest price threshold achieve for the NPV and Real Option approach. The utilization rate has a notable impact in reducing the price investment threshold. The second results showed a higher price thresholds when the price subsidy provision is uncertain. specially when the price subsidy provision probability is between 0.4 and 0.7. The price thresholds are higher when the subsidy size and utilization rate increase, as a result of higher opportunity costs to invest when the subsidy is absent (has not been provided).

1. Introduction

In December of 2022, the European Union (EU) parliament ratified the increase on the reduction of net greenhouse gas (GHG) emissions from 29% to 40% by 2030 with respect to the emissions levels registered in 2005 (Council, 2023). Aiming to achieve this ambitious goal for the following years, the EU authorities decided to deploy a series of policies. Since making wider the industries cover by the EU's Emissions Trading System (EU ETS), to conduct part of the EU ETS revenues collected to protect the most vulnerable population and productive sectors [1]. Therefore, the recent economic recovery path after the COVID-19 containment shock in the middle of the energy crisis (Due to Ukraine War) have been an opportunity to accelerate the GHG emission reduction Borrell (2021). Based on these new developments, climate change policies are constantly changing and adapting to the new public health, global economic and geopolitical uncertainties.

Given this scenario where mitigation targets and policies are tighter, the international agencies have suggested the Carbon Capture Technologies (CCT) as remarkable solution to achieve these climate goals for industrial and electric generation activities. According to Pathak et al. (2022), Carbon Capture and Storage (CCS) and Carbon Capture and Utilization (CCU) technologies are worthwhile tools to mitigate the GHG of industrial sectors such as: steel, cement, and concrete. More precisely, CCT are required to mitigate the remaining CO_2 emissions after energy efficient processes and make these industries net zero emissions completely. Precisely, the industrial sector emissions are barely under one-fourth of total GHG global emissions in 2021, and one-fifth of total European emissions during the third quarter of 2022 (eur, 2023). Likewise, regarding to the existing energy supply infrastructure and aiming to achieve a sustainable GHG reduction scenario for the following decades, the International Energy Agency (2021) recognized the CCT as an important tool to achieve it. Specially, the electric production feed by fossil fuel power (i.e., oil, natural gas, and coal) among other industrial processes that generates considerable GHG emissions. As of 2021, Pathak et al. (2022) report that the electric production emissions registered for around 23% of the global share, while eur (2023) records that the EU's total share was 21% in the third quarter of 2022. Therefore, reckoning that industries and electric production plants has significant unavoidable or harder abatement of their GHG production, the CCT are a feasible alternative to transform or storage it.

About the type of CCT technologies, there are plants that transport and storage underground and into other materials the CO_2 emitted (CCS plants). Additionally, other type of technologies use the CO_2 emissions to storage and generate added value into other products (CCUS plants). Usually, the CO_2 emissions are processed and recycled

[1] In detail, there will be cover the following sectors: cement, aluminum, fertilizers, electric energy production among others industries to get free CO_2 emissions allowances and preventing the sudden fleeing industries from Europe union. The full policy kit it is detailed in Council (2023).

as the result of the electric production of a coal-fired electric plant. These emissions are put it in another production and manufacturing processes as an input factor. Explicitly, there are two CO_2 emissions utilization divisions, Enhanced Oil Recovery (EOR) and non-EOR. The economic viability of EOR utilization approach has largely assessed for CCS plants, using the CO_2 emissions to extract oil as additional revenue (Assche and Compernolle, 2022). Additionally, industrial and food production purposes beyond non-EOR utilization have evaluated as well. The industrial applications are extensively conduct to produce many products such as: inorganic and organic chemicals, polymer materials, synthesis Urea from CO_2, among others. Besides, the CO_2 transformation for the food industry is also possible and involves the use of solid and liquid CO_2, as well as and additive for beer and beverages. According to Yang et al. (2019a), for non-EOR intentions, the transportation and storage costs are lower and revenues higher. However, its is harder to process, transform, and purify the CO_2 captured required to the food production, due to highly standard quality requirements. In this document is only considered a CCUS plant for non-EOR purposes. Only the CO_2 transformation, based on Yang et al. (2019a) work, conduct to produce food and industrial products is evaluated.

Further, the potential investors should contemplate CO_2 price policies, technological development, government subsidy schemes, tax exemption, and financial funding, as relevant sources of uncertainty at the time to invest in a CCUS plant. In that sense, the researchers have used the Real Option Theory methodology (ROT) to contemplate these additional considerations to evaluate an investment in CCT, rather than the standard and static net present value valuation. In detail, the studies mostly have been focused on modelling the uncertainty on the CO_2 prices as a Geometric Brownian Motion (GBM) process. Also, the uncertainties in costs and technological changes have been evaluated (Assche and Compernolle, 2022). According to Blyth et al. (2007), the CO_2 market price has market and regulation components. Further, Zhu and Fan (2011) proposed modeling CO_2 prices as a combination of a GBM process and a jump diffusion process[2], to consider the market and uncertainty policy factors. Finally, the current and unpredictable events relate to the climate change policies are made evident the importance of these regulations and policies.

Regarding to the ETS scheme jurisdiction in Europe only considers the CCS technologies as an effective CO_2 avoidance emission. Precisely, for CCS plants the CO_2 price is counted as a free CO_2 allowance because the CO_2 emission are stored (Commission, 2012). This is an important cashflow incentive for the potential investors because the increasing cost paid for each emission will be avoided for each ton of GHG emitted throughout all activity plant time. Nevertheless, the viability for the CCUS plants to count the CO_2 emission as a avoided has not been approved yet, mostly explained because there is not enough evidence that support the sufficient management of CO_2 emissions by these technologies (Commission, 2012). However, some of the CCUS tech-

[2]According to Mariani and Florescu (2019). The diffusion component is driving by the GBM process and the jump component by the Poisson process.

nologies can participate to be funded by public and private sources [3]. Recently, the EU ETS legislation included and assured a payment for carbon removal certification. In detail, there is a new income alternative for industries that use carbon removal technologies for made products which stored carbon for a long term. However, CCUS are not included, because these technologies store or directly recycle the fossil CO_2 emissions (Commission, 2022). Although CCU projects can receive public funding, the not explicit inclusion of the CO_2 allowances as a revenue stream is a notable disincentive to invest in Europe (Assche and Compernolle, 2022).

On the other hand, the United Kingdom (U.K.) and The People's Republic of China ("China") jurisdictions are distinct about the carbon price schemes and revenue incentive policies for CCT. The U.K. government introduced in 2013 the Carbon Price Floor Mechanism (CPFM) aiming to disincentive the energy production using fossil fuels. The mechanism consists of a carbon tax set as a determined trajectory for a certain time interval (Curtis et al. (2013)). Additionally, the CO_2 price counted as an avoided emission, which is a significant incentive for the CCU potential investors (Department for Business, Energy & Industrial Strategy (2021)). At the same time, there is additional financing supporting such as: CCUS innovation and Industrial Energy transformation funds, for the development of CCUS technologies, energy efficiency improvement and reduction of CO_2 emissions (Department for Business, Energy & Industrial Strategy (2019)). On the other hand, China launched the biggest CO_2 ETS market since 2021 that only covers the power generation sector and is based on carbon intensity of each plant operation[4] (Norris, 2022). Besides, the Chinese government has mainly focused its energy decarbonization policy through CCU technologies. Further, the CCU projects can obtain revenue streams via avoiding the CO_2 allowances payment and selling the captured and utilized CO_2 emissions (International Energy Agency, 2021). Likewise, the government has been provided financial support, such as: capital support, tax incentives among others revenue aids, to make the private investments attractive and financially viable (Asian Development Bank, 2021). In contrast to the EU jurisdiction, UK's and China's encourage the financial viability for these technologies through the price payment avoidance for CO_2 emissions stored or transformed. Meanwhile, keeping different financial funding channels such as cost stabilization and reduction, tax exemptions, among others. However, the three jurisdictions (EU, U.K., and China) agree in the financial support incentives to research and development in all type of low carbon emitting technologies.

Returning to EU climate policies, although the EU ETS market has been the most relevant, longest established, and effective climate policy around the world, the ETS market dynamic is strongly influenced by the EU policies actions and expectations. The ETS tool is responsible for bringing down more than 42% of the greenhouse gas emis-

[3] Concretely for these funding initiatives: Innovation Fund, Horizon 2020, and the LIFE program.

[4] Each emitter is assigned emission free allowances, and she can sell surplus allowances as long as reducing the carbon intensity of its operations (Norris, 2022).

sions since 2005 (European Commission, 2021b). The system operates by a 'Cap and trade' scheme. The total amount of greenhouse emissions (limit or 'cap') is set each year by the EU authority (Legal Information Institute, 2022). The emitter industries[5] obtain emission allowances[6] as result of auctioning allocation (European Commission, 2021a). The allowances scheme was defined by EU authorities in 4 phases out periods where the number of allowances available will be gradually decrease [7] to reduce the GHG allowed emissions by the economic activity (European Comission, 2023) . Alongside, additional policies have been implemented during the last 2 phases (3 and 4), such as: establishing the Market Stability Reserve (MSR), and carbon leakage preventing by the Carbon Boarder Adjustment Mechanism (CBAM) (European Commission, 2023). In detail, the MSR was a mechanism introduced in 2016 to mitigate the volatility in the CO_2 price market when the CO_2 allowances are under or over supplied in the market. In addition, in July of 2021 the phase 4 introduce the CBAM, which is a mechanism that taxed the import products [8] which have a high carbon footprint in its production process and prone to be imported in countries with less binding (absent) CO_2 prices or climate change policies. Recently, the expectations in the CO_2 market have been significantly affected by the COVID-19 pandemic and the 'Green Deal'. First, the unexpected downturn in economic activity, due to the COVID-19 pandemic, lead to a statistically significant negative shock on the demand side of the CO_2 prices. (Dong et al., 2022). On the other hand, the EU's green recovery plan is positive correlated with the CO_2 price, considering the funds available and targets set to reduce the GHG emissions (Dong et al., 2022). Finally, the CO_2 supply side is affected by introducing new mechanisms and rules for the quantities in the CO_2 allowances. Demand and supply sides are directly affected by economic and climate change policies, as well as the demand has drove by expectations in different issues.

As of yet, the CO_2 policy price component has not been considered by CCS and CCU(S) literature. First, although Blyth et al. (2007) and Zhu and Fan (2011) considered CO_2 as a GBM process with stochastic jumps, the former has not been explicitly formulated for a CCUS technology through the ROT, and the last one just only suggested the CO_2 prices from a jump diffusion process. Second, as explained earlier, the EU ETS market climate is directly impacted by the climate policies actions, at the same time, these polices are contingent to other fronts (economy, public health, climate policies implementation and geopolitical circumstances). Finally, the lack of approval with respect to free allowance as an important opportunity costs avoidance for CCUS tech-

[5]Power and heat generation sector, energy industrial and aviation sectors within Europe.

[6]According to the European Commission (2021b) each allowance gives to the right to emit one ton of carbon dioxide (CO2) or corresponding amount of other powerful greenhouse gases (nitrous oxide (N2O) and perfluorocarbons (PFCS)) by the holder.

[7]In detail the phase 3 (2013-2020) the number of allowances with respect to the previous phase 2 decreased linearly each year at 1.74%; meanwhile, the phase 4 established a linear reduction of 2.2% European Commission (2021b).

[8]This mechanism will be gradually introduced in the ETS. Currently, the CBAM will be initially apply to goods imports such as: cement, iron and steel, aluminum, fertilizers, and electricity (European Comission, 2023).

nologies is an additional factor of uncertainty. Based on these considerations. Hassett and Metcalf (1998) suggested to take this unpredictable policy factors as a jump process.

The main purpose of this document is to obtain the CO_2 threshold price when an electric producer has the option to invest in CCUS plant to store or transform the CO_2 emissions, under a price subsidy provision uncertainty. The hypothetical scenario for these subsidies provision is because the authorities have committed with tighter greenhouse gas reduction targets, a sudden increase in CO2 prices is expected. Meanwhile, the revenues collected by the CO_2 price system granting an additional investment cost aid to the CCUS technologies. In detail, the policy uncertainty follows a Poisson Stochastic jump for the provision of price subsidy policy. This policy is a known price subsidy fixed premium as an additional revenue fee proportional to the CO_2 prices per each CO_2 ton stored or transformed. First, The Real Option Value price thresholds for price and investment subsidy policies, when the CO_2 prices follows a GBM process, are obtained analytically and analyzed. These results are compared with respect the Net Present Value (NPV) approach. Further, the model when the price subsidy provision, as an additional source of uncertainty, is solved numerically and analyzed. At the end, the political recommendations are formulated in light of the results for the investment option in an hypothetical CCUS plant according to the parameter values given by **?** and Zhang and Liu (2019) .

In the following section, the literature review is presented with the most relevant results of the ROT for CCS and CCUS technologies. Next, the model for carbon capture technologies is set up when there is no policy uncertainty. Then, the price subsidy model is set up when there is uncertainty in its provision. Afterwards, the first numerical results compared NPV and ROT approaches with respect to uncertainty in the CO_2 prices. Then, the results are obtained when the price subsidy provision is included as an extra source of uncertainty under the ROT approach. Finally, the conclusions and summary of results are described, along with the limitations and future research work.

2. Review of the literature

In this section, the most relevant literature review findings are presented for investment in a coal-fired power plant with a CCT option. First, the assumptions of CO_2 prices and principal results of the investigators are explained for different types of carbon capture technology investment options. Subsequently, there are explicit differences in the GBM assumptions and results with respect to price schemes ETS and CPFM. Then, investigations of other types of government intervention are illustrated essentially for numerical results and other analytical results as well. Finally, the results and the Poisson model methodology applied for different policy uncertainties are reviewed.

To clarify, the literature review for CCS plants applies to CCU technologies. The ROT methodology has been used to obtain the optimal time to investment in CCT. However, the methods and assumptions used in previous research on CCS technologies are also valuable to solve CCU investment decision models Zhang and Liu (2019). Since CCS and CCU plants face similar uncertainties for CO_2 market prices, risk in technology development, subsidy schemes, among other sources of uncertainty.

Under the ROT, researchers have considered CO_2 prices as a GBM process, aiming at the CO_2 threshold price for real options investments for coal fired electric plants [9] with or without investment in CCT. Blyth et al. (2007) considered the GBM process for CO_2 prices as the result of the interaction between expected supply and demand. Abadie and Chamorro (2008) evaluated the volatility and maturity of allowance prices (CO_2 € per ton), from the five futures contracts observed between 2008 and 2012 [10], to suggest that the CO_2 allowance price follows a non-stationary stochastic process (GBM process). Likewise, Zhu and Fan (2011) described the EU ETS as a volatile carbon price mechanism, which is better modeled by a stochastic process that reflects changes and volatility trends in CO_2 prices. Additionally, Zhang and Liu (2019) supported the previous argument and took CO_2 prices as a GBM process for the ETS in China. In general, the results showed that the growth and volatility calculations for the GBM process are determinants of these results from the literature. To be specific, an increase in the average changes in CO_2 prices (drift rate) leads to a lower CO_2 threshold price to invest in a power plant with these types of technology. However, researchers have not found conclusive results from the effect of the volatility of CO_2 price on the threshold price. Abadie and Chamorro (2008), Blyth et al. (2007), and **?** Compernolle et al.(2017) found higher thresholds when volatility increases. Meanwhile, Zhang and Liu (2019) have not found a significant impact from the volatility of the CO_2 price on the threshold price. By contrast, Zhu and Fan (2011) found that market volatility in CO_2 prices is desirable [11] , the decision makers invest in CCT sooner resulting from volatility in CO_2 prices,

[9] Blyth et al. (2007) and Abadie and Chamorro (2008) found results for coal-fired power plants. Meanwhile, (Zhu and Fan (2011)) found these results for a thermos power plant.

[10] In this case, the prices were set by the ETS scheme.

[11] Indeed, the authors found the CO_2 price policy as the main factor to make CO_2 price threshold lower.

where the uncertainty of climate policy is represented by positive CO_2 price volatility.

Other studies found the investment implications of setting CO_2 prices between ETS and CPFM schemes. Following Walsh et al. (2014) and Assche and Compernolle (2022), the CO_2 prices formed by the ETS and CPFM schemes follow a GBM process for the CCS investment option in a coal-fired energy plant in Europe [12] . Concerning the volatility of the CO_2 price, the authors recognized that the ETS scheme has a volatility greater than zero whereas the CPFM has a volatility equal to zero [13] . This is because the ETS is a volatile allocation mechanism scheme compared to the deterministic CO_2 prices imposed by the CPFM. Furthermore, Walsh et al. (2014) found an earlier investment CO_2 threshold price under the CPFM rather than the ETS scheme, when investment costs are time dependent. On the contrary, Assche and Compernolle (2022) obtained an earlier CO_2 threshold price under the ETS, when the correlation between the volatility price processes is strong and individual volatilities are not appreciable. The main difference from these results is that Assche and Compernolle (2022) allow a correlation between CO_2 price, electricity price, and separable costs. As a result, the correlation between the uncertain prices is conveyed to acquire a diversification effect that reduces the overall uncertainty. Then, the CO_2 threshold price to invest is lower under the ETS scheme.

As mentioned previously about CO_2 market policies as a source of uncertainty under the ROT methodology, similarly, the literature has focused on evaluating the effects of incentive policies (different types of subsidies for the development of CCT). In general, government subsidies were considered additional sources of uncertainty for the evaluation of CCT. The results were calculated for different subsidies schemes for the whole CCUS operation (Yang et al. (2019b)), for a given range of R&D and generating costs subsidies (Zhu and Fan (2011)), subsidies for operational costs (Huang et al. (2021)), and subsidies for initial investment costs under the CPFM scheme Dong et al. (2022).

The most remarkable assessments about the government incentives have principally focused on evaluating different subsidies over investment timing in CCT for Coal Fired electricity. Zhu and Fan (2011) evaluated the investment option for thermal power with a CCS plant. Yang et al. (2019b) estimated the initial investment, electricity tariff, and CO_2 utilization subsidies [14] for a coal-fired power plant with CCUS as an investment option. Dong et al. (2022) acquired the findings for two environmental policy scenarios. First, the CCUS investment assessment was made concerning the subsidy for the initial investment cost; then, the same policy was estimated under the CPFM scheme. Zhu and Fan (2011) found that subsidizing the generating cost and R&D input would be convenient to make a profitable investment in CCS sooner. Regarding the results for CCUS technologies, Yang et al. (2019b) showed that subsidies over electricity tariff and

[12] As discussed previously, the Great Britain has the CPFM, and European Union has the ETS framework for CO_2 pricing.

[13] In contrast to Zhang et al. (2021) assumptions for the CPFM under binomial tree real option model. In this case, the assumption for CO_2 prices volatility is greater than zero under the CPFM scheme.

[14] This type of subsidies is known as Enhanced Oil Recovery (EOR).

CO_2 utilization subsidies were better schemes to incentive innovation on decarburization technologies and reduce CO_2 process capture costs, against investment subsidies. In contrast, Zhang et al. (2021) found evidence for their second policy scenario that if the subsidy was at least 33% of the total initial cost of investment, then a feasible minimum level of CPFM would be accurate to spur a Coal-fired plant investment with CCUS technology immediately. In general, subsidies drove to mitigate operational costs are the most convenient policies to make investments in CCT more profitable. In general, these and other subsidies analysis have been done by a sensitivity and scenario analysis (Assche and Compernolle (2022))

On the other hand, Huang et al. (2021) obtained analytical results when the operational subsidy is uncertain to be retracted. The authors evaluated the effects of technology, government, and market uncertainties on a CCS technology investment option. Their findings illustrated an earlier investment in CCS technologies when the volatility of the CO_2 price is higher. Likewise, the same result was previously found by Zhu and Fan (2011). Furthermore, the findings of Huang et al. (2021) suggested that a lower uncertainty around the subsidy policy retraction can result in reducing risk and promoting investments in CCS technologies. Contrasting with respect the sensite analysis made by the other authors, investigators have represented the uncertainty of the operational subsidy through the Poisson process. However, the CO_2 price still followed a standard GBM process. In contrast, in this document, a price subsidy provision is included as an additional uncertain source for the CO_2 prices as a GBM process.

About to consider other results related to solve dynamic investment problems when the policy uncertainty follows a Poisson process. Nagy et al. (2021) estimated optimal time and capacity size for the option to invest in a renewable energy project when a settled lump-sum investment subsidy has a withdrawal risk. Besides, Chronopoulos et al. (2016) analyzed the effect on investment time and capacity size with respect to the subsidy energy market price uncertainty to be retracted and/or provided of a tariff fee subsidy in energy prices for a renewable project. The results found by Nagy et al. (2021)) for a lump-sum subsidy retraction risk was lower investment time thresholds and capacity size for a renewal energy project. Likewise, the results where similar for Chronopoulos et al. (2016) for the time investment threshold and capacity when the tariff fee subsidy has a retraction risk and the opposite when the provision is unpredictable. For both cases, the capacity size was crucial to determine the time investmetn thresholds.

The findings of the former literature have only considered CO_2 prices as a GBM process, in addition to technology costs and uncertainties of policy incentives (such as operational subsidies, R&D subsidies, among others) and different market schemes. However, such approaches have not contemplated addressing CO_2 prices from its market and regulatory components at the same time, to assess the investment option in a power plant with CCUS technologies. Following Hassett and Metcalf (1993) tax parameters are dissimilar to normal prices processes (as GBM); this is because taxes would remain static during a certain period and change abruptly; as was mentioned in the introduction, the CO_2 prices mechanisms reflect this behavior as well. Similarly, Zhu and Fan (2011) recommended considering CO_2 prices as a GBM process with a jump diffusion process, given the policy uncertainty and design deficiencies of ETS [15] . Finally, although Blyth et al. (2005) suggested that CO_2 prices should have a discreet policy intervention described by a stochastic jump process , the authors did not contemplate the solution for an CCUS investment option.

For that reason, an additional source of uncertainty for the CO_2 prices, considering a price subsidy provision of a a price subsidy in the CO_2 prices a stochastic jump defined by the Poisson process. In detail, The jump is the result of a more stringent scenario for the CO_2 emissions, an a sudden increase is known but no when will be happened. In this sense, it is accurate to model this type of policy uncertainty for a subsidy price through the Poisson jump process. In the following section, the model is set up for the provision of price an investment subsidy under certainty. Afterwards, the model with an extra source of uncertainty in the price subsidy provision is incorporated and the numerical solution for this model is proposed.

[15] This is with respect to the ETS market in China. (On the other hand, I need to find a reference for this with respect to the EU ETS market.)

3. Model

In Subsection 3.1. Two scenarios of the subsidy policy are specified and analyzed: the price and the investment subsidy. First, the assumptions and technology description for the CCUS unit are stated. Then, the analytical results under the Real Option approach are obtained, under CO_2 prices uncertain from a GBM process. However, the price (investment) subsidy with and without the price (investment) subsidy provision is executed as a certainty.

Further, in Subsection 3.2., the same model is extended for an uncertainty price subsidy provision modeled by a Poisson process. Under the price subsidy scenario, it is expected a sudden increase in the CO_2 prices denoted by a subsidy fee (θ^p) provision, with probability ($\lambda_P dt$). First, the investment regions are explained. Then, a suggested solution for the model is proposed. Finally, the bounded conditions are specified to obtain the numerical solution in the next section.

3.1. Model specification during for CO_2 prices uncertainty and certainty for policies provision (Price and investment subisidies)

The investor is a coal-fired producer governed by the EU ETS. Then, for each CO_2 emission ton corresponds to a bought emission allowance. The agent has two options: pay the price for each CO_2 emission ton or invest in a CCUS plant. When investing, the producer has to pay the investment cost and operational costs related to transport, transformation, and storage of CO_2 emissions, while gaining revenue from avoiding CO_2 emissions and the sale of products made by transformation CO_2. It is assumed that the CO_2 captured emissions are permanently avoided for storage or use; because, the ETS legislation does not count as an emission. Regarding the actual application of these subsidies, the price subsidy fee in the CO_2 prices is a price increasing factor (θ_P) due to the execution of a phase out of CO_2 allowances in the ETS market. Meanwhile, the investment subsidy (θ_I) is a proportional discounted factor to the sunk investment costs which is provided by the government to promote the investments in CCUS technologies. In this case, both susidies are provided with certainty. For simplicity, the option of temporarily suspending when the CCUS operational costs are higher than the revenues is not considered. However, Compernolle et al.(2017) considered this additional decision option. In this case, it is not considered that the producer has this additional option. [16].

Furthermore, the production time frame is annual for revenues and operating costs. The evaluation of the investment option for the CCUS plant is on an infinite and con-

[16] In detail, Compernolle et al.(2017) determined the critical CO_2 price threshold (with respect to the avoided allowances or unit costs) is at least as greater than the operational costs entailed for the operation of the capture unit. The operation can be temporarily suspended when the price CO_2 is less than the operating costs.

tinuous time horizon. When the producer decides to invest in the CCUS plant, it is assumed that this will be built immediately. The CCUS plant does not improve oil recovery (as in Compernolle et al.(2017)), the CCUS only performs the transformation CO_2 from emissions to industrial and food goods, following Yang et al. (2019a). Cash flows are discounted by r interest applied to energy companies. In the assessment of the real option theory, the producer has the opportunity to wait for new information until the opportunity cost (value of holding the option) is equal to the net value of the CCUS plant (after the investment cost); there, the producer exercises the option. In this case, the source of uncertainty in CO_2 prices is only considered.

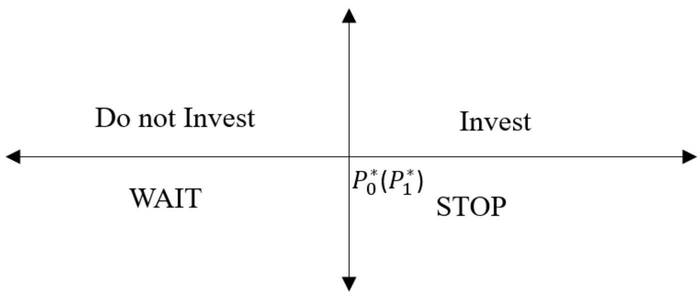
The price of carbon dioxide in any instantaneous time $P(t)_c = P_c$ follows a GBM process:

$$dP_c = \mu P_c d_t + \sigma P_c dz_t \tag{1}$$

Where μ is the drift or growth rate, σ is the volatility of the stochastic price, and dz_t is the delta of a Wiener process. Additionally, as stated in Dixit and Pindyck (1994) $r > \mu$, otherwise, it will be profitable to invest in time equal to infinite. The unit time interval is one year.

The investment model solution is brought about only under uncertainty with respect to the prices of P. Then the result of this model is contrasted with respect to the solution of the model when the price (investment) subsidy is provided permanently to all t. In this case, the investor has to decide only between two decision regions (waiting and stopping region).

Figure 1: Waiting and stopping regions



In Figure 1, P_0^* corresponds to the price threshold when the policy intervention is certainly not carried out, and P_1^* is the price threshold when the policy intervention is carried out. In the waiting region, the investor does not invest until the price is high enough to make the option to invest profitable. On the other hand, when the price is higher than the price thresholds P_i^*, the investor stops and invests. In this document, these price thresholds are considered as the time to invest. Specifically, at this price, the producer is indifferent between waiting (no investment) or stopping (investing).

Thus, the intertemporal maximization problem for the investment model when policy intervention does not (does) impact the investment value $V(P_0)(V(P_1))$ yields:

$$V(P_0) = V_0 = \max_T \quad E\left\{ \int_t^\infty e^{-rt} Q_c[P_{0c} + \eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}]dt - I(e^{-rT}) \right\} \quad (2)$$

Then since instant T on-wards, the net present of the investment option:

$$V(P_0) = V_0 = \int_0^\infty e^{-rt} Q_c[e^{\mu t} P_{0c} + \eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}]dt - I \quad (3)$$

In addition, the same happens when the subsidy price share (θ) with respect to the CO_2 price is already provided. Then, the investment value for P_{1c} is:

$$V(P_1) = V_1 = \int_0^\infty e^{-rt} Q_c[e^{\mu t} P_{1c}(1+\theta^P) + \eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}]dt - I \quad (4)$$

where P_c is the price of carbon dioxide per CO_2 ton set by the EU ETS. $\theta^P$is the price subsidy proportion between 0 and 1. Q_c is the amount of CO_2 emissions in tons generated by the electric producer. η is the utilization ratio of the CO_2 emissions, the remaining CO_2 emissions $(1-\eta)$ are transported and stored per each not used CO_2 ton. In particular $\eta \in (0,1)$, when $\eta = 0$ the total emissions of CO_2 are transported and stored in this case, the CCUS plant is a CCS, on the contrary, occurs when $\eta = 1$ and the CCUS is a CCU. Furthermore, C_{cs} corresponds to the total costs for transport and storage of the emission proportion $1-\eta$; $C_{o\&m}$ are the operating and maintenance costs for the total amount of CO_2 processed regardless of the purpose. The profit derived from the utilization of CO_2 emissions is π_u. Finally, I is the initial investment cost sunk.

The operational costs related to emissions storage are described by 5:

$$C_{cs} = (C_t - C_s) \quad (5)$$

C_T corresponds to transportation and C_S to storage, the $(1-\eta)Q_c$ remaining amount emission proportion.

Regarding utilization revenues and costs, the expression 7 yields the following:

$$\pi_u = (P_u - C_u) \quad (6)$$

Here, P_u is the weighted price of the goods per ton made by transformation CO_2 and C_u is the weighted cost to make it.

Further, the weighted prices and costs are given by the following expression:

$$P_u = w_f P_f + (1 - w_f) P_{in} \tag{7}$$

w_f is the proportion of food products sold and $(1 - w_f)$ is the remaining proportion of industrial products sold per each Q_C ton transformed.

Explicitly, the net present value for the two policy cases is presented in the following proposition.

Proposition 1. *The expected net present value of revenues when the price subsidy is not implemented V_0:*

$$V_0 = Q_c \left[\frac{P_{0c}}{(r-\mu)} + \frac{\eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}}{r} \right] - I \tag{8}$$

In addition, the expected net present value of revenues when the price subsidy is implemented V_1:

$$V_1 = Q_c \left[\frac{P_{1pc}(1+\theta^P)}{(r-\mu)} + \frac{\eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}}{r} \right] - I \tag{9}$$

Next, Proposition 2 specifies the suggested investment value and the value obtained for the waiting and stopping region (Figure 1), respectively.

Proposition 2. *The investment value $V(P_0)(V(P_1))$ when the price subsidy is not (is) executed is equal to*

$$V(P_0) = \begin{cases} A_0 P_{0c}^{\beta_1} & if\ P_c < P_0^* \\ Q_c \left[\frac{P_{0c}}{(r-\mu)} + \frac{\eta\pi_u-(1-\eta)C_{cs}-C_{o\&m}}{r} \right] - I & if\ P_c > P_0^* \end{cases} \tag{10}$$

Meanwhile, the investment value when the price subsidy policy is executed yields:

$$V(P_1) = \begin{cases} A_{1p} P_{1pc}^{\beta_1} & if\ P_c < P_1^* \\ Q_c \left[\frac{P_{1pc}(1+\theta^P)}{(r-\mu)} + \frac{\eta\pi_u-(1-\eta)C_{cs}-C_{o\&m}}{r} \right] - I & if\ P_c > P_1^* \end{cases} \tag{11}$$

$\beta(0)_1 > 1$ *corresponds one solution for the quadratic polynomial equation* $Q(0) = \sigma^2\beta(\beta-1)\frac{1}{2} + \mu\beta - r = 0$. *In Appendix A there is explicitly detailed the roots* $\beta(0)_1$ *and* $\beta(0)_2$.

Consequently, in Proposition 3 the coefficients A_0 and A_1, and the threshold values P^*_{0c} and P^*_{1pc} are depicted. These parameters satisfy the initial value, value matching, and smooth paste conditions for each policy scenario. In other words, for both cases, getting the price threshold when the benefit to invest is higher enough that it is not valuable to hold the option and invest later. In the appendix, all the propositions state in this document is proven.

Proposition 3. *The price threshold P^*_{0c} and constant A_0 , when the investment subsidy policy is not implemented, are:*

$$P^*_{0c} = \frac{(r-\mu)\beta(0)_1}{(\beta(0)_1 - 1)}\left[\frac{I}{Q_c} + \frac{(1-\eta)C_{cs}}{r} + \frac{C_{o\&m}}{r} - \frac{\eta\pi_u}{r}\right] \tag{12}$$

$$A_0 = \frac{Q_c}{(r-\mu)\beta(0)_1}\left(\frac{(r-\mu)\beta(0)_1}{(\beta(0)_1 - 1)}\left[\frac{I}{Q_c} + \frac{(1-\eta)C_{cs}}{r} + \frac{C_{o\&m}}{r} - \frac{\eta\pi_u}{r}\right]\right)^{(1-\beta(0)_1)} \tag{13}$$

Meanwhile, price threshold P^*_{1pc} and constant A_{1p}, when the investment subsidy policy is executed, are:

$$P^*_{1pc} = \frac{(r-\mu)\beta(0)_1}{(\beta(0)_1 - 1)}\left(\left[\frac{I}{Q_c} + \frac{(1-\eta)C_{cs}}{r} + \frac{C_{o\&m}}{r} - \frac{\eta\pi_u}{r}\right]\left(\frac{1}{1+\theta^P}\right)\right) \tag{14}$$

$$A_{1p} = \frac{Q_c}{(r-\mu)\beta(0)_1}\left(\frac{(r-\mu)\beta(0)_1}{(\beta(0)_1 - 1)}\left(\left[\frac{I}{Q_c} + \frac{(1-\eta)C_{cs}}{r} + \frac{C_{o\&m}}{r} - \frac{\eta\pi_u}{r}\right]\left(\frac{1}{1+\theta^P}\right)\right)\right)^{(1-\beta(0)_1)} \tag{15}$$

A direct result of Proposition 3 is $P^*_{0c} \geq P^*_{1pc}$. This is because the price subsidy θ^P, as a fraction between 0 and 1, increases revenues in any time interval d_t. In that sense, *ceteris paribus* the opportunity cost of waiting for the investment to occur later is greater, because the expected investment value increases rapidly. Then the decision maker will invest sooner.

On the other hand, in the next expressions 16- 17; and Propositions 4-6, the same results are detailed for the investment subsidy.

Then since instant T on-wards, the net present of the investment option:

$$V(P_0) = V_0 = \int_T^\infty e^{-rt} Q_c[e^{\mu t}P_{0c} + \eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}]dt - I \tag{16}$$

When the investment price share (θ^I) is discounted relative to the investment value I. Then, the investment value for P_{1c} is:

$$V(P_1) = V_1 = \int_T^\infty e^{-rt} Q_c[e^{\mu t} P_{1Ic} + \eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}]dt - I(1-\theta^I) \tag{17}$$

The main difference from 4 is that the investment value $(1-\theta^I)I$ is already known at time T. Thus, the size of the subsidy θ^I is not discounted by the interest rate r.

Proposition 4. *The expected net present value of revenues when the investment subsidy is not implemented V_0:*

$$Q_c\left[\frac{P_{0c}}{(r-\mu)} + \frac{\eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}}{r}\right] - I \tag{18}$$

Furthermore, the expected net present value for revenues when the investment subsidy is implemented V_1:

$$Q_c\left[\frac{P_{1Ic}}{(r-\mu)} + \frac{\eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}}{r}\right] - (1-\theta^I)I \tag{19}$$

Proposition 5. *The investment value $V(P_0)(V(P_1))$ when the investment subsidy policy is not (is) executed is equal to*

$$V(P_0) = \begin{cases} A_0 P_{0c}^{\beta_1} & if\ P_c < P_0^* \\ Q_c\left[\frac{P_{0c}}{(r-\mu)} + \frac{\eta\pi_u-(1-\eta)C_{cs}-C_{o\&m}}{r}\right] - I & if\ P_c > P_0^* \end{cases} \tag{20}$$

Meanwhile, the investment value when the investment subsidy policy is executed yields:

$$V(P_1) = \begin{cases} A_1 P_{1Ic}^{\beta_1} & if P_c < P_1^* \\ Q_c\left[\frac{P_{1Ic}}{(r-\mu)} + \frac{\eta\pi_u-(1-\eta)C_{cs}-C_{o\&m}}{r}\right] - (1-\theta^I)I & if P_c > P_1^* \end{cases} \tag{21}$$

$\beta(0)_1 > 1$ corresponds one solution for the quadratic polynomial equation $Q(0) = \sigma^2\beta(\beta-1)\frac{1}{2} + \mu\beta - r = 0$. In Appendix xx, the roots $\beta(0)_1$ and $\beta(0)_2$ are explicitly detailed.

Proposition 6. *The price threshold P^*_{0c} and constant A_0, when the investment subsidy policy is not taken place, are:*

$$P^*_{0c} = \frac{(r-\mu)\beta(0)_1}{(\beta(0)_1 - 1)}\left[\frac{I}{Q_c} + \frac{(1-\eta)C_{cs} - C_{o\&m}}{r} - \frac{\eta\pi_u}{r}\right] \tag{22}$$

$$A_0 = \frac{Q_c}{(r-\mu)\beta(0)_1}\left(\frac{(r-\mu)\beta(0)_1}{(\beta(0)_1 - 1)}\left[\frac{I}{Q_c} + \frac{(1-\eta)C_{cs} - C_{o\&m}}{r} - \frac{\eta\pi_u}{r}\right]\right)^{(1-\beta(0)_1)} \tag{23}$$

Meanwhile the price threshold P^*_{1Ic} and constant A^*_{1I}, when the investment subsidy policy is executed, are:

$$P^*_{1Ic} = \frac{(r-\mu)\beta(0)_1}{(\beta(0)_1 - 1)}\left[\frac{(1-\theta^I)I}{Q_c} + \frac{(1-\eta)C_{cs} - C_{o\&m}}{r} - \frac{\eta\pi_u}{r}\right] \tag{24}$$

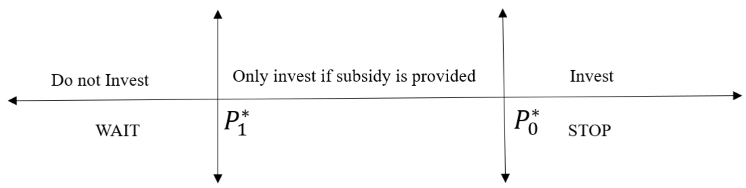
$$A_{1I} = \frac{Q_c}{(r-\mu)\beta(0)_1}\left(\frac{(r-\mu)\beta(0)_1}{(\beta(0)_1 - 1)}\left[\frac{(1-\theta^I)I}{Q_c} + \frac{(1-\eta)C_{cs} - C_{o\&m}}{r} - \frac{\eta\pi_u}{r}\right]\right)^{(1-\beta(0)_1)} \tag{25}$$

A direct result of Proposition 6 is $P^*_{0c} \geq P^*_{1Ic}$. This is because the price subsidy θ^I (between 0 and 1) lowers the investment cost and the total investment benefits to invest are higher in any time interval d_t. In that sense, the electric producer invests earlier. Finally, in the following section, the characterization and solution of the model are presented when the price subsidy provision is uncertain.

3.2. Model specification during uncertainty for CO_2 prices and price subsidy provision

In this case, the assumptions stated in subsection 3.2 are keep it, including the uncertainty of CO_2 prices as a GBM process. In this case, the policy uncertainty is about the provision of suddenly price subsidy fee (θ_P) in the CO_2 prices. About the actual application of this subsidy, it is like sudden increase in the CO_2 prices as consequence of a tighter phase-out pace of additional CO_2 allowances to be implemented in the ETS market, but it is uncertain when it will be executed. Then, the subsidy provision is added as another source of uncertainty that affected the expressions state in 10 and 11. To be precise, the electric producer is facing three decision regions.

Figure 2: Decision maker regions



In detail, the energy producer decides to wait or stop (not invest or invest) according to the price threshold given for each policy scenario (P^*_{ipc}). Then, the time frame decisions are as follows:

- $(0, P^*_{1pc})$ waiting time for the firm regardless of whether the policy is in effect or not
- (P^*_{1pc}, P^*_{0pc}) Invest if the policy is implemented. Otherwise, the potential investor will wait until the policy is executed
- (P^*_{0pc}, ∞) Invest because the revenues will be large enough that the uncertainty of the policy will not affect the decision to invest. Therefore, this region is called the stopping region.

In addition, the provision of the investment subsidy policy is uncertain and is provided with probability $\lambda_p dt$ in any time interval dt. On the contrary, $1 - \lambda_p dt$ is the probability that is not provided for the same time interval. In detail, the probability follows a Poisson process.

$$\epsilon_t^P = \begin{cases} 1 & if \quad price \quad subsidy \quad has \quad already \quad provided \quad at \quad time \quad t \quad or \quad earlier \\ 0 & otherwise \end{cases} \tag{26}$$

ϵ_t^P is identical and independent distributed variable, as well as with respect to P_c. The size of the subsidy θ^P is known between 1 and 0.

When the subsidy has not been provided, the NPV yields the following result.

Proposition 7. *The expected net present value of net revenues when the price subsidy has not been provided V_0:*

$$V_0 = Q_c \left[\frac{P_{0pc}(1 + \lambda_P \theta^P)}{r - \mu} + \frac{\eta \pi_u - (1 - \eta) C_{cs} - C_{o\&m}}{r} \right] - I \tag{27}$$

The main difference from 3 is the probability that the subsidy will be provided λ_p

Lets detail the expected value function when the option to defer is possible. The intertemporal maximization problem that aims at obtaining the optimal time threshold T to invest is:

$$V = \max_{T} \quad E\left\{ \int_{T}^{\infty} e^{-rt} Q_c [P_c (1+\theta_P * 1_{\epsilon^P_{(T)}}) + \eta\pi_u - (1-\eta) C_{cs} - C_{o\&m}] dt - I e^{-rT} \quad |\epsilon^P_0 = 0 \right\} \tag{28}$$

At time 0 the subsidy is not provided. Then, the probability that the price subsidy θ is provided in following interval time dt is $\lambda_p dt$, in this case, value function is 9 ($V_1(P+dP)$) . In contrast, when the investment has not been provided, $1-\lambda_P dt$, is the probability that the investment value continues without the price subsidy($V_0(P+dP)$) given by 27. The weighted value function associated with the subsidy provision yields the following.

$$V = e^{-rdt}[\lambda_p dt E(V_1(P+dP)) + (1-\lambda_p dt) E(V_0(P+dP)] \tag{29}$$

It is notable in 28 that compared to the case of the investment subsidy provision, the probability that the subsidy will be provided at any instant t remains after P^*_{0pc}. On the other hand, under uncertainty for the investment subsidy provision , the electric producer knows at the moment to invest T after P^*_{0pc} that the the subsidy θ^I has not been provided[17]. Otherwise, the electric producer had invested before P^*_{0pc}, when the subsidy is available (Figure 2).

The proposition 7 specifies the suggested value of the investment option for both policy cases:

Proposition 8. *The investment value $V(P_1)(V(P_0))$, when the policy is (is not) executed, is equal to*

$$V(P_1) = \begin{cases} A_1 P_c^{\beta(0)_1} & if \;\; P < P^*_{1pc} \\ Q_c \left[\frac{P_c(1+\theta^P)}{r-\mu} + \frac{\eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}}{r} \right] - I & if \;\; P > P^*_{1pc} \end{cases} \tag{30}$$

where A_1 is the constant parameter value when the policy subsidy is executed. Similarly, $\beta(0)_1 \geq 1$ is the positive root value of the quadratic polynomial 60. The value of these parameters are shown in Proposition 1.

Meanwhile, the investment value when the policy is not executed yields:

$$V(P_0) = \begin{cases} A_1 P_c^{\beta(0)_1} + C_1 P_c^{\beta(1)_1} & if \;\; 0 < P < P^*_{1pc} \\ B_1 P_c^{\beta(1)_1} + B_2 P_c^{\beta(1)_2} + Q_c \left[\frac{P_c \lambda_P (1+\theta)}{(r-\mu)(r+\lambda_P-\mu)} + \left(\frac{\lambda_P}{(r+\lambda_P)}\right) \frac{\eta\pi_u-(1-\eta)C_{cs}-C_{o\&m}}{r} \right] \\ -\frac{\lambda_P I}{r+\lambda_P} & if \;\; P^*_{1pc} \leq P < P^*_{0pc} \\ Q_c \left[\frac{P_c(1+\lambda_P\theta^P)}{(r-\mu)} + \frac{\eta\pi_u-(1-\eta)C_{cs}-C_{o\&m}}{r} \right] - I & if \;\; P^*_{0pc} \leq P < \infty \end{cases} \tag{31}$$

[17] In this case, V_0 had remained as stated it in the bottom equation 20

$\beta(1)_1 > 1$ and $\beta(1)_2 < 0$ correspond to the roots for the quadratic polynomial equation: $Q(1) = \sigma^2\beta(\beta-1)\frac{1}{2} + \mu\beta - (r+\lambda_p) = 0$ Besides, the parameter values for C_1, B_1, and B_2 are constant unknown.

The electricity producer is submitting to the price thresholds P^*_{1c} and P^*_{0c} to invest. The expression 31 specified that even when the subsidy is already provided, the price value P must be at least as high as P^*_{1pc} to stop and invest. Then, following Chronopoulos et al. (2016) the top side of 31 shows an additional factor $C_1 P^{\beta(1)_1}$ as an adjustment term for (V_0) when the investment subsidy is not yet available and the prices are lower than P_{1pc}. Next, in the middle expression of 31, P values between P^*_{1pc} and P^*_{0pc}, the expected net profit is given by the last to terms. The first term corresponds to the probability of investment if the subsidy is not provided, and the second term shows the probability that the price will drop below P^*_{1pc}. Finally, when the price is higher than at least P^*_{opc}, the electric producer invests.

Furthermore, in the bottom part of 31 is depicted the net expected revenues receive it by the electric producer after the investment, when P is greater than P^*_{opc}. This expected revenues are still affected by the price subsidy provision probability λ_P. Meanwhile, the third and fifth term of the middle part of 31 are similar as found by Huisman and Kort (2000) for the follower suggested solution when is waiting the Technology 2. In particular, this represents the discounted cash flows generated from the investment time onwards when the subsidy is available at the moment of invest. The denominator differences between these two terms is given by the subtracted μ because of the expected increase in P_c.

Proposition 9. *Given that the investment subsidy has not yet been provided. There are in total four equations, which correspond to four parameters. In detail, the threshold P^*_0 and 3 unknown constants (C_1, B_1 and B_2). The solution for the threshold P^*_0 is given for the value matching and smooth pasting conditions:*

- *Value Matching $V_0(P^*_0) = V_0(P^*_0)$:*

$$B_1 P_{0c}^{\beta(1)_1} + B_2 P_{0c}^{\beta(1)_2} + Q_c\left[\frac{P_{0c}\lambda_P(1+\theta)}{(r-\mu)(r+\lambda_P-\mu)} + \left(\frac{\lambda_P}{(r+\lambda_P)}\right)\frac{\eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}}{r}\right] - \frac{\lambda_1 I}{r+\lambda_P} = Q_c\left[\frac{P_{0c}(1+\lambda_P\theta^P)}{(r-\mu)} + \frac{\eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}}{r}\right] - I \quad (32)$$

- *Smooth Pasting $V_0'(P^*_0) = V_0'(P^*_0)$:*

$$B_1(\beta(1)_1) P_{0c}^{(\beta(1)_1 - 1)} + B_2(\beta(1)_2) P_{0c}^{(\beta(1)_2 - 1)} + \frac{Q_c\lambda_P(1+\theta)}{(r-\mu)(r+\lambda_P-\mu)} = \frac{Q_c(1+\lambda_P\theta^P)}{(r-\mu)} \quad (33)$$

Then, continuity and differentiable conditions have to be achieved for $V(0)$, when $P = P_1$:

- *Continuity (equaling top and middle part of 31)*

$$A_1 P_{1c}^{\beta(0)_1} + C_1 P_{1c}^{\beta(1)_1} = B_1 P_{1c}^{\beta(1)_1} + B_2 P_{1c}^{\beta(1)_2}$$
$$+Q_c \left[\frac{P_{1c}\lambda_P(1+\theta)}{(r-\mu)(r+\lambda_P-\mu)} + \left(\frac{\lambda_P}{(r+\lambda_P)} \right) \frac{\eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}}{r} \right] - \frac{\lambda_1 I}{r+\lambda_P} \tag{34}$$

- *Differentiable (differentiating with respect to P_1 and equalizing the top and middle part of 31):*

$$A_1(\beta(0)_1) P_{1c}^{\beta(0)_1 - 1} + C_1(\beta(1)_1) P_{1c}^{\beta(1)_1 - 1} =$$
$$B_1(\beta(1)_1) P_{1c}^{(\beta(1)_1 - 1)} + B_2(\beta(1)_2) P_{1c}^{(\beta(1)_2 - 1)} + \frac{Q_c \lambda_P (1+\theta)}{(r-\mu)(r+\lambda_P-\mu)} \tag{35}$$

In Proposition 8 the implicit solutions are obtained for the price threshold P_0^* and the remaining unknown parameters. It is necessary to satisfy these bounded conditions numerically, in order to obtain an explicit solution. In the following subsection, the sensitive analysis results for the models (3.1 and 3.2) specified previously are shown.

4. Results

This section presents the results of the investment analysis. First, the parameter values are depicted and explained to target it to a CCUS plant. Second, the real option analysis is compared with respect to the NPV approach. Price threshold differences are described and explained when price and investment subsidies are and are not provided with certainty. Likewise, a sensitive analysis for the price thresholds under the real option valuation approach is obtained with respect to θ, η, and σ. Third, the results are presented when the subsidy provision is unpredictable by the sensitive factor provision λ_p. Furthermore, a sensitive analysis is performed using a real option value approach with respect to λ_P,θ, η, and σ.

4.1 Parameter values explanation

To obtain numerical results, in Table 1 are detailed the parameters for a CCUS unit. Mostly of the parameters for the operational performance of the plant are taken from Compernolle et al.(2017) The goals of simplification, capture efficiency, annual operating cost, annual operating and maintenance cost, and investment cost for a CCUS unit are based on a CCS plant. Likewise the amount of emissions processed Q_c by the plant is based on a power producer who has two options for building a 'coal fueled super critical steam turbine power plant' electric plant, with or without the CCS unit. Then, the costs differences between these constructions are the costs of the CCS unit. Finally, the parameters obtained by Abadie and Chamorro (2008) were used to calculate the operation and maintenance costs of this plant, according to the amount of CO_2 emission captured from this CCS plant annually.

On the other hand, according to Zhang et al. (2021) it is considered the capture efficiency of a CCUS plant about 90% . However, other authors considered that this capturance efficiency can be lower 85% Compernolle et al. (2017) and 80% Abadie and Chamorro (2008) .

The parameters for the utilization technology are referred to Zhang and Liu (2019). In detail, the utilization rate and value of the transformed prices. For simplicity, it is assumed that the additional costs to transform CO_2 emissions into new products are zero. However, the utilization rate is limited to a maximum of 20% of the total emissions captured. In that sense, the marginal profits obtained from the sales of the transformed products given by 7 are P_u.

Finally, the values with respect to the GBM process for prices CO_2 for the growth rate (μ) and volatility (σ) are taken from Compernolle et al. (2017) based on Lukas and Welling (2014). And the discount rate (r) for electric plant projects is taken from the same authors, but based on Pershad et al.(2012).

4.2 Conventional Net Present Valuation approach vs real option investment valuation

Given that the electricity producer faces a valuable investment decision to make, an accurate valuation of these investments given the characteristics of the technology and available policies is decisive. Comparison of the price threshold between the NPV and ROT approach suggests a higher price to invest in these technologies considering the uncertainty in prices.

Table 3 shows the price threshold values by the net present value and the real option value, respectively, with respect to the utilization rate (η) and the subsidy proportion (θ). The results are obtained for $\eta = 0.1$ and $\eta = 0.2$ for the same price and an investment proportion subsidy of about 20%. First, the price thresholds P^*_{1pc} are lower relative to the price threshold without any subsidy provision. These results were previously stated in Propositions 3 and 6. In particular, the lowest price threshold is given by the investment subsidy (0.2) for $\eta = 0.2$. The investment subsidy case with an η greater than zero can be seen as a 'double' subsidy for a CCUS unit. First, by a higher proportion of the CO_2 amount transformed and sold, the income flow will increase at each time t with certainty. At the same time, increasing the total proportion used to make new products reduces the cost of transporting and storing the not used CO_2 emissions. Furthermore, for all cases, the threshold value using the NPV approach is lower than the value obtained using the option investment theory. NPV analysis suggests that the producer invests sooner without making an accurate assessment (or subestimation) of the volatility of the CO_2 prices.

Table 1: Parameter values

Variable	**Description and References**	**Parameter**	**Value**	**Unit**
Annual CO2 emission of a CCUS plant.	Total annual emissions of the electric producer. According to Compernolle et al. (20117) based on Piessens et al. (2012). The power plant produced 7013 (GWh) every year.	Q_c.	4590000	Ton
Capture efficiency of a CCUS plant.	Net emissions effectively captured by the carbon capture plant according to Zhang et al. (2021) based on IPCC (2014).	m	0.9	%
Utilization rate of a CCUS plant.	Proportion of the effectively captured emissions allocated to CO_2 transformation. According to Zhang et al. (2021), based on IPCC (2014).	η	0.1-0.2	%.
Weighted utilization prices.of a CCUS plant.	Weighted prices (60% food products and 40% industrial prices).	P_U.	57.	€/ton.
Industry prices.	Middle prices of the industrial material produced by transfroming the CO2 emissions. With reference to Zhang et al. (2021). Conversion exchange rate 7.45 RMB per 1 EUR for 2022 prices.	P_{in}	48	€/ton.
Food prices.	Middle prices of the food material produced by transforming the CO2 emissions. With reference to Zhang et al. (2021). Conversion exchange rate 7.45 RMB per 1 EUR at end of 2021 prices.	P_f	73	€/ton.
Annual operational Costs of a CCS operation	Annual operation costs for a CCS unit. According to Compernolle et al. (2017) based on Piessens et al. (2012).	C_{cs}	7.22	€/ton.
Annual operation and maintenance Costs of a CCUS plant	Annual operation and maintenance costs own calculations for a CCS unit. Based on Abadie and Chamorro (2008)	$C_{o\&m}$	1.87	€/ton.
Investment cost value of a CCUS operation	Investment cost for a CCS unit. According to Compernolle et al. (2017) based on Piessens et al. (2012).	I	1040	Million of €/ton
Growth rate of C_{O2} price.	According to Compernolle et al.(2017) based on Lukas and Welling (2014).	μ	0.05	%
Volatility of C_{O2} price.	According to Compernolle et al.(2017) based on Pershad et al.(2012)	σ	0.2	%
Interest rate return for electric plant projects.	Discounted rate for energy projects According to Compernolle et al. (2017) based on Lukas and Welling (2014).	r	0.1	%.

Table 3: NPV price vs Real Option Theory Valuation Theory when price subsidy provision is certain

	$\eta = 0.1$ and $\theta = 0.2$			$\eta = 0.2$ and $\theta = 0.2$		
	P^*_{0c}	P^*_{0pc}	P^*_{0Ic}	P^*_{0c}	P^*_{0pc}	P^*_{0Ic}
NPV-I	12.25	10.21	9.73	9.24	7.70	6.72
ROT	37.33	31.11	25.73	29.37	24.48	17.77

In detail, the price thresholds for each policy scenario that makes the investment profitable according to the NPV approach yields:

Net present value without subsidy

$$P^*_{npv1} = (r * I/Q_c) - \eta * P_u + (1-\eta) * C_{cs} + C_{o\&m} \tag{36}$$

Net present value with price subsidy

$$P^*_{npv2} = ((rI/Q_c) - \eta P_u + (1-\eta)C_{cs} + C_{o\&m})(1/(1+\theta)) \tag{37}$$

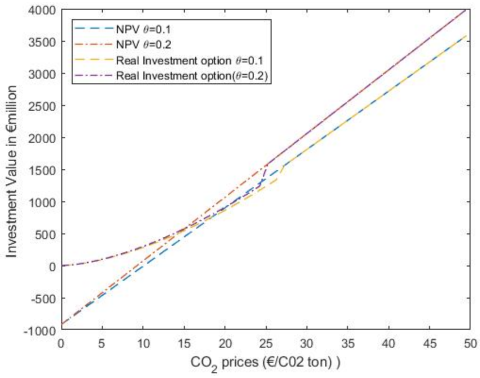
Net present value with investment subsidy

$$P^*_{npv3} = (r(1-\theta)I/Q_c) - \eta P_u + (1-\eta)C_{cs} + C_{o\&m} \tag{38}$$

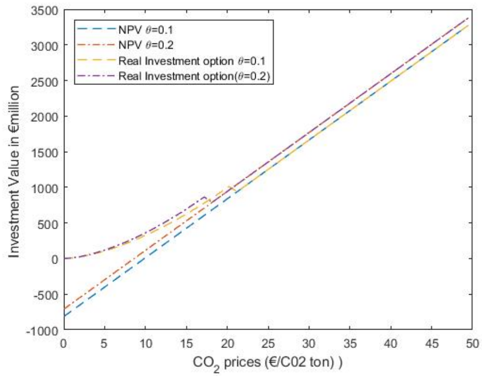
Additionally, the difference between both thresholds is not maintained equally. The percentage difference between the real valuation threshold with respect to NPV is the same for the thresholds without subsidies and the price subsidy for both values considered of η (this difference is around 32.2%). In contrast, for the investment subsidy price threshold, this gap decreases significantly, around 10.9% and 3.4% for $\eta = 0.1$ and $\eta = 0.2$, respectively. This suggests that participation in the total income of the CCUS through avoidance of prices CO_2 becomes lower when income from the sale of food and industrial products increases. Meanwhile, the investment cost is discounted by 20%. In other words, the opportunity cost to wait is higher and it is profitable to invest earlier at a lower CO_2 price. This makes the price threshold of the real option approach closer to that suggested by the NPV threshold.

In the end, these results reflect the powerful effect of using CO2 and transform it into valuable values. To be precise, Zhang and Liu (2019) found the same results through the Monte Carlo Simulation Method approach; the utilization rate approach for the CCUS technology proposed in this document is also based on these authors. Although the utilization rate is set to a maximum of 20% and production costs as zero for simplicity, it would be a significant reason for making profit per each CO_2 ton transformed.

Figure 1: Effect of the size subsidy (θ) on the value of the option to invest for the price (left) and the investment subsidy (right). The values of the remaining parameters are described in Table 1



(a) The real option investment valuation and CO_2 prices when price subsidy is provided



(b) The real option investment valuation and CO_2 prices when investment subsidy is provided

The discounting factor for the NPV in this case change and is $\frac{1}{r-\mu}$ which differs with respect to equations 36 - 38, but correspond to discounted factor for P as a GBM process.

4.1.1 Price threshold approach for price and investment subsidy analysis

Figure 1 shows the option value for the price and investment subsidy when theta values are 0.1 and 0.2. As described in Table 3, the price thresholds are lower for the investment subsidy case. Panel A describes the real option investment value for the price subsidy. The value of the NPV approach increases, as well as the value of the subsidy increases, keeping the same point of origin. Meanwhile, the NPV and the value of the investment option coincide at two points, satisfying the condition of matching values. However, the smooth pasting condition is satisfied only for the second point (27.27 for $\theta = 0.1$ and 25.25 for $\theta = 0.2$) to obtain the optimal point to exercise the investment option optimally. Furthermore, the price subsidy increases the opportunity cost of waiting and investing later at any time, when the CO_2 price is equal to the price threshold for each subsidy policy P^*_{1c}. Just after the investment time threshold, the additional revenue gain is given by the gap between the red and purple; and yellow and blue lines. Panel B represents the real option investment value for the investment subsidy. The NPV investment value approach for both theta values is separated and does not start from the same point as in panel A. In this case, the value of holding the option and investing later decreases significantly when the investment cost is discounted. This gap value is reflected between the purple and red and yellow and blue lines.

4.1.2 Price threshold dynamic with respect to volatility on CO_2 prices analysis

In panel (A) of Figure 2. The subsidy price threshold P^*_{1C} relate to the price threshold without subsidy provision P^*_{0c} is described, for the values of η and θ, between the volatility range σ 0.1 and 1. In general, the price threshold increases as long as CO_2 the volatility price does. This is because CO_2 price revenues with high uncertainty need a higher price threshold to make profitable the net gain of the investment in the CCUS unit. In addition, the price thresholds are lower when the price subsidy $\theta = 0.2$ and the utilization rate $\eta = 0.2$. Meanwhile, the difference is subtle between P^*_{1pc} with $\eta = 0$ and P^*_{0p} with $\eta = 0.2$.

In panel (B) of Figure 2. The same values are presented, but with respect to the investment subsidy provision. the same trend is presented for the threshold values P^*_{0p} and P^*_{1pc} for the parameters given. Nevertheless, it is remarkable the notable decreasing of the threshold value P^*_{1Ic} when $\theta = 0.2$ and $\eta = 0.2$. Suggesting a time threshold to invest below 20 /ton when σ is 0.1. It seems like the utilization rate increases the opportunity cost to wait and works a 'double' subsidy when the investment subsidy is provided at the same time. Likewise, the slope is less sensitive to changes in the volatility price for this lowest price threshold.

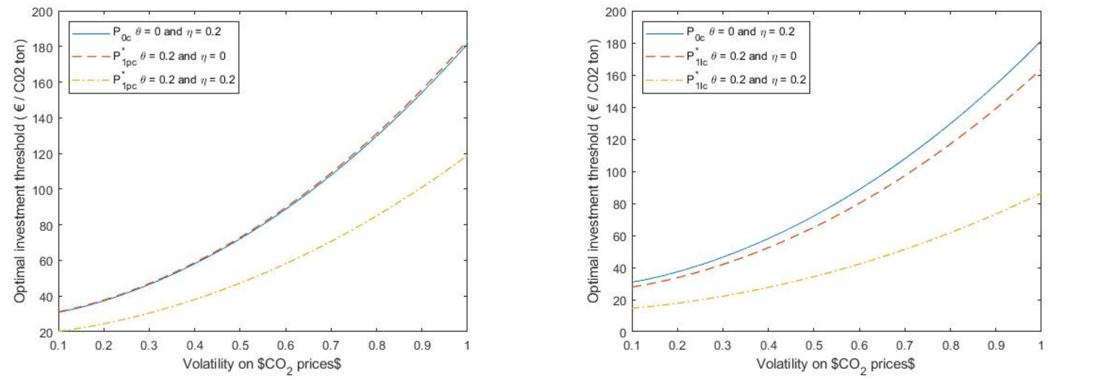
In summary, the strong effect associated with the utilization rate η is notable for making the investment in the CUS unit profitable. Moreover, making the price threshold less sensitive to changes in volatility CO_2 price σ. Taking together an increase in the utilization rate (η) has notable benefits in reducing the price thresholds, mostly explained by three factors: higher margin from sale of the transformed CO2 products (assuming 0 operational costs for its production), decreasing in transport and storage costs at the same time, and deterministic revenues/lower costs about these two factors.

4.2. Model under policy uncertainty of the price subsidy provision ($\lambda_p \geq 0$)

In this section, the results with respect to the price subsidy unpredictable provision are analyzed. First, the price thresholds results for volatility in CO_2 prices are analyzed. Second, sensitive analyzes with respect to changes in subsidy size (θ_p) and utilization rate (η). In general, the results suggest higher price thresholds, adding this additional source of uncertainty for the price subsidy provision.

Table 5 shows the values of the price thresholds P^*_{0c} in relation to the intensity of the subsidy provision λ_p and the volatility of the prices CO_2 σ. Similarly to Figure 2, it can be observed that the investment price increases as the volatility of the price CO_2 increases . Moreover, as the probability of subsidy provision increases, as well as the price threshold P^*_{0c}. This is because the electric producer expects higher future revenues and is willing to wait longer until the subsidy becomes available, and not is expecting to obtain revenues without the subsidy provision. This is in line with the Dixit and

Figure 2: CO_2 price volatility subsidy size (θ^I) and (θ^P) and utilization ratio (η) on the price threshold for the price subsidy (left) and the investment subsidy (right). The values of the remaining parameters are described in Table 1



(a) Price threshold when the price subsidy is in effect with respect to CO_2 price volatility (σ)

(b) Price threshold when the investment subsidy is in effect with respect to CO_2 price volatility (σ)

Pindyck (1994) findings in Chapter 9. When the probability of tax credit investment provision increases and the probability of retraction is zero , the price threshold P_0^* is higher. Additionally, as Chronopoulos et al. (2016) found, for small values of λ_P, the threshold value P_{0pc}^* has an increasing trend; while for large values of λ_P, P_{0c}^* has a decreasing trend. In detail, as the intensity of the subsidy provision approaches 1, the price threshold P_{0pc}^* decreases compared to the highest threshold values observed when the probability of provision is moderate, between 0.4 and 0.6. This oscillation with respect to the probability of the subsidy provision (λ_p) occurs at different rates, depending on the level of volatility (σ). Given Table 4, for parameters $\sigma = 0.1$ and $\lambda = 1$ is obtained the lower price threshold barely below 45 / ton.

Table 4: P_{0pc}^* with respect to the probability of price subsidy provision (λ_p) and volatility in prices (σ)

		λ_p									
		0.0	0.1	0.2	0.3	0.4	0.6	0.7	0.8	0.9	1.0
σ	0.1	30.74	59.81	125.45	286.61	284.49	162.77	100.90	75.01	59.28	44.94
	0.2	37.20	68.90	141.10	318.39	615.12	565.25	300.78	181.09	116.20	81.53
	0.3	46.33	80.25	159.71	354.33	689.37	621.07	342.16	195.25	124.75	87.21
	0.4	57.88	93.57	180.86	394.46	756.69	677.71	368.03	210.55	133.94	93.29
	0.5	71.80	108.88	204.59	438.83	784.51	739.23	401.98	226.99	143.77	99.78

The parameter values are indicated in Table 1. $\eta = 0.2$ and $\theta = 0.2$. The value for $\lambda_p = 0.0$ is 0.0001

Finally, in Appendix B, Table 5, the value of P_{1c}^* remains the same when λ_p increases. This is because the price subsidy has already been granted for P_{1c}. However, it increases as the volatility of the price increases, consistent with what is shown in Figure 2 for the analysis of P^*_{1pc}. In Appendix B, table 6 exactly the same happens for the price threshold P^*_{0c} when the subsidy is not provided.

Finally, higher price thresholds are obtained when λ_P are between 0.4 and 0.6. This is a problem. On the other hand, lower values would be got in the extremes close to zero or 1. For the last one, It is less probable that the government would set this type of policy scenarios, as was stated in chapter 9 of Dixit and Pindyck (1994).

Finally, the price threshold values P^*_{pc} are higher than P^*_{0c} and P^*_{1pc}. The higher price thresholds are obtained when λ_P are between 0.4 and 0.6. On the other hand, lower values would be got when λ_P is close to zero or 1. Given each volatility level, lower threshold values are given when it is less probable that the unpredictable policy is provided (ie., $\lambda_P = 0.0001$). It is similar to what was suggested in Chapter 9 ofDixit and Pindyck (1994) , but it is less probable that this type of policies being executed by any government.

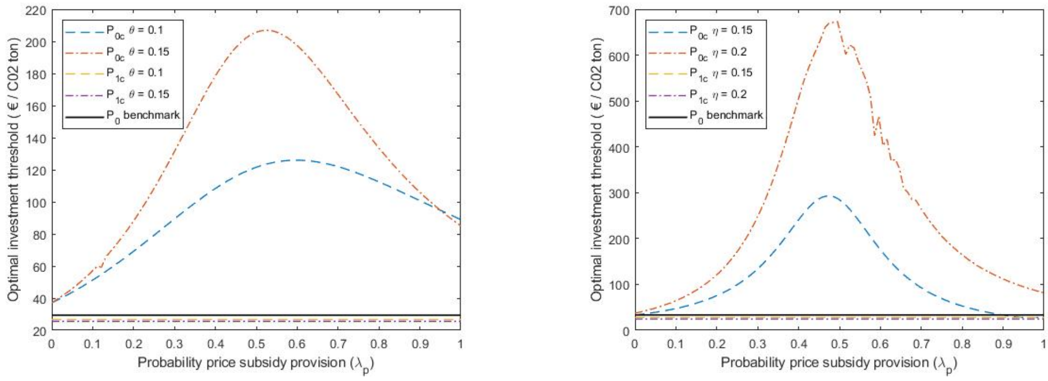
4.2.1 Prices thresholds with respect to subsidy price provision probability (λ_p)

In Figure 2, the behavior of Price thresholds is described with respect to changes in the subsidy size (Panel A) and transformation rate (Panel B). As mentioned above, the price threshold P_{0pc} has a similar trend as results obtained by Chronopoulos et al. (2016) for the fee subsidy provision under uncertainty in the energy prices.

In Panel (A) of Figure 3, the higher the probability of receiving the subsidy, the higher the expected value of the investment in the CCUS plant, so it is profitable waiting and invest later. To do this, it is necessary to postpone the investment until the carbon price is higher. However, as the probability of receiving the subsidy increases, the incentives to postpone the investment decrease, since the benefit of receiving the subsidy is very likely. As this probability increases, the price threshold for investing becomes smaller and the incentive to invest relatively early is more profitable. Meanwhile, the larger the subsidy size, the greater the opportunity cost of investing sooner, therefore, waiting for higher income given by a bigger subsidy size θ^p is a better option. Thus, the price threshold is higher when the subsidy size for λ_p (without extreme cases zero or one)

In panel (B) of Figure 3, the price threshold values are shown when the utilization rate η changes from 0.15 to 0.2, with a subsidy size of 0.2. As in the previous cases, the trend of P^*_{0pc} oscillates from increasing to decreasing, while the probability of providing the subsidy increases. Generally, the opportunity cost of investing is higher when the proportion of transformed and sold emissions (η) increases, similar to what happens when the size subsidy increases. The revenues that the electric producer would receive by investing in the CCUS plant would be higher with a greater sale of transformed goods and lower transportation and storage costs for the remaining CO_2 emissions. This translates into a higher price to invest or to maintain the option to invest and exercise it later when the price CO_2 is higher.

Figure 3: Probability provision λ_p effect on Price thresholds values with respect to subsidy size θ^p (left) and transformation rate η (right)



(a) Value of parameters are the same as described in Table 1 $\eta = 0.2$

(b) Value of parameters are the same as described in Table 1 $\theta = 0.2$

The numerical solutions in this model were obtained by *lsqnonlin*. Using the optimization toolbox Version 9.1 in MATLAB R2021a.

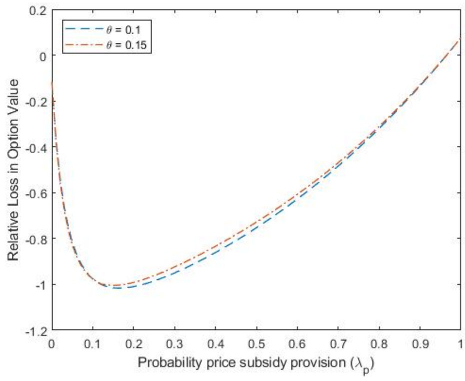
However, for values of $\eta = 0.15$, this trend does not hold and, even with a high probability of providing the subsidy approaching 1, P^*_{0pc} coincides with the value P^*_{1pc}, which is lower than the price threshold P^*_{0c} without the subsidy. The benefit of investing when the price is sufficiently higher, when the subsidy has not been provided, is greater than the incentive to invest when the subsidy is available. This can be seen in panel a of Figure 4, where the relative loss of value (RLV) is the difference between the middle and the bottom suggested by 31. In other words, RLV is the difference between the value of investing later (when the subsidy is available) and the value of investing earlier (when the subsidy will be available) with respect to λ_p for the two values of θ^p[18]. When the subsidy becomes certain to be provided (higher values of λ_p), the incentive to invest earlier increases, while the valuation for investing later decreases. The difference has a U-shaped movement from negative to positive when λ_P approaches 1. For $\theta = 0.15$, the difference is more negative for $\eta = 0.15$ than for $\eta = 0.2$. The reason is because the value of investment when the subsidy is available (between P^*_{1pc} and P^*_{0c}) is higher when the value of the revenues is higher.

However, when λ is close to 1, this difference is more positive when η is equal to 0.15. In this case, the incentive to invest later decreases substantially when the subsidy is close to being provided with certainty, obtaining a P^*_{0pc} equal to the price threshold P^*1pc. In other words, the real option valuation suggests a higher opportunity cost to invest earlier when the CCUS technology has a utilization rate of about 20%.

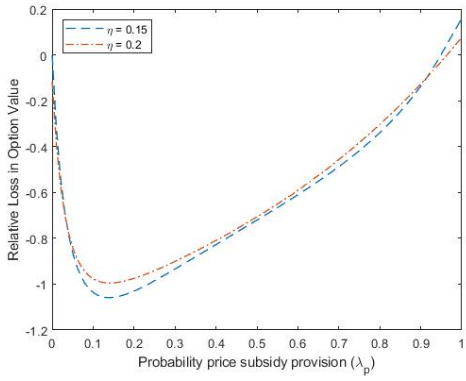
On the other hand, in panel (A) of Figure 4, the same RLV is depicted for the values

[18]The explicit formula is the relative difference of middle expression with respect to the bottom of 31, $\frac{V(P_0)middle - V(P_0)bottom}{V(P_0)bottom}$

Figure 4: Probability provision λ_p effect on Relative Loss in Option Value with respect to subsidy size θ^p (left) and transformation rate η (right)



(a) Value of parameters are the same as described in Table 1 $\eta = 0.2$



(b) Value of parameters are the same as described in Table 1 $\theta = 0.2$

θ. In this case, the U-shaped trend continues along all λ_p values. As mentioned above, the valuation to invest when the subsidy is provided is higher when the revenues are higher. Therefore, the RLV for $\theta = 0.15$ is higher than for $\theta = 0.1$. In the end, when λ_p approaches 1, the RLV for $\theta = 0.15$ is positive and subtle higher than the RLV when θ is 0.1. For this reason, in panel (B) of Figure 2, the price threshold is lower for $\theta = 0.15$ when the subsidy is almost guaranteed to be provided.

In general and similar to the results from the previous subsection, the price threshold P_0 is greater for an extra source of policy uncertainty. Even when the Price subsidy probability is very close to be provided, the price threshold P^*_{0pc} maintain substantially higher. Further, higher subsidy size (θ) and utilization rate (η) make greater the opportunity cost to invest sooner, then the time price threshold is higher to do it. The source of uncertainty from subsidy provision is harmful for a earlier investment in these type of technologies, even when the notable benefits from utilization rate of the CO_2 emissions increases.

5. Conclusions

In the present article, was investigated what is the CO_2 price threshold given by the investment option valuation for a CCUS plant, when the investor is a Coal Fired electric producer, under a price subsidy provision uncertainty. The price thresholds were higher when the uncertain in prices and price subsidy provision were considered. These results suggesting a significant impact discouraging a sooner investment given by the CO_2 prices for market volatility (or uncertainty) and even substantially higher when the subsidy provision uncertainty is considered.

Previous investigations focused on CO_2 prices as a GBM process non-considering additional sudden policy factors that impact the CO_2 prices at the same time. This provision analysis has not been done for these types of technologies for an infinity and continuous time frame under the Real Option approach. Although Similar studies have done for other technologies that involve the government policy risk as a Poisson process, these analyzes have mostly focused on evaluate the retraction risk of a lump-sum investment subsidy (See Nagy et al. (2019) and Huang et al. (2021)). On the other hand, Chronopoulos et al. (2016) considered retraction and provision separately and at the same time of a fixed premium on the electric prices. However, this model was made for a for a renewable energy project to obtain time price threshold and investment size. This document is the first to consider the CO_2 prices as a GBM process and the price subsidy provision uncertainty through a Poisson process at the same time, under a the real option investment model approach for a CCUS unit.

For the first analysis, where NPV thresholds were compared with respect to the ROT approach, the utilization rate was a noticeable factor to encourage the electric producer invest sooner. In the context of uncertainty in the CO_2 prices when the investment subsidy was provided certainly, the utilization rate work as a ´double´ subsidy and was important to set the lowest price threshold. The same was not obtained, when the price subsidy was provided. For future policy implications, this would suggest keeping the funding support for R&D in the CCUS technologies, as well as, backing subsidies for the sunk investment costs. Put it differently, making the utilization rate (η) and the investment subsidy (θ_I) higher. These implications are in line with the results found by Zhu & Fan (2011) with respect to the useful impact of the aids in R&D for making a sooner invest in a CCS plant. Likewise, Zhang et al. (2021) found favorable evidence about backing the investment costs to make appealing the investment in these technologies under the CPFM context. Finally, as was indicated previously, many jurisdictions have been promoting and implementing the supporting financing to R&D destination and alleviation the investment costs. At light of the result of these findings it goes as an accurate direction. However, the lacking of other legal incentives as the avoidance of the CO_2 allowances in the ETS system, from the utilization purposes of these emissions, is a pending task to solve in the future.

Regarding the results of price thresholds, they generally reflect an increasing the opportunity cost for investment , when the price subsidy provision is uncertain. Although at first glance one might think or deduce that the provision of a subsidy to the price of CO_2 is a wise policy to encourage investment in CCUS technologies. The unpredictable uncertainty surrounding its provision leads to the opposite result, holding the investment for higher price thresholds in the CCT. Furthermore, increasing the size of subsidies and the utilization rate factor is counterproductive, as both factors increase the opportunity cost of investments. Although the increase in the utilization rate in the first case was significantly to reduce the time price threshold when both subsidies are certain, it creates unwanted effects in view of the uncertain provision. Similarly, rising volatility in prices creates even higher time price thresholds. In this regard, the results suggest a policy in which the provision of such services is unlikely and small. As Dixit and Pyndick (1994) suggested, such policies are unlikely to be implemented. A desirable policy recommendation would therefore be to establish a deterministic path for pricing the CO_2 allocations for coal-fired power plants with the option of investing in a CCUS unit plant. Other studies have demonstrated the benefits of deterministic mechanisms, which have led to lower price thresholds under the CPFM system (Walsh et al. (2014), Comporneolle et al. (2017)).

This research has limitations about expanding the results to a most applicable context, the opportunity costs to process the CO_2 emissions instead of emitting them, and the robustness of the numerical results acquired. First, the parameter values used to get the numerical results are hypothetical aiming to obtain the numerical solution results and make sensitive analysis with respect the subsidy policies. It might need to be adapted to obtain an accurate valuation of a real project for a coal-fired power plant. Another limitation is about the assumption of considering profitable to invest in a CCUS plant, regardless evaluate the tradeoff between buy the CO_2 allowances to emit and take this emissions to transform or store it. This kind of analysis was done previously by Compernolle et al. (2017) and is relevant to apply in this context .Finally, To have a more robust and consistent analysis, other numerical estimation methods may be needed for the price thresholds P^*_{0pc}, especially when the probability of the price subsidy provision is either 0 or 1. In particular for the acquired result, only when η is 0.15, the price threshold approaches P^*oc when λP tends to zero, and to P^*1pc when λp tends to 1. Only for this situation the price threshold results are the same as Chronopulos et al. (2016), where all the price threshold values have this pattern. On possible reason is because the model needs to be solved with different initial values or a alternative algorithm (instead of (lsqnonlin) in MATLAB) needs to be used to obtain the numerical results. Additionally, the optimal investment size is not taken into account in this analysis, which would be a determinant factor leading to get this coincidence. Unfortunately, the existing literature on the Real Option theory does not usually indicate or explicitly provide the algorithm methods used to obtain the numerical results.

In the present article, the impact of the risk of the subsidy provision on the investment of real options in a CCUS plant was investigated. The time price threshold decreased when the subsidies were provided with certainty. Furthermore, the utilization factor was an important feature to make the investment in this option sooner . However, in an uncertain scenario of price subsidy provision, the utilization rate and size of the subsidy led to a postponement of investment. Taken together, this document offers a first perspective about the CO_2 prices involving an additional price subsidy provision factor. Future research may extend this work by other relevant unpredictable factors are incorporated to the real option analysis. Such as, optimal investment size based on the demand of the transformed goods, optimal subsidy size, among others. Finally, additional stochastic processes can be taking into account to capture policy scenarios transition in the climate change policies, as like as a stochastic jump transition from an ETS mechanism to a CPFM.

Appendix A

Proof of proposition 1

Given [2], the expected value when the decision maker decides to invest since T onward:

$$V(P_0) = V_0 = \int_0^\infty e^{-rt} Q_c[e^{\mu t} P_{0c} + \eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}]dt - I$$

$$V(P_0) = V_0 =$$
$$\int_0^\infty e^{-rt} Q_c e^{\mu t} P_{0c} dt + \int_0^\infty e^{-rt} Q_c \eta\pi_u dt - \int_0^\infty e^{-rt} Q_c (1-\eta) C_{cs} dt - \int_0^\infty e^{-rt} Q_c C_{o\&m} dt - I$$

Solving the integral gives:

$$V(P_0) = V_0 = \frac{Q_c P_{0c}}{r-\mu} + \frac{Q_c \eta\pi_u}{r} - \frac{Q_c(1-\eta)C_{cs}}{r} - \frac{Q_c C_{o\&m}}{r} - I$$

Factoring Q_C and r gives the expression [10]:

$$V(P_0) = V_0 = \frac{Q_c P_{0c}}{r-\mu} + \frac{Q_c \eta\pi_u}{r} - \frac{Q_c(1-\eta)C_{cs}}{r} - \frac{Q_c C_{o\&m}}{r} - I$$
$$V(P_0) = V_0 = Q_c[\frac{P_{0c}}{r-\mu} + \frac{\eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}}{r}] - I$$

Without loss of generality, from [3] it produces equivalent results when the subsidy size (θ^P) is added to the CO_2 price. This yields the expression [11]

$$V(P_{1pc}) = V_1 = \frac{Q_c P_{1pc}(1+\theta^P)}{r-\mu} + \frac{Q_c \eta\pi_u}{r} - \frac{Q_c(1-\eta)C_{cs}}{r} - \frac{Q_c C_{o\&m}}{r} - I$$
$$V(P_{1pc}) = V_1 = Q_c[\frac{P_{pc}(1+\theta^P)}{r-\mu} + \frac{\eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}}{r}] - I$$

Proof of proposition 2

Stopping region ($P \geq P^*_{0c}(P^*_{1pc})$) solution

The decision maker always invests when the price value is high enough to make the investment profitable in both cases $P^*_{0c}(P^*_{1pc})$. The investment value when the policy is (is not) executed given by [8]([9]).

Waiting region ($P < P^*_{0c}(P^*_{1pc})$) suggested solution

Under the waiting region, 8 should satisfies the bellman equation:

$$rV_0 = \pi + \lim_{dt \to 0} \tfrac{1}{dt} E(dV_0)$$

The investor decides to wait and resign to receive π instantaneously, then

$$rV_0 = \lim_{dt \to 0} \frac{1}{dt} E(dV_0) \tag{39}$$

According to Ito's lemma, dV_0 follows:

$$dV_0 = (V''(P_{0c})\sigma^2 P^2_{0c} d_t)\frac{1}{2} + V'(P_{0c})\mu P_{0c} d_t + V'(P_{0c})\sigma P_{0c} d_z \tag{40}$$

Because $E(dz) = 0$, the expected value of dV_0 is equal to:

$$E(dV_0) = ((P^2_{0c}\sigma^2 V''_0(P_{0c}))\frac{1}{2} + \mu P_{0c} V'_0(P_{0c}])d_t \tag{41}$$

Plugging 56 into 54, then

$$rV_0(P_{0c}) = (P_{0c})^2 \sigma^2 V''_0(P_{0c}))\tfrac{1}{2} + \mu P_{0c} V'_0(P_{0c})$$

$$(P_{0c})^2 \sigma^2 V''_0(P_{0c}))\frac{1}{2} + \mu P_{0c} V'_0(P_{0c}) - rV_0(P_{0c}) = 0 \tag{42}$$

Further, the general solution suggested for V_0 that satisfies the Bellman equation:

$$V_0 = A_0 P^{\beta(0)_1}_{0c} + B_0 P^{\beta(0)_2}_{0c} \tag{43}$$

Without prejudice to generality, a similar result holds when a price subsidy policy is taken place. In concrete, CO_2 price (P_{0c}) times the price subsidy factor $(1+\theta^P)$ replaces P_{1pc}. Then, the function value when policy of price subsidy is already applied yields:

$$V_1 = A_{1p} P^{\beta(0)_1}_{1c} + B_{1p} P^{\beta(0)_2}_{1c} \tag{44}$$

Considering that 58 is a second-order homogeneous differential equation. Therefore, the solution for V_0 ($V(1)$) is equal to a linear combination of two independent solutions. Furthermore, the solution is followed by the quadratic polynomial equation for 55:

$$Q(0) = \frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0 \tag{45}$$

The solutions for $Q(0)$ roots are given by:

$$\beta(0)_1 = \frac{((0.5*\sigma^2) - \mu)}{\sigma^2} + \frac{\sqrt{(\mu - 0.5.*\sigma.^2)^2 + (2*\sigma^2).*(r)}}{\sigma^2}$$
$$\beta(0)_2 = \frac{((0.5*\sigma^2) - \mu)}{\sigma^2} - \frac{\sqrt{(\mu - 0.5.*\sigma.^2)^2 + (2*\sigma^2).*(r)}}{\sigma^2}$$

Equations [58]and [59] give the solutions for [8] and [9], respectively.

Proof of proposition 3

Waiting region ($P < P^*_{0c}(P^*_{1pc})$) solution

The bounded conditions to obtain V_0 solving for A_0, B_0, and P^*_{0c} are:

- Initial condition: $V(0) = 0$
- Value Matching condition: $V(P^*_{0c}) = V(P^*_{0c})$
- Smooth pasting condition $V'(P^*_{0c}) = V'(P^*_{0c})$

Given that $\beta(0)_1 > 1$, $\beta(0)_2 < 0$, and the initial condition $V(0) = 0$,then $B_1 P^{\beta(0)_2}$ goes to infinite when $P_{0c} = 0$, thus $B_0 = 0$. The value-matching condition, equaling 58 and 2:

$$A_0(P^*_{0c})^{\beta(0)_1} = Q_c\left[\frac{P_{0c}}{r-\mu} + \frac{\eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}}{r}\right] - I \tag{46}$$

Deviating the smooth pasting condition:

$$A_0(\beta(0)_1)(P^*_{0c})^{\beta(0)_1 - 1} = \left(\frac{Q_c}{r-\mu}\right) \tag{47}$$

In total, the system is made up of 2 equations (61 and 62) and 2 unknowns (A^*_0 and P^*_{0c}). Solving the system, the values yields:

$$P^*_{0c} = \frac{(r-\mu)\beta(0)_1}{(\beta(0)_1 - 1)}\left[\frac{I}{Q_c} + \frac{(1-\eta)C_{cs} - C_{o\&m}}{r} - \frac{\eta\pi_u}{r}\right] \tag{48}$$

Given P^*_{0c}, the solution for A_0 is equal:

$$A^*_0 = \frac{Q_c}{(r-\mu)\beta(0)_1}\left(\frac{(r-\mu)\beta(0)_1}{(\beta(0)_1 - 1)}\left[\frac{I}{Q_c} + \frac{(1-\eta)C_{cs} - C_{o\&m}}{r} - \frac{\eta\pi_u}{r}\right]\right)^{(1-\beta(0)_1)} \tag{49}$$

Given P^*_0 and A_0, the investment value V^*_0 is equal:

$$V^*_0 = A^*_0(P^*_{0c})^{\beta(0)_1} \tag{50}$$

Without prejudice to generality, a similar result holds when a price subsidy policy is provided. As mentioned above, the price CO_2 (P_{0c}) is replaced by $P_{1pc}(1+\theta^P)$. Then, the unknowns P^*_{1Ic} and A^*_{1I}:

$$P^*_{1Ic} = \frac{(r-\mu)\beta(0)_1}{(\beta(0)_1 - 1)}\left[\frac{I}{Q_c} + \frac{(1-\eta)C_{cs} - C_{o\&m}}{r} - \frac{\eta\pi_u}{r}\right]\left(\frac{1}{(1+\theta^P)}\right) \tag{51}$$

$$A_{1p}^{*}=\frac{Q_c}{(r-\mu)\beta(0)_1}\left(\frac{(r-\mu)\beta(0)_1}{(\beta(0)_1-1)}\left(\left[\frac{I}{Q_c}+\frac{(1-\eta)C_{cs}}{r}+\frac{C_{o\&m}}{r}-\frac{\eta\pi_u}{r}\right]\left(\frac{1}{1+\theta^P}\right)\right)\right)^{1-\beta(0)_1} \tag{52}$$

Finally, the investment value (V_1^*), given the previous parameters ,in this case yields:

$$V_1^{*}=A_{1p}^{*}(P_{1pc}^{*})^{\beta(0)_1} \tag{53}$$

Proof of proposition 4

Given [16], the expected value when the decision maker decides to invest since T onwards:

$$V(P_0) = V_0 = \int_0^\infty e^{-rt} Q_c[e^{\mu t} P_{0c} + \eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}]dt - I$$

$$V(P_0) = V_0 =$$
$$\int_0^\infty e^{-rt} Q_c e^{\mu t} P_{0c} dt + \int_0^\infty e^{-rt} Q_c \eta\pi_u dt - \int_0^\infty e^{-rt} Q_c (1-\eta) C_{cs} dt - \int_0^\infty e^{-rt} Q_c C_{o\&m} dt - I$$

Solving the integral gives:

$$V(P_0) = V_0 = \frac{Q_c P_{0c}}{r-\mu} + \frac{Q_c \eta\pi_u}{r} - \frac{Q_c(1-\eta)C_{cs}}{r} - \frac{Q_c C_{o\&m}}{r} - I$$

Factoring Q_C and r gives the expression [18]:

$$V(P_0) = V_0 = \frac{Q_c P_{0c}}{r-\mu} + \frac{Q_c \eta\pi_u}{r} - \frac{Q_c(1-\eta)C_{cs}}{r} - \frac{Q_c C_{o\&m}}{r} - I$$
$$V(P_0) = V_0 = Q_c[\frac{P_{0c}}{r-\mu} + \frac{\eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}}{r}] - I$$

Without loss of generality, from [17] it produces equivalent results when the investment sunk cost I is discounted by the subsidy size θ^I. This yields the expression [19]:

$$V(P_{1pc}) = V_1 = \frac{Q_c P_{1Ic}}{r-\mu} + \frac{Q_c \eta\pi_u}{r} - \frac{Q_c(1-\eta)C_{cs}}{r} - \frac{Q_c C_{o\&m}}{r} - (1-\theta^I)I$$
$$V(P_{1pc}) = V_1 = Q_c[\frac{P_{1Ic}}{r-\mu} + \frac{\eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}}{r}] - (1-\theta^I)I$$

Proof of proposition 5

Stopping region ($P \geq P^*_{0c}(P^*_{1Ic})$) solution

The decision maker always invests when the price value is high enough to make the investment profitable in both cases $P^*_{0c}(P^*_{1Ic})$. The investment value when the policy is (is not) executed given by [16]([17]).

Waiting region ($P < P^*_{0c}(P^*_{1Ic})$) suggested solution

Under the waiting region, 8 should satisfies the bellman equation:

$$rV_0 = \pi + \lim_{dt \to 0} \tfrac{1}{dt} E(dV_0)$$

The investor decides to wait and resign to receive π instantaneously, then

$$rV_0 = \lim_{dt \to 0} \frac{1}{dt} E(dV_0) \tag{54}$$

According to Ito's lemma, dV_0 follows:

$$dV_0 = (V''(P_{0c})\sigma^2 P_{0c}^2 d_t)\frac{1}{2} + V'(P_{0c})\mu P_{0c} d_t + V'(P_{0c})\sigma P_{0c} d_z \tag{55}$$

Because $E(dz) = 0$, the expected value of dV_0 is equal to:

$$E(dV_0) = ((P_{0c}^2 \sigma^2 V_0''(P_{0c}))\frac{1}{2} + \mu P_{0c} V_0'(P_{0c}])d_t \tag{56}$$

Plugging 56 into 54, then

$$rV_0(P_{0c}) = (P_{0c})^2 \sigma^2 V_0''(P_{0c}))\tfrac{1}{2} + \mu P_{0c} V_0'(P_{0c})$$

$$(P_{0c})^2 \sigma^2 V_0''(P_{0c}))\frac{1}{2} + \mu P_{0c} V_0'(P_{0c}) - rV_0(P_{0c}) = 0 \tag{57}$$

Further, the general solution suggested for V_0 that satisfies the Bellman equation:

$$V_0 = A_0 P_{0c}^{\beta(0)_1} + B_0 P_{0c}^{\beta(0)_2} \tag{58}$$

Without prejudice to generality, a similar result holds when investment subsidy policy is taken place. In concrete, investment sunk cost I is replaced by $(1 - \theta^I)I$. Then, the function value when policy of investment subsidy is already applied yields:

$$V_1 = A_{1I} P_{1Ic}^{\beta(0)_1} + B_{1I} P_{1Ic}^{\beta(0)_2} \tag{59}$$

Considering that 58 is a second-order homogeneous differential equation. Therefore, the solution for V_0 ($V(1)$) is equal to a linear combination of two independent solutions. Furthermore, the solution is followed by the quadratic polynomial equation for 55:

$$Q(0) = \frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0 \tag{60}$$

The solutions for $Q(0)$ roots are given by:

$$\beta(0)_1 = \frac{((0.5*\sigma^2)-\mu)}{\sigma^2} + \frac{\sqrt{(\mu-0.5.*\sigma.^2)^2+(2*\sigma^2).*(r)}}{\sigma^2}$$
$$\beta(0)_2 = \frac{((0.5*\sigma^2)-\mu)}{\sigma^2} - \frac{\sqrt{(\mu-0.5.*\sigma.^2)^2+(2*\sigma^2).*(r)}}{\sigma^2}$$

The equations [58]and [59] give the solutions for [18] and [19], respectively.

Proof of proposition 6

Waiting region ($P < P^*_{0c}(P^*_{1c})$) solution

The bounded conditions to obtain V_0 solving for A_0, B_0, and P^*_{0c} are:

- Initial condition: $V(0) = 0$
- Value Matching condition: $V(P^*_{0c}) = V(P^*_{0c})$
- Smooth pasting condition $V'(P^*_{0c}) = V'(P^*_{0c})$

Given that $\beta(0)_1 > 1$, $\beta(0)_2 < 0$, and the initial condition $V(0) = 0$,then $B_1 P^{\beta(0)_2}$ goes to infinite when $P_{0c} = 0$, thus $B_0 = 0$. The value-matching condition, equaling 58 and 2:

$$A_0(P^*_{0c})^{\beta(0)_1} = Q_c\left[\frac{P_{0c}}{r-\mu} + \frac{\eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}}{r}\right] - I \tag{61}$$

Deviating the smooth pasting condition:

$$A_0(\beta(0)_1)(P^*_{0c})^{\beta(0)_1 - 1} = \left(\frac{Q_c}{r-\mu}\right) \tag{62}$$

In total, the system is made up of 2 equations (61 and 62) and 2 unknowns (A^*_0 and P^*_{0c}). Solving the system, the values yields:

$$P^*_{0c} = \frac{(r-\mu)\beta(0)_1}{(\beta(0)_1 - 1)}\left[\frac{I}{Q_c} + \frac{(1-\eta)C_{cs} - C_{o\&m}}{r} - \frac{\eta\pi_u}{r}\right] \tag{63}$$

Given P^*_{0c}, the solution for A_0 is equal:

$$A^*_0 = \frac{Q_c}{(r-\mu)\beta(0)_1}\left(\frac{(r-\mu)\beta(0)_1}{(\beta(0)_1 - 1)}\left[\frac{I}{Q_c} + \frac{(1-\eta)C_{cs} - C_{o\&m}}{r} - \frac{\eta\pi_u}{r}\right]\right)^{(1-\beta(0)_1)} \tag{64}$$

Given P^*_0 and A_0, the investment value V^*_0 is equal:

$$V^*_0 = A^*_0(P^*_{0c})^{\beta(0)_1} \tag{65}$$

Without prejudice to generality, a similar result holds when an investment subsidy policy is provided. As was mentioned before, the investment sunk cost I is replaced by $(1-\theta^I)I$. Then, the unknowns P^*_{1Ic} and A^*_{1I}:

$$P^*_{1Ic} = \frac{(r-\mu)\beta(0)_1}{(\beta(0)_1 - 1)}\left[\frac{(1-\theta^I) * I}{Q_c} + \frac{(1-\eta)C_{cs} - C_{o\&m}}{r} - \frac{\eta\pi_u}{r}\right] \tag{66}$$

$$A_{1I}^{*} = \frac{Q_c}{(r-\mu)\beta(0)_1}\left(\frac{(r-\mu)\beta(0)_1}{(\beta(0)_1 - 1)}\left[\frac{(1-\theta^I)I}{Q_c} + \frac{(1-\eta)C_{cs} - C_{o\&m}}{r} - \frac{\eta\pi_u}{r}\right]\right)^{(1-\beta(0)_1)} \quad (67)$$

Finally, the investment value (V_1^*), given the previous parameters ,in this case yields:

$$V_1^* = A_{1I}^*(P_{1Ic}^*)^{\beta(0)_1} \quad (68)$$

Proof of proposition 7

Given [27], the expected value when P is greater than P^*_{0pc}:

$$V_0 = E[e^{-rt}[(Q_c(P_c(1+\theta_P * 1_{\epsilon^P_{(t}}) + \eta\pi_u - (1-\eta)C_{cs} - C_{o\&m})]|\epsilon^P_0 = 0] - I$$

$$V_0 = Q_cE[e^{-rt}((P_c(1+\theta_P * 1_{\epsilon^P_{(t)}})] + \int_0^\infty e^{-rt}Q_c[\eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}]dt - I$$

$$V_0 = Q_cE[e^{-rt}((P_c(1+\theta_P * 1_{\epsilon^P_{(t)}})] + \frac{Q_c[\eta\pi_u - (1-\eta)C_{cs} - C_{o\&m}]}{r} - I \quad (69)$$

Following the properties of expected value, the first term follows:

$$Q_cE[e^{-rt}((P_c(1+\theta_P * 1_{\epsilon^P_{(t)}})] = \int_0^\infty e^{-rt}(Q_c[E(P_c)] + Q_c\theta_P[E(P_c * 1_{\epsilon^P_{(t)}})])dt$$

$$Q_cE[e^{-rt}((P_c(1+\theta_P * 1_{\epsilon^P_{(t)}})] = \int_0^\infty e^{-rt}(Q_c[E(P_c)] + Q_c\theta_P[E(P_c * 1_{\epsilon^P_{(t)}})])dt \quad (70)$$

$P_c \sim GBM(\mu,\sigma)$ process and $\epsilon^P_t \sim Poisson(\lambda_P)$. As was mentioned above, both processes are independent. Then, the covariance between them is zero:

$$E[(P_c - E(P_c))(\epsilon^P - E(\epsilon^P))] = 0$$

$$E[P_c\epsilon^P] - E(P_c)E(\epsilon^P) = 0$$

$$E[P_c\epsilon^P] = E(P_c)E(\epsilon^P) \quad (71)$$

Plugging [71] into [70] :

$$\int_0^\infty e^{-rt}(Q_c[E(P_c)] + Q_c\theta_P E(P_c)E(1_{\epsilon^P_{(t)}}))dt$$

$$\int_0^\infty e^{-rt}(Q_ce^{\mu t}P_c + Q_ce^{\mu t}\theta_P\lambda_pP_c))dt \quad (72)$$

Plugging in [72] into [69]:

$$V_0 = \int_0^\infty e^{-rt}(Q_ce^{\mu t}P_c + Q_ce^{\mu t}\theta_P\lambda_pP_c)) + \frac{Q_c[\eta\pi_u-(1-\eta)C_{cs}-C_{o\&m}]}{r} - I$$

$$V_0 = \frac{Q_c(P_c+\theta_P\lambda_pP_c)}{r-\mu} + \frac{Q_c[\eta\pi_u-(1-\eta)C_{cs}-C_{o\&m}]}{r} - I$$

At the end is the same as [27]

$$V_0 = Q_c\left[\frac{P_{0pc}(1+\lambda_P\theta^P)}{r-\mu} + \frac{\eta\pi_u-(1-\eta)C_{cs}-C_{o\&m}}{r}\right] - I$$

Proof of proposition 8

Stopping region ($P \geq P^*_{0pc}$) solution

The decision maker always invests, when the price value is higher enough to make the investment profitable for both the thresholds value P^*_{0pc}. The investment value when the policy is provided, and $P^*_{1pc} \geq P$, is given by Equation 9 . On the other hand, the investment value when the subsidy is not available, and $P^*_{0pc} \geq P$, is given by equation 27.

Waiting region ($0 < P < P_1$) suggested solution

Following the proposed solution established by Dixit and Pindyck (1994) in chapter 9 with respect to investment option model when an investment tax credit policy is uncertained.

Under the continuation region the equation **??** should satisfy the Bellman equation for V_0:

$$rV_0 = \lim_{dt \to 0} \frac{1}{dt} E(dV_0) \tag{73}$$

Following Ito's lemma, $E(dV_0)$ yields the following:

$$E(dV_0) = (1 - \lambda_p dt)[(P^2\sigma^2 V_0''(P))\tfrac{1}{2}] + \mu P V_0'(P)] + \lambda_p dt[V_1(P) - V_0(P)]$$

Plugging $E(dV_0)$ into 73.

$$rV_0 = (\lim_{dt \to 0})\tfrac{1}{dt}[(1 - \lambda_p)dt[(P^2\sigma^2 V_0''(P)\tfrac{1}{2} + \mu P V_0'(P))] + \lambda_p dt[V_1(P) - V_0(P)]]$$

After rearranging:

$$(r + \lambda_p)V_0 = (P^2\sigma^2 V_0''(P))\tfrac{1}{2} + \mu P V_0'(P) + \lambda_p V_1(P)$$

Finally,

$$(P^2\sigma^2 V_0''(P))\frac{1}{2}) + \mu P V_0'(P) - (r + \lambda_p)V_0(P) + \lambda_p V_1(P) = 0 \tag{74}$$

The equation 74 established the homogeneous and non-homogenous solutions for this region.

- Homogeneous part for the first V_0'',V_0', and V_0 factors of the LHS equation 74 Follows:

$$V_0^h = C_1 P^{\beta(0)_1} + D_1 P^{\beta(2)_2} \tag{75}$$

  The parameter values $\beta(1)_1$ and $\beta(1)_2$ are the result of the quadratic polynomial equation 74 for the homogeneous solution part:

$$Q(1) = \frac{1}{2}\sigma^2\beta(\beta-1) + \mu\beta - (r+\lambda_p) = 0 \tag{76}$$

The solutions for Q_1 roots are given by:

$$\beta(1)_1 = \frac{((0.5*\sigma^2)-\mu)}{\sigma^2} + \frac{\sqrt{(\mu-0.5.*\sigma.^2)^2+(2*\sigma^2).*(r+\lambda_p)}}{\sigma^2}$$
$$\beta(1)_2 = \frac{((0.5*\sigma^2)-\mu)}{\sigma^2} - \frac{\sqrt{(\mu-0.5.*\sigma.^2)^2+(2*\sigma^2).*(r+\lambda_p)}}{\sigma^2}$$

Given that $\lambda_p \in (0,1)$, then $\beta(1)_2 < 0$ and $\beta(1)_1 > 1$. Additionally, The roots relation between the roots given by 76 and 60 is:

$\beta(1)_1 > \beta(0)_1 > 1 > 0 > \beta(1)_2$

- Non-homogeneous part for the remaining factor $\lambda_p V_1(P)$. When the policy is provided, This is the suggested solution:

$$V_0^p = d_1 A_{1p} P^{\beta(0)_1} \tag{77}$$

Here, C_1, D_1, and d_1 are constant unknowns to find. Meanwhile, A_{1p} is a constant given by 15.

The whole solution, homogeneous plus particular yields:

$$V_0 = V_0^h + V_0^p = C_1 P^{\beta(1)_1} + D_1 P^{\beta(1)_2} + d_1 A_{1p} P^{\beta(0)_1} \tag{78}$$

Further, the suggested non-homogenous part solution in the continuation region for V_0^p and V_1 given by equations 77 and 68 [19], respectively. Plugging these suggested solutions in 74 yields:

$$\mu P(d_1\beta(0)_1 A_1 P^{\beta(0)_1-1}) + P^2\sigma^2\tfrac{1}{2}(d_1(\beta(0)_1)(\beta(0)_1-1)A_1 P^{\beta(0)_1-2}) - (r+\lambda_p)(d_1 A_1 P^{\beta(0)_1}) + \lambda_p(A_1 P^{\beta(0)_1}) = 0$$

Factoring terms for $A_1 P^{\beta(0)_1}$ as a common factor,the previous expression yields:

$$A_1 P^{\beta(0)_1}(\mu d_1 + \sigma^2\tfrac{1}{2}(d_1(\beta(0)_1)(\beta(0)_1-1)) - (r+\lambda_p)d_1 + \lambda_p) = 0$$

Multiplying both sides by $\frac{1}{A_0 P^{\beta(0)_1}}$, then:

$$\mu d_1 + \sigma^2\tfrac{1}{2}(d_1(\beta(0)_1)(\beta(0)_1-1)) - (r+\lambda_p)d_1 + \lambda_p = 0$$

Then, factorizing d_1 and λ_p as common factors:

$$d_1(\mu + \sigma^2\frac{1}{2}(\beta(0)_1)(\beta(0)_1-1) - r) + \lambda_p(1-c_1) = 0 \tag{79}$$

In order to satisfy 79, given the conditions: $\lambda_p \neq 0$ and $\sigma^2\frac{1}{2}(\beta(0)_1)(\beta(0)_1-1) - r + \mu = 0$. Then, $d_1 = 1$.

Therefore, the solution for $V_0 = V_0^h + V_0^p$ between region $(0, P_{1pc}^*)$ is:

[19] Given $\beta(0)_2 < 0$, then, $B_1 = 0$

$$V_0 = C_1 P_c^{\beta(1)_1} + D_1 P_c^{\beta(1)_2} + A_{1p} P_c^{\beta(0)_1}$$

Satisfying the initial condition $V_0(0) = 0$ and $\beta(1)_2 < 0$, then $D_1 = 0$. Finally, the suggested solution yields the following.

$$V_0 = C_1 P_c^{\beta(1)_1} + A_{1p} P_c^{\beta(0)_1} \tag{80}$$

Waiting region ($P_{1pc} \leq P < P_{0pc}$) suggested solution

The solution for the waiting region when the price is greater than P_{1pc} but less than P_{0pc}. Since P_{1pc} is already known, then $V(1)$ is known as well.

Following the procedure prescribed by Dixit and Pindyck (1994). Equation 74 established the homogeneous and non-homogenous solutions for this region.

- Homogeneous part for the first V_0'',V_0', and V_0 factors of the LHS equation 74 Follows:

$$V_0^h = B_1 P^{\beta(1)_1} + B_2 P^{\beta(1)_2} \tag{81}$$

  The parameter values $\beta(1)_1$ and $\beta(1)_2$ are the roots of the quadratic polynomial equation 76.

- Non-homogeneous part for the remaining factor $\lambda_p V_1(P)$. When the policy is provided, This is the suggested solution:

$$\lambda_p V_1 = V_0^p = c_2 P + c_3 \tag{82}$$

Here, B_1, B_2, C_1 and c_2 are constant unknowns to find. The whole solution, homogeneous plus particular solution yields:

$$V_1 = V_1^h + V_1^p = B_1 P^{\beta(2)_1} + B_2 P^{\beta(2)_2} + c_2 P + c_3 \tag{83}$$

The suggested non-homogenous part solution in the continuation region for V_0^p and V_1 given by equations2 and 3, respectively. Plugging these suggested solutions in 74 yields the following.

$$\mu P c_2 - (r + \lambda_p)(c_2 P + c_3) + \lambda_p \left(Q_c [\tfrac{P_{0c}(1+\theta^p)}{r-\mu} + \tfrac{\eta \pi_u - (1-\eta) C_{cs} - C_{o\&m}}{r}] - I \right) = 0$$

Reorganizing terms for P as a common factor and the remaining constants c_2 and c_3 as well , thus:

$$P\left((\mu - (r+\lambda_p))c_2 + \left(\frac{\lambda_p Q_c (1+\theta^p)}{r-\mu} \right) \right) - (r+\lambda_p)c_3 - \lambda_p I + \lambda_p Q_c \left[\frac{\eta \pi_u - (1-\eta) C_{cs} - C_{o\&m}}{r} \right] = 0 \tag{84}$$

Solving for c_2 and c_3, according to satisfy 84, thus:

$$c_2 = \frac{\lambda_p Q_c (1+\theta^p)}{(r-\mu)(r+\lambda_p-\mu)}$$

$$c_3 = \frac{\lambda_p Q_c}{r+\lambda_p}\left(\frac{\eta\pi_u-(1-\eta)C_{cs}-C_{o\&m}}{r}\right) - \frac{\lambda_p I}{r+\lambda_p}$$

In summary, equation 83 ended up in the following expression:

$$V_0(P) = B_1 P^{\beta(1)_1} + B_2 P^{\beta(1)_2} + c_2^* P + c_3^*$$

$$V_0(P_c) = B_1 P_c^{\beta(1)_1} + B_2 P_c^{\beta(1)_2} + \frac{\lambda_p Q_c (1+\theta^p)}{(r-\mu)(r+\lambda_p-\mu)} + \frac{\lambda_p Q_c}{r+\lambda_p}\left(\frac{\eta\pi_u-(1-\eta)C_{cs}-C_{o\&m}}{r}\right) - \frac{\lambda_p I}{r+\lambda_p} \quad (85)$$

Appendix B

Table 5: P^*_{1pc} with respect to the probability of price subsidy provision (λ_p) and volatility in prices (σ)

		λ_p									
		0.0	0.1	0.2	0.3	0.4	0.6	0.7	0.8	0.9	1.0
σ	0.1	20.23	20.23	20.23	20.23	20.23	20.23	20.23	20.23	20.23	20.23
	0.2	24.48	24.48	24.48	24.48	24.48	24.48	24.48	24.48	24.48	24.48
	0.3	30.49	30.49	30.49	30.49	30.49	30.49	30.49	30.49	30.49	30.49
	0.4	38.09	38.09	38.09	38.09	38.09	38.09	38.09	38.09	38.09	38.09
	0.5	47.30	47.30	47.30	47.30	47.30	47.30	47.30	47.30	47.30	47.30

The parameter values are indicated in Table 1. $\eta = 0.2$ and $\theta = 0.2$. The value for $\lambda_p = 0.0$ is 0.0001

Table 6: P^*_{0c} when subsidy provision has not been provided with respect to the probability of price subsidy provision (λ_p) and volatility in prices (σ)

		λ_p									
		0.0	0.1	0.2	0.3	0.4	0.6	0.7	0.8	0.9	1.0
σ	0.1	24.27	24.27	24.27	24.27	24.27	24.27	24.27	24.27	24.27	24.27
	0.2	29.37	29.37	29.37	29.37	29.37	29.37	29.37	29.37	29.37	29.37
	0.3	36.58	36.58	36.58	36.58	36.58	36.58	36.58	36.58	36.58	36.58
	0.4	45.71	45.71	45.71	45.71	45.71	45.71	45.71	45.71	45.71	45.71
	0.5	56.76	56.76	56.76	56.76	56.76	56.76	56.76	56.76	56.76	56.76

The parameter values are indicated in Table 1. $\eta = 0.2$ and $\theta = 0.2$. The value for $\lambda_p = 0.0$ is 0.0001

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