TILBURG UNIVERSITY

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MASTER'S THESIS IN FINANCE

Factor Investing using Trend Following strategy

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1 Abstract

The study looks into a trend-following methodology for asset allocation within factor investing. It uses the standard mean-variance optimization model using the non-constant mean model ARMA. The non-constant volatility models are analyzed using the multivariate dynamic conditional correlation within generalized autoregressive conditional heteroscedasticity (DCC-GARCH). To examine the effects of trend-following strategies in factor investing, we use back-testing and several benchmarks: market performance, the buy-and-hold equally-weighted multiple-factors portfolio, and the standard mean-variance portfolio computed using rolling average returns and the rolling average covariance matrix. This thesis focuses on long-term investment performance. Including the cyclical nature of the factors turns out to achieve substantially better risk-adjusted returns and lower sensitivity to the business cycle. Furthermore, it delivers higher and more consistent outperformance of the market-cap. These techniques help address issues with the sequencing of returns, which can be severe for financial planning. However, the estimated covariance matrix from the DCC-GARCH can be suboptimal if the amount of data is insufficient; choosing a sufficient amount of data provides a remedy to this problem.

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2 Introduction

Over the past ten years, the concept of factor investing has rapidly emerged in asset management. Risk factor investment (RFI) is a method of investing which is guided by the existing risk aspects in the capital distribution. The main aim of RFI is to capture risk premiums, corresponding to various risk factors. Factor investment knows a relatively short history. The financial crisis of 2007-2009 and the research of Ang, Goetzmann, and Schaefer (2009) on a leading Norwegian fund draw attention to this investing strategy. They presented the first study to recommend factor investing explicitly. Later, Ang (2014) expanded the findings from his previous research on the performance of the factor investment. According to the research from 2014, the Norwegian fund reduced its risk profile significantly, while keeping the returns from active management positive.

The capital asset pricing model (CAPM) introduced the first risk factor, the market. This model has significantly contributed to the development of contemporary financial theory in the 1960s. The CAPM model was published by Sharpe (1964) and also by Mossin, Lintner, and Treynor in 1966, 1965, and 1961, respectively. The CAPM model was further advanced by incorporating unique features to improve its accuracy and accommodate the ever-changing financial sector. Fama and French (1996) discovered two additional factors and introduced a new three-factor model, which includes market, size, and book-to-value (value) factors. Also, Carhart (1997) adjusted the three-factor model by modeling the momentum premium. Ang, Hodrick, Xing, and Zhang (2006) argued that the low volatility anomaly exists and should be considered as a risk factor next to the market, value, size, and momentum. Recently, Fama and French (2016) demonstrated the five-factor model by including investment and profitability factors to their three-factor model.

Over time, many risk factors affect the assets, and exposure to those risks is systematic, and therefore, it cannot be diversified away. Exposure to the unsystematic risk can be reduced or even completely removed in large and well-diversified portfolios. Contemporary risk professionals use the RFI model as an efficient tool to allocate capital efficiently. Investor preferences and specific considerations guide this approach. The factor investing concept focuses on analyzing the features of single factors and allocating the capital accordingly. The existing literature examines the characteristics of different factor indexes, and it is evident that the factor premium exists. Kouzmenko and Nagy (2009) mention that the factor premiums are not constant through time and appear to be cyclical. Integrating this aspect into the factor portfolio can substantially improve the risk profile of the portfolio and better our comprehension of the underlying dependency between the different factors as argued by Ang (2014).

The main aim of this thesis is to analyze the performance of a trend-following strategy in a factor portfolio. The research is conducted on six MSCI factor indexes. Our findings suggest that a dynamic approach to the factor portfolio provides superior results compared to our benchmarks. A trend-following strategy outperforms the market, na´ive multi-factor portfolio and the factor portfolio excluding trend in terms of risk-adjusted returns and outperformance of the market. However, this is true only for a sufficient number of observations. The content of the thesis is presented in five sections, including the present Introduction. The rest of the paper is organized as follows. Section 3 provides an overview of the related literature. Section 4 describes data and methodology. Section 5 discusses the empirical findings of the portfolio analysis, and Section 6 concludes the research.

3 Theoretical framework

3.1 The CAPM

Factor investing is the systematic use of factor models to invest in market segments that achieve better returns than those in other sectors. The perception that returns could be associated with quantifiable factors occurred in the postwar period as an attempt to overcome an arbitrary approach for the determination of financial prices and expected returns. Value investing was the prevailing school of thought in those days.

The capital asset pricing model (CAPM) is the oldest factor-model. The model is a foundation of modern financial theory in the 1960s and was published by Sharpe (1964). It was also published by Treynor (1961), Lintner (1965) and Mossin (1966). This model may be viewed as an equilibrium model on the Markowitz (1952) mean-variance framework. The CAPM securities have two main drivers: systematic risk and idiosyncratic risk.

The market factor, also known as market beta, represents the asset's systematic risk. Market beta is used to measure the sensitivity of individual assets to changes in the market's performance. Contrary to idiosyncratic risk, it cannot be diversified away. Therefore, investors get compensated for holding this risk. However, numerous studies indicate that the excess return on an asset is not proportional to the market factor. For example, Black, Jensen, and Scholes (1972) demonstrated the non-empirical evidence of the CAPM.

3.2 Fama-French model

Fama and French (1992, 1993) extended the traditional CAPM model and created a unique model, that has become standard in asset management. Fama and French (1993) explain the US equity market returns with three factors: the market factor, the size factor, and the value factor. The size effect is that assets with a small market cap earn higher returns than assets with a large market cap. The value effect is the superior performance of assets with a low price to book compared to assets with a high price to book.

Recently, Eugene Fama and Kenneth French (2016) presented a five-factor model to describe excess stock returns by adding two new factors to their traditional three-factor model. These new factor effects are investment and profitability. Profitability premium comes from the outperformance of the stocks with high operating profitability. Investment premium originates from the perception that companies with high total asset growth have below-average returns. The five-factor model seems to outperform the 3-factor model according to research performed by Chiah, Chai, Zhong, and Li (2016). This research was conducted in Australian equities between 1982 and 2013. Chiah, Chai, Zhong, and Li's (2016) study indicates that the five-factor model has more explanatory power in asset pricing anomalies than the three-factor model and other competing asset pricing models.

Nevertheless, the five-factor model ignores momentum; Titman and Jegadeesh (1993) connote that, this factor depicts irregular earnings, which are not subject to its delayed stock price reactions to common factors or systematic risk. This aspect illustrates that French and Fama ignored the low volatility factor in their 5-factor model. Low volatility factor is another widely excepted factor, for instance. Haugen and Heins (1972) and Ang, Hodrick, Xing, and Zhang (2006) argued that size, book-to-market, momentum, and liquidity effects could not account for the low average returns earned by stocks with high exposure to systematic volatility risk.

3.3 Factor Investing

The concept of the CAPM model is based on the idea that systematic market risk is the only determinant driving stock prices. The factor theory proposes the opposite. It suggests that there are underlying risk factors that affect all assets in varying levels and cannot be diversified away as idiosyncratic risk. Factor investing is based on the existence of factors that have earned positive risk premiums for exposure to these factors. The factors proposed by Fama and French in 2016 can be considered as such.

Nevertheless, the five-factor model ignores momentum; Titman and Jegadeesh (1993) connote that, this factor depicts irregular earnings, which are not subject to its delayed stock price reactions to common factors or systematic risk. Furthermore, French and Fama excluded the low volatility factor from their five-factor model. Low volatility factor is another widely excepted factor, for instance. The conventional CAPM model holds the assumption of higher returns for higher risk; this postulation contradicts the low volatility anomaly. Other common factors are liquidity discussed by, for example, Pastor and Stambaugh (2003), quality, e.g., Asness, Frazzini, and Pedersen (2014), and carry, e.g., Koijen, Moskowitz, Pedersen, and Vrugt (2012).

Haugen and Heins (1972) and Ang, Hodrick, Xing, and Zhang (2006) argued that size, book-to-market, momentum, and liquidity effects could not account for the low average returns earned by stocks with high exposure to systematic volatility risk. However, contrary to Fama and French, Haugen (1994) argues that the market is not efficient. The interpretation of the risk premiums causes controversy. In the perspective of Fama and French (1995), the value risk premium is compensation for the risk of financial distress. However, Lakonishok, Shleifer, and Vishny (1994) argue that the risk premium for value exists due to investor overreaction.

The unsystematic risk of the assets can be reduced or even eliminated through diversification. Diversification is the approach of allocating capital in such a way that it reduces the exposure to unsystematic risk. In general, it happens by allocating investment risk over different asset classes. The basic approach is to allocate the capital between equity and fixed-income investments. Nevertheless, an investor can diversify within asset classes; for example, by allocating the capital to various industries. The factor investing uses a different approach to diversification. The allocation happens based on exposure to a specific risk factor. In other words, the underlying risk factors cannot be diversified away because the goal is to find the optimal level of exposure to each factor.

The objective of this thesis focusses on the time-varying behavior of risk factors, which we believe can better our comprehension of the underlying causal dependency between changes in risk factor conditions and changes in asset performance as argued by Ang (2014). The correlations between different factor indexes are also essential for diversification. The lower the correlation, the better are the diversification benefits investor can achieve. When assets are not perfectly correlated, combining them into portfolios will improve the risk-adjusted performance due to the reduction of the unsystematic risk. In the case of two perfectly negatively correlated assets, the diversification will yield the maximum risk reduction, corresponding to these assets. In a recent review, Koedjik, Slager, and Stork (2016) found that the correlations between different factor portfolios are less than 1. Therefore, they found significant diversification benefits by combining the factor portfolios into one multi-factor portfolio. Besides, they found that the factor portfolio returns remain stable over time and across U.S., European, and other markets. The investor's time horizon must be considered while using factor investing. The factor investing can be computed for both short-term and long-term asset allocation purposes.

3.4 The Cyclical Nature of Risk Factors

One of the main points of interest of factor investing is that all factors demonstrate irregular cycles of outperforming and underperforming market-cap-weighted indexes. Therefore, when constructing a multi-factor portfolio, it should be investigated what trends the factors follow and how long these trends last. Kouzmenko and Nagy (2009) argued that some of the factors demonstrate a stronger defensive nature, for example, Low Volatility. The other factors, for example, Momentum, shows offensive behavior. Kouzmenko and Nagy also indicate that there is evidence that factor risk premia are not consistent through time.

Moreover, according to Kouzmenko and Nagy, the covariance matrices are not constant through time. Various studies provide evidence of increased correlations in global asset returns throughout the bear markets, for example, a study of increased correlation during the bear market by Campbell, Koedijk and Kofma (2002). A bear market occurs when market prices drop 20% or more. The covariance matrix depends on the state of the business cycle. This property gave rise to numerous studies on conditional portfolio composition, whereby the selection of the factors in the portfolio depends upon the state of the economy, as argued by Ang and Bekaert (2002).

Factor selection should also take into account the correlations between factors that affect portfolio-level risk. The research conducted by Pantchev and Kahra (2017) illustrates that there is significant volatility clustering present: periods of low volatility tend to be followed by low volatility and periods of high volatility by high volatility. Finally, this volatility clustering occurs at the same moment for each factor. Therefore, reliable and dynamic volatility modeling is essential when computing the risk factor portfolio.

The irregular cycles of single factors provide opportunities for trend following strategies. The aim is to combine the individual factors into a multi-factor portfolio and modify the weights to accumulate the cyclicality. Naik, Devarajan, Nowobilski, Page, and Pedersen (2016) documented the business cycle sensitivity of the average returns and the covariance matrices of significant risk factors, which is especially useful during the weak economic conditions.

The implementing of the cyclical nature of the factor indexes into the portfolio is the central theme of this thesis. Modeling trends and cycles in time series has a long history in empirical economics. In this research, we will investigate the cyclical nature of the six MSCI factors using the autoregressive–moving-average (ARMA) model with multivariate dynamic conditional correlation generalized autoregressive conditional heteroscedasticity (DCC-GARCH) adjusted covariance matrix to adjust the price and covariance expectations. The replication of the portfolio is conducted using standard mean-variance optimization.

3.5 Portfolio Construction

The Markowitz (1952) mean-variance optimization is a traditional approach to portfolio optimization. It is based on the premise that when allocating the capital between various assets, the investor is interested in the risk-adjusted return, rather than solely the expected returns. It originates from the portfolio theory whose fundamental aim is to allocate an investor's investments between varying assets. It is a quantitative financial application tool that facilitates such an allocation through comparison and consideration of the trade-off likely to be incurred through risk and return. In other words, the ultimate intent of mean-variance optimization is to maximize the anticipated returns based on the identified risk. Mean-variance optimization can compute both single- and multi-period optimization. The optimization happens by combining various assets into a portfolio. Then, an investor can allocate the capital between the resulting mean-variance efficient portfolio and a risk-free asset, such as T-bill, to maximize the expected utility based on his risk aversion. This allocation is subject to the restriction imposed by his budget constraint.

The mean-variance optimization (MVO) has been extensively criticized in various studies, for example, by Michaud (1989). The stability of the mean-variance allocation is an open issue. The process of capital allocation leads to estimation errors, which may have a tremendous impact on portfolio weights. Recently, Bruder (2013) reviewed some of the methods, which aim to stabilize the mean-variance allocation. The mean-variance model depends on expected returns and correlation of the assets. Therefore, MVO is highly sensitive to input errors. However, this model simplifies the portfolio selection problem and quantifies risk reduction.

3.6 Research Question

The literature review has demonstrated the existence of factor premiums. This advanced approach of weighting indexes based on fundamental information has contributed significantly to the construction of factor indexes by MSCI and other index providers. MSCI reports also discuss the existence of the cyclical nature of various factors. Still, the literature about the factor investing model has not provided insights on the effect of a dynamic approach to capture the cyclicality of the factors. This thesis aims to bring light to this topic. For this purpose, the benchmarks for this thesis are the market index, equally-weighted factor portfolio, and mean-variance factor portfolio. The research restricts the model only to the long-only portfolio due to the high implementations costs of the long-short portfolio. The current research illustrates MSCI six factors. It includes Value, Size, Volatility, Dividend, Quality, and Momentum. The thesis concepts are presented in two parts: the first part discusses modeling the trend using the ARMA model with the DCC-GARCH covariance matrix. The second part discusses the constructing of the portfolio using the mean-variance optimization. Therefore, this study is guided by the following research question;

How does multi-factor portfolio with trend adjusted returns and covariance perform compared to the benchmarks?

In the course of answering the main question, we will consider a model without the transaction costs. The equally-weighted factor portfolio, market index, and mean-variance factor portfolio will be used as benchmarks to analyze the performance of the trend adjusted mean-variance factor portfolios.

4 Data

The dataset considered in the thesis contains monthly observations of factor indexes prices dating as far back as November 1998. We choose monthly data because daily and weekly data are available only from 2014. This research is based on factor indexes provided by the MSCI. The MSCI factor indexes are designed to capture the return of factors which have historically demonstrated excess market returns over the long run. The MSCI described its factor indexes as follows: "...rules-based, transparent indexes targeting stocks with favorable factor characteristics are designed for simple implementation, replicability, and use for both traditional passive and active investment." The MSCI constructed a broad selection of factor indexes depending on the risk driver, market, and investability. The pure factors (i.e., the Fama–French factors) are the theoretical factors that one may wish to capture, but due to the limited nature of the risk driver, it is not achievable in the real-life scenarios. MSCI factor indexes are more investability-oriented than pure factors, and therefore, it provides feasible opportunities. However, they are less investability-oriented than the MSCI factor tilt indexes. According to MSCI, the factor tilt indexes are closer to market capitalization-weighted indexes. These are indexes that hold all the stocks in the parent index but tilt the market cap weights toward the desired factor. The factor indexes, also referred to as High Exposure Indexes, hold a subset of names in the parent index and employ more aggressive weighting mechanisms, compared to the factor tilt indexes. For this study, we select the MSCI factor indexes because they provide a balance between investability and exposure to the pure factor. MSCI provides an extensive selection of factor indexes. For this study, we select the following common factors: Value Weighted (Value), Equal Weighted (Size), Minimum Volatility (Volatility), High Dividend Yield (Dividend), Quality, and Momentum. [1] For this thesis, we use ACWI MSCI indexes, which include 23 developed markets countries and 26 emerging markets countries. The table 8 in Appendix F provides additional information on the selected MSCI factor indexes. The U.S. 3-month treasury bill represents the risk-free rate. [2]

In order to choose the appropriate forecast model, we tested the data for its stationarity. The observed logarithmic returns series seem to be stationary, as the augmented Dickey-Fuller test (ADF) rejects the null hypothesis of a unit root for up to 10 lag for every factor tilt index based on the full data sample from December 1998 to June 2019. Each factor portfolio rejects the null hypothesis of unit root at 5% significance level.

4.1 Factor Performance

This part briefly reveals the general characteristics of the single factor performance. We will discuss returns, volatilities, and correlations from approximately 21 years of data.

Let $P_{i,t}$ be the price[3] of a factor index i at time t. Thus, the simple return $R_{i,t}$ is defined as follows:

$$R_{i,t} = \frac{P_{i,t}}{P_{i,t-1}} \tag{1}$$

[1] https://www.msci.com/end-of-day-data-search

[2] The treasury bill data is obtained from Datastream using "USGBILL3" ticker

[3] The price excludes dividends due to the absence of total return information on MSCI factor indexes

	Value	Size	Dividend	Low volatility	Momentum	Quality	Market
Value	1.00	0.97	0.91	0.84	0.81	0.87	0.94
Size	0.97	1.00	0.92	0.87	0.82	0.89	0.95
Dividend	0.91	0.92	1.00	0.92	0.78	0.89	0.94
Low volatility	0.84	0.87	0.92	1.00	0.82	0.87	0.89
Momentum	0.81	0.82	0.78	0.82	1.00	0.89	0.88
Quality	0.87	0.89	0.89	0.87	0.89	1.00	0.97
Market	0.94	0.95	0.94	0.89	0.88	0.97	1.00

This table reports the correlation matrix of the excess returns of the individual assets. Dividend, Momentum, Quality, Size, Value, and Volatility are the MSCI ACWI Factor indexes. The market is the MSCI ACWI equity index. The risk-free rate is the 3-month treasury bill from the Datastream. The estimation is based on the data between December 31 1998 and June 29 2019.

Table 1: Correlation matrix of the assets.

Where $i = 1, \cdots, N$ with N as the amount of factors that we analyze, in this case six different MSCI factors. And $t = 1, \cdots, T$ with T the number of factor index data observed.

It is common to modify the data in a way that it becomes justified to assume that the data follow a stationary process—that is, a time-independent mean and variance. In practice, for the price time series $P_{i,t}$ the stationarity cannot be assumed, but after transformation to simple returns and subsequent application of logarithmic transformation, the stationarity can be assumed, and the test supports this assumption. The logarithmic return $r_{i,t}$ is defined as following:

$$r_{i,t} = ln(R_{i,t}) \tag{2}$$

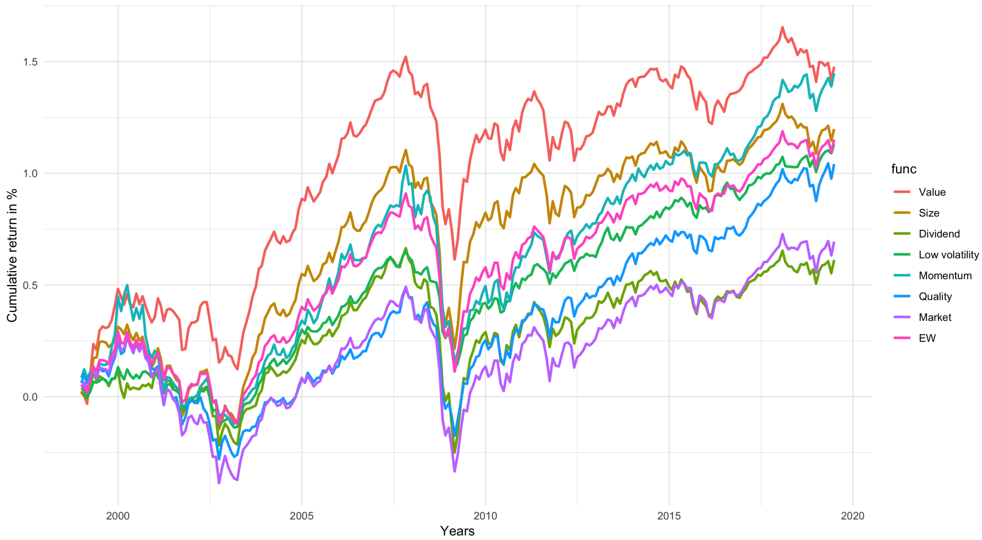
The excess return of the asset is defined as the asset's achieved rate of return less the risk-free rate of return over the same period, $r_{i,t} - rf_t$ The total portfolio return can be defined by $R_p = w'R$, where R is the vector of returns and w the vector of weights.

Table 1 reflects a high correlation under the different factors and its high correlation with the market index. However, all of the correlations are under 1. Therefore, there is room to improve the performance by combining multiple factors in the multiple-factor portfolio. Figure 1 below is the graphical representation of the cumulative logarithmic returns of the single-factor and market-cap indexes. From this graph, it is clear that the different factors follow the same pattern, as the market. Both the market and the factor portfolios exhibit a significant drawdown during the recessions of 2000–2001 and 2007–2009. The correlation between the different factors is between 0.78 and 0.97. Overall, the factor indexes have high correlations with each other and the market, The Momentum factor has the lowest correlation with the market cap: 0.88. The high correlation is not an unexpected result due to the long-only construction of the indexes, which means that they are expected to be sensitive to the movements of the market. Table 2 displays the summary statistics of the six single-factor portfolios and the market portfolio; all factors except Dividend show superior results in terms of risk-adjusted return compared to the market.

	Value	Size	Dividend	Low volatility	Momentum	Quality	Market
Annualized Return	7.41	5.98	3.02	5.63	7.27	5.18	3.42
Annualized Excess Return	5.55	4.14	1.22	3.79	5.40	3.35	1.62
Annualized Volatility	18.32	17.47	15.08	10.26	16.57	14.03	15.50
Annualized Sharpe Ratio	0.30	0.24	0.08	0.37	0.33	0.24	0.10
Annualized Information Ratio	0.52	0.39	-0.06	0.37	0.45	0.50	
Maximum Drawdown	62.69	62.39	62.63	42.73	61.81	50.64	58.94
Skewness	-0.59	-0.95	-0.86	-1.12	-0.81	-0.79	-0.89
Kurtosis	1.66	3.90	3.14	3.12	1.99	1.63	2.35
Normality	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Minimum	-23.36	-27.27	-21.62	-15.55	-18.98	-18.25	-22.20
Maximum	15.51	16.46	13.50	6.44	16.48	9.07	10.88

This table reports the summary statistics of the single factors. All the calculations are based on the logarithmic returns. The sample period is December 31, 1998 - June 29, 2019. The annualized return is the arithmetic mean of the annual returns. The annualized excess return is the annualized return minus the arithmetic mean of the risk-free return. The annualized volatilities are the annual standard deviations of the monthly returns. The annualized Sharpe ratio is calculated as the annualized excess return divided by the standard deviation of the excess return. The annualized Information ratio is calculated as the annualized active return divided by the standard deviation of active return. The maximum drawdown is the lowest return over the period, measured in percent. Skewness is the degree of distortion of returns from the symmetrical bell curve. Normality is the p-value from the Jarque-Bera test, H_0 is the normality. Kurtosis is a measure of the "tailedness". Dividend, Momentum, Quality, Size, Value, and Volatility are the MSCI ACWI Factor indexes, Table 8 in Appendix A contains information on the MSCI indexes. The market is the MSCI ACWI equity index. The risk-free rate is the 3-month treasury bill from the Datastream with a ticker "USGBILL3".

Table 2: Summary statistics of the assets



(a) Assets cumulative returns

Figure 1: The cumulative returns of the single factors.

5 Methodology

This section discusses the stages of analysis, including the calculation of stock returns, expected return modeling, volatility modeling, and portfolio optimization.

The analysis is carried out on a "rolling" basis:

- For each time, t, the previous k periods of the logarithmic returns are used as a window for fitting an optimal ARMA and GARCH model.
- The combined model is used to predict the next period return and variance.
- estimation of the predicted efficient frontier
- The calculation of the optimal weights for the tangency portfolio
- Back-testing of the portfolio results

5.1 Prediction of the prices with ARMA(p,q)

Knowledge of the trends is essential for drawing up appropriate policies in the financial fields. A time-series automatic forecast is beneficial for this purpose. The automatic forecast algorithm will determine an appropriate time series model, estimate the parameters of the model, and compute the forecast. For this thesis, we will use one of the most used automatic forecast algorithms based on the ARMA(p,q) model.

In this section, we introduce univariate autoregressive moving average (ARMA) processes, which provide a model for predicting the dynamics of a univariate time series based on the dependency of the values of a time series on its past. Engle and Bollerslev (1986) and Baillie and Bollerslev (1992) consider predictions from an ARMA model with GARCH errors, which are similar to the model used in this thesis.

The notation AR(p) refers to the autoregressive model of order p. The AR(p) model can be described by the following equation:

$$r_{i,t} = c_i + \phi_{1,i} r_{i,t-1} + \phi_{2,i} r_{i,t-2} + \cdots + \phi_{p,i} r_{i,t-p} + \epsilon_{i,t} \tag{3}$$

where $\phi_{1,i}, \phi_{2,i}, \cdots, \phi_{p,i}$ are the parameters of the model for asset i, the constant c_i is the constant and $\epsilon_{i,t}$ is white noise term.

The notation MA(q) refers to the moving average model of order q:

$$r_{i,t} = \mu_i + \theta_{1,i}\epsilon_{i,t-1} + \theta_{2,i}\epsilon_{i,t-2} + \cdots + \theta_{q,i}\epsilon_{i,t-q} + \epsilon_{i,t} \tag{4}$$

where $\theta_{1,i}, \theta_{2,i}, \cdots, \theta_{q,i}$ are the parameters of the model for asset i, μ_i is the expectation of the asset i and the $\epsilon_{i,t}, \epsilon_{i,t-1}, \cdots, \epsilon_{i,t-q}$ are white error terms.

An $ARMA(p,q)$ process includes both autoregressive and moving average terms:

$$r_{i,t} = c + \phi_{1,i} r_{i,t-1} + \phi_{2,i} r_{i,t-2} + \cdots + \phi_{p,i} r_{i,t-p} + \theta_{1,i}\epsilon_{i,t-1} + \theta_{2,i}\epsilon_{i,t-2} + \cdots + \theta_{q,i}\epsilon_{i,t-q} + \epsilon_{i,t} \tag{5}$$

Where $\epsilon_{i,t} \sim N(0, \sigma_i^2)$, so the residuals $\epsilon_{i,t}$ are normally distributed white noise with a mean 0 and variance σ_i^2 Provided that the roots of

$$1 - \phi_{1,i} z - \phi_{2,i} z^2 - \cdots - \phi_{p,i} z^p = 0 \tag{6}$$

Based on the ADF test described in section data, we assume that the data satisfies the condition of $r_{i,t}$ being stationary at every point in time $t = 1, ..., T$ and ARMA(p,q) model is an appropriate model for estimating the expected return. To fit data to an ARMA model, we use the Akaike Information Criterion (AIC), as proposed by Brockwell and Davis (1991), across a subset of values for p, q to find the model with minimum AIC. we run the model for $p \leq 5$ and $q \leq 5$. The logarithmic returns data will be used to estimate the mean model using the 100, 124, and 148 months rolling window. Based on the AIC, the tentative models can be specified for each factor return.

5.2 GARCH

In order to construct an optimal portfolio, we need a reasonable estimation of the expected covariance matrix. It is prevailing to estimate the expected covariance matrices based on equally weighted historical returns, but this requires an assumption of volatility being constant over time, which is unlikely in general. The volatility is not directly observable. Therefore, the computing of the optimal portfolio requires an attractive model to estimate the expected covariance matrix. The most common model that captures time-varying properties is the General Autocorrelated Conditional Heteroscedasticity (GARCH) model introduced by Bollerslev (1986). The GARCH model is the generalization of autoregressive conditional heteroscedasticity (ARCH) model introduced by Engle (1982) as a convenient way of modeling time-dependent conditional heteroscedasticity. This model has shown to be successful in estimating and predicting volatility changes, for example, Berra and Higgins (1993) demonstrate it in their study. Following the Engle's model, multiple modifications of conditionally heteroscedastic models (e.g., component GARCH, asymmetric power ARCH and more) have been presented, creating an extensive ARCH family.

We will take the following steps to model volatility for each application:

- The estimated of appropriate ARMA (p,q) model
- Test of univariate GARCH(1,1) effects
- Estimation of DCC-GARCH(1,1) model
- Test of the fitted model

We assume that the variance is not constant over time and to relax the assumption of a constant variance a dynamic conditional correlation (DCC) GARCH as describes an article of Engle (2002) will be applied. This method is chosen over the regular multivariate GARCH due to its simplicity and flexibility. Furthermore, the DCC-model includes conditions that make the covariance matrix positive definite at any time t and the process covariance stationary. These conditions make the estimation of the covariance matrix more robust.

There are many methods to get reliable estimates of correlations between financial variables. There are simple techniques such as rolling historical correlations and exponential smoothing, which are widely used. We will use the rolling historical correlations for our benchmark. There are more sophisticated methods, such as varieties of multivariate generalized autoregressive conditional heteroscedasticity (GARCH). In most cases, the number of parameters in large models is too large for straightforward optimization. These functions entail the estimation and forecasting of large covariance matrices. In this thesis, the number of assets is six, but the DCC-GARCH provides the flexibility of univariate GARCH but not the complexity of standard multivariate GARCH. DCC-GARCH, which parameterize the conditional correlations directly, is easily estimated in two steps— a series of univariate GARCH estimates and the correlation estimate. This method has a computational advantage over the multivariate GARCH model in that the number of parameters to be estimated in the correlation process is independent of the number of series to be correlated.

The dynamic conditional correlation (DCC) estimator is a generalization of Bollerslev's (1990) constant conditional correlation estimator. It parameterizes the conditional correlations directly and is estimated in two steps – a series of univariate GARCH estimates and the correlation estimate. Since the number of the estimated parameters in the correlation process is independent of the number of series to be correlated, the methods have clear computational advantages over multivariate GARCH models. Correlation matrices can, therefore, be feasibly estimated.

For the backtesting and after the estimation of the mean model ARMA(p,q), GARCH $(1,1)$ gets estimated where p and q are chosen differently for each time t in order to minimize the AIC of the model.

For this model, a few assumptions are made. The multivariate normal distribution of the errors is the first assumption. The proof by Jeantheau (1998) typically justifies this assumption of the strong consistency of the Gaussian quasi-maximum likelihood for multivariate GARCH models.

DCC - GARCH model is described by Engle (2002) as following:

$$r_{i,t} = \phi_{0,i} + \phi_{1,i} r_{i,t-1} + \phi_{2,i} r_{i,t-2} + \cdots + \phi_{p,i} r_{i,t-p} + \epsilon_{i,t} + \theta_{1,i}\epsilon_{i,t-1} + \theta_{2,i}\epsilon_{i,t-2} + \cdots + \theta_{q,i}\epsilon_{i,t-q} + \mu_{i,t} \tag{7}$$

$$H_t = D_t R_t D_t \tag{8}$$

$$\mu_{i,t} \sim N(0, H_t) \tag{9}$$

In this model r_t is a vector of log returns and it is specified as an ARMA(p,q) model, where the conditional mean is a function of lagged returns and moving average. $mu_{i,t}$ is the error term vector and it is the normally distributed with a mean of zero and variance of H_t.

H_t is the conditional variance N by N matrix. D_t is the diagonal N by N matrix of the conditional standard deviations and is defined as following:

$$D_t = \begin{pmatrix} \sqrt{h_{1,t}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{h_{N,t}} \end{pmatrix} \tag{10}$$

The conditional variances, and $h_{i,t}$, which can be estimated separately, can be written in vector form based on GARCH(1,1) models Where h_t can be defined by the following equation:

$$h_t = \omega + \sum_{i=1}^{r} A_i \varepsilon_{t-i} \odot \varepsilon_{t-i} + \sum_{i=1}^{q} B_i h_{t-i} \tag{11}$$

Where $\omega \in R^n$, A_i and B_i are NxN diagonal matrices and $\odot$ denotes the Hadamard operator.

R_t is the conditional correlation N by N matrix of the standardized disturbances u_t and it is defined as following:

$$R_t = diag(Q_t)^{-1/2} Q_t diag(Q_t)^{-1/2} \tag{12}$$

where Q_t has a GARCH(1,1) specification:

$$Q_t = (1 - \alpha - \beta)\overline{Q} + \alpha z_{t-1} z_{t-1}^T + \beta Q_{t-1} \tag{13}$$

where α and β are non-negative scalars, with the condition that $\alpha + \beta < 1$ imposed to ensure stationarity and positive definiteness of Q_t, z_t are the standard residuals and are defined as following:

$$z_t = \frac{\mu_{i,t}}{\sqrt{h_{i,t}}} \tag{14}$$

α and β are the unknown scalars, $\overline{Q}$ is the unconditional covariance matrix and can be described by the following equation:

$$\overline{Q} = \frac{1}{T}\sum_{t=1}^{T}\begin{pmatrix} z_{1,t}^2 & z_{1,t}z_{2,t} & \cdots & z_{1,t}z_{N,t} \\ z_{2,t}z_{1,t} & z_{2,t}^2 & \cdots & z_{2,t}z_{N,t} \\ \vdots & \vdots & \ddots & \vdots \\ z_{N,t}z_{1,t} & z_{N,t}z_{2,t} & \cdots & z_{N,t}^2 \end{pmatrix} \tag{15}$$

5.3 Forecasting

Because of the nonlinearity of the DCC evolution process, the multi-step ahead forecast of the correlation cannot be directly solved, and is instead based on the approximation suggested in Engle and Sheppard (2001). Consider the one-step ahead evolution of the proxy process Q_{t+1}:

$$Q_{t+1} = (1-\alpha-\beta)\overline{Q} + \alpha E_t[z_t z_t^T] + \beta Q_t \tag{16}$$

where $E_t[z_t z_t^T] = R_t$

5.4 DCC-GARCH Estimation

DCC-GARCH parameters can be estimated by maximum likelihood. For this sample with $t = 1, ..., T$, the log-likelihood function of multivariate GARCH model is defined as following:

$$log\ L = \frac{1}{T}\sum_{t=1}^{T} logL_t = \frac{1}{T}\sum_{t=1}^{T} log f(r_{1,t}, r_{2,t}, ..., r_{N,t}) \tag{17}$$

where $f(r_{1,t}, r_{2,t}, ..., r_{N,t})$ is an N-dimensional multivariate probability distribution, which is defined by the following definition:

$$f(r_{1,t}, r_{2,t}, ..., r_{N,t}) = (\frac{1}{2\pi})^{N/2}|H_t|^{-1/2} e^{-0.5u_t^T H_t^{-1} u_t} \tag{18}$$

where u_t is the N by 1 vector of errors at time t and they are calculated as following: $u_t = r_t - \mu_t$, where r_t is a N by 1 vector of returns and μ_t is the N by 1 vector of conditional means of the returns, as mentioned by Hurn, Martin, Philips and Yu (2015).

Assuming multivariate conditional normality, the log-likelihood function is defined by the following equation:

$$log\ L_t = log f(r_{1,t}, r_{2,t}, ..., r_{N,t}) = -0.5N log(2\pi) - 0.5 log|H_t| - 0.5u_t^T H_t^{-1} u_t \tag{19}$$

5.5 Mean-Variance portfolio

Markowitz (1952) proposed a model for constructing an optimal portfolio, where optimal is under the assumption that investors are rational and seek a high return and corresponding low risk. The objective is to find an optimal trade-off between the risk and return.

Consider constructing a portfolio consisting of the M factor indexes. Suppose that r_{it} is factor return i at time t, where $i = 1, \cdots, N$ with N is the number of factor indexes that were analyzed and $t = 1, \cdots, N$ with T the amount of factor data observed. Suppose that $w^T = (w_1, \cdots, w_N)$ is the weight vector of the amount of wealth distributed in the different factors of the portfolio, $r^T = (r_{1t}, \cdots, r_{Nt})$, and $e^T = (1 \cdots, 1)$ is the unit vector. Porfolio return can be expressed by $r_p = w^T r$ with $w^T e = 1$. Suppose $\mu^T = (\mu_{1t}, \cdots, \mu_{Nt})$ is the expectations of portfolio μ_p expressed by:

$$\mu_p = E[r_p] = w^T \mu \tag{20}$$

Suppose that given covariance matrix $\Sigma = (\sigma_{ij}\ i, j = 1, \cdots, N)$ where $\sigma_{ij} = Cov(r_{it}, r_{jt})$. Variance of the total portfolio return will be expressed by the following equation:

$$\sigma_p^2 = w^T \Sigma w \tag{21}$$

Suppose that the given covariance matrix $\Sigma = (\sigma_{ij})\ i, j = 1, \cdots, N$, where $\sigma_{ij} = Cov(r_{it}, r_{jt})$. Variance of the portfolio return can be expressed by the following equation:

$$\sigma_p^2 = w^T \Sigma w \tag{22}$$

A portfolio $p*$ can be called mean-variance efficient if and only if there is no portfolio P with $\mu_p \geq \mu_{p*}$ and $\sigma_p^2 < \sigma_{p*}^2$ To estimate an efficient portfolio we use the following objective to maximize:

$$2\tau\mu_p - \sigma_p^2,\ \tau \geq 0 \tag{23}$$

where the parameters of the investor's risk tolerance. It means, for investor with risk tolerance τ, where $\tau \geq 0$, the portfolio problem is estimated as following:

$$Max\ \{2\tau w^T \mu - w^T \Sigma w\} \tag{24}$$

$$s.t.:\ w^T e = 1 \tag{25}$$

The completion of the following maximization problem will form a complete set of efficient portfolios for $\tau \in [0, \infty)$. Set of all points in the diagram (μ_p, σ_p^2) relate to the efficient portfolio, efficient frontier. The equation number (26) gives the optimization problem of quadratic convex.The book by Shapiro (1979b) marks the first appearance of the term Lagrangian relaxation in a textbook and describes it. The function of Lagrangian multiplier of portfolio optimization problem is given by:

$$L(w, \lambda) = 2\tau w^T \mu - w^T \Sigma w + \lambda(w^T e - 1) \tag{26}$$

Using the Kuhn-Tuckler theorem, the optimality condition of equation is $\frac{\partial L}{\partial w} = 0$ and $\frac{\partial L}{\partial \lambda} = 0$. Completing these two conditions in the Lagrangian multiplier, the optimal weight can be calculated as following:

$$w* = \frac{1}{e^T \Sigma^{-1} e} \Sigma^{-1} e + \tau\{\Sigma^{-1}\mu - \frac{e^T \Sigma^{-1} \mu}{e^T \Sigma^{-1} e} \Sigma^{-1} e\} \tag{27}$$

5.6 Backtests

We use the mean-variance optimization strategy to implement backtests. The backtests are built using a rolling-sample approach by rebalancing the portfolios every month based on a one-step-ahead forecast. The rebalancing dates correspond to the last trading day of the month. The initial rolling window of 100 months will be used to give a better insight into trend-following strategies for a long holding period. We choose for long rolling windows to make the estimation of GARCH covariance more robust due to the data being monthly as when the rolling window has less than 100 observations, the results will not be robust. For each application, we will compute the (compound) annual return, the volatility, the Sharpe ratio, information ratio, the maximum drawdowns, the minimum and maximum return over the full period. Moreover, we will estimate the same model using the 124- and 148-months rolling window.

As mentioned earlier the mean and volatility models will be used to calculate the prediction terms of mean and volatility, $\hat{\mu}_{it} = \hat{r}_{ih}(l)$ and $\hat{\sigma}^2_{it} = \hat{\sigma}^2_{ih}(l)$, for 1-period ahead of the starting point prediction h. The prediction results of mean $\hat{\mu}_t = \hat{r}_{ih}(1)$ and volatility $\hat{\sigma}^2_{it} = \hat{\sigma}^2_{ih}(1)$, will be used for optimization calculations.

After estimating the expected returns and the expected covariance we apply the Ljung-Box test with the lag value 20 to determine if a good fit has been achieved, for particular values of ARMA(p,q) and DCC-GARCH$(1,1)$ at time t. We run this test for each estimation separately. If the p-value of the test is higher than the 5% significance level, we can conclude that the residuals are independent and white noise.

5.7 Benchmarks

Performance should always be interpreted compared to a certain benchmark. This way, the results can be placed in perspective. We choose the following benchmarks for this research:

- The equally-weighted factor portfolio
- The global market proxy portfolio
- The simple mean-variance factor portfolio without the time-varying effects.

Equally weighted portfolios are widely used in practice, and this na'ive approach has been proved efficient (e.g., DeMiguel, Garlappi and Uppal (2009)) The equally-weighted (EW) factor portfolio weights are calculated as follows:

$$w_i^{EW} = \frac{1}{N} \tag{28}$$

where w_i^{EW} is the weight of asset i in the EW portfolio, and N is the number of assets in the portfolio, in this thesis, there are six factors. Therefore, each factor's weight will be set to $\frac{1}{6}$ in the portfolio.

The global market proxy portfolio is represented by the ACWI MSCI Standard, as mentioned in the Data section.

The simple mean-variance factor portfolio is calculated using the rolling historical returns and correlations. Therefore, the weights are represented by the equation (27), where

$\widehat{\mu^T} = \frac{1}{T}\sum_{t=1}^{T}(\mu_{1t},\cdots,\mu_{Nt})$ is the expected return on the factors represented by the average returns on the factor during the rolling window and $\widehat{\Sigma} = \frac{1}{T}\sum_{t=1}^{T}\Sigma_t$ is the expected covariance matrix represented by the average covariance matrix during the length of the rolling window. This mean-variance portfolio will help to investigate the effect of the time-varying approach of the ARMA DCC-GARCH model.

5.8 Performance Measures

The main aim of investing is to generate a substantial return. The most straightforward performance measure is the active return. The active return reveals the return on an investment relative to a benchmark. For this study, we use the market portfolio as a benchmark for active return. The active return is calculated as follows:

$$\alpha_p = R_p - R_b \tag{29}$$

where R_p is the annualized return of portfolio p, R_b is the annualized return of benchmark b, in this thesis the market portfolio, α_p represents the relative return of portfolio p to the benchmark b.

The other measurement computed is the Information ratio. The information ratio is calculated using a tracking error. The tracking error is defined by the standard deviation of the active return. The lower the tracking error, the more the portfolio (or index) replicates the benchmark. The tracking error is the standard deviation of the return differences (active returns); $\sigma_{R_p - R_b}$. The information ratio combines the active return and the tracking error to describe how consistent an investor outperforms his benchmark. A high information ratio suggests positive active returns (outperformance) and high consistency due to the ability to follow the price behavior of the benchmark. The Information ratio is defined as follows:

$$IR = \frac{R_p - R_b}{\sigma_{R_p - R_b}} = \frac{\alpha_p}{\sigma_{\alpha_p}} \tag{30}$$

where R_b is the annualized return of benchmark b, in this thesis the market portfolio, α_p represents the annualized relative return of portfolio p to the benchmark b, σ_{α_p} is the standard deviation of the annualized relative return of the portfolio p.

In order to compare different portfolios, it is important to examine the performance by adjusting for its risk. The Sharpe (1994) ratio provides a measure for the risk-adjusted returns. The higher Sharpe ratio indicates a better return yielding capacity for every additional unit of risk taken. The annualized Sharpe ratio is defined as follows:

$$SR = \frac{R_p - R_f}{\sigma_{R_p - R_f}} = \frac{R_p^e}{\sigma_{R_p^e}} \tag{31}$$

where R_p is the annualized return of portfolio p, R_f is the annualized risk-free rate, R_p^e is the annual portfolio excess return, and $\sigma_{R_p^e}$ is the standard deviation of the annualized portfolio excess returns.

The maximum drawdown (MDD) is the maximum observed loss during the sample period. MDD is a measure of the downside risk of a portfolio or an asset. There are several approaches to calculate the maximum drawdown, depending on the purpose. For this thesis, we use the most common approach. The MDD tells the percentage loss from peak value to trough the value of the asset. MDD is defined as $MDD = \frac{TroughValue - PeakValue}{PeakValue}$. Furthermore, a time series of drawdowns can be used to reveal the recovery period after a substantial financial loss. For this purpose, the drawdowns are calculated as a time series of losses from the cumulative return time series and as a maximum during the full estimation period.

Another risk measure that we use is the monthly asset turnover of the portfolio. We calculate it to investigate the stability of the portfolio. The monthly asset turnover is defined as the mean of the absolute weight changes:

$$Turnover_t = \sum_{i=1}^{N} w_{t,i} - w_{t-1,i} \tag{32}$$

6 Results

This section analyzes the performance of the individual factor portfolios and the trend-following strategy. It is divided in three subsections. The first subsection discusses the performance and properties of single-factor portfolios. The second subsection discusses the performance of the trend-following strategy and compares it to the benchmarks. The last subsection discusses the portfolio composition of the trend-following strategy.

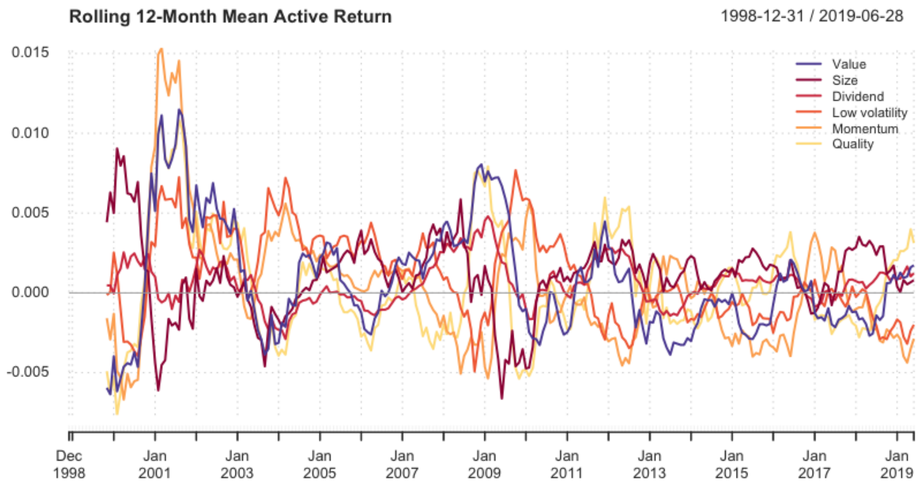
It should be noted that in order to implement the volatility modeling based on DCC-GARCH, we use R package "mgarch". This DCC-GARCH model requires 100 or more observations to run. However, when using the minimum number of observations, minor convergence problems can occur, and the estimated covariance matrix is not optimal for some iterations. Merton (1980) argues that a higher amount of observations, for example, 2500 observations is more appropriate to estimate a multivariate GARCH model, but using a time series over 20 years with lower frequency data such as the data available for this study does not give an opportunity to increase the number of observations to this level. According to Hwang (2006), multivariate GARCH estimates from the low-frequency data could suffer from the temporal aggregation problem.

6.1 Factor Performance

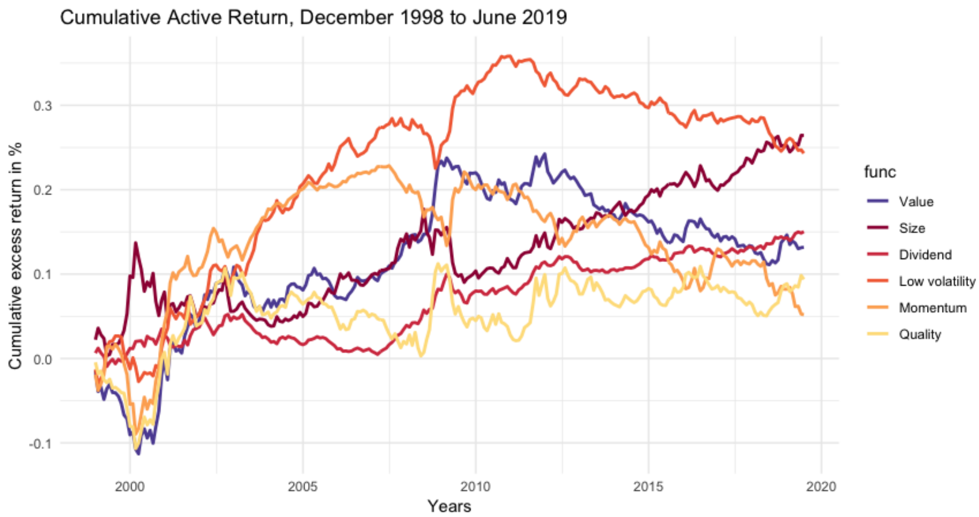
In this subsection, we discuss the performance of single factors to demonstrate how single-factor portfolios perform in terms of cyclicality. The purpose of this comparison is to determine whether there are benefits of combining factor indexes into multi-factor portfolios and applying the trend-following strategy.

To investigate possible benefits, we review the relative returns on assets. Figure 2(a) shows the annualized active returns relative to the market-cap-weighted index (MSCI ACWI Index). It presents the time series of 12-month average active performance for each asset. For this analysis, we use the full sample period from December 1998 to June 2019. Figure 2(b) reveals cumulative active returns relative to the market; from Figure 2 one can draw conclusions about the cyclicality of the relative performance during the last approximately 21 years. Factor returns have historically been highly cyclical. Each of the factor indexes has undergone at least one period of two consecutive years of inferior or superior performance. Some factors experienced even more extended periods: the Low Volatility factor, for example, went through eight years of underperformance from 2011 to 2019. On the contrary, the Size factor has been outperforming the market for an even more extended period: from 2010 to June 2019.

During 1999–2001, only Size and Dividend factors outperformed the market. This period corresponds to the recessions that affected the European Union and the United States during 1999 and 2001. During the financial crisis of 2007–2009, the factors of Size, Value, and Dividend outperformed the market. Based on the historical performance, we can conclude that the factors Size, Dividend, and possibly Value behave defensively during recessions.



(a) Average Relative (Active) Returns



(b) Cumulative Relative (Active) Returns

Figure 2: Asset's relative (active) returns

6.2 Portfolio Analysis

This subsection analyzes the properties and performance of the trend-following strategy. As mentioned in Section 5, the research is conducted using three different rolling windows of 100, 124, and 148 months. To make the results comparable, we analyze performance from April 2011 to June 2019 based on the 148-month rolling window. Moreover, we investigate performance during the financial crisis of 2007–2009. To do this, we review the performance of the portfolio with a 100-month rolling window from April 2007 to April 2011. The portfolios are rebalanced monthly, and we do not consider any transaction costs.

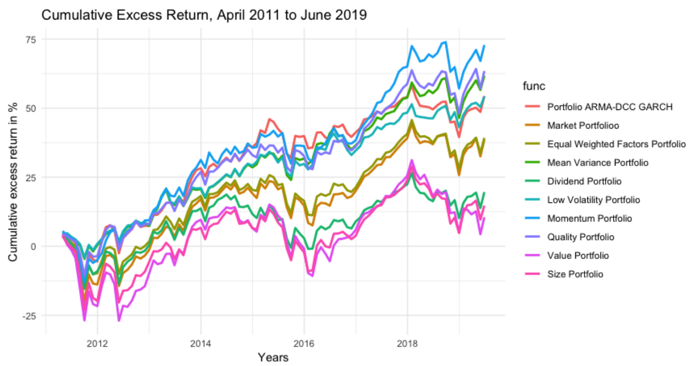
For every time t between April 2007 and June 2019, the new mean-variance portfolio is estimated based on the expected return and covariance matrix from the ARMA DCC-GARCH model. Diagnostic tests for these models are conducted using the residual data correlogram and Ljung-Box test hypotheses. The test results show that the residuals of the models are white noise in 90.88% of cases when using the 5% significance level of the Ljung Box test and lag of 20 months. In a 100-month rolling window model, 81 of 888 of the residuals are below the significance level of 5%. In the same model with 124- and 148-month windows, respectively, 89.11% and 90.5% pass the white noise condition. Appendix G shows the corresponding tables containing the p-values for each asset and each iterations for 100-, 124- and 148-month windows.

6.2.1 Results excluding the financial crisis 2007-2009

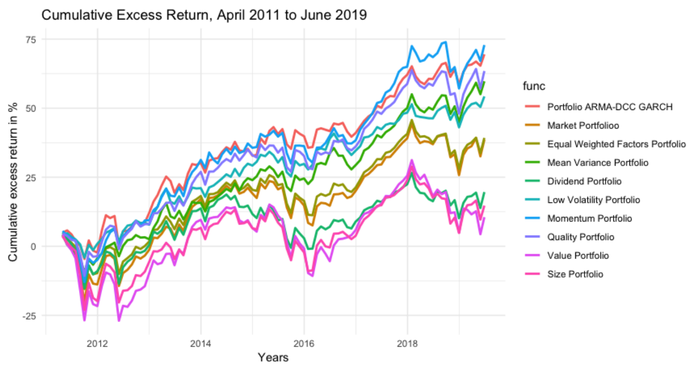
100-month rolling window Table 3 in Appendix A.1 reports the summary statistics of the portfolios with a 100-month rolling window and its benchmarks. The returns of this model are normal on a 5% significance level with slightly negative skewness. In terms of the Sharpe ratio, the ARMA DCC-GARCH (1,1) mean-variance portfolio outperforms the market-cap-weighted index and equally weighted factor portfolio. The Sharpe ratio of 0.6 is substantially higher than the Sharpe ratio of the market-cap-weighted index and equally weighted factor portfolio, which are, respectively, 0.37 and 0.42. It also outperforms these benchmarks in terms of absolute returns (7.42%, 5.38%, and 5.47%) and information ratio (0.47 and 0.12). This trend-following strategy also outperforms the single factors Value and Size. Figure 7 (a) in Appendix C plots the rolling Sharpe ratio through the estimation period; it shows that this model demonstrates a much lower Sharpe ratio in 2016–2019 than other benchmarks. Figures 3 (a) and 6 (a) in Appendix B support the finding: in 2016–2019, this trend-following strategy underperforms the market cap index.

This trend-following model performs poorly compared to the other benchmarks. It has a substantially lower Sharpe ratio (0.6) than a simple mean-variance model (0.78) and the following single factor indexes: Low Volatility (0.79), Momentum (0.77), and Quality (0.68). Additionally, the model with this rolling window shows poor performance in terms of maximum drawdown. Figure 8 (a) in Appendix D displays the maximum drawdowns through the time. The trend-following strategy persistently underperforms in terms of drawdowns throughout the full sample period.

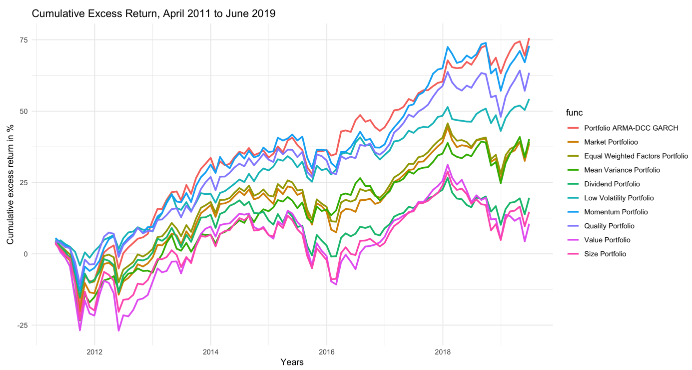
Unfortunately, the ARMA DCC-GARCH (1,1) model with a 100-month rolling window fails to capture most trends within factor investing. It appears to be inferior to several single-factor portfolios: Low Volatility, Momentum, and Quality. Moreover, it is also inferior



(a) Cumulative Returns with Rolling Window 100 months



(b) Cumulative Returns with Rolling Window 124 months



(c) Cumulative Returns with Rolling Window 148 months

Figure 3: Portfolio's cumulative returns

to the simple mean-variance model, which does not model the trend. We can conclude that ARMA DCC-GARCH (1,1) with a 100-month rolling window makes severe errors in estimating the expected return and covariance. The problem with the 100-month rolling window hides behind the construction of the DCC-GARCH (1,1). In order to converge to the right optimum, it needs to estimate many parameters, and such a low amount of data in some cases causes convergence problems. The next paragraph demonstrates that using even a slightly greater amount of data improves the results. However, the 100-month rolling window allows one to research the performance during the financial crisis of 2007–2009, which we discuss in subsection 6.2.2.

124-months rolling window Table 4 in Appendix A.2 reports the summary statistics of the portfolios with a 124-month rolling window and its benchmarks. The returns of this model are normal on a 5% significance level, with slightly negative skewness and slightly long tails. In terms of the Sharpe ratio, the ARMA DCC-GARCH (1,1) mean-variance portfolio outperforms each of the benchmarks, including the single factor indexes. The Sharpe ratio of 0.8 is substantially higher than the Sharpe ratio of the market-cap-weighted index (0.37), the equally weighted factor portfolio (0.42), Dividend (0.21), Quality (0.68), Value (0.09), and Size (0.13). This model also slightly outperforms the simple mean-variance model and substantially outperforms it in terms of information ratio; with 0.76 to 0.45, respectively. This result reflects how well the multiple factors diversify each other in the trend-following strategy.

The trend-following strategy with a 124-month rolling window also shows a slightly superior Sharpe ratio to the single factor indexes of Momentum (0.77) and Low Volatility (0.79). However, it demonstrates inferior performance in terms of the information ratio when compared to these indexes. This strategy outperforms the market cap, but Low Volatility and Momentum indexes perform slightly better if considering the outperformance of the market. However, the trend-following strategy with a 124-month rolling window has a much lower maximum drawdown (11.14%) than Momentum (17.35%). Figure 7 (b) in Appendix C plots the rolling Sharpe ratio through the estimation period. It demonstrates a low Sharpe ratio in 2017–2018 compared to the other benchmarks. Figures 3 (b) and 6 (b) in Appendix B support the finding: in mid-2016–2018, the model underperforms the market cap index. However, it performs well in 2011–mid-2012, 2016, and 2018. Figure 7 (b) demonstrates a low Sharpe ratio compared to the other benchmarks during 2018. Except for 2018, this plot exhibits the same or slightly better performance of this trend-following model in terms of the Sharpe ratio. Figure 8 (b) in Appendix D demonstrates a persistent and low drawdown through the estimation window.

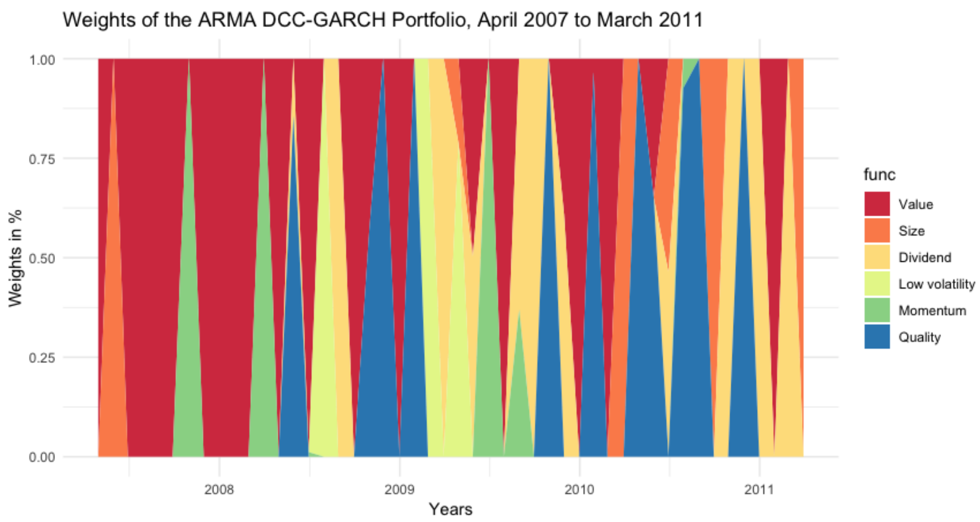
The ARMA DCC-GARCH (1,1) model with a 124-month rolling window demonstrates superior results to each benchmark in terms of risk-adjusted-performance. This model displays persistent outperformance of the market cap and low downside risk. It also displays far fewer convergence issues than the model with 100-month rolling windows. Thus, even a slight increase in the rolling window from the minimum can substantially improve the obtained results. The model with a 124-month rolling window succeeds in capturing the cyclical nature of the factor indexes in this sample.

148-months rolling window Table 5 in Appendix A.3 reports the summary statistics of the portfolios with a 148-month rolling window. The returns of this model are non-normal, negatively skewed, and show long tails. It returns similar results to the model discussed in the previous paragraph. The model with a 148-month rolling window shows the same risk-adjusted-performance. As a result of this, it performs better, in terms of the Sharpe ratio, than any other benchmark. This model differs from the model based on the 124-month rolling window in terms of the information ratio; the information ratio is 1.0 for a 148-month rolling window. By this means, the current model exhibits a higher and more consistent outperformance of the market cap than any other benchmarks. However, this strategy has a much higher maximum drawdown (20.81%) than the model with a 124-month rolling window (11.14%). Figure 7 (c) in Appendix C plots the rolling Sharpe ratio through the estimation period; it shows that this model demonstrates a lower Sharpe ratio in 2017–2018 compared to the other benchmarks. It is also lower than the Sharpe ratio of a model with a 124-month rolling window in 2017–2018. However, the model with a 148-month rolling window performs better in terms of risk-adjusted returns during 2013 and 2018–2019. Figures 3 (c) and 6 (c) in Appendix B support the finding. In mid-2016–2018, the model underperforms the market cap index. However, it performs well in 2011–mid-2012, 2016, and 2018. However, Figures 7 (c) and 6 demonstrate poor performance during 2018 in terms of risk-adjusted and relative returns. This performance is very similar to the performance of the model with a 124-month rolling window.

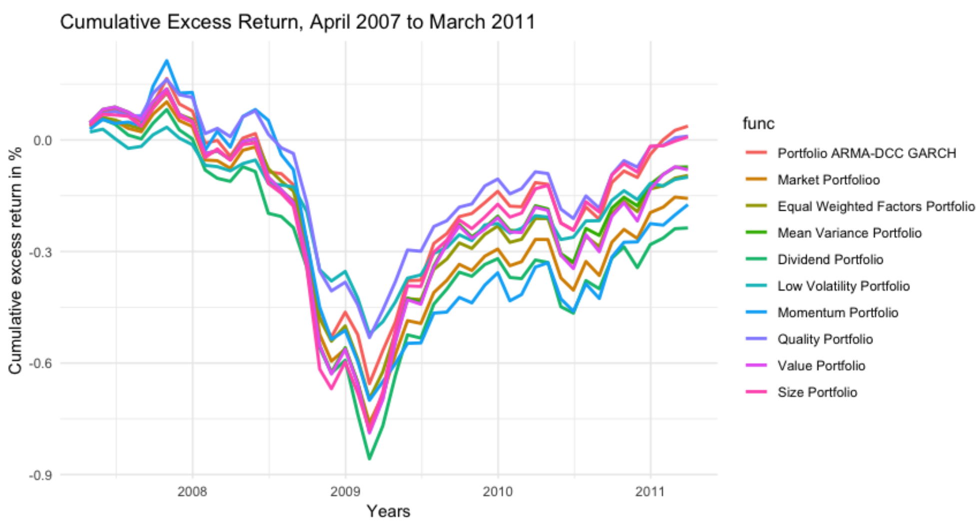
The ARMA DCC-GARCH (1,1) model with a 148-month rolling window demonstrates superior results to each benchmark in terms of risk-adjusted performance and outperformance of the market cap. The model results in much higher downside risk, but the remaining statistics suggest that this model captures the trend. Also, this model displays far fewer convergence issues than the model with a 100-month rolling window. From the results, we can conclude that the model with a 148-month rolling window succeeds in capturing the cyclical nature of the factor indexes in this sample.

6.2.2 Results including the financial crisis of 2007–2009

100-month rolling window Only the model with a 100-month rolling window was able to capture a part of the financial crisis of 2007–2009 due to the length of the estimation period. Table 7 in Appendix A.4 reports the summary statistics of this model from April 2007 to March 2011. The returns of this model are non-normal and negatively skewed and show long tails. The 100-month rolling window has a poor performance from April 2011 to June 2019. However, during the recession and till March 2011, this model outperforms each of the other benchmarks. This portfolio exhibits superior performance in terms of the Sharpe ratio and the (active) return. Thus, the model with a 100-month rolling window may suffer from convergence issues, but it still outperforms every other benchmark during the crisis and up to March 2011. Figure 4 (a) shows the cumulative returns during this estimation period. The cumulative return of the trend-following model follows the same pattern as the other assets in this period but runs slightly smoother. Due to the short length of the sample period, the rolling drawdowns and Sharpe ratio are not plotted for this part.



(a) Weights



(b) Cumulative excess returns

Figure 4: Portfolio's cumulative returns and weights. April 2007 - March 2011

6.3 Portfolio Composition

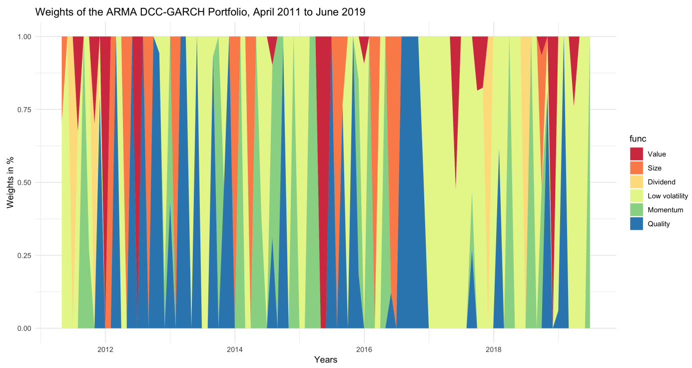
This subsection discusses the allocation of the ARMA DCC-GARCH (1,1) multi-portfolios with 100-, 124- and 148-month rolling windows. The main point of interest is to see whether some factors are overweighted relative to the other factors. Moreover, we discuss how the weightings change over the sample period, with a particular interest in the financial crisis of 2007–2009. Figures 5 (a, b, and c) display the time-series of the portfolio weights of the ARMA DCC-GARCH (1,1) mean-variance portfolios between April 2011 and June 2019. Figure 4 (a) plots the time-series of the portfolio weights from April 2007 to April 2011. The allocation shows a dependency on the rolling window so that we discuss all three rolling windows separately.

6.3.1 Results Excluding the Financial Crisis of 2007–2009

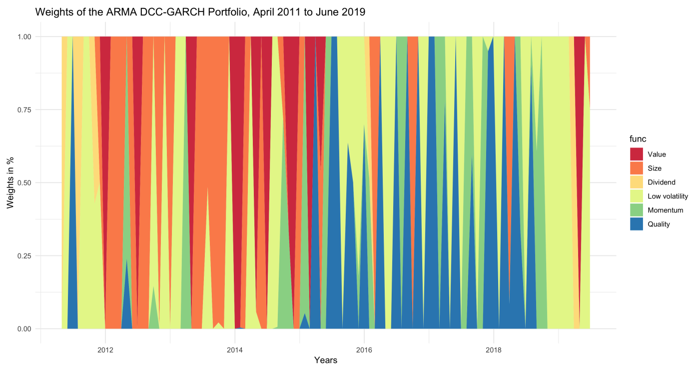
100-months rolling window Figure 5 (a) plots the time-series of the portfolio weights estimated using the trend-following strategy and a 100-month rolling window. As follows from the previous subsection, this model fails to capture the trend. The plot of the portfolio weights supports these findings. In 2012–2014, 2015, and 2017 the model frequently allocates more than 75% to the Quality factor. However, Figures 4(a) and 4(b) show that the Quality factor performed well in 2012 and 2016. In 2017–2019 the strategy with a 100-month rolling window frequently allocated 50–100% of the capital to the Low Volatility factor. This factor showed a superior performance until 2009, but in 2011–2019 the Low Volatility factor underperforms the market cap. Therefore, capital allocation fails to take advantage of the cyclicality of the factors. On the contrary, it does the opposite, which is in line with the findings from the previous subsection 6.2.1.

124-month rolling window Figure 5 (b) plots the time-series of the portfolio weights estimated using the trend-following strategy and a 124-month rolling window. This model displays a different allocation. The allocation is mainly distributed in 2011–2014 between the factors Size and Low Volatility and in 2014–2015 between Size, Low Volatility, and Value. Figures 4(a) and 4(b) show that the Size factor was outperforming during this period. After 2015, this model frequently allocates more than 75% to the Quality factor. In 2019 the model assigns the weight to an underperforming Low Volatility factor, which supports the low Sharpe ratio during this period.

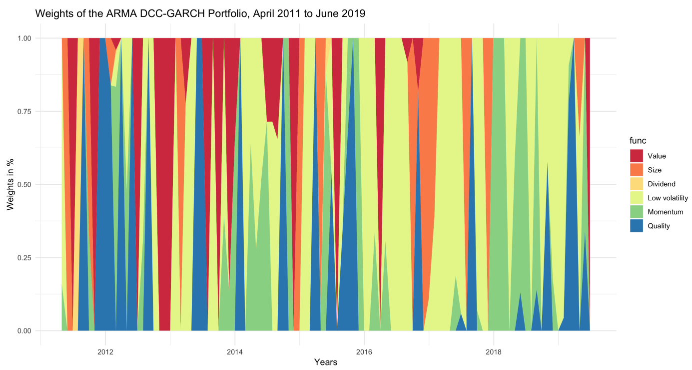
148-month rolling window Figure 5 (c) plots the time-series of the portfolio weights estimated using the trend-following strategy and a 148-month rolling window. The allocation in 2011–2014 is mainly distributed between factors Quality (2011–end 2012), Value, Size, and Low Volatility (2013). As mentioned in the previous paragraph, the Size factor was outperforming during this period. This model captures the outperformance of the Quality factor during 2011–2012. In 2014–2016 this model allocated weight to various factors, with no overweight in particular factors. In 2016–2018 it frequently allocated more than 75% to the Low Volatility, Momentum, or Dividend factor. In 2019, this model assigns a considerable weight to the Quality factor, which exhibits outperformance starting in 2019, as displayed by Figures 4(a) and 4(b).



(a) Weights with rolling window 100 months



(b) Weights with rolling window 124 months



(c) Weights with rolling window 148 months

Figure 5: Portfolio's weights

6.3.2 Results including the Financial crisis of 2007–2009

Figure 4 (a) plots the time-series of the portfolio weights estimated using the trend-following strategy and a 100-month rolling window from April 2007 to March 2011. As follows from the previous subsection, this model fails to capture the trend. However, during 2007–2009, it allocates the capital to the best-performing factor, Value.

6.3.3 Portfolio Turnover

Figures 9 (a and b) and 10 (a) in Appendix E plot the time-series of the portfolio monthly turnover for 100-, 124-, and 148-month rolling windows, respectively. All three models exhibit very high asset turnover: 143.67%, 148.55%, and 135.51% for the 100-, 124-, and 148-month rolling windows, respectively. The model based on a 148-month rolling window shows the most stable portfolio of these three models; this supports the findings from the previous sections. However, a turnover this high will result in high transaction costs. Figure 10 (b) plots the time-series of the monthly turnover for 100-month rolling window from April 2007 to March 2011. During this period the model exhibits even higher turnover: 156.63%.

7 Conclusion

The results presented in this paper are based on the attempt to quantitatively capture the cyclical nature of the factors in the multiple-factor portfolio. We find that when multiple factor indexes are combined into a single portfolio using a dynamic approach, the model shows superior results. When multiple factor indexes are combined into a single trend-following multi-factor index, diversification across factors leads to better risk-adjusted performance compared to the benchmarks. The model outperforms the market and the naïve diversification approach substantially. However, this is only true for the 128- and 148-month rolling windows. The same model with a 100-month rolling window performs poorly in terms of risk-adjusted-performance. However, the trend-following strategy with a 100-month rolling window demonstrates superior performance during the recession. Therefore, we can conclude that our model exhibits less regime dependency upon business cycles.

Nevertheless, rolling windows of 100 months seem not to be sufficient to estimate a reliable covariance matrix. The ARMA DCC-GARCH model benefits from a higher number of observations, which improves the estimation of the modeled covariance matrix. In this research, even a slightly higher amount of data improves the results drastically. The usage of more frequent data will improve the robustness of the estimated covariance matrix. Therefore, research with a higher and more frequent number of observations will bring this strategy further to light.

However, the mean-variance portfolio is expensive to replicate due to the high transaction costs from extremely high asset turnover. This characteristic of the estimated model makes it not directly applicable to the real-life investment strategy. The issue with mean-variance optimization is that the estimated covariance matrix is often estimated with an error. It implies that the extreme asset allocations in such an estimated matrix tend to take on extreme values, due to a maximizing effect of error in the input assumptions, as argued by Michaud (1989). Other models with less excessive factor allocations can provide a more practical implementation of a trend-following strategy. The Black-Litterman model could offer a compromise between exposure to the cyclicality of the factors while putting the constraints to the factor weights. The Black-Litterman framework allows users to specify their subjective views on the market in a consistent and tractable manner.

There are various modifications of the mean-variance model, which penalize the high asset turnover and therefore keep the transaction costs within the limit. Kourtis (2015) discusses some of the stabilization approaches on the mean-variance model and suggests that these models show superior results in the presence of the transaction costs. Further research can help analyze the performance of the trend-following strategy in the presence of the transaction costs.

Another possible solution is to investigate the optimal rebalancing frequency. The performance of the single factors reveals that each factor premium exhibits at least one period of two consecutive years of inferior or superior performance — the periods of under- or outperformance last in general for more than one year. Therefore, the monthly rebalancing frequency may be not optimal for factor investing in the presence of transaction costs. Finding the right approach to the rebalancing frequency can significantly improve the performance in the presence of transaction costs.

Factor investing and trend-following strategies provide a broad spectrum of possibilities for managing the sensitivity to the business cycle. The trend-following strategy in factor investing can be used for tactical and more strategic asset allocation and risk management purposes. Though, the MSCI factor indexes do not provide an accurate representation of the performance of pure factors. What is more, MSCI factors are more concentrated than MSCI tilt factors. Consequently, this makes factor strategy less investable for large institutional investors due to the limited presence of the factors. For large investors, narrow factor MSCI indexes may not deliver sufficient liquidity and volume due to their concentrated nature.

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9 Appendix

A Summary statistics

A.1 The 100-month rolling window

	ARMA-DCC GARCH	Market	Equally Weighted Factors	Mean Variance	Dividend	Low Volatility	Momentum	Quality	Value	Size
Annualized Return	7.42	5.38	5.47	8.37	3.00	7.39	9.83	8.59	1.87	2.39
Annualized Excess Return	6.80	4.77	4.86	7.75	2.40	6.78	9.20	7.97	1.28	1.80
Annualized Active Return	1.94	0.00	0.08	2.85	-2.27	1.92	4.24	3.06	-3.34	-2.85
Annualized Volatility	11.24	12.73	11.54	9.96	11.67	8.63	11.91	11.75	15.05	13.84
Annualized Sharpe Ratio	0.60	0.37	0.42	0.78	0.21	0.79	0.77	0.68	0.09	0.13
Annualized Information Ratio	0.47		0.12	0.49	-0.63	0.34	0.89	0.91	-0.85	-0.90
Maximum Drawdown	15.98	22.14	18.60	13.58	18.70	8.75	17.35	14.64	29.20	24.35
Skewness	-0.14	-0.50	-0.48	-0.61	-0.28	-0.47	-0.68	-0.46	-0.48	-0.47
Kurtosis	0.61	0.76	0.51	0.89	0.15	-0.17	0.77	0.43	0.64	0.88
Normality	0.32	0.03	0.07	0.01	0.46	0.15	0.00	0.10	0.05	0.02
Minimum	-8.12	-10.15	-8.80	-8.66	-9.67	-5.49	-10.53	-8.12	-13.13	-12.68
Maximum	8.90	10.09	8.37	7.93	8.24	5.62	8.46	8.70	10.43	9.81

This table reports the summary statistics of the single factors. All the calculations are based on the logarithmic returns. The estimation period is April 30, 2011 - June 29, 2019. The annualized return is the arithmetic mean of the annual returns. The annualized excess return is the annualized return minus the arithmetic mean of the risk-free return. The annualized active return is the annualized return minus the annualized return of the market. The annualized volatilities are the annual standard deviations of the monthly returns. The annualized Sharpe ratio is calculated as the annualized excess return divided by the standard deviation of the excess return. The annualized Information ratio is calculated as the annualized active return divided by the standard deviation of active return. The maximum drawdown is the lowest return over the period, measured in percent. Skewness is the degree of distortion of returns from the symmetrical bell curve. Normality is the p-value from the Jarque-Bera test, H_0 is the normality. Kurtosis is a measure of the "tailedness". Dividend, Momentum, Quality, Size, Value, and Volatility are the MSCI ACWI Factor indexes, Table 8 in Appendix A provides more information on the MSCI indexes. The market is the MSCI ACWI equity index. The risk-free rate is the 3-month treasury bill from the Datastream with a ticker "USGBILL3".

Table 3: Summary statistics, 100-month rolling window

A.2 The 124-month rolling window

	ARMA-DCC GARCH	Market	Equally Weighted Factors	Mean Variance	Dividend	Low Volatility	Momentum	Quality	Value	Size
Annualized Return	9.38	5.38	5.47	8.11	3.00	7.39	9.83	8.59	1.87	2.39
Annualized Excess Return	8.76	4.77	4.86	7.49	2.40	6.78	9.20	7.97	1.28	1.80
Annualized Active Return	3.82	0.00	0.08	2.60	-2.27	1.92	4.24	3.06	-3.34	-2.85
Annualized Volatility	10.92	12.73	11.54	9.50	11.67	8.63	11.91	11.75	15.05	13.84
Annualized Sharpe Ratio	0.80	0.37	0.42	0.79	0.21	0.79	0.77	0.68	0.09	0.13
Annualized Information Ratio	0.76		0.12	0.45	-0.63	0.34	0.89	0.91	-0.85	-0.90
Maximum Drawdown	11.14	22.14	18.60	12.84	18.70	8.75	17.35	14.64	29.20	24.35
Skewness	-0.39	-0.50	-0.48	-0.46	-0.28	-0.47	-0.68	-0.46	-0.48	-0.47
Kurtosis	0.81	0.76	0.51	-0.37	0.15	-0.17	0.77	0.43	0.64	0.88
Normality	0.05	0.03	0.07	0.14	0.46	0.15	0.00	0.10	0.05	0.02
Minimum	-10.90	-10.15	-8.80	-6.41	-9.67	-5.49	-10.53	-8.12	-13.13	-12.68
Maximum	7.98	10.09	8.37	5.91	8.24	5.62	8.46	8.70	10.43	9.81

This table reports the summary statistics of the single factors. All the calculations are based on the logarithmic returns. The estimation period is April 30, 2011 - June 29, 2019. The annualized return is the arithmetic mean of the annual returns. The annualized excess return is the annualized return minus the arithmetic mean of the risk-free return. The annualized active return is the annualized return minus the annualized return of the market. The annualized volatilities are the annual standard deviations of the monthly returns. The annualized Sharpe ratio is calculated as the annualized excess return divided by the standard deviation of the excess return. The annualized Information ratio is calculated as the annualized active return divided by the standard deviation of active return. The maximum drawdown is the lowest return over the period, measured in percent. Skewness is the degree of distortion of returns from the symmetrical bell curve. Normality is the p-value from the Jarque-Bera test, H_0 is the normality. Kurtosis is a measure of the "tailedness". Dividend, Momentum, Quality, Size, Value, and Volatility are the MSCI ACWI Factor indexes, Table 8 in Appendix A provides more information on the MSCI indexes. The market is the MSCI ACWI equity index. The risk-free rate is the 3-month treasury bill from the Datastream with a ticker "USGBILL3".

Table 4: Summary statistics, 124-month rolling window

A.3 The 148-month rolling window

	ARMA-DCC GARCH	Market	Equally Weighted Factors	Mean Variance	Dividend	Low Volatility	Momentum	Quality	Value	Size
Annualized Return	10.19	5.38	5.47	5.59	3.00	7.39	9.83	8.59	1.87	2.39
Annualized Excess Return	9.56	4.77	4.86	4.98	2.40	6.78	9.20	7.97	1.28	1.80
Annualized Active Return	4.58	0.00	0.08	0.20	-2.27	1.92	4.24	3.06	-3.34	-2.85
Annualized Volatility	11.94	12.73	11.54	11.42	11.67	8.63	11.91	11.75	15.05	13.84
Annualized Sharpe Ratio	0.80	0.37	0.42	0.44	0.21	0.79	0.77	0.68	0.09	0.13
Annualized Information Ratio	1.00		0.12	0.06	-0.63	0.34	0.89	0.91	-0.85	-0.90
Maximum Drawdown	20.81	22.14	18.60	24.83	18.70	8.75	17.35	14.64	29.20	24.35
Skewness	-0.35	-0.50	-0.48	-0.69	-0.28	-0.47	-0.68	-0.46	-0.48	-0.47
Kurtosis	1.09	0.76	0.51	1.19	0.15	-0.17	0.77	0.43	0.64	0.88
Normality	0.02	0.03	0.07	0.00	0.46	0.15	0.00	0.10	0.05	0.02
Minimum	-10.98	-10.15	-8.80	-10.75	-9.67	-5.49	-10.53	-8.12	-13.13	-12.68
Maximum	10.43	10.09	8.37	9.05	8.24	5.62	8.46	8.70	10.43	9.81

This table reports the summary statistics of the single factors. All the calculations are based on the logarithmic returns. The estimation period is April 30, 2011 - June 29, 2019. The annualized return is the arithmetic mean of the annual returns. The annualized excess return is the annualized return minus the arithmetic mean of the risk-free return. The annualized active return is the annualized return minus the annualized return of the market. The annualized volatilities are the annual standard deviations of the monthly returns. The annualized Sharpe ratio is calculated as the annualized excess return divided by the standard deviation of the excess return. The annualized Information ratio is calculated as the annualized active return divided by the standard deviation of active return. The maximum drawdown is the lowest return over the period, measured in percent. Skewness is the degree of distortion of returns from the symmetrical bell curve. Normality is the p-value from the Jarque-Bera test, H_0 is the normality. Kurtosis is a measure of the "tailedness". Dividend, Momentum, Quality, Size, Value, and Volatility are the MSCI ACWI Factor indexes, Table 8 in Appendix A provides more information on the MSCI indexes. The market is the MSCI ACWI equity index. The risk-free rate is the 3-month treasury bill from the Datastream with a ticker "USGBILL3".

Table 5: Summary statistics, 148-month rolling window

A.4 The 100-month rolling window including crisis

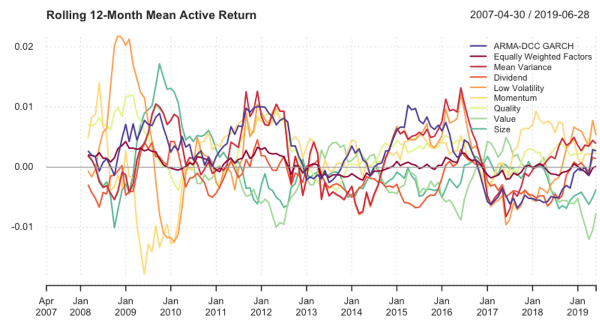
Table 6: Summary Statistics, April 2007 ot March 2011. 100-months rolling window

	ARMA-DCC GARCH	Market	Equally Weighted Factors	Mean Variance	Dividend	Low Volatility	Momentum	Quality	Value	Size
Annualized Return	2.09	-2.75	-1.23	-0.67	-4.67	-1.35	-3.16	1.41	-0.86	1.36
Annualized Excess Return	0.93	-3.86	-2.36	-1.80	-5.76	-2.47	-4.26	0.26	-1.99	0.21
Annualized Active Return	4.97	0.00	1.56	2.14	-1.97	1.44	-0.41	4.27	1.94	4.22
Annualized Volatility	23.33	22.73	21.99	25.52	24.09	14.87	24.40	19.86	26.21	26.40
Annualized Sharpe Ratio	0.04	-0.17	-0.11	-0.07	-0.24	-0.17	-0.17	0.01	-0.08	0.01
Annualized Information Ratio	0.87		0.81	0.28	-0.42	0.31	-0.08	1.01	0.20	0.52
Maximum Drawdown	58.55	60.24	58.39	62.97	63.30	43.92	62.78	52.04	63.14	62.96
Skewness	-0.84	-0.87	-0.84	-0.68	-0.63	-1.17	-0.82	-0.92	-0.58	-0.92
Kurtosis	0.61	1.09	0.83	0.81	0.59	1.98	0.18	0.92	0.49	2.11
Normality	0.03	0.01	0.02	0.05	0.11	0.00	0.05	0.01	0.15	0.00
Minimum	-20.50	-22.27	-20.71	-23.36	-21.72	-15.64	-19.09	-18.36	-23.36	-27.30
Maximum	11.24	10.93	10.61	15.58	13.56	6.50	11.33	9.26	15.58	16.52

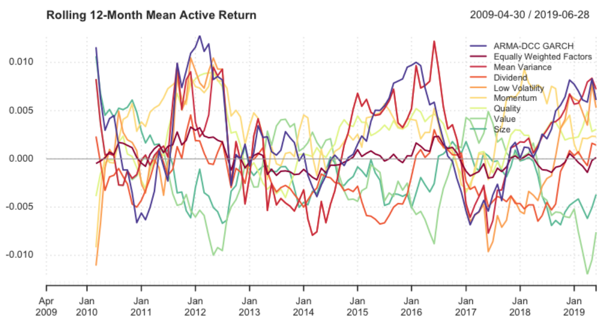
This table reports the summary statistics of the single factors. All the calculations are based on the logarithmic returns. The estimation period is April 30, 2007 - March 29, 2011. The annualized return is the arithmetic mean of the annual returns. The annualized excess return is the annualized return minus the arithmetic mean of the risk-free return. The annualized active return is the annualized return minus the annualized return of the market. The annualized volatilities are the annual standard deviations of the monthly returns. The annualized Sharpe ratio is calculated as the annualized excess return divided by the standard deviation of the excess return. The annualized Information ratio is calculated as the annualized active return divided by the standard deviation of active return. The maximum drawdown is the lowest return over the period, measured in percent. Skewness is the degree of distortion of returns from the symmetrical bell curve. Normality is the p-value from the Jarque-Bera test, H_0 is the normality. Kurtosis is a measure of the "tailedness". Dividend, Momentum, Quality, Size, Value, and Volatility are the MSCI ACWI Factor indexes, Table 8 in Appendix A provides more information on the MSCI indexes. The market is the MSCI ACWI equity index. The risk-free rate is the 3-month treasury bill from the Datastream with a ticker "USGBILL3".

Table 7: Summary statistics, 100-month rolling window, April 2007 to March 2011

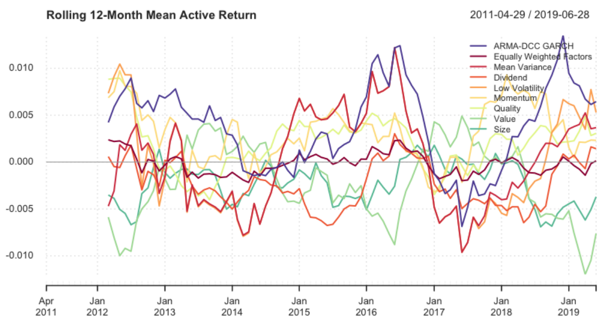
B Portfolio average relative returns



(a) Average Relative (Active) Returns with Rolling Window 100 months



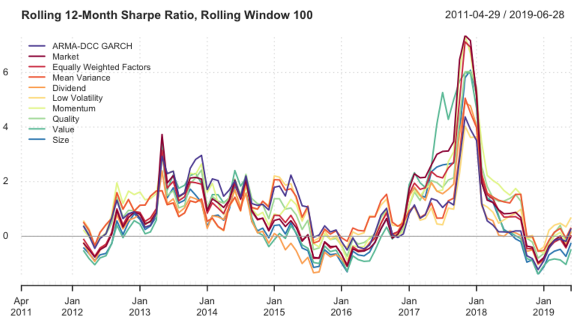
(b) Average Relative (Active) Returns with Rolling Window 124 months



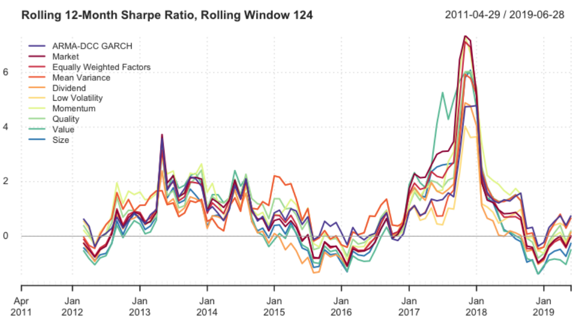
(c) Average Relative (Active) Returns with Rolling Window 148 months

Figure 6: Portfolio's average relative (active) returns

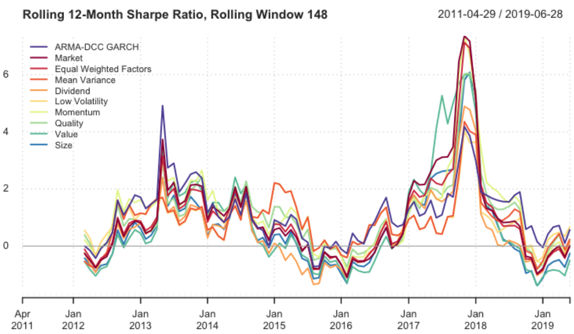
C Portfolio rolling sharpe ratio



(a) Rolling Sharpe Ratio with Rolling Window 100 months



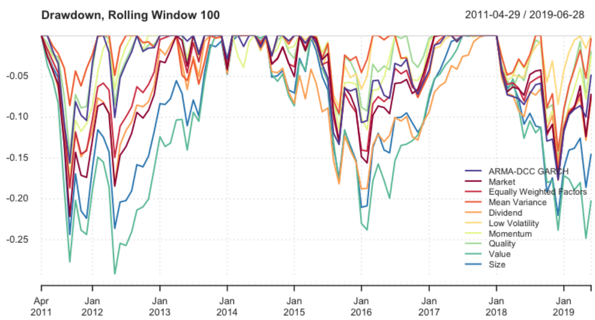
(b) Rolling Sharpe Ratio with Rolling Window 124 months



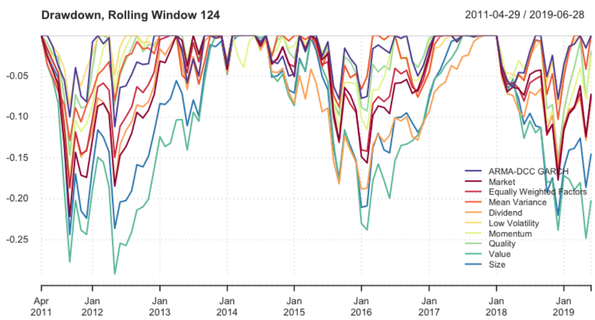
(c) Rolling Sharpe Ratio with Rolling Window 148 months

Figure 7: Portfolio's rolling Sharpe ratio

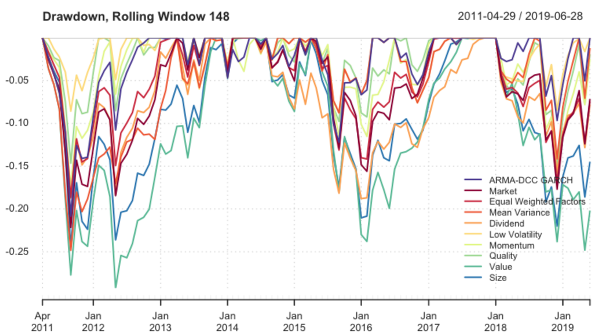
D Portfolio rolling drawdown



(a) Drawdowns with Rolling Window 100 months



(b) Drawdowns with Rolling Window 124 months

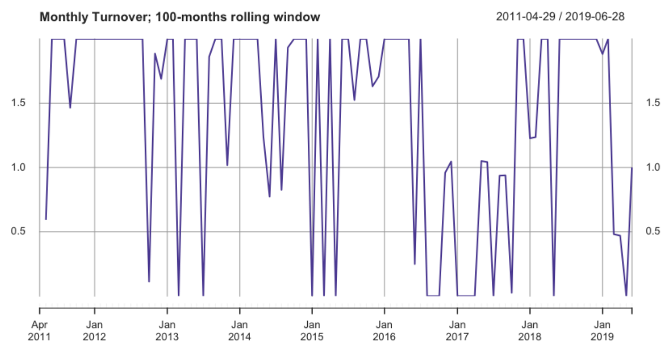


(c) Drawdowns with Rolling Window 148 months

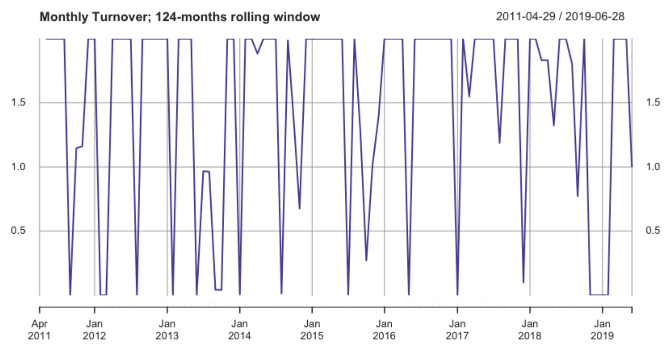
Figure 8: Portfolio's drawdown

E Portfolio's asset turnover

E.1 100- and 124-month rolling window



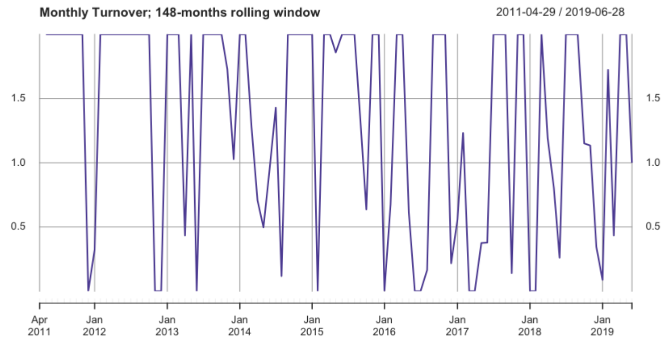
(a) Monthly Asset Turnover; 100-months Rolling Window



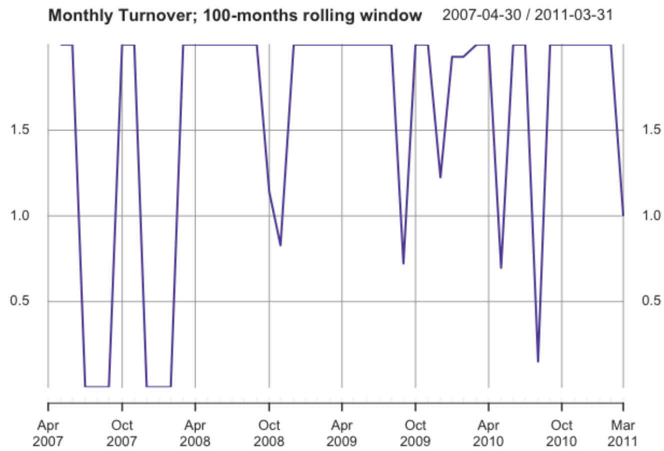
(b) Monthly Asset Turnover; 124-months Rolling Window

Figure 9: Portfolio's asset turnover: 100- and 124-months rolling window

E.2 148-month rolling window and 100-month rolling window from April 2007 to March 2011



(a) Monthly Asset Turnover; 124-months Rolling Window



(b) Monthly Asset Turnover; 148-months Rolling Window

Figure 10: Portfolio's asset turnover: 148-months rolling window and 100-month rolling window from April 2007 to March 2011

F MSCI description

Factor	Index name	Short description
Low Volatility	ACWI MINIMUM VOLATILITY	"The MSCI ACWI Minimum Volatility (USD) Index aims to reflect the performance characteristics of a minimum variance strategy. The index is calculated by optimizing the MSCI ACWI Index, its parent index, in USD for the lowest absolute risk (within a given set of constraints)."**
Dividend	ACWI HIGH DIVIDEND YIELD	"The MSCI ACWI High Dividend Yield Index is based on MSCI ACWI. The index is designed to reflect the performance of equities in the parent index (excluding REITs) with higher dividend income and quality characteristics than average dividend yields that are both sustainable and persistent."**
Quality	ACWI QUALITY	"The MSCI ACWI Quality Index is based on the MSCI ACWI Index. The index aims to capture the performance of quality growth stocks by identifying stocks with high quality scores based on three main fundamental variables: high return on equity (ROE), stable year-over- year earnings growth and low financial leverage."**
Momentum	"ACWI MOMENTUM	The MSCI ACWI Momentum Index is based on MSCI ACWI. It is designed to reflect the performance of an equity momentum strategy by emphasizing stocks with high price momentum, while maintaining reasonably high trading liquidity, investment capacity and moderate index turnover."**
Value	ACWI ENHANCED VALUE	"The MSCI ACWI Enhanced Value Index captures large and mid-cap representation across 23 Developed Markets (DM) countries and 26 Emerging Markets (EM)* countries exhibiting overall value style characteristics. The index is designed to represent the performance of securities that exhibit higher value characteristics relative to their peers within the corresponding GICS sector."**
Size	ACWI EQUAL WEIGHTED	"The MSCI ACWI Equal Weighted Index represents an alternative weighting scheme to its market cap weighted parent index, MSCI ACWI. However, at each quarterly rebalance date, all index constituents are weighted equally, effectively removing the influence of each constituent's current price (high or low)."**
Market	ACWI Standard	"The MSCI ACWI captures large and mid cap representation across 23 Developed Markets (DM) and 26 Emerging Markets (EM) countries*. With 2,844 constituents, the index covers approximately 85% of the global investable equity opportunity set."**

"*DM countries include: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Israel, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, the UK and the US. EM countries include: Argentina, Brazil, Chile, China, Colombia, Czech Republic, Egypt, Greece, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Qatar, Russia, Saudi Arabia, South Africa, Taiwan, Thailand, Turkey and United Arab Emirates."**

**Source: https://www.msci.com/factor-indexes

Table 8: MSCI factor descriptions

G Ljung-Box test results, p-values

G.1 100-month rolling window

	Value	Size	Dividend	Low volatility	Momentum	Quality
2007-03-30	0.95	0.93	0.90	0.92	0.74	0.77
2007-04-30	0.96	0.93	0.81	0.89	0.93	0.63
2007-05-31	0.93	0.89	0.82	0.86	0.31	0.66
2007-06-29	0.93	0.90	0.85	0.92	0.95	0.68
2007-07-31	0.66	0.90	0.90	0.91	0.72	0.58
2007-08-31	0.91	0.85	0.83	0.90	0.74	0.47
2007-09-28	0.95	0.90	0.86	0.94	0.96	0.64
2007-10-31	0.98	0.91	0.75	0.92	0.68	0.66
2007-11-30	0.99	0.93	0.82	0.93	0.85	0.74
2007-12-31	0.98	0.94	0.82	0.93	0.81	0.74
2008-01-31	0.93	0.97	0.81	0.95	0.95	0.80
2008-02-29	0.92	0.98	0.83	0.94	0.87	0.64
2008-03-31	0.95	0.98	0.91	0.97	0.98	0.83
2008-04-30	0.97	0.99	0.87	0.98	0.85	0.76
2008-05-30	0.96	0.98	0.86	0.99	0.90	0.95
2008-06-30	0.85	0.95	0.83	0.99	0.88	0.70
2008-07-31	0.90	0.92	0.98	0.98	0.91	0.85
2008-08-29	0.94	0.92	0.97	0.97	0.97	0.52
2008-09-30	0.98	0.98	0.87	0.99	0.99	0.85
2008-10-31	0.96	0.99	0.97	0.99	1.00	1.00
2008-11-28	0.96	0.96	0.97	0.99	1.00	0.99
2008-12-31	0.98	0.98	0.99	1.00	0.99	1.00
2009-01-30	0.97	0.91	0.91	0.95	0.99	0.99
2009-02-27	0.95	0.79	0.91	0.88	0.98	0.94
2009-03-31	0.99	0.95	0.61	0.93	0.98	0.98
2009-04-30	0.94	0.80	0.83	0.85	1.00	0.97
2009-05-29	0.97	0.89	0.94	0.97	1.00	0.99
2009-06-30	0.95	0.91	0.92	0.97	1.00	0.99
2009-07-31	0.50	0.79	0.72	0.92	1.00	0.94
2009-08-31	0.16	0.78	0.62	0.93	1.00	0.94
2009-09-30	0.62	0.54	0.63	0.92	1.00	0.95
2009-10-30	0.75	0.54	0.96	0.88	1.00	0.93
2009-11-30	0.85	0.52	0.69	0.84	0.99	0.83
2009-12-31	0.66	0.50	0.01	0.86	0.99	0.67
2010-01-29	0.08	0.24	0.60	0.70	0.99	0.00
2010-02-26	0.15	0.00	0.78	0.68	0.97	0.86
2010-03-31	0.33	0.36	0.20	0.62	0.96	0.75
2010-04-30	0.18	0.57	0.16	0.61	0.95	0.26
2010-05-31	0.61	0.03	0.13	0.22	0.82	0.24

2010-06-30	0.10	0.28	0.94	0.17	0.84	0.26
2010-07-30	0.86	0.84	0.92	0.56	0.84	0.35
2010-08-31	0.86	0.54	0.87	0.55	0.99	0.23
2010-09-30	0.10	0.11	0.02	0.05	0.95	0.60
2010-10-29	0.17	0.27	0.09	0.20	0.94	0.55
2010-11-30	0.14	0.02	0.36	0.06	0.94	0.52
2010-12-31	0.20	0.19	0.14	0.06	0.93	0.34
2011-01-31	0.01	0.07	0.20	0.38	0.91	0.56
2011-02-28	0.16	0.01	0.12	0.05	0.87	0.44
2011-03-31	1.00	0.88	0.57	0.77	0.98	0.61
2011-04-29	0.97	0.89	0.17	0.80	0.98	0.67
2011-05-31	0.98	0.97	0.92	0.74	0.98	0.86
2011-06-30	0.98	0.94	0.92	0.90	0.99	0.87
2011-07-29	0.01	0.00	0.20	0.48	0.88	0.09
2011-08-31	0.56	0.00	0.07	0.11	0.82	0.24
2011-09-30	0.28	0.03	0.18	0.65	0.80	0.28
2011-10-31	0.15	0.20	0.06	0.02	0.80	0.25
2011-11-30	0.07	0.21	0.11	0.71	0.79	0.20
2011-12-30	0.83	0.90	0.64	0.89	0.94	0.52
2012-01-31	0.07	0.34	0.01	0.60	0.80	0.09
2012-02-29	0.59	0.31	0.02	0.10	0.81	0.08
2012-03-30	0.06	0.00	0.02	0.11	0.81	0.08
2012-04-30	0.95	1.00	0.60	0.99	0.92	0.55
2012-05-31	0.36	0.24	0.51	0.13	0.83	0.18
2012-06-29	0.31	0.46	0.06	0.15	0.84	0.20
2012-07-31	0.96	0.98	0.93	0.88	0.92	0.75
2012-08-31	0.49	0.46	0.06	0.25	0.78	0.17
2012-09-28	0.28	0.43	0.07	0.16	0.77	0.17
2012-10-31	0.01	0.01	0.07	0.04	0.78	0.18
2012-11-30	0.32	0.41	0.08	0.27	0.79	0.18
2012-12-31	0.48	0.46	0.08	0.03	0.80	0.17
2013-01-31	0.42	0.29	0.05	0.35	0.80	0.17
2013-02-28	0.15	0.15	0.06	0.12	0.80	0.17
2013-03-29	0.06	0.11	0.40	0.02	0.76	0.16
2013-04-30	0.98	0.98	0.96	0.74	0.88	0.76
2013-05-31	0.98	0.97	0.96	0.51	0.90	0.76
2013-06-28	0.95	0.93	0.94	0.42	0.85	0.66
2013-07-31	0.95	0.95	0.94	0.82	0.85	0.62
2013-08-30	0.96	0.92	0.92	0.76	0.81	0.53
2013-09-30	0.04	0.14	0.03	0.19	0.69	0.13
2013-10-31	0.87	0.97	0.90	0.33	0.79	0.52
2013-11-29	0.91	0.96	0.89	0.74	0.97	0.51
2013-12-31	0.01	0.30	0.01	0.05	0.65	0.13
2014-01-31	0.86	0.95	0.94	0.84	0.82	0.64
2014-02-28	0.93	0.93	0.96	0.35	0.83	0.65

2014-03-31	0.34	0.00	0.00	0.17	0.49	0.14
2014-04-30	0.37	0.08	0.13	0.17	0.48	0.14
2014-05-30	0.87	0.90	0.33	0.30	0.78	0.60
2014-06-30	0.90	0.90	0.33	0.29	0.76	0.59
2014-07-31	0.01	0.33	0.00	0.14	0.47	0.14
2014-08-29	0.38	0.35	0.04	0.02	0.49	0.12
2014-09-30	0.03	0.21	0.01	0.12	0.41	0.12
2014-10-31	0.88	0.85	0.93	0.21	0.68	0.41
2014-11-28	0.02	0.00	0.02	0.12	0.49	0.12
2014-12-31	0.87	0.85	0.94	0.22	0.71	0.46
2015-01-30	0.88	0.88	0.94	0.21	0.69	0.41
2015-02-27	0.60	0.05	0.01	0.07	0.33	0.07
2015-03-31	0.72	0.94	0.82	0.56	0.66	0.40
2015-04-30	0.79	0.94	0.25	0.56	0.59	0.38
2015-05-29	0.00	0.01	0.03	0.11	0.26	0.05
2015-06-30	0.00	0.02	0.04	0.14	0.24	0.05
2015-07-31	0.03	0.03	0.03	0.24	0.24	0.03
2015-08-31	0.06	0.00	0.04	0.01	0.29	0.04
2015-09-30	0.88	0.93	0.30	0.69	0.71	0.42
2015-10-30	0.93	0.98	0.37	0.97	0.83	0.58
2015-11-30	0.50	0.05	0.00	0.96	0.79	0.02
2015-12-31	0.00	0.00	0.04	0.76	0.02	0.01
2016-01-29	0.97	0.99	0.96	0.99	0.79	0.75
2016-02-29	0.93	0.07	0.00	0.19	0.01	0.06
2016-03-31	0.43	0.01	0.00	0.06	0.04	0.02
2016-04-29	0.96	0.93	0.93	0.86	0.79	0.60
2016-05-31	0.95	0.91	0.93	0.62	0.97	0.83
2016-06-30	0.95	0.92	0.92	0.56	0.97	0.85
2016-07-29	0.92	0.95	0.91	0.51	0.98	0.87
2016-08-31	0.93	0.96	0.91	0.52	0.36	0.82
2016-09-30	0.92	0.96	0.92	0.46	0.53	0.84
2016-10-31	0.99	0.97	0.95	0.76	0.90	0.81
2016-11-30	0.00	0.01	0.00	0.09	0.01	0.00
2016-12-30	0.01	0.68	0.01	0.08	0.21	0.01
2017-01-31	0.96	0.64	0.04	0.28	0.02	0.00
2017-02-28	0.79	0.94	0.68	0.79	0.12	0.29
2017-03-31	0.96	0.96	0.81	0.66	0.14	0.66
2017-04-28	0.95	0.93	0.96	0.68	0.13	0.55
2017-05-31	0.94	0.98	0.97	0.74	0.51	0.87
2017-06-30	0.96	0.98	0.86	0.75	0.84	0.84
2017-07-31	0.98	1.00	0.64	0.64	0.71	0.67
2017-08-31	0.97	0.97	0.95	0.66	0.81	0.88
2017-09-29	0.76	0.48	0.95	0.67	0.41	0.59
2017-10-31	0.88	0.56	0.94	0.66	0.44	0.50
2017-11-30	0.94	0.96	1.00	0.74	0.87	0.75

2017-12-29	0.80	0.92	1.00	0.75	0.86	0.74
2018-01-31	0.89	0.99	1.00	0.72	0.92	0.82
2018-02-28	0.55	0.97	0.91	0.70	0.96	0.82
2018-03-30	0.86	0.97	0.90	0.64	0.96	0.84
2018-04-30	0.58	0.97	0.91	0.68	0.97	0.85
2018-05-31	0.43	0.92	0.72	0.36	0.72	0.60
2018-06-29	0.94	0.99	1.00	0.39	0.73	0.62
2018-07-31	0.97	1.00	1.00	0.98	0.67	0.61
2018-08-31	0.86	0.99	1.00	0.97	0.69	0.57
2018-09-28	0.60	0.12	0.95	0.56	0.93	0.85
2018-10-31	0.87	1.00	0.97	0.53	0.97	0.90
2018-11-30	0.95	1.00	1.00	0.49	0.94	0.83
2018-12-31	0.88	0.99	0.95	0.97	0.93	0.81
2019-01-31	0.88	0.92	0.93	0.83	0.98	0.85
2019-02-28	0.54	0.59	0.42	0.59	0.80	0.42
2019-03-29	0.82	0.65	0.39	0.57	0.80	0.45
2019-04-30	0.89	0.96	0.88	0.44	0.94	0.73
2019-05-31	0.97	0.96	0.90	0.54	0.95	0.94
2019-06-28	0.98	0.99	0.97	0.80	0.99	0.85

Table 9: Ljung-Box test results, reported p-value. df = 20. 100-month rolling window

G.2 124-month rolling window

	Value	Size	Dividend	Low volatility	Momentum	Quality
2009-03-31	0.99	0.98	0.90	0.97	0.99	0.96
2009-04-30	0.98	0.88	0.83	0.96	0.98	0.95
2009-05-29	0.84	0.91	0.86	0.98	1.00	0.99
2009-06-30	0.87	0.92	0.85	0.98	0.99	0.98
2009-07-31	0.99	0.94	0.96	0.97	1.00	0.98
2009-08-31	0.01	0.68	0.53	0.97	1.00	0.95
2009-09-30	0.03	0.64	0.79	0.97	1.00	0.92
2009-10-30	0.01	0.65	0.51	0.96	1.00	0.94
2009-11-30	0.74	0.65	0.71	0.91	1.00	0.83
2009-12-31	0.18	0.63	0.74	0.97	1.00	0.80
2010-01-29	0.03	0.40	0.38	0.96	1.00	0.82
2010-02-26	0.95	0.49	0.40	0.97	1.00	0.82
2010-03-31	0.01	0.56	0.00	0.95	1.00	0.75
2010-04-30	0.90	0.53	0.50	0.95	0.98	0.80
2010-05-31	0.74	0.35	0.16	0.61	0.96	0.45
2010-06-30	0.75	0.57	0.13	0.79	0.86	0.43
2010-07-30	0.43	0.50	0.72	0.62	0.92	0.44
2010-08-31	0.05	0.35	0.59	0.55	0.99	0.31
2010-09-30	0.20	0.34	0.40	0.61	0.98	0.76
2010-10-29	0.07	0.30	0.34	0.62	0.95	0.71
2010-11-30	0.51	0.36	0.53	0.54	0.97	0.73
2010-12-31	0.97	0.64	0.79	0.98	0.99	0.92
2011-01-31	0.01	0.37	0.69	0.61	0.85	0.73
2011-02-28	0.01	0.06	0.49	0.61	0.97	0.68
2011-03-31	0.02	0.35	0.51	0.61	0.96	0.66
2011-04-29	0.42	0.35	0.31	0.09	0.98	0.54
2011-05-31	0.49	0.37	0.28	0.39	0.98	0.53
2011-06-30	0.61	0.45	0.60	0.10	0.99	0.58
2011-07-29	0.08	0.01	0.13	0.51	0.99	0.65
2011-08-31	0.55	0.09	0.42	0.46	0.97	0.67
2011-09-30	0.13	0.31	0.27	0.55	0.97	0.72
2011-10-31	0.33	0.34	0.31	0.59	0.93	0.45
2011-11-30	0.06	0.04	0.27	0.57	0.91	0.40
2011-12-30	0.60	0.27	0.13	0.59	0.91	0.23
2012-01-31	0.63	0.06	0.35	0.54	0.87	0.15
2012-02-29	0.08	0.01	0.18	0.51	0.87	0.13
2012-03-30	0.19	0.26	0.18	0.45	0.87	0.16
2012-04-30	0.20	0.18	0.22	0.44	0.87	0.16
2012-05-31	0.28	0.34	0.30	0.30	0.87	0.25
2012-06-29	0.05	0.19	0.25	0.23	0.80	0.27
2012-07-31	0.47	0.15	0.06	0.39	0.81	0.23

2012-08-31	0.39	0.05	0.02	0.39	0.89	0.23
2012-09-28	0.12	0.28	0.32	0.33	0.88	0.24
2012-10-31	0.32	0.02	0.25	0.43	0.86	0.24
2012-11-30	0.31	0.08	0.08	0.06	0.79	0.47
2012-12-31	0.39	0.31	0.18	0.28	0.77	0.46
2013-01-31	0.22	0.11	0.19	0.12	0.69	0.49
2013-02-28	0.96	0.96	0.29	0.79	0.89	0.62
2013-03-29	0.06	0.00	0.15	0.25	0.69	0.34
2013-04-30	0.12	0.25	0.13	0.03	0.70	0.33
2013-05-31	0.09	0.25	0.19	0.08	0.75	0.29
2013-06-28	0.19	0.16	0.19	0.18	0.74	0.25
2013-07-31	0.04	0.16	0.07	0.09	0.75	0.12
2013-08-30	0.07	0.07	0.12	0.27	0.72	0.13
2013-09-30	0.00	0.18	0.05	0.40	0.72	0.12
2013-10-31	0.12	0.01	0.03	0.36	0.68	0.11
2013-11-29	0.07	0.05	0.02	0.02	0.68	0.11
2013-12-31	0.21	0.21	0.01	0.02	0.70	0.11
2014-01-31	0.06	0.05	0.05	0.01	0.72	0.13
2014-02-28	0.04	0.05	0.12	0.02	0.73	0.14
2014-03-31	0.09	0.00	0.12	0.02	0.73	0.13
2014-04-30	0.98	0.95	0.94	0.92	0.89	0.55
2014-05-30	0.89	0.91	0.94	0.93	0.90	0.56
2014-06-30	0.22	0.16	0.04	0.01	0.70	0.10
2014-07-31	0.97	0.98	0.97	0.91	0.88	0.57
2014-08-29	0.96	0.97	0.97	0.90	0.99	0.57
2014-09-30	0.86	0.89	0.96	0.88	0.88	0.53
2014-10-31	0.26	0.89	0.06	0.00	0.68	0.08
2014-11-28	0.87	0.97	0.96	0.77	0.91	0.54
2014-12-31	0.07	0.00	0.06	0.00	0.85	0.06
2015-01-30	0.07	0.17	0.01	0.09	0.76	0.07
2015-02-27	0.15	0.09	0.01	0.02	0.71	0.07
2015-03-31	0.01	0.08	0.01	0.08	0.68	0.06
2015-04-30	0.93	0.98	0.96	0.80	0.89	0.55
2015-05-29	0.93	0.98	0.96	0.76	0.89	0.55
2015-06-30	0.08	0.00	0.00	0.05	0.85	0.07
2015-07-31	0.90	0.90	0.94	0.74	0.99	0.49
2015-08-31	0.08	0.10	0.03	0.00	0.83	0.06
2015-09-30	0.89	0.94	0.93	0.70	1.00	0.48
2015-10-30	0.08	0.00	0.06	0.00	0.88	0.10
2015-11-30	0.94	0.90	0.96	0.88	1.00	0.62
2015-12-31	0.03	0.10	0.00	0.12	0.71	0.08
2016-01-29	0.93	0.92	0.96	0.88	1.00	0.67
2016-02-29	0.93	0.90	0.95	0.86	0.92	0.66
2016-03-31	0.95	0.92	0.94	0.83	0.93	0.64
2016-04-29	0.62	0.00	0.00	0.22	0.53	0.09

2016-05-31	0.93	0.93	0.95	0.82	0.88	0.53
2016-06-30	0.93	0.93	0.95	0.61	0.86	0.53
2016-07-29	0.00	0.02	0.00	0.06	0.47	0.08
2016-08-31	0.95	0.91	0.92	0.78	0.81	0.54
2016-09-30	0.95	0.90	0.92	0.74	0.76	0.51
2016-10-31	0.37	0.10	0.01	0.22	0.40	0.06
2016-11-30	0.38	0.00	0.00	0.07	0.42	0.07
2016-12-30	0.90	0.89	0.93	0.82	0.77	0.55
2017-01-31	0.88	0.87	0.93	0.82	0.75	0.53
2017-02-28	0.41	0.00	0.09	0.08	0.32	0.07
2017-03-31	0.28	0.13	0.05	0.30	0.27	0.06
2017-04-28	0.60	0.00	0.07	0.13	0.25	0.04
2017-05-31	0.94	0.85	0.91	0.56	0.67	0.43
2017-06-30	0.94	0.86	0.91	0.53	0.67	0.44
2017-07-31	0.90	0.93	0.84	0.50	0.66	0.38
2017-08-31	0.00	0.01	0.01	0.06	0.25	0.02
2017-09-29	0.16	0.02	0.01	0.40	0.24	0.02
2017-10-31	0.94	0.96	0.88	0.92	0.68	0.44
2017-11-30	0.94	0.85	0.90	0.96	0.64	0.50
2017-12-29	0.93	0.94	0.90	0.74	0.96	0.50
2018-01-31	0.96	0.97	0.91	0.88	0.61	0.62
2018-02-28	0.08	0.01	0.00	0.13	0.04	0.08
2018-03-30	0.00	0.20	0.00	0.06	0.04	0.04
2018-04-30	0.96	0.86	0.88	0.82	0.95	0.57
2018-05-31	0.86	0.81	0.84	0.41	0.95	0.80
2018-06-29	0.87	0.83	0.83	0.37	0.96	0.79
2018-07-31	0.85	0.82	0.82	0.35	0.49	0.75
2018-08-31	0.11	0.09	0.00	0.00	0.07	0.00
2018-09-28	0.11	0.00	0.00	0.00	0.00	0.00
2018-10-31	0.98	0.98	0.89	0.41	0.91	0.72
2018-11-30	0.95	0.73	0.85	0.22	0.39	0.69
2018-12-31	0.93	0.72	0.88	0.53	0.83	0.73
2019-01-31	0.97	0.98	0.98	0.94	0.79	0.64
2019-02-28	0.95	0.94	0.43	0.64	0.31	0.58
2019-03-29	0.93	0.93	0.18	0.61	0.27	0.60
2019-04-30	0.71	0.93	0.22	0.55	0.22	0.48
2019-05-31	0.96	0.99	0.48	0.66	0.66	0.74
2019-06-28	0.99	1.00	0.86	0.73	0.90	0.85

Table 10: Ljung-Box test results, reported p-value. df = 20. 124-month rolling window

G.3 148-month rolling window

	Value	Size	Dividend	Low volatility	Momentum	Quality
2011-03-31	0.44	0.50	0.48	0.25	0.99	0.67
2011-04-29	0.63	0.42	0.24	0.12	1.00	0.67
2011-05-31	0.63	0.17	0.43	0.13	0.99	0.66
2011-06-30	0.05	0.48	0.71	0.77	1.00	0.68
2011-07-29	0.95	1.00	0.92	0.81	1.00	0.84
2011-08-31	0.19	0.18	0.44	0.74	0.99	0.67
2011-09-30	0.24	0.26	0.24	0.76	1.00	0.65
2011-10-31	0.37	0.45	0.23	0.73	1.00	0.58
2011-11-30	0.06	0.47	0.20	0.73	1.00	0.56
2011-12-30	0.48	0.52	0.22	0.76	1.00	0.54
2012-01-31	0.62	0.55	0.22	0.74	0.93	0.53
2012-02-29	0.41	0.07	0.20	0.83	1.00	0.64
2012-03-30	0.55	0.50	0.18	0.84	0.98	0.74
2012-04-30	0.59	0.53	0.26	0.80	0.98	0.23
2012-05-31	0.64	0.58	0.24	0.67	0.97	0.80
2012-06-29	0.53	0.59	0.23	0.67	0.97	0.56
2012-07-31	0.43	0.40	0.21	0.52	0.95	0.54
2012-08-31	0.50	0.35	0.18	0.66	0.90	0.47
2012-09-28	0.50	0.36	0.20	0.68	0.89	0.44
2012-10-31	0.75	0.34	0.18	0.43	0.87	0.61
2012-11-30	0.60	0.34	0.21	0.44	0.90	0.63
2012-12-31	0.63	0.16	0.19	0.49	0.88	0.65
2013-01-31	0.56	0.47	0.18	0.51	0.80	0.41
2013-02-28	0.67	0.31	0.17	0.50	0.79	0.40
2013-03-29	0.58	0.48	0.19	0.57	0.84	0.58
2013-04-30	0.52	0.45	0.21	0.17	0.90	0.52
2013-05-31	0.60	0.36	0.21	0.23	0.85	0.51
2013-06-28	0.66	0.36	0.24	0.62	0.92	0.40
2013-07-31	0.97	0.54	0.33	0.67	1.00	0.72
2013-08-30	0.62	0.32	0.24	0.55	0.98	0.47
2013-09-30	0.37	0.32	0.19	0.58	0.92	0.36
2013-10-31	0.41	0.36	0.20	0.63	0.92	0.38
2013-11-29	0.41	0.30	0.19	0.62	0.91	0.37
2013-12-31	0.43	0.29	0.18	0.58	0.91	0.26
2014-01-31	0.16	0.18	0.17	0.56	0.78	0.15
2014-02-28	0.12	0.00	0.15	0.57	0.73	0.15
2014-03-31	0.19	0.19	0.16	0.29	0.69	0.15
2014-04-30	0.30	0.14	0.16	0.53	0.75	0.15
2014-05-30	0.18	0.08	0.15	0.49	0.67	0.14
2014-06-30	0.25	0.11	0.15	0.46	0.69	0.32
2014-07-31	0.22	0.04	0.15	0.16	0.74	0.13

2014-08-29	0.12	0.16	0.15	0.48	0.71	0.14
2014-09-30	0.10	0.37	0.14	0.49	0.70	0.14
2014-10-31	0.21	0.17	0.17	0.01	0.57	0.14
2014-11-28	0.05	0.14	0.12	0.07	0.63	0.12
2014-12-31	0.08	0.09	0.11	0.02	0.59	0.10
2015-01-30	0.88	0.88	0.41	0.86	0.99	0.60
2015-02-27	0.14	0.00	0.08	0.09	0.60	0.08
2015-03-31	0.17	0.03	0.06	0.09	0.58	0.06
2015-04-30	0.02	0.02	0.09	0.07	0.59	0.07
2015-05-29	0.07	0.09	0.05	0.05	0.59	0.07
2015-06-30	0.13	0.09	0.06	0.07	0.59	0.06
2015-07-31	0.00	0.02	0.06	0.01	0.06	0.06
2015-08-31	0.09	0.14	0.06	0.02	0.84	0.09
2015-09-30	0.13	0.11	0.07	0.17	0.71	0.06
2015-10-30	0.97	0.98	0.93	0.94	1.00	0.59
2015-11-30	0.98	0.98	0.91	0.94	1.00	0.58
2015-12-31	0.21	0.23	0.02	0.06	0.84	0.06
2016-01-29	0.87	0.97	0.94	0.95	0.96	0.64
2016-02-29	0.01	0.13	0.02	0.04	0.77	0.08
2016-03-31	0.03	0.00	0.05	0.28	0.81	0.09
2016-04-29	0.84	0.98	0.96	0.93	0.96	0.55
2016-05-31	0.95	0.99	0.96	0.94	0.96	0.55
2016-06-30	0.99	0.96	0.96	0.93	0.95	0.56
2016-07-29	0.96	0.95	0.97	0.92	0.95	0.61
2016-08-31	0.98	0.95	0.96	0.88	0.94	0.62
2016-09-30	0.98	0.95	0.96	0.89	0.94	0.62
2016-10-31	0.27	0.01	0.02	0.14	0.72	0.05
2016-11-30	0.23	0.02	0.01	0.33	0.74	0.05
2016-12-30	0.00	0.00	0.06	0.25	0.74	0.05
2017-01-31	0.06	0.00	0.09	0.13	0.74	0.04
2017-02-28	0.18	0.00	0.05	0.07	0.72	0.05
2017-03-31	0.97	0.97	0.97	0.89	0.93	0.65
2017-04-28	0.97	0.96	0.97	0.89	0.94	0.66
2017-05-31	0.96	0.96	0.96	0.86	0.94	0.64
2017-06-30	0.56	0.56	0.03	0.14	0.64	0.07
2017-07-31	0.91	0.93	0.92	0.79	1.00	0.57
2017-08-31	0.17	0.01	0.04	0.17	0.02	0.06
2017-09-29	0.91	0.98	0.84	0.80	1.00	0.57
2017-10-31	0.01	0.00	0.06	0.14	0.60	0.05
2017-11-30	0.87	0.89	0.91	0.82	0.99	0.55
2017-12-29	0.00	0.00	0.00	0.26	0.57	0.03
2018-01-31	0.88	0.96	0.92	0.79	1.00	0.59
2018-02-28	0.94	0.87	0.92	0.72	1.00	0.60
2018-03-30	0.91	0.86	0.91	0.71	1.00	0.59
2018-04-30	0.74	0.00	0.01	0.05	0.34	0.06

2018-05-31	0.93	0.86	0.91	0.67	0.99	0.56
2018-06-29	0.93	0.86	0.91	0.67	0.79	0.54
2018-07-31	0.94	0.96	0.91	0.66	0.99	0.57
2018-08-31	0.08	0.38	0.00	0.22	0.26	0.08
2018-09-28	0.40	0.28	0.00	0.08	0.20	0.07
2018-10-31	0.04	0.00	0.00	0.07	0.24	0.07
2018-11-30	0.03	0.06	0.00	0.01	0.31	0.08
2018-12-31	0.92	0.91	0.87	0.83	0.78	0.51
2019-01-31	0.88	0.88	0.91	0.80	0.69	0.65
2019-02-28	0.86	0.86	0.91	0.81	0.98	0.61
2019-03-29	0.39	0.13	0.00	0.01	0.17	0.10
2019-04-30	0.11	0.03	0.00	0.09	0.17	0.07
2019-05-31	0.87	0.95	0.90	0.78	0.99	0.64
2019-06-28	0.89	0.97	0.93	0.62	0.72	0.69

Table 11: Ljung-Box test results, reported p-value. df = 20. 148-month rolling window