The Size Effect to Low Volatility Anomaly

Master Thesis

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Abstract

The theoretical Capital Asset Pricing Model predicts a positive relationship between risk and return. However, the fact shows flatter or even negative risk and return relationship, which known as low volatility anomaly. This study focuses on the size effect as a driver in the low volatility anomaly. The first finding shows that low volatility still exists in the US stock market by using updated data. Later, this study also shows that the anomalous effect exists in small stocks, however not in big stocks. The result is in line with finding by Bali and Cakici (2008) that firm size matters for the volatility anomaly.

1. Introduction

The capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965), and Black (1972) is one of the important milestones in finance. The CAPM implies the mean-variance analysis of modern portfolio theory by Markowitz (1959). Based on CAPM, there is a positive linear relationship between risk and return. Investors who bear a higher risk asset class should be rewarded by a higher expected return. However, the validity and reliability of CAPM in asset pricing theory have become a debatable topic among scholars. Some papers report that CAPM has a flaw in explaining the positive relationship between risk and return. Black et al. (1972) explain that the relationship between risk and return is flatter. They show that high beta stocks have lower return results than CAPM prediction. Furthermore, Fama and French (1992, 1993) argued that the relation between market beta and average return is flat. They adjusted the initial CAPM model by adding size and value as additional factors in determining expected return. The Fama French three factor model develops better result in predicting expected stock return than traditional CAPM.

As discussed above, in fact, the relationship between risk and return can be flat or even negative, which a lower risk asset class experiences a higher return than higher risk asset class. This puzzle is known as "low volatility anomaly". However, low volatility anomaly is not a new phenomenon, which is just discovered recently. Haugen and Heins (1975) found low volatility anomaly in the United States (US) stock market from 1926 to 1971, which low beta stocks gain higher returns compare to high beta stocks. In more recent research, Ang et al. (2006) report a similar result that the US stocks with high volatility experience low average return during the period 1963 to 2000. Moreover, Ang et al. (2009) found the same effect in developed markets other than the US.

Some scholars do not only focus on providing conclusions regarding the existence of low volatility anomaly, but also explaining the drivers behind the anomaly. Baker, Bradley, and Wugler (2011) explain from behavioral side. They argue that the irrational behavior of investors trigger exaggerated demand on risky assets, which make returns on these risky assets are decreasing. They also argue that the persistent existence of the anomaly is then explained by the limit to arbitrage. Benchmarking activity limits the opportunity of institutional investors for taking more profit from mispriced assets. Institutional investors have to stick along their benchmark, and accordingly they are not able to take benefit from mispricing caused by irrational investors. On the other hand, other scholars proposed some ideas from rational side to explain the reason for this anomalous phenomenon.

Ang et al. (2006) show that size, book-to-market, momentum, and liquidity effects cannot account for unexplained excess returns of stocks with low volatility. However, Bali and Cakici (2008) show a different result, which is the negative relation identified by Ang et al. (2006) does not hold once excluding the smallest, least liquid, and lowest priced stocks from the data. They conclude that the result found by Ang et al. (2006) is mainly due to stocks with small size and illiquid characteristics.

This thesis builds on Ang et al. (2006) work and specifically focusses on clarifying whether low volatility anomaly is able to ignore when we remove stocks with the small size. Previous papers investigated the presence of low volatility by using data until the 2000s era only. This thesis will extend the investigation in order to examine whether the low volatility anomaly still exists in the US stock market even by using the updated data, which is from July 1963 to December 2018. The existence of low volatility anomaly will be verified if the portfolio with low volatile stocks could generate better expected return than its riskier counterpart. Further, this study will also examine whether the firm size is a driver for low volatility anomaly. In this study, the data will be sorted by size, then the existence of low volatility anomaly will be checked in every level of size. So, we could see whether in small size stocks or big size stocks that the anomalous effect is present. By knowing the size a driver of low volatility, institutional investors could utilize this opportunity to obtain a better risk-adjusted return on their asset under management. For institutional investors who are evaluating factor allocations, probably the most critical part of the process for acquiring an investment method is forming a view or belief about what drives the factors. Individual investors also should take advantage of this phenomenon by considering to construct their portfolio consisting of the low volatile stocks. The individual investors could evaluate and rebalance their portfolio by buying low volatile stocks and sell stocks which no longer categorized as low volatility.

This thesis consists of two stages of researches. In the first stage, the existence of low volatility anomaly will be investigated. Then, the purpose of the second stage is to verify the existence of volatility anomaly after omitting the small size stocks. Result analysis of this research is based on the value of average unexplained excess return which gathered from the CAPM model and Fama French three factor model. The results for the low volatility anomaly are similar to those of Ang et al. (2006) in the full set of data. Later on, the insignificant relationship found only for the portfolio which contains the biggest size of stocks. This thesis contributes to the understanding of low volatility anomaly and the driver to the anomaly.

In this study, the first finding is that the low idiosyncratic volatility anomaly still exists in the US stock market. The outperformance of low volatility stocks can explain the low volatility effect through negative unexplained excess return (alpha) of long-short portfolio of the most volatile stocks minus the least volatile stock. The second finding indicates that firms size drives the low volatility anomaly. These results show that the low volatility anomaly exists in small stocks, however not in big stocks. This finding is in line with a paper by Bali and Cakici (2008) that firm size matters for the volatility anomaly.

Previously, based on behavioral explanation that investor tends to over demand on risky assets that turn those securities become overpriced (Baker, Bradley and Wugler, 2011). So then, by exploiting the opportunity of getting a better risk-adjusted return from low volatility phenomenon, the market player should put more attention to low volatility stocks. Then, asymmetric information in the market could be diminished by reducing stress on higher risk assets and compensate it to lower risk assets.

The rest of this thesis is organized as follows. Chapter 2 elaborates on the literature review regarding the low volatility anomaly and the stock size as a driver of the anomalous phenomenon. Chapter 3 gives the hypotheses of this thesis. Chapter 4 provides information regarding the data and methodology used in this thesis. Chapter 5 show results of this thesis. Chapter 6 contains the conclusion of this thesis.

2. Literature Review

The more, the merrier, that is what proverb says. Meanwhile, in investing activity, the more risk in an asset class, the higher the expected return should be. That is the general rule of financial market should work. The higher expected return is compensation for investors who are willing to bear a higher risk. It is a basic knowledge we could get from an investment or financial analysis class. The most popular theory to explain the relationship between risk and return is the Capital Asset Pricing Model (CAPM). However later, some studies argue the reliability of CAPM to explain the risk and return relationship. Some researchers show that the risk and return relationship is flatter or even negative. When the risk and return have a negative relationship, this means lower risk asset class gives a higher expected return compared to its respective counterpart. This phenomenon is called low volatility anomaly. This chapter will explain regarding the critics to CAPM, papers that show the existence of low volatility anomaly and drivers of the anomalous phenomenon.

2.1 Risk-Return Relationship and the Critics

The classical approach to risk and return relationship is CAPM, which became a foundation of modern financial theory. The CAPM model grew during the 1960s by Sharpe (1964) and Lintner (1965). The model predicts expected return of a risky asset by single factor only, which is market return. Two main drivers in CAPM are systematic risk and idiosyncratic risk. The systematic risk in the CAPM is the risk that measure of market risk and is captured by beta. The beta explains the sensitivity of a stock's return to the market risk. Investors are compensated with returns for bearing systematic risk because the idiosyncratic risk would be diversified to many stocks. Covariance between stock and market controls the stock's expected return. Therefore, based on the model, the relationship between systematic risk and expected return of a stock is positive. Some assumptions are underlying the CAPM, there are: Individual investors are price takers, the investment horizon is in single-period, investments are limited to traded financial assets, investors can buy and sell financial assets without taxes or transaction costs, information is costless and available to all investors, investors are rational mean-variance optimizers, and all investors have homogeneous expectations regarding the return's parameter.

Later, Ross (1976) had a different approach to explain the driver behind stock returns. "Arbitrage pricing theory" (APT) became the early approach of the multifactor model in predicting the expected return of an asset. The APT model includes the function of various macroeconomic factors or market indexes. Unlike the CAPM, APT did not determine what factors should be included in the model. The number and nature of the factor may vary across market and change over time.

Around a decade after the introduction of CAPM, Black et al. (1972) came with a new study that challenged the CAPM in explaining the relationship between risk and return. They show an alternative hypothesis of the pricing of financial assets by relaxing one of the assumptions of the CAPM, which is risk-free borrowing and lending opportunity. That relaxation of assumption leads to the formulation of two-factor model, which is more robust than previous models. They indicated that the risk and return relationship is much flatter than CAPM predicted, that high-beta stocks have negative alphas and low-beta stocks have positive alphas.

The reliability of CAPM is also argued by other scholars. Haugen and Heins (1975) found an inverted risk and return relationship. The result shows that stock portfolio with lower risk gives more returns than higher risk stock portfolio in long-term. Moreover, Miller and Scholes (1972), Fama and MacBeth (1973), and Tinic and West (1986) found similar results that argue the CAPM prediction in risk and return relationship.

Later, Fama and French (1992, 1993) introduced a new model as an improvement of the traditional CAPM model. The new model captured cross-sectional variation in US equity market returns with two additional factors. Those factors are the market factor (based on traditional CAPM model), the size factor, and the value factor. The extra factors become additional axes in explaining the exposure of systematic risk. The size factor is denoted as SMB (Small-Minus-Big), which is the outperformance of small size portfolio from big size portfolio. Then, the value factor is denoted as HML (High-Minus-Low), which explains the outperformance of portfolio with high book-to-market ratio (value portfolio) from portfolio with low book-to-market ratio (growth portfolio). Their results from period 1963-1990 do not support the traditional CAPM prediction that average returns have a positive relationship with market betas. They concluded that the multifactor model (market, size, and value) predicted stock's average returns more accurately.

The development of the multifactor model was not stopped in three factor model, one of notable later development is Carhart (1997) four factor model. Carhart constructs the four factor model using Fama and French's (1993) three factor model and an additional factor from Jegadeesh and Titman's (1993) one-year momentum anomaly. The momentum factor captured the outperformance of previous period winner stocks from previous period loser stocks. This model may be interpreted as a performance attribution model (Cahart, 1997).

2.2 Existence of the Low Volatility Anomaly

According to the Efficient Market Hypothesis, investors who are holding risk-free asset will not obtain an excess return. There is a motivation to gain excess return by bearing additional risk. It is the only way that investors can do to experience return, which is better than the market return. The basic principle is based on the traditional CAPM model that high volatility and high beta stocks will give a higher return than low volatility and low beta stocks.

However, the reality is not in line with the traditional CAPM model explanation. Some papers address some deficiencies in risk and return positive relationship as predicted by the CAPM model. Haugen and Heins (1975) documented and inverted risk and return relationship in the US stock market for period 1926 to 1971. Their results indicate that, over the long run, stocks with lower volatility in monthly returns have experienced higher average returns than their “riskier” counterparts.

Some anomalies become weaker or disappear when time passes, while low volatility anomaly becomes persistent over time. Ang et al. (2006) examined whether the volatility of the market is a priced risk factor and estimate the price of aggregate volatility risk using the cross-section of stock returns. They find a statistically significant negative price of risk of approximately -1% per annum. This result is supported by economic theory. For example, Bakshi and Kapadia (2003) explain that assets with a high correlation with market volatility risk provide hedge against market downside risk. Therefore, the demand for an asset with high systematic volatility loadings will increase and lowers their average return. Finally, stocks that perform poorly during volatility rises tend to have negatively skewed returns, and vice versa, stocks that do well during volatility increases tend to have positively skewed returns.

Ang et al. (2006) also examined the cross-sectional relationship between idiosyncratic volatility and expected returns. They defined the idiosyncratic volatility according to Fama and French (1993) model. Some papers argue that an investor who is holding undiversifiable risk need to be compensated by higher expected returns. The motivation comes from investors that demand a premium for holding securities with high idiosyncratic risk, the undiversifiable risk. However, this idea is not in line with Ang et al. (2006) result. They find that US stocks with high idiosyncratic volatility have low average returns. The quintile portfolio with the lowest idiosyncratic volatility stocks outperforms the quintile portfolio with the highest idiosyncratic volatility stocks by 1.06% per month. Therefore, they conclude that the low volatility anomaly is not caused by small stocks. They also perform another testing by controlling for size. After controlling for size, they still find outperformance of the lowest idiosyncratic volatility stocks as before by 1.04% per month. Thus, they conclude that market capitalization does not explain the low returns to high idiosyncratic volatility stocks.

A related study with Ang et al. (2006), Blitz and van Vliet (2007) presents empirical evidence for the low volatility anomaly. They find that US stocks with low volatility earn high risk-adjusted returns between the years 1985 and 2006. To reach that conclusion, they created decile portfolios that were ranked of stocks on historical return volatility. Based on this method, they find that the portfolios with the lowest historical volatility can generate returns that slightly above average. In addition, they find that the low risk portfolios underperform the market during up market months while outperforming the market during down market months. However, the underperformance during up months is smaller than the outperformance during down months (59%, versus 41% down months). The high risk portfolios exhibit precisely the opposite behavior, outperformance during up months, but not enough to offset the underperformance during down months.

Furthermore, Blitz and van Vliet (2007) argue that the low volatility anomaly cannot be explained by other effects such as value and size. They also observed that their findings apply to both global and regional stock markets. The annual alpha spread of global low versus high volatility decile portfolios amount to 12%. They also observe that the existence of low volatility anomaly is not only in the US market but also in the European and Japanese market.

Ang et al. (2009) extended their analysis to 23 developed markets. The low volatility effect is individually significant in each G7 country. Based on those strong international results, it is more likely that there is a driver behind the anomalous effect rather than a small-sample problem. Further, the negative spread in returns between stocks with high and low idiosyncratic volatility in international markets strongly correlate with the US returns with high and low idiosyncratic volatilities. The large commonality in correlation suggests that broad, which is not easily diversifiable factors lie behind this effect. Finally, they rule out explanations for the high idiosyncratic volatility and low average returns relation based on market frictions, information dissemination, and option pricing in US market data. Thus, they conclude that the existence of low volatility anomaly by employing more extended US sample and international equity markets. However, the underlying economic factors of this phenomenon require further investigation.

Later, Blitz, Pang, and van Vliet (2013) find that the empirical relation between risk and return in emerging markets are flat, or even negative. The negative risk and return relationship is even stronger when volatility instead of beta is used to measure risk. The existence of low volatility anomaly in emerging market is similar to the result in the US and other developed equity markets (Blitz and van Vliet, 2007). Specifically, they find that the top quintile portfolio, based on the past three year volatility, outperforms the bottom quintile portfolio by 4.4% per annum over sample period between 1989 and 2010. The empirical evidence of the existence of low volatility anomaly in emerging market is also relevant for robustness effect. The empirical robustness effect can counter some critiques related insufficiency data mining issue in this research area.

2.3 Low Volatility Anomaly Driver

The other group of researchers has interest in giving explanation regarding the underlying idea behind the low volatility anomaly, instead of proving its existence only. The volatility effect can be explained from two point of views, which are behavioral biases and rational economics. Based on MSCI Research Insight (Bender et al., 2013), low volatility can be explained by behavioral biases, such as lottery effect, representativeness, overconfidence, agency issue, and asymmetric behaviors in bull and bear markets. Finally, low volatility anomaly can also be explained by the size effect.

In the MSCI Research Insight (Bender et al. 2013), it explained that the low volatility anomaly clearly contradicts the Efficient Market Hypothesis and assumption of the CAPM. The first explanation is the “lottery effect”, i.e., people tend to take bets with small expected loss but a large expected win, even though the probability of a loss is much higher than the win, and the expected payoff may be negative. According to Barberis and Huang (2008), investors overweight the positive event of gaining a huge return and underweight the negative event of incurring a loss. Hence, investors view stocks as lottery game may overprice the high volatility stocks.

Another example of a behavioral explanation is called representativeness, which is explained by an experiment in Tversky and Kahneman (1983). In investment context, the success story of a few and well publicized high volatility stocks make all volatile stocks seem a good option for investment. Further, investors tend to neglect the speculative nature of such stocks.

Overconfidence is also behavioral explanation for low volatility anomaly. Investors tend to overestimate their ability to forecast the future, and they tend to overestimate their belief or predictions. Based on the MSCI report (Bender et al., 2013), in the practical world, it is easier and more practical to express a positive view (buy) rather than a negative view (short sell). This condition leads the pessimists cannot adequately express their view of volatile stocks, leaving only the optimists driving up the stock price. As a result, risky stocks become overpriced.

Low volatility stocks are less likely to obtain attention and support in terms of research and trading. This reason leads to agency issue for asset management to have tendency to avoid low volatility stocks. The avoidance of low volatility stocks causes increase demand for high volatility stocks and become overpriced.

Asymmetric behaviors during bull and bear markets occur due to investors behave differently in bull markets compare to bear markets. During down markets, the low volatility portfolio will have much lower beta, since low volatility and high volatility portfolios have a huge dispersion of beta. Therefore, the low volatility portfolio will have much less drawdown compare to the high volatility portfolio. The low volatility portfolio needs to go up much less than high volatility portfolio to recover back to its original value. On the other hand, during up markets, the beta dispersion is not so great. Thus the low volatility portfolio underperforms only slightly than the high volatility. The net result of down and up markets, the low volatility portfolio still experiences better returns in the long run (Sefton et al., 2011).

Finally, the size effect is nonbehavioral driver that can explain the low volatility anomaly. The size effect was first discovered by Banz (1981) that showed smaller companies listed in the New York Stock Exchange on average experienced higher risk-adjusted returns than larger companies for period between 1963 and 1975. Later, Fama and French (1992, 1993) include the size effect in Fama French three factor model. Bali and Cakici (2008) propose a theory that the size effect is the underlying factor behind low volatility anomaly. They argue that the negative cross-sectional relation between idiosyncratic volatility and expected stock returns is driven by small-caps. To obtain that result, they test the robustness of the findings of Ang et al. (2006) by controlling the size, price, and liquidity. After excluding the smallest, least liquid, and lowest priced stocks, they find no relationship between idiosyncratic (high) volatility and expected returns. The low volatility effect found by Ang et al. (2006) disappears after a screen for size, price, and liquidity, implying that the effect is mainly driven by small and illiquid stocks.

3. Hypotheses

The low volatility anomaly is not a new phenomenon. Even Haugen and Heins (1975) found a negative relationship between risk and return in a decade after Sharpe (1964) and Lintner (1965) introduced CAPM. Later, in the 2000s era, Ang et al. (2006) show that the US stocks with high volatility experience low average return during period 1986 to 2000. Blitz and van Vliet (2007) found the same phenomenon in the US, Europe, and Japan market over the 1986-2006 period. Furthermore, Ang et al. (2009) show that the effect also exists in developed markets other than the US based on sample period from 1980 to 2003. Those papers covered sample period until early 2000s. Therefore, this study will try to prove the existence of low volatility anomaly phenomenon in the US market using updated data. This leads to the first hypothesis:

H1: The low idiosyncratic volatility anomaly still exists in the United States stock market.

Decile portfolio, which is obtained from Kenneth French's website, sorted by residuals variance will be used in this testing. The outperformance of the least volatile stocks compare to the most volatile stocks would be tested by regressing CAPM and Fama French three factor model.

The failure of CAPM to explain the relationship between risk and return triggers some scholars to prove that relationship based on factual data. Black et al. (1972) show that the relationship between risk and return is flatter compare to result which predicted by the CAPM. Later, Fama and French (1992) find that the relationship between risk and return is flat, leading to consensus that beta is dead. Historical data show that lower risk asset outperforms the riskier counterpart. In this thesis, I would examine the existence of low volatility anomaly in the United States stock market for period from July 1963 to December 2018.

By knowing the existence of low volatility, an institutional investor can consider this phenomenon when setting the investment policy. The investor can consider whether the low volatility anomaly is a factor that should be included in the investor's view or belief. Moreover, individual investors can also include the phenomenon into account when constructing a portfolio. They can use the existence of low volatility anomaly to gain better risk-adjusted return and do not to overweight a portfolio with high volatility in order to aim high expected return.

The second hypothesis of this thesis will discuss the exposure of low volatility portfolios to other factors. Baker et al. (2014) explain that the portfolios obtain benefit from small cap and value premium. Ang et al. (2006) find significant interaction between idiosyncratic volatility and size. Bali and Cakici (2008) follow Ang et al. (2006) approach to examine that interaction with modification. Bali and Cakici exclude the smallest size decile of stocks from the portfolio. The result shows that the idiosyncratic volatility effect reported by Ang et al. disappears after that screening. This leads to the second hypothesis of this study.

H2: The low idiosyncratic volatility anomaly still exists in the United States stock market even after omitting the small size stocks

For this testing, I will use 25 portfolios double sorted by size and residuals variance, which also obtained from Kenneth French’s website. Later on, the low volatility anomaly will be assessed through long-short portfolio of the least volatile stocks minus the most volatile stocks with the same level of size.

The anomalous returns supposed to be caused by a factor that affects the relationship between risk and return. This thesis tries to figure out whether the size factor is a driver behind the anomalous returns. The long-short portfolio strategy will be performed for five quintile portfolios sorted by size. Therefore, the result is able to be observed in every level of stock’s size.

From investor or equity strategist point of view, the existence of low volatility anomaly phenomenon is not only attractive and also brings opportunity. The anomaly can be utilized to obtain a better risk-adjusted return compare to standard benchmark indices. According to Eduardo Lecubarri, global head of small and mid cap equity strategy at JPMorgan, small cap has only 4.4 analysts who research it, compared with 23.2 analysts for mega cap (Jasinski, 2019). Therefore, the coverage of bigger size stocks is much better in terms of quality and quantity. Investors have more access to measure the risk profile of big cap than that of small cap. Based on that fact, investors are able to utilize the low volatility anomaly and consider size factor as driver of the anomaly.

4. Data and Methodology

4.1 Data

The primary data of this thesis are obtained from Kenneth French's website. I use value-weighted portfolio from that website in providing low volatility portfolio. The data is collected for the period from July 1963 through December 2018.

For the first hypothesis, I use ten portfolios formed on residual variance. These portfolios are formed monthly on the variance of the residuals from the Fama French three factor model (RVar) using NYSE breakpoints. RVar is estimated using 60 days (minimum 20) of lagged returns. The portfolios for month t (formed at the end of month t-1), include NYSE, AMEX, and NASDAQ stocks. The portfolios are sorted from the least volatile stocks (portfolio 1) to the most volatile portfolio (portfolio 10).

In order to test the second hypothesis, I use double sorted 25 portfolios formed on size and residual variance. These portfolios are formed monthly. The portfolios are the intersections of five portfolios formed on size (market equity, ME) and 5 portfolios formed on the variance of the residuals from the Fama French three factor model (RVar). The monthly size breakpoints are the NYSE market equity quintiles. RVar is estimated using 60 days (minimum 20) of lagged returns. The RVar breakpoints are NYSE quintiles. The portfolios for month t (formed at the end of month t-1), include NYSE, AMEX, and NASDAQ stocks. Below table explains the sorting of the portfolios.

Residuals Variance	Size	ME1 SMALL	ME2	ME3	ME4	ME5 BIG
RVAR1	LOW	Portfolio 1	Portfolio 6	Portfolio 11	Portfolio 16	Portfolio 21
RVAR2		Portfolio 2	Portfolio 7	Portfolio 12	Portfolio 17	Portfolio 22
RVAR3		Portfolio 3	Portfolio 8	Portfolio 13	Portfolio 18	Portfolio 23
RVAR4		Portfolio 4	Portfolio 9	Portfolio 14	Portfolio 19	Portfolio 24
RVAR5	HIGH	Portfolio 5	Portfolio 10	Portfolio 15	Portfolio 20	Portfolio 25

The excess return on market and Fama French factors, which are Small Minus Big (SMB) and High Minus Low (HML), are also obtained from Kenneth French's website. The excess return on the market is value-weight return of all CRSP firms incorporated in the United States and listed on the NYSE, AMEX, or NASDAQ at the beginning of month t minus the one-month Treasury bill rate (from Ibbotson Associates). SMB is the excess return of small portfolios minus big portfolios. HML is the excess return value portfolios (high book-to-price ratio) minus growth portfolios (low book-to-price ratio).

4.2 Methodology

The method used to test the volatility effect is the CAPM model and Fama French three factor model. The first set of portfolios, 10 portfolios form on residual variance, is used to verify the existence of low volatility anomaly. Then, it can be observed if the least volatile stocks outperforms the most volatile stocks by doing portfolio 10 minus portfolio 1 long-short arrangement. For the next part, the double sorted 25 portfolios formed on size and residual variance is used to verify the size effect to low volatility anomaly phenomenon. The same long-short procedure will be used; however, the outperformance will be measured for each level of size. The most volatile portfolio will be subtracted by the least volatile portfolio from the same level of size. Therefore, this computation will be done for five times, from the smallest size stocks to the biggest size stocks.

By using two sets of portfolios, I would report the average unexplained excess return (Jensen's Alpha) together with the market beta by employing the CAPM model, which is:

$$R_i - R_f = \alpha_i + \beta_m(R_m - R_f) + \varepsilon \qquad (1)$$

$R_i - R_f$ is the excess return of portfolio *i,*
$R_m - R_f$ is the excess return of market,
α_i is the unexplained excess return of the portfolio *i,*
β_m is the beta of portfolio *i* with respect to market,
ε is the idiosyncratic return.

Given the weakness of CAPM to explain the relationship of risk and return, I would also run Fama French three factor regression for all sets of portfolio:

$$R_i - R_f = \alpha + \beta_m(R_m - R_f) + \beta_s SMB + \beta_h HML + \varepsilon \qquad (2)$$

SMB is the excess return of small size stocks over big size stocks,
HML is the excess return of value stocks over growth stocks.

SMB is calculated from the average return on the three small portfolios minus the average return on the three big portfolios.

SMB = 1/3 (Small Value + Small Neutral + Small Growth)
- 1/3 (Big Value + Big Neutral + Big Growth)

HML is calculated from the average return on the two value portfolios minus the average return on the two growth portfolios.

HML = 1/2 (Small Value + Big Value) – 1/2 (Small Growth + Big Growth)

Fama French three factor model is an extension of the CAPM model. In CAPM, there is only single factor, which is beta (β_{m}), to compare a portfolio to market as a whole. Meanwhile, Fama French three factor model includes two more additional factors, which are SMB and HML. The two extra factors give two additional axes, therefore the regression model can predict portfolio return with better r-squared value.

Each factor in Fama French three factor model represents variable that affect portfolio's excess return as the dependent variable. The first factor is market risk premium. It is a compensation to investor for bearing an additional risk of return above the risk free rate. The second factor is SMB. Small Minus Big captures size effect based on market capitalization by controlling Book-to-Market ratio. The third factor is HML. High Minus Low captures value premium effect based on Book-to-Market ratio by controlling size. Once SMB and HML are identified, the corresponding coefficients, beta of SMB (β_s) and beta of HML (β_h), can be determined by linear regression. Those coefficients can take positive or negative values.

This thesis will show the average unexplained excess return, or alpha, which obtained from CAPM and Fama French three factor model regression. To examine the first hypothesis, the set of 10 portfolios will be employed. The existence of low volatility anomaly phenomenon can be identified if the least volatile portfolio (portfolio 1) outperform the most volatile portfolio (portfolio 10). The outperformance can be proved by the alpha of long-short portfolio 10-1 with negative and significant result. Later, the set of 25 portfolios will be used to examine the second hypothesis. As well as in the first part, the long-short method will also be implemented to examine the second hypothesis. However, the outperformance will be examined for five times on every level of size. Therefore, we can identify whether the low volatility anomaly exists in every level of size or not.

After obtaining the results from regression (1) and (2), I will examine if the findings are robust to various measurement. First, I will continue the previous long-short portfolio testing based on 25 portfolios by omitting the most volatile portfolios. Therefore the long-short portfolio will be consist of the fourth quintile of volatile minus the least volatile portfolio from each level of size (e.g., portfolio 4-1). Then, I will examine the impact of falling or rising interest rate, by splitting the sample into two periods: falling interest rate (July 1963 to December 1985) and rising interest rate (January 1986 to December 2018). In the last, I will examine all of previous testing using equally-weighted portfolios.

5. Results

Table 1 depicts the descriptive statistic and CAPM and Fama French three factor model regression results for the first set of portfolios (ten portfolios). The table shows the results for the low volatility strategy. The portfolio with the least volatile stocks (portfolio 1) has a monthly excess return of 0.93% with a t-statistic of 6.77. While the portfolio with the most volatile stocks (portfolio 10) has a monthly excess return of only 0.33% with a t-statistic of 1.00. The difference in monthly excess return between portfolio 1 and portfolio 10 is 0.60%. If we put focus on the difference based on variance, the comparison of Sharpe ratio is interesting to see. From this, it can be observed that the portfolio with the least volatile stocks, portfolio 1, has the highest Sharpe ratio by 0.71. On the other hand, the portfolio with the most volatile stocks, portfolio 10, has the lowest Sharpe ratio by -0.04. This means that the least volatile portfolio has the best trade-off between return and risk, and in contrary, the most volatile portfolio experiences the worst trade-off. At the same level of risk, portfolio 1 has the highest return, while portfolio 10 has the lowest return. Based on the value of monthly excess returns and Sharpe ratios for each portfolio, it is presented that the low idiosyncratic volatility still exists in the US stock market.

Table 1
Main characteristics of ten portfolios sorted in volatility

The table below shows several characteristics of ten portfolios sorted on residual volatility. It shows the average monthly excess return, the standard deviation of the returns, the Sharpe ratio, and the t-statistic for means of the portfolio. The next last three columns show the average unexplained excess return (Jensen's Alpha), the CAPM market beta, and the unexplained excess return based on the Fama French three factor model. The sample period is from July 1963 to December 2018. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Portfolio	Mean	Std	Sharpe Ratio	t (mean)	CAPM α	CAPM β	FF3 α
1	0.93	3.54	0.71	6.77	0.18***	0.71***	0.15***
2	0.96	4.08	0.57	6.11	0.14**	0.87***	0.13***
3	0.91	4.40	0.46	5.35	0.05	0.94***	0.01
4	1.00	4.73	0.46	5.44	0.10	1.02***	0.01
5	1.02	5.00	0.42	5.24	0.08	1.08***	0.02
6	1.17	5.43	0.43	5.54	0.18***	1.17***	0.15**
7	0.98	5.90	0.30	4.31	-0.05	1.27***	-0.08
8	1.17	6.52	0.33	4.62	0.08	1.37***	0.04
9	0.92	7.32	0.18	3.25	-0.23*	1.50***	-0.26***
10	0.33	8.43	-0.04	1.00	-0.87***	1.59***	-0.92***
10 - 1	-0.60	7.06	-0.82	-2.20	-1.05***	0.88***	-1.07***

The last three columns consist of average unexplained excess return (Jensen's Alpha or CAPM alpha), the CAPM market beta and the unexplained excess return based on the Fama French three factor model (FF3 alpha). The results show a similar pattern for CAPM alpha and FF3 alpha. The alpha is decreasing gradually between the first and ninth decile and more extreme for portfolio 10. The long short portfolio 10-1 has negative FF3 alpha of -1.07. The FF3 alpha has 1% significance level. This means the long-short portfolio 10-1 strategy will give unfavorable return due to the outperformance of the least volatile portfolio over the most volatile portfolio. The results strengthen the conclusion that the low idiosyncratic volatility anomaly still exists in the US stock market. The results are in line with findings on the low volatility anomaly by Ang et al. (2006). The result of table 1 suggests that we fail to reject the first hypothesis (H1), that the low idiosyncratic volatility anomaly still exists in the United States stock market. It suggests that investors may utilize this anomalous phenomenon to obtain a better risk-adjusted return.

Small Size Stocks as a Driver of Low Volatility Anomaly

Similar to Table 1, Table 2 depicts the descriptive statistic and CAPM and Fama French three factor model regression result for the second set of portfolios (25 portfolios). The portfolios are double sorted in size and volatility. Portfolio 1 until portfolio 5 are constructed from the smallest size quintile, and then the biggest size quintile spreads in portfolio 21 until portfolio 25. In every level of size, the bottom portfolio is the least volatile portfolio (e.g., portfolio 1 and portfolio 21), and the top portfolio is the most volatile portfolio (e.g., portfolio 5 and portfolio 25). For group of portfolio of smallest size quintile, the portfolio with the least volatile stocks (portfolio 1) has a monthly excess return of 1.39% with a t-statistic of 8.64 and the most volatile stocks (portfolio 5) has a monthly excess return of 0.16% with a t-statistic of 0.47. Then for group of portfolio of biggest size quintile, the lowest risky portfolio (portfolio 21) has monthly excess return of 0.87% with a t-statistic of 6.18 and the greatest risky portfolio (portfolio 25) has monthly excess return of 0.84% with a t-statistic of 3.47. The rest groups of portfolio also have a similar pattern for monthly excess return. For the same level of size, the least volatile portfolio exhibits better performance than the most volatile portfolio.

From the last three columns in Table 2, it shows the CAPM alpha and beta and FF3 alpha of 25 portfolios. Once again, the results show similar pattern for CAPM alpha and FF3 alpha. For every level of size, the alpha is decreasing gradually from the first and fourth portfolio and more discrepancy for the fifth portfolio. Looking to FF3 alpha, there is a unique pattern which the most volatile portfolio has negative alpha with 1% significance level, except the portfolio 25 (biggest size and most volatile portfolio) is not statistically significant. The negative alpha forces the investor to experience underperformance result to bear idiosyncratic risk in portfolios with the most volatile stocks. Other than those portfolios, portfolio 4 and portfolio 24, the rest of portfolio has positive alpha with various significance level. The results of Table 2 itself is not sufficient to examine the second hypothesis. The second hypothesis will be covered by the long-short portfolio strategy of 25 double sorted portfolios in Table 3.

Table 2
Main characteristics of 25 portfolios double sorted in size and volatility

The table below shows several characteristics of 25 portfolios double sorted on size and residual volatility. The portfolios are the intersections of 5 portfolios formed on size and 5 portfolios formed on the residuals variance. It shows the average monthly excess return, the standard deviation of the returns, and the t-statistic for means of the portfolio. The next last three columns show the average unexplained excess return (Jensen's Alpha), the CAPM market beta, and the unexplained excess return based on the Fama French three factor model. The sample period is from July 1963 to December 2018. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Portfolio	Mean	Std	t (mean)	CAPM α	CAPM β	FF3 α
1	1.39	4.15	8.64	0.63***	0.73***	0.36***
2	1.52	5.57	7.06	0.61***	1.04***	0.32***
3	1.38	6.41	5.57	0.38***	1.22***	0.09
4	1.14	7.43	3.96	0.05	1.38***	-0.21**
5	0.16	8.89	0.47	-1.00***	1.52***	-1.25***
6	1.28	4.12	8.04	0.51***	0.77***	0.27***
7	1.40	5.24	6.88	0.49***	1.02***	0.23***
8	1.40	5.87	6.18	0.43***	1.16***	0.19***
9	1.29	6.73	4.94	0.21*	1.36***	0.01
10	0.58	8.31	1.79	-0.63***	1.62***	-0.72***
11	1.11	3.82	7.52	0.36***	0.73***	0.15**
12	1.27	4.79	6.86	0.40***	0.97***	0.19***
13	1.26	5.34	6.10	0.32***	1.09***	0.12*
14	1.26	6.08	5.36	0.24**	1.25***	0.08
15	0.71	7.64	2.39	-0.46***	1.54***	-0.48***
16	1.09	3.81	7.41	0.34***	0.72***	0.18**
17	1.14	4.46	6.59	0.29***	0.91***	0.15**
18	1.13	5.08	5.76	0.20***	1.07***	0.07
19	1.14	5.61	5.24	0.14*	1.19***	0.05
20	0.78	7.24	2.78	-0.37***	1.49***	-0.35***
21	0.87	3.63	6.18	0.11*	0.73***	0.10*
22	0.87	3.63	6.18	0.08	0.86***	0.11**
23	0.90	4.48	5.17	0.02	0.96***	0.02
24	0.87	4.93	4.56	-0.06	1.08***	-0.06
25	0.84	6.26	3.47	-0.22**	1.32***	-0.10

From Table 3 it shows the regression results for the low volatility strategy of 25 double sorted portfolios. In this section, the long-short strategy is employed in order to verify the existence of low volatility in the second set of portfolios. The method is similar to method which is applied in previous set of portfolios. However, in this part the long-short strategy is applied in five level of size quintiles. Therefore, we can see the size effect on low volatility anomaly.

The first row of Table 3 shows the result of long-short portfolio of the smallest size quintile, i.e., portfolio 5 minus portfolio 1 (portfolio P5-P1). The portfolio P5-P1 has negative FF3 alpha of -1.62. The rest of long-short portfolio also experience negative FF3 alpha. Those alphas have 1% significance level, except for portfolio P25-P21, the long-short portfolio of the biggest size quintile. The FF3 alpha of P25-P21 is not statistically significant. Looking at anomaly effect in the whole portfolio, I average across the five size quintiles and present it in the last row of Table 3 as Mean (col). The Mean portfolio also has a negative FF3 alpha of -0.79 at 1% significance level.

Table 3
Main characteristics of 5 long-short portfolios based on quintile 1 and 5 for volatility

The table below shows several characteristics of 5 long-short portfolios each based on quintile 1 and 5 for idiosyncratic volatility. The portfolios are constructed by showing the outperformance the least volatile portfolio over the most volatile portfolio with the same size quintile. It shows the average monthly excess return, the standard deviation of the returns, and the t-statistic for means of the portfolio. The next last three columns show the average unexplained excess return (Jensen's Alpha), the CAPM market beta, and the unexplained excess return based on the Fama French three factor model. The sample period is from July 1963 to December 2018. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Portfolio	Mean	Std	t (mean)	CAPM α	CAPM β	FF3 α
P5 – P1	-1.23	6.24	-5.08	-1.63***	0.78***	-1.62***
P10 – P6	-0.71	5.79	-3.15	-1.14***	0.85***	-0.99***
P15 – P11	-0.41	5.56	-1.88	-0.82***	0.81***	-0.64***
P20 – P16	-0.32	5.49	-1.49	-0.71***	0.77***	-0.52***
P25 – P21	-0.03	4.54	-0.15	-0.33**	0.59***	-0.20
Mean (col)	-0.54	5.13	-2.70	-0.93***	0.76***	-0.79***

Based on results in Table 3, the long-short strategy will give suboptimal return due to the outperformance of the lowest volatile portfolio over the most volatile portfolio, in every level of size except the biggest size quintile. These results are not in line with findings by Ang et al. (2006) that the smallest and largest quintiles are not statistically significant at the 5% level. In contrary, this paper shows that the smallest quintile is statistically significant at the 1%

level, although the biggest quintile is not statistically significant. It implies that the low volatility anomaly exists in small stocks, however not in big stocks. Therefore, these results confute Ang et al. (2006) conclusion that small stocks are not the driver of low volatility anomaly. It indicates that we cannot confirm the second hypothesis (H2), that the low idiosyncratic volatility anomaly still exists in the US stock market even after omitting the small size stocks. The results confirm the finding of Bali and Cakici (2008) that firm size matters for the volatility anomaly. For institutional investors, these results may assist them in evaluating factor allocations. They may use the results in forming a view or belief about underlying factor which drives their assumption, specifically in low volatility strategy.

Robustness Tests

As the robustness test in this study, I do several testings, such as omitting the most volatile quintile (Table 4 and 10) and impact of bull and bear market (Table 5, 6, 11, and 12). Later, I repeat all testings by using equally-weighted portfolio (Table 7-12).

Table 4
Main characteristics of 5 long-short portfolios based on quintile 1 and 4 for volatility

The table below shows several characteristics of 5 long-short portfolios each based on quintile 1 and 4 for idiosyncratic volatility. The portfolios are constructed by showing the outperformance the least volatile portfolio over the fourth quintile of volatile portfolio with the same size quintile. It shows the average monthly excess return, the standard deviation of the returns, and the t-statistic for means of the portfolio. The next last three columns show the average unexplained excess return (Jensen's Alpha), the CAPM market beta, and the unexplained excess return based on the Fama French three factor model. The sample period is from July 1963 to December 2018. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Portfolio	Mean	Std	t (mean)	CAPM α	CAPM β	FF3 α
P4 – P1	-0.25	4.30	-1.49	-0.58***	0.65***	-0.57***
P9 – P6	0.01	3.61	0.05	-0.30***	0.59***	-0.26***
P14 – P11	0.15	3.38	1.15	-0.12	0.52***	-0.08
P19 – P16	0.04	3.17	0.36	-0.20**	0.47***	-0.13
P24 – P21	0.00	2.85	0.03	-0.18*	0.35***	-0.16*
Mean (col)	-0.01	3.03	-0.07	-0.27***	0.52***	-0.24***

The robustness test by omitting the most volatile quintile generates results which strengthen the previous finding in Table 3. Compared to the test in Table 3, the robustness test in Table 4 exhibits a similar conclusion, that firm size matters for the volatility anomaly. The smallest quintile portfolio (portfolio P4-P1) still has FF3 alpha with 1% significance level. This means, even by omitting the most volatile quintile, the low volatility effect still exists in the smallest size quintile. While the biggest size quintile has lower magnitude of FF3 alpha and it is significant at 10% level (less statistical power).

Compared to results in Table 2, the results in Table 5 have a similar pattern. The FF3 alpha of the most volatile portfolio in every level of size has negative alpha with 1% significance level, except the portfolio 25 (not statistically significant) and portfolio 20 during falling interest rates (significant at 5% level). The results imply that, in general, the bull or bear market does not give any impact in each portfolio FF3 alpha.

Table 5
Main characteristics of 25 portfolios double sorted in size and volatility, split to two periods

The table below shows several characteristics of 25 portfolios double sorted on size and residual volatility. The portfolios are the intersections of 5 portfolios formed on size and 5 portfolios formed on the residuals variance. By splitting the data set in two periods (before and after 1985) to get insight in the impact of falling (bull market) or rising interest rate (bear market). It shows the average monthly excess return, the standard deviation of the returns, and the t-statistic for means of the portfolio. The next last three columns show the average unexplained excess return (Jensen's Alpha), the CAPM market beta, and the unexplained excess return based on the Fama French three factor model. The sample period is from July 1963 to December 2018. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Portfolio	Rising interest rates						Falling interest rates					
	Mean	Std	t (mn)	CAPM α	CAPM β	FF3 α	Mean	Std	t (mn)	CAPM α	CAPM β	FF3 α
1	1.62	4.33	6.16	0.81***	0.81***	0.38***	1.23	4.03	6.09	0.53***	0.68***	0.39***
2	1.88	6.24	4.95	0.93***	1.23***	0.43***	1.28	5.05	5.05	0.42***	0.92***	0.29***
3	1.72	7.03	4.02	0.72***	1.38***	0.17**	1.15	5.95	3.86	0.17	1.11***	0.06
4	1.41	7.79	2.97	0.38	1.50***	-0.24**	0.96	7.19	2.66	-0.15	1.31***	-0.20
5	0.45	8.61	0.86	-0.59*	1.54***	-1.33***	-0.04	9.08	-0.08	-1.27***	1.50***	-1.26***
6	1.39	4.25	5.38	0.57***	0.84***	0.19**	1.21	4.03	5.97	0.48***	0.72***	0.35***
7	1.67	5.67	4.83	0.74***	1.16***	0.32***	1.21	4.93	4.90	0.35**	0.93***	0.22**
8	1.66	6.45	4.23	0.68***	1.34***	0.30***	1.23	5.43	4.50	0.29*	1.05***	0.17*
9	1.40	7.31	3.16	0.37*	1.52***	0.04	1.21	6.31	3.82	0.14	1.25***	0.06
10	0.41	7.98	0.85	-0.65***	1.59***	-1.01***	0.69	8.54	1.60	-0.63***	1.63***	-0.56***
11	1.16	3.97	4.78	0.35***	0.80***	0.03	1.08	3.72	5.81	0.38***	0.68***	0.26***
12	1.43	5.00	4.68	0.54***	1.04***	0.21**	1.17	4.64	5.01	0.31***	0.92***	0.19**
13	1.49	5.67	4.33	0.56***	1.20***	0.25***	1.10	5.11	4.30	0.18	1.02***	0.07
14	1.37	6.52	3.46	0.38**	1.39***	0.11	1.19	5.78	4.10	0.17	1.16***	0.10
15	0.72	7.37	1.61	-0.31	1.50***	-0.53***	0.70	7.83	1.78	-0.57***	1.56***	-0.48***
16	1.13	4.04	4.60	0.31***	0.82***	0.04	1.07	3.65	5.83	0.38***	0.66***	0.27***
17	1.18	4.63	4.18	0.31***	0.99***	0.11	1.11	4.35	5.09	0.29***	0.86***	0.19**
18	1.15	5.30	3.57	0.23**	1.16***	0.07	1.12	4.93	4.53	0.20*	1.02***	0.11
19	1.26	5.84	3.53	0.29***	1.28***	0.23**	1.06	5.46	3.86	0.06	1.14***	-0.01
20	0.71	6.80	1.71	-0.31*	1.44***	-0.44***	0.83	7.53	2.18	-0.42**	1.52***	-0.32**
21	0.76	3.55	3.51	-0.04	0.75***	0.01	0.94	3.68	5.10	0.22**	0.72***	0.17**
22	0.76	3.55	3.51	0.01	0.89***	0.14**	0.94	3.68	5.10	0.14	0.84***	0.11
23	0.78	4.54	2.83	-0.09	1.00***	0.00	0.98	4.45	4.37	0.11	0.94***	0.06
24	0.79	4.84	2.67	-0.11	1.07***	0.01	0.93	5.00	3.70	-0.04	1.09***	-0.07
25	0.86	6.03	2.34	-0.11	1.30***	0.01	0.83	6.42	2.57	-0.29**	1.33***	-0.18

Table 6

Main characteristics of 5 long-short portfolios based on quintile 1 and 5 for volatility, split to two periods

The table below shows several characteristics of 5 long-short portfolios each based on quintile 1 and 5 for idiosyncratic volatility. The portfolios are constructed by showing the outperformance the least volatile portfolio over the most volatile portfolio with the same size quintile. By splitting the data set in two periods (before and after 1985) to get insight in the impact of falling (bull market) or rising interest rate (bear market). It shows the average monthly excess return, the standard deviation of the returns, and the t-statistic for means of the portfolio. The next last three columns show the average unexplained excess return (Jensen's Alpha), the CAPM market beta, and the unexplained excess return based on the Fama French three factor model. The sample period is from July 1963 to December 2018. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Portfolio	Rising interest rates						Falling interest rates					
	Mean	Std	t (mean)	CAPM α	CAPM β	FF3 α	Mean	Std	t (mean)	CAPM α	CAPM β	FF3 α
P5 – P1	-1.17	5.13	-3.75	-1.40***	0.73***	-1.71***	-1.27	6.91	-3.66	-1.80***	0.82***	-1.65***
P10 – P6	-0.98	4.91	-3.28	-1.21***	0.75***	-1.20***	-0.52	6.32	-1.64	-1.11***	0.91***	-0.91***
P15 – P11	-0.43	4.80	-1.49	-0.66***	0.71***	-0.56***	-0.39	6.04	-1.27	-0.95***	0.88***	-0.74***
P20 – P16	-0.42	4.25	-1.64	-0.62***	0.62***	-0.48***	-0.24	6.20	-0.78	-0.80***	0.87***	-0.59***
P25 – P21	0.10	3.83	0.43	-0.07	0.55***	0.00	-0.11	4.97	-0.45	-0.51**	0.62***	-0.35**
Mean (col)	-0.58	4.17	-2.29	-0.79***	0.67***	-0.79***	-0.51	5.69	-1.77	-1.04***	0.82***	-0.85***

Based on results in Table 6, the findings show different condition during bull market and bear market. During rising interest rates or bear market, the findings show the same result as in Table 3, which is the smallest quintile is statistically significant at the 1% level and the biggest quintile is not statistically significant. This means the rising interest rate does not give any impact to low volatility anomaly phenomenon. However, during the falling interest rate, the findings show a different result. The smallest quintile still exhibits the same result. However, the biggest quintile is statistically significant at the 5% level. Therefore, this is an indication that the small stocks do not become a driver of low volatility anomaly when the interest rate is falling or bull market. Only the bull market affect the size and low volatility anomaly relationship.

The main data of this paper is using value-weighted portfolio. Then, for robustness test, I employ equally-weighted portfolio and re-run all the six previous testings. In overall, the robustness test results show findings which strengthen the main analysis. Below explanation will discuss further regarding results based on value-weighted and equally-weighted portfolio. Testing table 1 will be tested with new set of portfolio in testing Table 7. Then the robustness test results are ordered as the same as benchmark testing. Therefore, Table 12 shows results with the same method as for Table 6.

Results in Table 7 show consistent results in Table 1. The portfolio 10-1 shows negative FF3 alpha of -0.70 with 1% significance level. Meanwhile, the FF3 alpha in Table 7 is lower, in absolute value, than that of in Table 1, the result shows the existence of low volatility anomaly.

Table 7
Main characteristics of ten portfolios sorted in volatility, using equal-weighted portfolio

The table below shows several characteristics of ten equal-weighted portfolios sorted on residual volatility. It shows the average monthly excess return, the standard deviation of the returns, and the t-statistic for means of the portfolio. The next last three columns show the average unexplained excess return (Jensen's Alpha), the CAPM market beta, and the unexplained excess return based on the Fama French three factor model. The sample period is from July 1963 to December 2018. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Portfolio	Mean	Std	t (mean)	CAPM α	CAPM β	FF3 α
1	1.08	3.04	9.21	0.41***	0.56***	0.25***
2	1.17	3.77	8.02	0.40***	0.76***	0.22***
3	1.23	4.11	7.70	0.41***	0.84***	0.23***
4	1.27	4.45	7.33	0.42***	0.91***	0.21***
5	1.29	4.79	6.97	0.41***	0.97***	0.20***
6	1.39	5.08	7.05	0.48***	1.03***	0.27***
7	1.36	5.44	6.46	0.42***	1.10***	0.20***
8	1.37	5.94	5.94	0.38***	1.18***	0.17***
9	1.29	6.59	5.05	0.25*	1.28***	0.04
10	0.87	8.37	2.70	-0.23	1.40***	-0.45***
10 - 1	-0.21	6.76	-0.80	-0.64***	0.84***	-0.70***

For the second testing, the robustness test results in Table 8 and Table 9 show similar result with the benchmark analysis. In Table 8, the unique pattern as in Table 2 is occurred, that the most volatile portfolio has negative FF3 alpha. However, the test in Table 8 generates a slightly different in terms of significance level. Portfolio 5 has 5% significance level, portfolio 25 has 10% significance level, and Portfolio 10, 15, 20 have 1% significance level, which is the same as benchmark results. Then, in Table 9, the portfolio P5-P1 has negative

FF3 alpha of -0.85 with 1% significance level. The figure is similar to the benchmark but with slightly lower FF3 alpha. In absolute value. The portfolio P25-P21 also has negative FF3 alpha of -0.22 with 10 % significance level. Those results show low volatility anomaly existence in small and big stocks. However, for the big stocks portfolio, the low volatility cannot be verified if we increase the critical value to 5% or 1 %. Based on this limitation, it indicates that the low idiosyncratic volatility anomaly still exists after omitting the small size stocks by using equally-weighted portfolio at the 5% significance level.

Table 8
Main characteristics of 25 portfolios double sorted in size and volatility, using equal-weighted portfolio

The table below shows several characteristics of 25 equal-weighted portfolios double sorted on size and residual volatility. The portfolios are the intersections of 5 portfolios formed on size and 5 portfolios formed on the residuals variance. It shows the average monthly excess return, the standard deviation of the returns, and the t-statistic for means of the portfolio. The next last three columns show the average unexplained excess return (Jensen's Alpha), the CAPM market beta, and the unexplained excess return based on the Fama French three factor model. The sample period is from July 1963 to December 2018. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Portfolio	Mean	Std	t (mean)	CAPM α	CAPM β	FF3 α
1	1.38	3.90	9.16	0.67***	0.65***	0.41***
2	1.58	5.22	7.83	0.72***	0.94***	0.44***
3	1.51	6.07	6.44	0.57***	1.10***	0.28***
4	1.35	6.97	5.01	0.34**	1.24***	0.07
5	0.91	9.02	2.62	-0.17	1.37***	-0.44**
6	1.29	4.12	8.11	0.52***	0.77***	0.28***
7	1.41	5.23	6.95	0.51***	1.02***	0.24***
8	1.42	5.88	6.21	0.44***	1.17***	0.19***
9	1.29	6.69	5.00	0.22*	1.35***	0.02
10	0.60	8.32	1.86	-0.61***	1.62***	-0.71***
11	1.13	3.81	7.62	0.37***	0.73***	0.16**
12	1.29	4.80	6.96	0.42***	0.97***	0.20***
13	1.26	5.35	6.10	0.32***	1.09***	0.12*
14	1.27	6.09	5.37	0.24**	1.25***	0.07
15	0.72	7.68	2.41	-0.45***	1.54***	-0.48***
16	1.09	3.79	7.44	0.34***	0.72***	0.17**
17	1.13	4.47	6.55	0.28***	0.91***	0.14**
18	1.15	5.07	5.87	0.22***	1.07***	0.09
19	1.14	5.63	5.22	0.14*	1.19***	0.04
20	0.77	7.23	2.74	-0.38***	1.49***	-0.36***
21	0.89	3.55	6.46	0.16**	0.68***	0.06
22	0.89	3.55	6.46	0.16**	0.85***	0.11**
23	0.98	4.41	5.76	0.12**	0.94***	0.05
24	0.92	4.81	4.93	0.00	1.05***	-0.06
25	0.84	6.34	3.40	-0.23**	1.34***	-0.15*

Table 9

Main characteristics of 5 long-short portfolios based on quintile 1 and 5 for volatility, using equal-weighted portfolio

The table below shows several characteristics of 5 long-short equal-weighted portfolios each based on quintile 1 and 5 for idiosyncratic volatility. The portfolios are constructed by showing the outperformance the least volatile portfolio over the most volatile portfolio with the same size quintile. It shows the average monthly excess return, the standard deviation of the returns, and the t-statistic for means of the portfolio. The next last three columns show the average unexplained excess return (Jensen's Alpha), the CAPM market beta, and the unexplained excess return based on the Fama French three factor model. The sample period is from July 1963 to December 2018. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Portfolio	Mean	Std	t (mean)	CAPM α	CAPM β	FF3 α
P5 – P1	-0.47	6.53	-1.85	-0.84***	0.72***	-0.85***
P10 – P6	-0.70	5.76	-3.11	-1.13***	0.85***	-0.98***
P15 – P11	-0.41	5.58	-1.89	-0.82***	0.81***	-0.64***
P20 – P16	-0.33	5.45	-1.55	-0.72***	0.76***	-0.54***
P25 – P21	-0.05	4.87	-0.28	-0.39**	0.66***	-0.22*
Mean (col)	-0.39	5.18	-1.95	-0.78***	0.76***	-0.65***

I also repeat the robustness test of omitting the most volatile quantile (Table 4) and impact of bull and bear market (Table 5 and 6) using equally-weighted portfolio. The findings have various outcome. In Table 10, the results show all negative FF3 alpha for every level of size. However, all of them is not statistically significant. Therefore, by doing this test, we cannot confirm that the size effect is the low volatility driver.

Table 10

Main characteristics of 5 long-short portfolios based on quintile 1 and 4 for volatility, using equal-weighted portfolio

The table below shows several characteristics of 5 long-short equal-weighted portfolios each based on quintile 1 and 4 for idiosyncratic volatility. The portfolios are constructed by showing the outperformance the least volatile portfolio over the fourth quintile of volatile portfolio with the same size quintile. It shows the average monthly excess return, the standard deviation of the returns, and the t-statistic for means of the portfolio. The next last three columns show the average unexplained excess return (Jensen's Alpha), the CAPM market beta, and the unexplained excess return based on the Fama French three factor model. The sample period is from July 1963 to December 2018. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Portfolio	Mean	Std	t (mean)	CAPM α	CAPM β	FF3 α
P4 – P1	-0.03	3.98	-0.19	-0.33***	0.58***	-0.34
P9 – P6	0.00	3.57	0.00	-0.30***	0.58***	-0.26
P14 – P11	0.14	3.37	1.09	-0.13	0.53***	-0.09
P19 – P16	0.04	3.12	0.37	-0.20**	0.47***	-0.14
P24 – P21	0.03	2.68	0.29	-0.16*	0.37***	-0.12
Mean (col)	0.04	2.97	0.33	-0.22***	0.51***	-0.19

The unique pattern in Table 11 is similar with explained in Table 5, except the portfolio 5 during falling interest rates (not significant). Furthermore, the results in Table 12 show the same conclusion as in Table 6, that bull market affects size and low volatility anomaly relationship, while the bear market does not. Thus, the results imply that, in general, the equally-weighted portfolio does not give significant deviation in explaining the impact of bull and bear market to size effect and low volatility anomaly relationship.

Table 11
Main characteristics of 25 portfolios double sorted in size and volatility, using equal-weighted portfolio and split to two periods

The table below shows several characteristics of 25 equal-weighted portfolios double sorted on size and residual volatility. The portfolios are the intersections of 5 portfolios formed on size and 5 portfolios formed on the residuals variance. By splitting the data set in two periods (before and after 1985) to get insight in the impact of falling (bull market) or rising interest rate (bear market). It shows the average monthly excess return, the standard deviation of the returns, and the t-statistic for means of the portfolio. The next last three columns show the average unexplained excess return (Jensen's Alpha), the CAPM market beta, and the unexplained excess return based on the Fama French three factor model. The sample period is from July 1963 to December 2018. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Port-folio	Rising interest rates						Falling interest rates					
	Mean	Std	t (mn)	CAPM α	CAPM β	FF3 α	Mean	Std	t (mn)	CAPM α	CAPM β	FF3 α
1	1.64	4.28	6.32	0.85***	0.76***	0.41***	1.21	3.61	6.64	0.57***	0.58***	0.43***
2	1.99	6.09	5.36	1.06***	1.15***	0.52***	1.31	4.53	5.76	0.53***	0.80***	0.41***
3	1.93	6.93	4.57	0.96***	1.30***	0.34***	1.23	5.40	4.55	0.34**	0.97***	0.24**
4	1.68	7.63	3.62	0.68**	1.39***	0.00	1.13	6.48	3.47	0.14	1.13***	0.09
5	1.16	8.95	2.12	0.13	1.48***	-0.76***	0.75	9.08	1.64	-0.36	1.30***	-0.33
6	1.41	4.27	5.43	0.58***	0.85***	0.21**	1.22	4.02	6.03	0.49***	0.72***	0.36***
7	1.66	5.68	4.81	0.74***	1.16***	0.31***	1.24	4.91	5.02	0.38***	0.93***	0.24***
8	1.68	6.49	4.25	0.70***	1.34***	0.30***	1.24	5.43	4.53	0.29**	1.05***	0.18*
9	1.42	7.30	3.19	0.38**	1.51***	0.04	1.21	6.24	3.86	0.15	1.24***	0.06
10	0.45	8.03	0.91	-0.62**	1.60***	-0.99***	0.70	8.52	1.65	-0.61**	1.63***	-0.54***
11	1.17	3.97	4.86	0.37***	0.79***	0.05	1.09	3.71	5.86	0.39***	0.68***	0.27***
12	1.46	5.02	4.77	0.57***	1.04***	0.23***	1.18	4.64	5.07	0.33***	0.91***	0.21**
13	1.51	5.69	4.34	0.57***	1.20***	0.26***	1.10	5.09	4.29	0.18	1.02***	0.07
14	1.39	6.53	3.50	0.40***	1.39***	0.12	1.18	5.78	4.07	0.17	1.16***	0.09
15	0.74	7.38	1.65	-0.29	1.51***	-0.52***	0.70	7.88	1.78	-0.57***	1.57***	-0.48***
16	1.13	4.02	4.62	0.31***	0.82***	0.05	1.07	3.64	5.86	0.38***	0.66***	0.27***
17	1.18	4.64	4.18	0.31***	1.00***	0.11	1.10	4.35	5.04	0.28**	0.86***	0.18*
18	1.17	5.30	3.61	0.24**	1.16***	0.08	1.15	4.92	4.64	0.23**	1.01***	0.13
19	1.27	5.87	3.55	0.30***	1.29***	0.21**	1.05	5.46	3.83	0.06	1.13***	-0.02
20	0.69	6.84	1.67	-0.32**	1.45***	-0.47***	0.82	7.50	2.17	-0.42**	1.51***	-0.32**
21	0.76	3.61	3.48	-0.03	0.75***	-0.08	0.97	3.52	5.51	0.30***	0.64***	0.20**
22	0.76	3.61	3.48	0.08	0.89***	0.11	0.97	3.52	5.51	0.22**	0.82***	0.14*
23	0.90	4.43	3.32	0.03	0.98***	0.03	1.04	4.40	4.73	0.19**	0.92***	0.10
24	0.81	4.83	2.77	-0.08	1.07***	-0.06	0.99	4.81	4.10	0.06	1.04***	-0.01
25	0.79	6.00	2.17	-0.18	1.31***	-0.14	0.87	6.57	2.62	-0.28**	1.37***	-0.18

Table 12
Main characteristics of 5 long-short portfolios based on quintile 1 and 5 for volatility, using equal-weighted portfolio and split to two periods

The table below shows several characteristics of 5 long-short equal-weighted portfolios each based on quintile 1 and 5 for idiosyncratic volatility. The portfolios are constructed by showing the outperformance the least volatile portfolio over the most volatile portfolio with the same size quintile. By splitting the data set in two periods (before and after 1985) to get insight in the impact of falling (bull market) or rising interest rate (bear market). It shows the average monthly excess return, the standard deviation of the returns, and the t-statistic for means of the portfolio. The next last three columns show the average unexplained excess return (Jensen's Alpha), the CAPM market beta, and the unexplained excess return based on the Fama French three factor model. The sample period is from July 1963 to December 2018. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Portfolio	Rising interest rates						Falling interest rates					
	Mean	Std	t (mean)	CAPM α	CAPM β	FF3 α	Mean	Std	t (mean)	CAPM α	CAPM β	FF3 α
P5 – P1	-0.49	5.41	-1.48	-0.72***	0.72***	-1.18***	-0.46	7.21	-1.26	-0.92***	0.72***	-0.77***
P10 – P6	-0.96	4.91	-3.23	-1.20***	0.75***	-1.20***	-0.51	6.28	-1.62	-1.10***	0.91***	-0.90***
P15 – P11	-0.43	4.78	-1.49	-0.66***	0.71***	-0.56***	-0.39	6.06	-1.28	-0.96***	0.89***	-0.74***
P20 – P16	-0.44	4.22	-1.69	-0.64***	0.63***	-0.51***	-0.25	6.16	-0.82	-0.81***	0.85***	-0.59***
P25 – P21	0.03	3.80	0.12	-0.15	0.56***	-0.05	-0.11	5.49	-0.39	-0.58**	0.73***	-0.37**
Mean (col)	-0.46	4.18	-1.81	-0.67***	0.68***	-0.70***	-0.34	5.77	-1.19	-0.87***	0.82***	-0.68***

6. Conclusion

One of the most notable work in modern portfolio theory, CAPM explains that risk and return have a positive relationship. The CAPM model defines that investors need to be rewarded for bearing risk by higher expected return. However later, many papers argue the reliability of CAPM in predicting the relationship between risk and return. Black et al. (1972) find flatter risk and return relationship. Moreover, Fama and French (1992, 1993) find the relation between market beta and average return is flat.

Flat or negative risk and return relationship inspire scholars to investigate this phenomenon. The negative risk and return relationship puzzle are known as “low volatility anomaly”. Ang et al. (2006) find that high idiosyncratic volatility stocks experience low average returns based on sample in the US market. The quintile portfolio with the lowest idiosyncratic volatility stocks outperforms the quintile portfolio with the highest idiosyncratic volatility stocks by 1.06% per month. Later, Ang et al. (2009) extended their analysis to 23 developed markets. They find that the low volatility effect is not only present in the US market, but also in G7 country. Based on MSCI Research Insight (Bender et al., 2013), the low volatility anomaly contradicts the Efficient Market Hypothesis and assumption of the CAPM.

After some papers proved the existence of volatility anomaly, then some scholar tried to explain the underlying idea behind the anomalous effect. Bali and Cakici (2008) find that the firm size is a driver of low volatility anomaly. They argue that the negative relation between idiosyncratic volatility and expected stocks return is driven by small-caps.

Given those perspectives, this study examines the existence of low volatility anomaly in US stocks market. Moreover, this study also analyses size as an underlying factor that caused the low volatility anomaly. Since previous paper using data until the 2000s, this study employs updated data, which is from July 1963 to December 2018. All data is downloaded from Kenneth French’s website.

To conduct this research, the method used to test the volatility effect is the CAPM model and Fama French three factor model. From the CAPM regression, the alpha and beta will be reported. Then, FF3 alpha is also reported. The volatility effect will be observed through the alphas of each portfolio, specifically the comparison between the least volatile portfolio and the most volatile portfolio. The low volatility phenomenon exists when the least volatile portfolio outperforms the most volatile portfolio.

The first result of this study shows that the low idiosyncratic volatility anomaly still exists in the US stock market. It can be observed from Table 1, the long short portfolio of the most volatile stocks minus the least volatile stocks has negative FF3 alpha of -1.07 with 1% significance level. This means that we fail to reject the first hypothesis (H1). Then, this study also shows that the low volatility anomaly exists in small stocks, however not in big stocks. The result is in line with finding by Bali and Cakici (2008) that firm size matters for the volatility anomaly. Therefore, we cannot confirm the second hypothesis (H2), that the low idiosyncratic volatility anomaly still exists in the US stock market even after omitting the small size stocks.

This study contributes to the literature about the existence of low volatility anomaly in US stocks market based on recent data period. Besides, this study tries to identify a driver of the low volatility anomaly, which is size effect. This results of this study support the previous study by Bali and Cakici (2008). Further, based on the results, it can be recommended to individual investor to utilize the low volatility phenomenon for obtaining a better risk-adjusted return. Then, for institutional investor, the results could assist them in evaluating factor allocation, specifically when they build a view for their assumptions. They should consider what factor that drives their assumptions. In low volatility strategy, size is one example of a driver for volatility effect.

The existence of low volatility anomaly had been proven by some papers for equity markets worldwide (Ang et al., 2009; Blitz and van Vliet, 2007, 2013). For future improvement, there are some points to consider, such as the extended sample from other regions. It would be interesting to investigate whether the firm size as volatility driver is robust to region other than the US. Then, research of low volatility anomaly in other securities has a small number. Therefore, it would also be interesting to check the anomaly in other financial assets class, such as corporate bonds, government bonds, and derivatives. Future research could examine a model that could distinguish other drivers for the anomaly and explain the presence of low volatility anomaly in other financial asset markets.

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