The Altman Z-score model and financial distress prediction: The case of Dutch SMEs

Master Thesis

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Preface

This thesis has been written to fulfill the graduation requirements for the master Finance at Tilburg University. The study is performed at the request of HKB, where I undertook an internship. The company, located in Veldhoven, provides business advice for small and medium sized firms.

Hereby, I would like to thank my supervisor dr. P.C. de Goeij for the feedback, advice and the personal touch. Additionally, I would like to thank drs. Ing. R.W.J. Bierens MBV for accompaniment and the internship opportunity.

Eindhoven, 07-19-2018

ABSTRACT

In this paper I test to what extend the revised Altman Z-score model (1983) provides accurate predictions for Dutch SMEs. The empirical analysis is conducted using a sample for the years 2010-2015 consisting of 2.632 firms of which 3,53% went bankrupt in the sample. The empirical findings indicate that the Altman Z-score model predicts correctly approximately 71% of bankruptcies one year prior. According to the Altman Z-score for the sample used in this paper, the accurate classification percentage decreases for both bankrupt and non-bankrupt firms two years prior to bankruptcy, compared to one statement prior to bankruptcy. All individual ratios of the model significantly contribute to the prediction of bankruptcies for Dutch SMEs. In addition, inspired by the Altman (1983) model, I estimate a new bankruptcy model specially for Dutch SMEs. This new ZL-score model works better than the Altman Z-score model as it predicts approximately 82% of bankrupt firms correctly. The empirical findings of this paper can be used by stakeholders of Dutch SMEs when making financing related strategic decisions.

Table of contents

1. Introduction

For decades, researchers intensively work on models that are able to predict bankruptcies. The predictability is important for managers, investors, creditors, and other stakeholders. The current economic circumstances are excellent, but new upcoming crises will come with certainty. Is bankruptcy prediction possible, based on annual statements and ratios? What is the risk entrepreneur's bear? Investigators point out that a few years before bankruptcy, actions can be undertaken like the fulfillment of a turnaround plan to prevent bankruptcy (Pompe, and Bilderbeek, 2005). Therefore, a model that is able to predict bankruptcies is extremely useful.

Some studies focus on predicting financial distress of firms by the Altman Z-score model (Altman, 1968). That model, published in 1968, is the first multivariate technique for predicting bankruptcies (Altman, 1968). The model is linear and consists of five weighted coefficients belonging to financial ratios. The weights are multiplied with the ratios and added up, resulting in a score. That score is used as a cut-off and classifies firms into bankrupt or non-bankrupt. The main idea of the model is that firms that go bankrupt in the future present significant different financial ratios than firms that continue their operations in the future (Altman, 1968; Caouette, Altman, & Narayanan, 1998). Even though the model is developed exactly 50 year ago, it is still widely used both in research and in practice (Altman et al., 2017; Almamy, Aston, & Ngwa, 2016; Bellovary, Giacomino, & Akers, 2007).

Until now, financial studies about the Altman Z-score model are mainly based on large enterprises, publicly-traded firms, countries like France, Greece and the United States, and countries in Asia[1]. To my knowledge, the model is not tested for Dutch small, and medium sized enterprises (SMEs). In most countries of the world –including the Netherlands, SMEs form an important part of the economy (99%). They stimulate innovation, contribute to economic growth, and create jobs (Audretsch and Keilbach, 2004). Therefore, it is important to investigate the predictability of bankruptcies for SMEs. By gaining insights on whether and how the Altman Z-score model is applicable for Dutch SMEs, tools and scans can be developed in order to apply analyses, and to improve the financial viability of companies. The model can be used as a warning signal, and actions can be undertaken to prevent a bankruptcy.

[1] Investigated by: Altman et al, 1968, 2000, 2017; Begley et al., 1996; Chiaramonte, Poli, & Zhou (2016); Gerantonis, Vergos and Christopoulos, 2009; Grice and Ingram, 2001; Mselmi et al, 2017; Rezende et al., 2017; Tian and Yu., 2017

Noteworthy, due to the improved economic circumstances in recent years, worldwide bankruptcies decreased last years. According to the Global Insolvency Index 2017[2], the Netherlands is an outlier as regards to decreases in bankruptcies, since Dutch bankruptcies decreased more than bankruptcies of other countries worldwide. Irrespective of the number of bankruptcies, studying financial distress will always be important, because business cycles are patterns of economic fluctuations, resulting in expansions and recessions. This implies new crises and bankruptcies come with certainty.

The financial literature shows that the Altman Z-score model is discussed and tested widely. However, to my knowledge, Dutch SMEs have not been investigated so far. Therefore, this study firstly provides insight into the predictable value of the Altman Z-score for Dutch SMEs. Note that the original model (1968) is developed for publicly traded companies only. Due to the private nature of most Dutch SMEs, a revised formula (1983), developed for private firms is used to test for the predictable value of the model. The revised model replaces market value by book value (Altman, 1983), and therefore, allocates different weights to the five financial ratios. Furthermore, since researchers increasingly use logistic regression analysis (LRA) to predict bankruptcies, a comparison between classification results of the Altman Z-score model and of the LRA is made. Besides the comparison, the LRA helps to understand the individual significance of the included ratios for Dutch SMEs for predicting bankruptcies.
Second, this thesis provides a new model, called the ZL-score model, which optimizes bankruptcy prediction for Dutch SMEs by changing the weights to each ratio and the cut-off score, but restricting the included ratios to those of the revised model. Considering the above, this thesis will answer the next main research question:

> *To what extent is the revised Altman Z-score model able to predict bankruptcies for Dutch SMEs?*

To answer this research question, three hypotheses are formulated, which are tested in this study. The first hypothesis indicates a positive relationship between the revised Altman Z-score model (1983) and predicting bankruptcies for Dutch SMEs. The second hypothesis indicates that all ratios of the Altman Z-score model help to predict bankruptcies for Dutch SMEs. Last, the optimal time horizon for predicting bankruptcy for Dutch SMEs based on the Altman Z-

[2] Owned by credit insurer 'Euler Hermes'

score model is two statements prior to bankruptcy. All obtained results are used to generate the ZL-score model.

The remainder of the study is organized as follows. Chapter 2 discusses a literature review, wherein the topic financial distress is discussed, a description of the Altman Z-score model is given, findings of other researchers regarding the Altman Z-score model are examined, and limitations of the existing model are discussed. In chapter 3, the dataset and methodologies are described. The methodologies explain how I test for the effectiveness of the existing model, the individual significance of the included ratios, the working method of the LRA, and how I build the ZL-score model. Chapter 4 discusses results of the revised Altman Z-score (1983), the LRA and of the new ZL-score model. Finally, the last chapter describes conclusions, a discussion, and recommendations for future research.

2. Literature review

Financial ratios are generally used measures to predict the financial viability of enterprises (Altman 2000; Barnes, 1987). Among other appliances, they may be used to forecast future profits and earnings, to predict sales and bankruptcies, and to calculate credit ratings. Therefore, it is not surprising that financial ratios have been studied intensively. Researchers agree about the relationship between firms that go bankrupt and firms that do not go bankrupt, and their financial ratios (Altman 1968; Balcaen, and Ooghe, 2006; Barnes 1987; Caouette, Altman, & Narayanan, 1998). They conclude that a few years prior to an actual bankruptcy, firms deal with different ratios in terms of solvency, liquidity, and profitability than firms that do not go bankrupt. Forecasts of financial ratios are useful to test for different suspicions about the company (Tian, Yu, 2017). Two useful examples: the forecasts may be used to advice about safety of firms, and therefore about the credit risks and lending decisions, and test whether default risk is priced-in in stock returns (Tian, Yu, 2017). This study focuses on the Altman Z-score model, which is based on five financial ratios, measuring: activity, leverage, profitability, liquidity and solvability.

2.1 Financial distress

First, it is necessary to clarify the concept financial distress. Investigators deviate in their researches regarding the discussed term. For example, Mselmi, Lahiani, & Hamza (2017) investigate financial distress prediction for SMEs in France, and they differentiate between distressed and non-distressed firms. In their research, a firm is labeled as distressed if it has a "*statement of event to the Judicial tribunal of commerce*" (Mselmi, Lahiani, & Hamza, 2017). Moreover, Rezende et al. (2017) classify a firm as financial distressed in times "*the EBITDA*[3] *is lower than its financial costs for two serial statements*". Additionally, the initial work of Altman (1968) labels the bankrupt group as so in case firms "*submitted a petition covering bankruptcy (National Bankruptcy Act) during 1946-1965*". In the Netherlands, according to 'faillissementsdossier'[4] (2018) a firm is bankrupt in case *"the obligor is unable to meet his financial obligations or terminated to pay his debt.*

[3] Earnings before interest, taxes, depreciation and amortization.
[4] Faillissementdossier is the most up-to-date Dutch database which provides insights into bankruptcies, insolvencies, suspensions and debt changes of Dutch firms. Additionally, the database offers options like to enrich debtor data or match it with the bankruptcy register.

Researchers use different techniques to predict bankruptcies and financial distress. The most used techniques in the financial academic literature are: multiple discriminant analysis (MDA), the logit model (LM), neural network (NW), contingent claims (CC), and statistical and machine learning techniques (Altman, 2017; Mselmi, Lahiani & Hamza, 2017). This thesis focuses on the MDA and LM techniques, which are discussed in chapter 3.

2.2 Development of the Altman Z-score model

Until the mid-1960's, investigators mainly focus on univariate analysis (Beaver, 1966; Bellovary et al., 2007). This analysis implies that the focus is on examining single ratios, one at a time (Beaver, 1966). A well know univariate study is that of Beaver (1966). He divulges a univariate analysis by comparing the mean values of several financial ratios. Additionally, he tests the predictable value of the ratios by classifying firms into bankrupt and non-bankrupt (Beaver, 1966). Beaver (1966) discriminates between matched samples of bankrupt and non-bankrupt firms. In 1966, the investigator already points out that future research should focus on multivariate analysis, since the financial ratios together may result in a higher predictable value.

The first multivariate study for predicting bankruptcies is developed by Altman (1968). The model is linear and consists of five weighted coefficients belonging to financial ratios (Altman, 1968). The weights are multiplied with the ratios and added up, resulting in a score. That score is used as a cut-off and classifies firms into bankrupt or non-bankrupt. The multivariate ratio analysis is especially used to provide insight into the performance of the enterprise, and to predict financial distress within firms, but other reasons exist. Pindado et al. (2008), investigate that the model may also be used to provide insights into type of debt (bank, private or public), and expenses and gains of covenants in bonds. However, the main idea of this approach is that firms that go bankrupt in the future show significant different financial ratios than firms that continue their operations in the future (Altman, 1968; Caouette, Altman, & Narayanan, 1998).

The original Altman model (1968) restricts 22 potential essential financial ratios, based on solvability, leverage, profitability, activity and liquidity, to only five by including solely the ratios with the highest power. In other words, including the ratios with the highest probability of correctly rejecting the null hypothesis (Nieuwenhuis, 2009). The power of a test can be affected by: sample size, significance level and the 'true value' of the parameter that is tested (Nieuwenhuis, 2009). The five ratios included in the Altman Z-score model (1968) are confirmed to be significant in different cases, and so may help to predict future bankruptcies. The initial research of Altman (1968) labels 94% of bankrupt companies as such one year before bankruptcy for the within-sample, and 96% for the holdout sample. Altman (1968) tests this based on a sample of 66 manufacturing firms, and uses the MDA, which is a model based on ordinary least squares (OLS). The aim of this MDA is to generate a weighted function of the included Altman Z-score ratios (equation 1) that discriminates optimal between two different groups (Altman 1968; Lord, Weech-Maldonado, & Davlaov, 2017).

Note that the original model of Altman (1968) uses market value of equity as input variables of X_4 and is developed for public companies:

$$Z - score = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5 \tag{1}$$

Where: X_1 = *working capital/total assets;*
X_2 = *retained earnings/total assets;*
X_3 = *earnings before interest and taxes/total assets;*
X_4 = *market value of equity/book value of total liabilities;*
X_5 = *sales/ total assets* (Altman, 1968).

Percentages should be entered as 0.25 (for 25%). Note that equation (1) does not include a constant or an intercept. The Z-score is used as a cut-off and divides firms into financially distressed and financially solvent. The cut-off score used in the original research is 2.67 for financial solvent firms and scores smaller than or equal to 1,81 represent financial distressed firms (Altman, 1968). The scores in between 1,81 and 2,67 are considered as the grey area, which is the zone of ignorance (Figure 1). The outcome of the model is easy to interpret, making it possible for stakeholders to quickly obtain insights into firms' financial distress.

High risk of bankruptcy (insolvent firms) ZL ≤ 1,81	Results are uncertain Grey area 1,81 < ZL > 2,67	Low risk of bankruptcy (solvent firms) ZL ≥ 2,67

Figure 1

X_1 = *working capital/total assets* - liquidity

Working capital is calculated by taking the difference between the current assets of firms (e.g. cash and account receivables), and current liabilities (e.g. accounts payables). Notably, working capital determines a snapshot, since the value can change daily. This liquidity ratio measures the difference between current assets and current liabilities compared to the total assets. Total assets represent the total capitalization of the firm. This ratio is preferred over other liquidity ratios like the current ratio and quick ratio because the value of those ratios can be counterproductive for firms that face financial distress (Altman, 2000).

X_2 = *retained earnings/total assets* - solvency

Retained earnings measure the amount of money that is not paid out to shareholders but retained in the firm to be reinvested. This solvency ratio calculates the amount of money retained in the firm compared to the total capitalization. A high number implies that a company retains profits into the company and is financed through retention instead of debt. A low number indicates that money is paid out to shareholders and/or the company is financed through debt.

X_3 = *earnings before interest and taxes/total asset* - profitability

Earnings before interest and taxes (EBIT) is used to measure the firms' earnings independent of any tax or leverage factors. EBIT compared to total capitalization provides insight into the productivity of the company's assets (Altman, 2000). Research shows that this profitability ratio is intuitive since it is an important factor for the continuation of firms (Altman, 2000). A low ratio implies that the company may face financial distress, since the continuation of the firm is in danger, and the profitability compared to the total capitalization of the operations is low.

X_4 = *market value of equity/book value of total liabilities* - leverage

This leverage ratio measures how quickly a company can be insolvent. In other words: how quickly are the firms' liabilities going to exceed the firms' assets? In case the market value of equity is twice as much as the book value of total liabilities, the firm can experience two-thirds decrease in market value of equity before it becomes insolvent. Noteworthy, on the occasion that firms' operations generate higher rates of returns then the interest costs subsequently from loans, debt is helping to make growth possible. However, uncontrolled debt levels may be dangerous for firms.

X_5 = *sales/ total assets* - activity

This activity ratio is called the 'capital turnover' and measures the proportion of sales compared to the total assets. The parameter indicates the amount of revenue of a firm compared to the capitalization. Generally, the higher the ratio, the better the performance of the company, since that indicates that the firm is generating more earnings per euro of asset on the balance sheet.

The original model is developed for publicly traded companies only, because X_4 requires information about stock prices. In 1983, due to the private nature of a significant part of firms of the worldwide economy, the original model (equation 1) is revised. The revised model replaces market value of equity by book value of equity (Altman, 1983). Theoretically, book value of equity is the amount stakeholders would receive if all liabilities are subtracted from the assets. Market value of equity is based on the current stock price and is calculated by multiplying the current firms' stock price by the number of shares outstanding. The next formula, which attaches different weights to the ratios, is generated:

$$Z - score = 0.717X_1 + 0.847X_2 + 3.107X_3 + 0.420X_4 + 0.998X_5 \qquad (2)$$

Altman (1983) investigates that the prediction accuracy for the revised model is the highest by considering 1 year prior to failure. In that case, 90,9% of bankrupt firms is classified correctly. Due to lack of data, Altman (1983) was unable to test the model on a secondary sample. Note that the cut-off score for the revised model is slightly different from the original model. Scores smaller than 1,81 represent financial distressed firms and scores bigger than 2,99 represent financially solvent firms. The scores in between 1,81 and 2,99 are considered as the grey area, which is the zone of ignorance.

2.3 Prior research

Nowadays, the Altman Z-score model is an intensively investigated, and well-known model. Summarizing prior research, the Altman Z-score model works well for large enterprises, publicly-traded firms, countries like France, Greece and the United States, and countries in Asia (Altman 1968; 1983; 2000; Altman, et al. 2017; Balcaen and Ooghe, 2006; Gerantonis, N., Vergos, K., and Christopoulos, A., 2009; Grice and Ingram, 2001). However, investigators suggest that it is difficult to generalize the results of the Altman Z-score model. Not all countries, asset classes and firm sizes have already been examined. To my knowledge, previous studies have not extended the Altman Z-score in the case of Dutch SMEs. Prior research mainly focuses on the predictable value of the *original* Altman Z-score model, and this research contributes to existing literature by examining the *revised* model.

Begley et al (1996) conclude that the *original* model works less optimal in the US during 1990-1995 compared to 1980-1990. They include 65 bankrupt and 1300 non-bankrupt firms in their sample and report a 78% accuracy rate for the total period. Grice and Ingram (2001) publish similar results, since they conclude that the original model is less useful for recent years compared to previous years. In their research, the power of the model decreases throughout the years. In contrast to Begley et al (1996) and Grice and Ingram (2001), Altman (2000) investigates that the model retains high accuracy rates in the last decade.

Taking prior research into account, this research adds value to the existing literature, because, to my knowledge, the Altman Z-score model is not tested for Dutch SMEs so far. This particular size of companies features less available data and therefore is more difficult to investigate. As a result of the generalization problems, it is important to enhance knowledge about this particular group. Mainly because a large percentage of total companies is covered by SMEs. Additionally, most studies focus on samples from the 1980's or 1990's, this study uses a sample from the 2010-2015 period. Furthermore, developing a new model, specially for Dutch SMEs is relevant since prior research suggests to re-estimate the discriminant coefficients of the model using current samples with the aim to increase accuracy rates (Grice and Ingram, 2001).

2.4 Limitations of the Altman Z-score model

Researchers discuss several limitations regarding the Altman Z-score model. This paragraph discuses limitations of the MDA assumptions, the weak intuitive interpretation of a cut-off score, macro-economic factors that may have an influence on the accuracy of the model, and the importance of carefully analyzing small companies.

First, the MDA starts with strict assumptions, which are discusses in detail in paragraph 3.3. One assumption of the technique is that both groups (bankrupt and not bankrupt) must have indistinguishable variance-co-variance matrices (Mselmi, Lahiani, & Hamza, 2017). This is due to the multivariate approach and implies that the matrices of the two groups should be identical. In case of unequal matrices, a quadratic model should be used. Nevertheless, those models are extremely complex and seem to outperform linear models only when a large sample is used (Balcaen and Ooghe, 2006). Therefore, investigators usually transform the data to roughly equal matrices and then use linear models (Balcaen and Ooghe, 2006). Not surprisingly, several researchers point out that assumptions of the MDA technique are often violated (e.g. Barnes, 1982; Karels and Prakash, 1987; Mcleay and Omar, 2000; Premachandra, Bhabra, & Sueyoshi, 2009). Considering these violations, researchers use the LRA. This technique regards a model non-linear in parameters to estimate the probability of bankruptcy to happen. The LRA permits to evaluate the financial ratios of the Z-score model independently, by reporting the statistical significance of the ratios individually. Previous studies conclude accurate classification results within-samples by using the LRA (Platt and Platt, 1990; Premachandra, Bhabra, & Sueyoshi, 2009). However, the accuracy results for holdout samples is significantly poor. This implies that the LRA results are difficult to generalize. Though, the LRA helps to evaluate individual parameters, and does not start with the strict assumptions MDA does.

Second, the technique of calculating a cut-off score is a reason for a weak intuitive interpretation (Mselmi, Lahiani, & Hamza, 2017). This implies that one can immediately perceive the status of the firm, without reasoning, knowledge or previous experience about bankruptcies of firms. The score may strengthen to no longer being critical about the values of the included ratios. One outlier ratio may decrease or increase the total Z-score tremendously, possibly resulting in a wrong classification. By providing that score, a chance exists that the binary classification is not investigated further, which may lead to wrong conclusions.

Additionally, macro-economic factors can have a major influence on the accuracy of the model because the cut-off point between bankrupt and non-bankrupt firms can be disturbed by different stages of the business cycles (Mselmi, Lahiani, & Hamza, 2017). For example, international wars may influence the ratios, due to damage of property or reallocation of recourses. Additionally, cyclical macro-economic factors like higher prices may suppress households to spend, affecting revenues of firms. Barnes (1987) points out that a model for predicting bankruptcy is only useful if the parameters used are stable over time. Dombolena and Khoury (1980) investigate that the parameters become more instable over time as the firm nears bankruptcy. In their research, the instability in ratios is measured by their standard deviations.

Lastly, it is important to analyze very small companies (assets smaller than €1 million) carefully, since the data of that group may be of poor quality due to missing values and instable ratios (Balcaen, Ooghe, 2006). The financial ratios are instable, because the deviations from the means are large for small firms, limiting the possibility to compare the ratios.

3. Data and Methodology

3.1 European law for SME

I consider the European law[5] definition for SMEs (Table 1). Therefore, a medium sized organization has a maximum of 249 employees, and/or annual revenue less than €50 million, and/or a total balance sheet of less than €43 million. Small companies have a maximum of 49 employees, and/or annual revenue less than €10 million, and/or a total balance sheet of less than €10 million.

Table 1 Definitions of European SMEs according to the European Commission[1]

	Micro enterprise	**Small enterprise**	**Medium enterprise**
Employees	1-9	10-49	50-249
Annual revenue	€2 million	€10 million	€50 million
Total balance sheet	€2 million	€10 million	€43 million

3.2 Sample selection

The data for this thesis are obtained from Orbis, which is widely used for financial company data, ratios, and to compare companies for decision making. The database provides several key financial variables that are necessary to calculate the Altman Z-score.

Private SMEs are included in the sample. To accomplish this, the number of employees is restricted from a minimum of 10 to a maximum of 249. The minimum is set to exclude micro organizations. Micro organizations usually have low, unstable financial ratios, which can affect the Z-score (Balcaen, Ooghe, 2006). Furthermore, I use the revised Altman Z-score equation (2), which replaces market value of equity by book value of equity since most Dutch SMEs are held privately. Dutch SMEs generally obtain money by banks as lenders, or by investors who hold debt securities, which are not freely tradable on the stock exchange (Altman et al., 2017). This restriction is achieved by only selecting unlisted companies via database Orbis.

Investors commonly use matched sampling for the MDA technique (Altman, 2000; Beaver, 1966; Begley et al. 1996; Mselmi, Lahiani, & Hamza, 2017; Mossman et al., 1998). This implies that assets, and/or number of employees, and/or sales of the bankrupt firms are matched with those characteristics of the non-bankrupt firms. The idea of this approach is to detect the origin

[5] Definition of SME according to the European Commission, retrieved from: http://ec.europa.eu/growth/smes/business-friendly-environment/sme-definition_nl

of the difference between the bankrupt and non-bankrupt groups, and to compare homogenous groups as much as possible. However, a reason why matched sampling is not optimal for all researches is that over selection bias may occur. This implies that the sample of non-bankrupt firms is selected in such a way that randomization of the group is not accomplished. In that case, the sample of non-bankrupt firms is not representative for the population of non-bankrupt firms. To prevent this bias, random sampling if often used. This approach implies that all firms of the population have an equal chance of being elected in the sample group (Marshall, 1996). The pitfall of random sampling is random sampling error, which means that components of the sample deviate from the population. This error may be solved by including a large sample size (Marshall, 1996). Therefore, I use all available non-bankrupt firms that contain all relevant variables to calculate the ratios to represent the non-bankrupt group.

Data is gathered for the years 2010-2015. The years 2014 and 2015 are available via the usual Orbis database, older data (2010-2013) is gathered from a remote desktop connection. Orbis (2018) divides active firms into 6 subclasses (*active, active rescue plan, active default of payment, active insolvency proceedings, active reorganization, and active dormant*), and inactive companies in 8 subclasses *(in liquidation, bankruptcy, dissolved merger or take-over, dissolved demerger, dissolved liquidation, dissolved bankruptcy, dissolved, inactive no precision*). Orbis uses the 'IP data stamp' issued by their information providers to classify firms (Orbis, 2018). This thesis considers firms labeled in database Orbis as 'bankruptcy', to represent bankrupt firms, and 'active' firms are selected to present active firms. My selection contrasts with research of Altman et al. (2017), since they additionally include firms labeled as 'active insolvency proceedings' to represent bankrupt firms due to data restrictions. I try to avoid ambiguity and therefore only include firms labeled as 'bankrupt' to represent the bankrupt firms. Due to the strict restriction that all firms need to contain all variables to calculate the ratios of the Altman Z-score model, data is limited. All parameters of the Altman Z-score are a requisite to calculate the particular score. In case of one missing variable, ratios cannot be calculated, and therefore the score is not representative. A sample of 93 bankrupt firms that provide their last statements in between 2010 and 2015 is collected and extended by all available non-bankrupt firms.

Due to data limitations, only the last statement plus two statements before bankruptcy are included in the database. Noteworthy, only statements from most recent years are included in the dataset, because research shows that macro-economic factors can have major influence on the ratios and disturb the cut-off point between bankrupt and non-bankrupt (Begley et al., 1996;

Grice and Ingram, 2001; Mselmi, Lahiani, & Hamza, 2017). So, by additionally including, for example, the financial crisis of 2008, the ratios may be majorly disturbed, and therefore, the model may predict distorted scores.

According to CBS[6], in 2016, more than half of the Dutch bankruptcies for SME's regard proprietorships. Additionally, 36% of bankruptcies are small sized organizations, while 1,6% of bankruptcies are medium sized firms. Bankruptcies peaked in 2013, and from then on, they start decreasing. Comparing 2016 to 2015, bankruptcies decreased for almost all firm sizes and firm ages. Last years, only companies that exists three to five years face increasing bankruptcy rates.

Table 2 shows the number of included firms per year for the bankrupt and non-bankrupt sample. All firms included in Table 2 contain all relevant variables to calculate the ratios, and test for the Altman Z-score model one and two statements prior to bankruptcy. The Table already excludes outliers. Data is checked for outliers by plotting the data and deleting surprising data points.

Table 2 Total sample per year

Year	**Bankrupt sample**	**Non-bankrupt sample**
2010	15	648
2011	23	424
2012	24	482
2013	5	676
2014	17	100
2015	6	209
Total Sample	**93**	**2.539**

The non-bankrupt sample is easier to collect since more firms continue their operations than go bankrupt. The Table above shows that the total sample consist of 2.632 firms and contains 3,53% bankrupt firms. My total sample includes in percentage slightly more bankrupt firms than 'the real world' does. This is due to data limitations for the non-bankrupt sample. It is important to include unique numbers, and again all variables for all firms need to be available in order to calculate the ratios and the Z-score. I also checked not to consider the same firm multiple times and made sure to include firms of different ages.

[6] Dutch organization for statistics (Centraal Bureau voor Statistiek)

3.3 Research hypotheses and methodology

The Altman Z-score model is still an intensively investigated, and well-known model. However, not all countries, asset classes and firm sizes have already been examined. Prior research mainly focuses on publicly traded firms, specific countries like France and Greece, and countries in Asia[7]. To my knowledge, Dutch SMEs have not been examined so far. Therefore, the next hypothesis will be tested: *H_1: A positive relationship exists between the revised Altman Z-score model and predicting bankruptcy for Dutch SMEs.* To calculate the Altman Z-score for Dutch SMEs, the revised equation (2) is used, since my dataset of Dutch SMEs considers book values instead of market values. The equation concerns MDA, which is a generally used technique for predicting bankruptcies (Altman 1968; 1983; 2000; Altman et al. 2017; Bilderbeek 1979; Eisenbeis, 1977; Grice & Ingram, 2001). The technique allows researchers to study the differences between bankrupt and non-bankrupt firms with respect to the included ratios and is based on ordinary least squares (OLS). The ratios are transformed into a single discriminant score Z, which is used to classify the firms (Gu, 2002). The model is itemized as follow:

$$Z - score = D_1X_1 + D_2X_2 + D_nX_n \tag{3}$$

Where: $D_{1}, D_{2,\ldots\ldots} D_{n} =$ *discriminant coefficients*

$X_{2}, X_{2, \ldots} X_{n} =$ *financial ratios*

The independent financial ratios, measured on a ratio scale, discriminate the dependent status of the firm, and so the groups of interest (bankrupt / non-bankrupt). Note that the MDA starts with assumptions, which are summarized in Table 3 (Balcaen and Ooghe, 2006). In case of violation of the assumptions, the results should not be generalized (Balcaen and Ooghe, 2006). Multicollinearity is an important factor since -among other factors, the amount of assets may predict bankruptcy (Beaver, McNichols, & Rhie 2005).

[7] Investigated by: Altman et al, 1968, 1983, 2000, 2017; Begley et al., 1996; Chiaramonte, Poli, & Zhou (2016); Gerantonis, Vergos, & Christopoulos, 2009; Grice and Ingram, 2001; Mselmi et al, 2017; Rezende et al., 2017; Tian, and Yu., 2017

Table 3 Assumptions of the MDA

Assumption MDA	**Explanation**
Dichotomous dataset	The two groups (bankrupt and non-bankrupt) should be non-overlapping and identifiable.
Equal variance-co-variance matrices	Equal variance-co-variance matrices between the bankrupt and non-bankrupt group.
Multivariate normally distributed independent variables	Tests for significance may be biased in case of no normal distribution.
Multicollinearity	In case of multicollinearity, part of the relationship between the included financial ratios and bankruptcy is due to the mutual correlations.
Homoscedasticity	The error term between bankrupt and non-bankrupt firms need to be the same across all values.
Linearity	A linear relationship is a requirement because in case of non-linearity it is impossible to calculate a score and to attach weights to the coefficients.

Next, many researchers point out that two assumptions of the MDA technique; multivariate normally distributed independent variables and equal variance-co-variance matrices, are often violated (e.g. Barnes, 1982; Karels and Prakash, 1987; Mcleay and Omar, 2000). Considering these violations, researchers additionally perform LRA. This technique uses a model non-linear in parameters to estimate the probability of bankruptcy to happen. The analysis is mostly used for large sample sizes, while the MDA is generally used for small sample sizes like the original sample of Altman (1968). Logit models assume a logistic distribution. That, in contrast to probit models, which assume a cumulative normal distribution. A benefit of this technique is that it does not start with the strict assumptions that the MDA does (Table 3). The assumptions that the LRA does not start with, compared to the MDA, is the necessity for equal variance-co-variance matrices, linearity, homoscedasticity, and the multivariate normally distributed independent variable (Premachandra, Bhabra, & Sueyoshi 2009). Additionally, another benefit of the LRA is that the model allows to evaluate the financial ratios of the Z-score model independently, by reporting the statistical significance of the ratios individually (Premachandra, Bhabra, & Sueyoshi 2009). The LRA helps to explain the relationship between the two groups, and the different independent ratios of the Altman Z-score model. The logistic regression model relies on maximum likelihood estimation (MLE), which evaluates observations of individual firms, instead of the total group of firms. Considering the above mentioned, the next hypothesis is tested by this type of analysis: *H_2: Ratios of the Altman Z-score model help to predict bankruptcies.* The dependent variable is transformed into a discrete, binary dependent variable (Bernoulli distribution).

Y=1 if the firm is not bankrupt, Y=0 is the firm is bankrupt. The LRA is created and constructs equation (4):

$$\ln \frac{P}{1-P} = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 \quad (4)$$

Where: α = *intercept*

X_1 = *Working capital/ Total assets*

X_2 = *Retained earnings/ Total assets*

X_3 = *Earnings before interest and taxes/ Total assets*

X_4 = *Value of equity/ Book value of total liabilities*

X_5 = *Sales/ Total assets*

Furthermore, Mselmi, Lahiani, & Hamza (2017) show that for French SMEs the model predicts two financial statements prior to bankruptcy most optimal. Therefore, the next hypothesis will be tested: *H_3: The optimal time horizon for predicting bankruptcy is two statements prior to bankruptcy.*

To develop the ZL-score model, further understanding of the MDA is needed. The analysis helps to determine the relative importance of several financial ratios as they contribute to the prediction of bankruptcies. The aim of the new model, based on Dutch SMEs, is to find a predictive equation for classifying individual Dutch SMEs into groups (bankrupt / non-bankrupt) with the highest accuracy rates possible. First, a selection of ratios to include must be determined. That selection is based on t-values, Wilks' Lambda, and the LRA results. Additionally, the adjusted R-square will be compared by changing the composition of the financial ratios. The adjusted R-square value only increases if the included parameter improves the model more than can be expected by chance. In case of pour quality of the parameter, the term decreases.

A type of bias that may occur when predicting bankruptcies with the MDA is search bias. This type of bias arises when investigators are trying every subset of variables or possible outcomes and evaluate which one works best for the selected sample (Frank, Massy, & Morrison, 1965).

Not surprisingly, the model will work for the used dataset, but the model cannot be generalized. An approach to eliminate this bias is to split the sample into training and testing (Frank, Massy & Morrison, 1965). The aim of the training subsample is to optimally predict bankruptcies for Dutch SMEs, and the test sample is used to evaluate the obtained model and is not used to achieve the model. In imitation of Mselmi, Lahiani, & Hamza, 2017, who investigate bankruptcies for French SMEs, 70% of the sample is used as the training sample, and 30% is used as the test sample.

When using the MDA, it is important that the number K of discriminant functions needs to be both less than the number of groups (bankrupt and non-bankrupt), and less than the number of included ratios (maximum of five) J: K ≤ min {G-1, J} (Walde, 2018). Note that assumptions of the MDA (discussed in Table 3) are still applicable for the ZL-score model. This implies, I assume: dichotomous dataset, equal variance-co-variance matrices, multivariate normally distributed independent variables, multicollinearity, homoscedasticity and linearity. A new discriminant model can be built by STATA, and for each observation, the classification rate is optimized by Excel Solver. To compare the results of the Altman Z-score, the LRA and the ZL-score, new classification accuracy is provided.

4. Empirical Results

In this chapter I discuss the empirical findings. First, I discuss the descriptive statistics, and the results of the revised Altman Z-score model (1983) by using my dataset of Dutch SMEs. Then, I discuss results of the LRA, and lastly, I discuss the new ZL-score model. The ZL-score model is a best fitted model to predict bankruptcies, especially for, and based on Dutch SMEs.

4.1 Descriptive statistics

The descriptive statistics and t-value of the bankrupt and non-bankrupt samples are shown in Table 4 and 5 below.

Table 4 Descriptive statistics one statement before bankruptcy

Ratio	Working capital / Total assets	Retained earnings / Total assets	EBIT / Total assets	Equity/ Liabilities	Sales / Total assets	**Z-score**
Non-bankrupt firms						
Mean	21,00%	34,26%	6,80%	138,04%	2,20 ×	3,43
Std. Dev.	0,50	0,76	0,27	2,94	1,88	2,45
Min.	-8,16	-14,26	-3,66	-6,67	-0,12	-12,37
Max.	4,02	3,25	1,88	43,49	19,34	18,51
Bankrupt firms						
Mean	1,45%	-40,43%	-5,53%	41,24%	1,61 ×	1,27
Std. Dev.	1,76	1,44	0,38	1,16	1,82	2,40
Min.	-8,50	-8,55	-1,21	-1,02	0,01	-6,99
Max.	13,50	0,87	2.23	6,46	15,14	9,73
t-value	-3,11***	-8,95***	-4,21***	-3,19***	2,96***	-8,34***

Note: Working capital/ Total assets; Retained earnings/ Total assets; Earnings before interest and taxes/ Total assets, and Equity/ Liabilities are percentages. Sales/ Total assets considers a multiplication factor (×). *** $p \leq 0.01$, ** $p \leq 0.05$, * $p \leq 0.$

The Table shows that on average all the ratios are larger for non-bankrupt firms, which is in line with expectations. Altman (2000) publishes similar results as all ratios of the non-bankrupt firms are on average higher than the bankrupt firms. The average of ‘Equity / Liabilities’ for non-bankrupt firms seem high, but shows that, on average, the assets can decline in value before the liabilities exceed the assets. Note that the maximum of ‘Equity / Liabilities’ of non-bankrupt firms seem high due to the diverging amount of equities and liabilities. I checked for outliers, by plotting my dataset and deleting surprising data points. However, firms with a high ratio of ‘Equity/ Liabilities’ are not necessarily outliers. Altman results in his research a mean of 247,7% for the ratio.

The financial ratios of the bankrupt firms are as expected, except for the ratio ‘Working capital / Total assets’. The average of that ratio is positive, and I expected the average of working capital to be negative one statement before bankruptcy, since the average current liabilities are most likely to exceed the average current liabilities just before bankruptcy, resulting in average negative working capital. However, the mean of ‘Retained earnings / Total assets’ for bankrupt firms is as expected and negative, because retained earnings are expected to be negative. The firm probably faces some financial distress one year before bankruptcy, which may lead to greater cumulative losses than cumulative profits. The amount of retained earnings is on a randomly bases checked by hand and turns out to be correct The average standard deviation of ‘Equity/ Liabilities’ is quite high for the non-bankrupt group, which implies the ratio is volatile. Not surprisingly, the mean average of ‘Earnings before interest and taxes / Total assets’ for bankrupt firms is negative, because the firms possibly already face some downturn, and therefore negative earnings before interest and taxes.

Additionally, the t-values of differences in means tests between bankrupt and non-bankrupt firms are presented. All financial ratios are different between bankrupt and non-bankrupt.

Finally, the averages of the Z-scores are as expected for the non-bankrupt firms, since the mean score of the non-bankrupt sample is above the cut-off point for non-bankrupt firms (2,99), and the mean for the bankrupt sample is below that point (1,81). This implies that the average firms would be classified correctly according to the Altman Z-score model. Moreover, the standard deviations, minima and maxima of the scores are as expected.

Table 5 Descriptive statistics two statements before bankruptcy

Ratio	Working capital / Total assets	Retained earnings / Total assets	EBIT / Total assets	Equity/ Liabilities	Sales / Total assets	**Z-score**
Non-bankrupt firms						
Mean	31,10%	28,30%	7,51%	107,15%	2,16 ×	3,10
Std. Dev.	0,65	0,70	0,24	1,51	1,76	2,15
Min.	-9,61	-13,63	-2,31	-3,19	0,05	-9,86
Max.	2,26	10,09	2,22	19,56	16,59	14,88
Bankrupt firms						
Mean	-0,62%	-0,16%	-0,38%	42,09%	1,61 ×	1,53
Std. Dev.	0,92	1,19	0,33	0,92	1,31	2,51
Min.	-6,23	-7,82	-1,33	-1,12	-2,51	-11,36
Max.	2,78	0,82	1,01	4,87	6,16	6,64
t-value	-4,52***	-5,78***	-4,43***	-2,45**	-2,99***	6,89***

Note: Working capital/ Total assets; Retained earnings/ Total assets; Earnings before interest and taxes/ Total assets, and Equity/ Liabilities are percentages. Sales/ Total assets considers a multiplication factor (×). *** $p \leq 0.01$, ** $p \leq 0.05$, * $p \leq 0$.

Table 5 presents that two statements prior to bankruptcy all means of bankrupt ratios are below the means of non-bankrupt ratios. Moreover, in contrast to one year prior to bankruptcy, the mean of 'Working capital / Total assets' for bankrupt firms is negative, as I expected. Evidently, average current liabilities exceed average current liabilities two statements before bankruptcy, resulting in average negative working capital. Again, the means of the Z-scores are as expected for both bankrupt and non-bankrupt firms, since the mean scores are above and below the cut-off points. The average Z-score for bankrupt firms increases, compared to one statement prior to bankruptcy, implying the accuracy classification on average decreases two statements prior.

Finally, t-values of the differences in means tests between bankrupt and non-bankrupt firms are presented. Again, and not surprisingly, all ratios differ from each other.

4.2 Revised Z-score model results

This paragraph discusses the empirical results when estimating a revised Altman Z-score model (1983), using my dataset for Dutch SMEs. Studies about the Altman Z-score model are mainly based on large enterprises. Financial ratios of firms of the investigated firm size may deviate from the ratios of Dutch SMEs as that size of firms is dealing with significantly different ratios than big companies (Altman and Sabato, 2007). Even though this study includes, at this point, the same ratios as included in the revised Altman Z-score model (1983), results of the grouping ratios, and the two groups may deviate from the original research, because the ratios may deviate. To test for this, the value of Wilks' Lambda is calculated. That test statistic is regularly used in multivariate studies and measures the contribution of the included ratios to the discriminant model (Walde, 2018). Small proportion variance implies that a small proportion of the total variance is explained by the grouping financial ratios (Walde, 2018). This results in no effect from the grouping ratios, and the two groups: bankrupt and non-bankrupt (Walde, 2018). Discrimination is achieved by maximizing the between-group variance, compared to the within-group variance (Walde, 2018). The value of Wilks' Lambda (^) is calculated as follow:

$$\text{Wilks}'\text{Lambda}\ (\wedge) = \frac{SS_w}{SS_b + SS_w} \tag{5}$$

Where: SS_b = *explained deviation, calculated by the sum of the squared deviations between-groups*

SS_w = *remaining deviation, calculated by the sum of the squared deviations within-groups* (Walde, 2018)

The value of Wilks' Lambda lies in between 0 and 1. The results of the Wilks' Lambda (^) test show that the statistic for equality of two group means is *0,6130 (Prob>F = 0.0000),* implying that 61,30% of the variance is not explained from the grouping ratios, and the groups. At first sight, the number seems high. However, it shows that an effect exists between the grouping ratios and the groups, and that they have different mean values. This is consistent with t-value results of Table 4 and 5. Furthermore, given the p-value *(0,0000),* I conclude that the model is highly significant at a 5% significance level of the ratios. Thus, for my dataset for Dutch SMEs a significant proportion of the variance is explained by the independent ratios. Unfortunately, Altman (1968) does not report the value of Wilks' Lambda for the original sample.

The revised Z-score model is tested on my dataset for Dutch SMEs. Table 7 shows classified results of my dataset by using equation (2), which computes the revised Z-score. The results show that one year prior to bankruptcy, 70,97% of bankrupt firms is classified correctly, and 55,30% is classified correctly as non-bankrupt. The result of Altman (1983) shows higher classification accuracy, as 90,9% of bankrupt firms is classified correctly one year prior. Moreover, Table 7 shows a decrease in correctly classifying firms two years prior to bankruptcy, compared to one year prior for both bankrupt (56,99%) as non-bankrupt firms (49,15%). Additionally, one year prior to bankruptcy, 13,98% of bankrupt firms is classified as non-bankrupt, which is wrong. Two years prior 19,35% of firms that go bankrupt in two years is classified as non-bankrupt. The percentage of wrong classifications increases for both bankrupt and non-bankrupt firms two years prior.

In general, the percentage of correct classifications is slightly lower than results of other investigators who use the Altman Z-score model (Altman, 1983; 2000; Altman et al., 2017; Gerantonis et al., 2009; Grice & Ingram, 2001; Tian and Yu, 2017). However, they mainly examine the original Altman Z-score model and focus on large public enterprises which are featured by high total assets, turnovers and shareholders capital.

Table 7 Classification results revised Z-score

		Bankrupt firms			
	# firms	*Correctly classified*	*Grey area*	*% correct*	*% incorrect*
One year prior	93	66	16	70,97	13,98
Two years prior	93	53	22	56,99	19,35
		Non-bankrupt firms			
	# firms	*Correctly classified*	*Grey area*	*% correct*	*% incorrect*
One year prior	2.539	1.404	588	55,30	21,54
Two years prior	2.539	1.248	656	49,15	25,00

Note: Z-score <1,81 represents bankrupt firms, and Z-score>2,99 represent financial soundness. The scores in between 1,81 and 2,99 are considered as the grey area, which is the zone of ignorance. The bankrupt sample consists of 93 firms and the non-bankrupt sample comprises 2.539 firms. Results of one year prior, and two years prior to bankruptcy are presented.

4.3 Logistic Regression results

For comparison purposes and because researchers point out that assumptions regarding the MDA are often violated, the LRA findings are presented and discussed (e.g. Barnes, 1982; Karels and Prakash, 1987; Mcleay and Omar, 2000). I transformed the dependent variable into a discrete, binary, dependent variable. Therefore, it may explain the relationship between the two groups, and the different independent ratios of the Altman Z-score model.

The LRA is majorly influenced by the proportionate number of data points regarding the bankrupt and non-bankrupt groups (Hauser and Booth, 2011). My dataset is highly tilted towards non-bankrupt firms, as only 3,53% of my total sample comprises bankrupt firms, and 96,47% includes non-bankrupt firms. Thus, non-bankrupt firms appear more often in my dataset and in the 'real world', making it in general easier to predict non-bankrupt firms. Taking that into account, I report two models regarding the LRA: one of the total sample, and one of two equal groups. Following Mselmi, Lahiani, & Hamza (2017), and Laitinen (2000), the equal groups consist of 70% of the total bankrupt sample, resulting in 65 bankrupt firms, and 65 non-bankrupt firms. The firms are randomly selected from the total sample via the random selection generator of STATA. The approach of random selecting is repeated 10 times. Note that the chance of a bankrupt firm to be selected twice or more times is quite high, since 65 of 93 firms are selected every approach.

Table 8 shows results of the LRA by restricting the independent variables to the five financial ratios of the Altman Z-score model. The numbers reported are based on the last available statements prior to bankruptcy. I report odds ratios and marginal effects. Essentially, odds ratios reflect changes in odds for every increment on the predictive variables by calculating the ratio of the numbers of times that bankruptcy happens to the number of times that bankruptcy does not happen. The odds ratios help to investigate the effects of the financial ratios on the relationship between the status of firm (bankrupt / non-bankrupt). My independent ratios are continuous, meaning STATA computes the odds ratios in terms of one-unit change in the included ratios. An odds ratio of exactly one implies that the relationship between the ratio and status of firm (bankrupt / non-bankrupt) is not affected. A ratio less than one indicates a negative relationship between the ratio and status of the firm. Thus, an outcome of 0,5 implies that an included ratio negatively contributes the status of firms, meaning that if the ratio increases, the odds of being a bankrupt firm increases. A parameter bigger than one implies a positive relationship between the ratio and the status of the firm. This implies that if the odds ratio is

2,0, and a financial ratio increases, the odds of non-bankrupt firms increases. Under the logit model, the odds ratio comprises the next simplified formula (e.g. for ratio X_1):

$$Odds\ ratio = \frac{\exp(\beta_0+\beta_1+\beta_2+\beta_3+\beta_4+\beta_5 \times \boldsymbol{x})}{\exp(\beta_0+\beta_2+\beta_3+\beta_4+\beta_5 \times \boldsymbol{x})} = \exp(\beta_1) \qquad (6)$$

Odds ratios are hard to interpret and understand. Therefore, instead of odds ratios, economist generally prefer calculating marginal effects. Another reason for using marginal effects is that doubling a probability of 0,00001 and interpreting the odds ratio of that probability may not be interesting for this thesis. Marginal effects indicate the expected change in the status of firms (bankrupt / non-bankrupt) as a function of change in a certain included ratio, while keeping all other covariates constant. Interpreting the marginal effects is a requirement in order to interpret the effect of my ratios on the status of firms. The next simplified formula is used to calculate the marginal effects (e.g. for ratio X_1):

$$Marginal\ effects = \frac{\delta\, \mathrm{P(Y=1)}}{\delta\, x_1} = P(y=1) \times P\,(y=0) \times \beta_1 \qquad (7)$$

Note that the outcomes of the marginal effects are multiplied by the standard deviations of the independent ratios, since economists prefer interpreting changes in standard deviations instead of units. Therefore, the marginal effects help to understand the increase or decrease in the predicted probability of non-bankruptcy by changing the independent ratios by one standard deviation. The reported odds ratios and marginal effects for the equal groups is an average of the ten random selected samples. Note that the significance level for the odds ratios equals the significance level for the marginal effects.

Table 8 Logistic regression results

Altman Z-score ratios	Total sample		Equal groups	
	Odds ratio	Marginal effects	Odds ratio	Marginal effects
Working capital / Total assets	1,34 (0,142)**	0,55%	1,29 (0,870)*	2,84%
Retained earnings / Total assets	1,45 (0,110)***	9,44%	2,98 (0,787)**	2,32%
Earnings before interest and taxes / Total assets	2,60 (0,305)***	0,84%	3,99 (0.764)**	5,03%
Value of equity / Book value of total liabilities	1,32 (0,106)**	22,50%	1,54 (0.249)*	10,45%
Sales / Total assets	1,49 (0,079)***	2,40%	1,44 (0,132)***	1,01%
Constant	2,27 (0,171)***		0,35 (0,462)***	
N	2.632		130	
Log likelihood	-357,67		-53,98	

*Note: logistic regression odds ratios and marginal effects with standard errors in parentheses. *** $p \leq 0.01$, ** $p \leq 0.05$, * $p \leq 0$. Dependent variable is status of firm (1 = not bankrupt, 0 = bankrupt). Independent variables are the ratios included in the Altman Z-score model. The total sample consists of 2.632 firms, and the equal groups sample of 65 bankrupt and 65 non-bankrupt firms. LRA for the equal groups is repeated 10 times, averages are reported*

Table 8 shows that all included parameters are significant for as well as the total sample as for the equal groups sample. Evidently, all financial ratios significantly contribute to the prediction of bankruptcies for Dutch SMEs in this particular model. Parameters ‘Working capital/ Total assets’ and ‘Value of equity/ Book value of total liabilities’ are individually least significant. However, they contribute significantly to the total model, and therefore may be added to a new model. This result is in contrast to the research of Altman (1968), since he investigates the ratio ‘Sales / Total assets’ as least significant.

Odds ratios show how many times more likely the odds of finding an exposure in a bankrupt firm is compared to finding the exposure in a non-bankrupt firm. My results show that all odds ratios are above 1, implying there is a higher chance of non-bankruptcy with exposure to the financial ratios. Put differently, all individual ratios positively contribute to financial healthy firms. The ratio 'EBIT / Total assets' is most important in this relationship as the odds ratio is the highest for as well as the total sample as for the equal groups sample. The odds of being a non-bankrupt firm for the ratio 'EBIT / Total assets' are respectively 2,60 and 3,99 for the total sample and for the average of the equal groups sample. This implies that the ratio 'EBIT / Total assets' is more than 3 times higher for non-bankrupt firms than for bankrupt firms. Additionally, the ratio 'Retained earnings / Total assets' is important in this relationship, since the odds of being a non-bankrupt firm are respectively 1,45 and 2,98 for the total sample and for the average of the equal groups sample. Non-bankrupt firms for the total sample face ratio 'Working capital / Total assets' about 1,34 times higher than bankrupt firms.

With the marginal effects, I try to explain the difference between the means of bankrupt and non-bankrupt firms. The total sample presents a marginal effect of 0,55% for the ratio 'Working capital / Total assets', meaning that for one standard deviation increase, the chance of being a non-bankrupt firm increases by 0,55%. Additionally, the total sample shows a marginal effect of 22,50% for the ratio 'Value of equity / Book value of total liabilities'. So, for one standard deviation increase in that ratio, the predicted probability of being a non-bankrupt firm increases by 22,50%. One standard deviation increase in 'EBIT / Total assets' increases on average the predicted probability of a non-bankrupt firm by 0,84%. Additionally, one unit increase in 'Retained earnings / Total assets' increases on average the predicted probability of a non-bankrupt firm by 9,44%. Finally, one unit increase in 'Sales / Total assets' increases on average the predicted probability of a non-bankrupt firm by 2,40%. Not surprisingly, the equal groups present different marginal effects. Notably, the odds ratios between the total sample and the equal groups sample does not deviate extremely, but the marginal effects deviate tremendously between the two different samples. The results show that sample selection is crucial when conducting LRA. The classification results, which are discussed next, help to obtain insights into which sample to select to best predict bankruptcies for Dutch SMEs.

I cannot compare the log likelihood of the models, because I cannot compare different samples with different sample size.

Then, I classified the firms again, based on the LRA. For the purpose of this study, following Hauser and Booth (2011), I classified a prediction as correct in case the probability of bankruptcy for bankrupt firms is greater than 50%. Random selection for equal groups (65 firms per group) is repeated 10 times. Reported estimates for the equal groups is an average of the 10 random selected samples. Figure 6.4 in the Appendix presents individual results per random sample for the equal groups.

Table 9 evidences that in case the probability of bankruptcy for bankrupt firms for the total sample is greater than 50%, one year prior to bankruptcy, 16,67% of bankrupt firms is classified correctly, and 99,80% of non-bankrupt firms is classified correctly. This implies that 83,33% of bankrupt firms is classified as non-bankrupt, and thus incorrect. The results show that only 0,20% of the non-bankrupt firms is classified as bankrupt. Additionally, the Table shows that for the equal group case, one year prior to bankruptcy, on average 71,85% of bankrupt firms is classified correctly, and on average 73,54% of non-bankrupt firms is classified correctly. This implies that for the equal groups sample, 28,15% of bankrupt firms is classified as non-bankrupt, and 26,46% of non-bankrupt firms is classified as bankrupt one statement prior. Two statements prior to bankruptcy, the correct classification for as well as bankrupt firms as non-bankrupt firms decreases for the total sample. The correct classification for the equal groups sample decreases on average for bankrupt firms (70,31%), and increases on average for non-bankrupt firms (74,15%).

To summarize the empirical LRA results, the total sample is highly tilted towards non-bankrupt firms in classifying correctly, and the accuracy results for the two equal groups are approximately equal for bankrupt and non-bankrupt firms. However, the LRA accuracy results for the equal groups are slightly higher for non-bankrupt firms, compared to bankrupt firms. Comparing the LRA with the revised Altman Z-score results for the total sample, results present amelioration in correctly classifying non-bankrupt firms, and a deterioration in correctly classifying bankrupt firms. The LRA for the equal groups sample classifies both bankrupt and non-bankrupt firms less accurate, compared to results of the Altman Z-score model.

Table 9 Classification results of the Logistic Regression Analysis for the total sample and for two equal groups

Actual	Classification one statement prior to bankruptcy		Classification two statements prior to bankruptcy	
Total sample				
	Bankrupt	Non-bankrupt	Bankrupt	Non-bankrupt
Bankrupt	16,67%	83,33%	15,33%	84,67%
Non-Bankrupt	0,20%	99,80%	3,40%	96,60%
Equal groups sample				
	Bankrupt	Non-bankrupt	Bankrupt	Non-bankrupt
Bankrupt	71,85%	28,15%	70,31%	29,69%
Non-Bankrupt	26,46%	73,54%	25,85%	74,15%

Note: This table reports the percentage of correct and non-correct classified firms one and two statements prior to bankruptcy using LRA. 50% is the percentage correct prediction. E.g. if the probability of going bankrupt is greater than 50%, the firm is classified as correct. The total sample consists of 2.632 firms, and classification for the equal groups is an average of 10 randomly drawn samples of 65 bankrupt firms, and 65 non-bankrupt firms.

4.4 ZL-score results

Taking into account the findings of the previous paragraphs, I build a new model for Dutch SMEs, called the ZL-score model, based on the work of Altman (Altman 1968; 1983; 2000). The model is built with STATA and optimized with Microsoft Excel. The aim is to best predict in which group firms belong. By developing a bankruptcy model, an important trade-off exists between predicting future bankrupt firms correctly, and the accuracy of predicting future financial solvent firms. By increasing the accurate predictability of bankrupt firms, the amount of correctly classified non-bankrupt firms decreases.

Investigators point out that the procedure of selecting variables is difficult since there is no one best method for examining the relative importance of the variables (Altman, 2000). The t-test results show that the included financial ratios differ from each other. Moreover, LRA results show that all ratios significantly contribute to the prediction of bankruptcies for Dutch SMEs. Thus, all five parameters contribute to the complete model. This is due to the unique relationship of the parameter with the other ratios of the ZL-score model. Therefore, the five financial ratios of the Altman Z-score model (1983) are included in the ZL-score model.

To prevent search bias, the total dataset is randomly split into two subsamples: training and testing. The random selection is obtained by the random sample generator of STATA. Following Mselmi, Lahiani, & Hamza (2017) I split the total sample and use 70% of the sample as training sample, and 30% to test the model. Thus, the training sample consists of 1.842 firms, of which 65 bankrupt firms. The test sample comprises the remaining firms: 762 non-bankrupt firms and 28 bankrupt firms. The aim of the training subsample is to optimally predict bankruptcies for Dutch SMEs, and the test sample is used to evaluate the obtained model and is not used to achieve the model.

Following Thai et al. (2014), who revisit the Altman Z-score model for listed companies in Bursa Malaysia, unstandardized canonical discriminant function coefficients are calculated to recalibrate the model. Just as before, the new ZL-score equation does not include a constant term, and assumptions regarding the MDA are still applicable. The coefficients of the parameters of the ZL-score model are:

$$ZL - score = 0.6394X_1 + 1.3768X_2 + 3.1134X_3 + 0.414X_4 + 0.8126X_5 \qquad (8)$$

Where: X_1 = *Working capital/ Total assets*

X_2 = *Retained earnings/ Total assets*

X_3 = *Earnings before interest and taxes/ Total assets*

X_4 = *Value of equity/ Total liabilities*

X_5 = *Sales/ Total assets*

Comparing equation (8) with the revised equation (2), T-test results show that the discriminant coefficients are significantly the same for 'EBIT/ Total assets' and 'Value of equity/ Total liabilities', and significantly lower for the three other ratios. Group ZL-scores are statistically significant different from each other (bankrupt/ non-bankrupt). Paragraph 4.2 discusses the value of Wilks' Lambda of the model. Note that the included ratios do not deviate from the revised research of Altman (1983). Thereby, the value of Wilks' Lambda for the Altman Z-score equals the value for the ZL-score.

The new cut-off scores are found by optimizing the accuracy rate for bankrupt firms (true positive classifications) and thereby considering the false negative and false positive classifications. This means, I also analyze the number of wrong classified bankrupt and non-bankrupt firms (see Figure 6.3 of the Appendix). Bankrupt firms classified as non-bankrupt firms or visa versa results in major problems in practice, and therefore it is important to take into account. Note that the bandwidth of the grey area equals the original interval (1,18). The ZL-score labels firms with a score higher than or equal to 2,58 as non-bankrupt, and firms with a score lower than or equal to 1,40 are labeled as bankrupt (Figure 2). This implies that the new grey area lies in between 1,40 and 2,58. Figure 6.1 of the Appendix shows the trade-off for the cut-off scores without a grey zone graphically, and Figure 6.2 of the Appendix shows classification results with grey zones.

High risk of bankruptcy (insolvent firms) ZL ≤ 1,40	Results are uncertain Grey area 1,40 < ZL > 2,58	Low risk of bankruptcy (solvent firms) ZL ≥ 2,58

Figure 2

Table 10 presents descriptive statistics of the ZL-score for the total sample one and two statements prior to bankruptcy. This implies that data of the training and test sample is merged. Note that the descriptive statistics of the individual ratios are not reported, since the same ratios are included as in the Altman model. The mean for bankrupt firms is for both one and two statements prior to bankruptcy below the cut-off score of 1,40, and the mean for non-bankrupt firms is both statements above the cut-off score of 2,58. Note that the ZL-means are lower than the Z-means, since the discriminant coefficients for ratio's 'Working capital/ Total assets', 'Retained earnings/ Total assets' and 'Sales/ Total assets' are significantly lower. The lower coefficients additionally result in lower minima and lower maxima of both bankrupt and non-bankrupt firms. Naturally, the minima and maxima of the bankrupt and non-bankrupt firms do not deviate tremendously from those of the revised Altman Z-score model since the new parameters are only slightly different.

Table 10 Descriptive statistics of the ZL-score

ZL-score	**One statement prior**		**Two statements prior**	
	Bankrupt	Non-bankrupt	Bankrupt	Non-bankrupt
Mean	0,76	3,16	1,14	2,86
Std. Dev.	2,79	2,38	2,92	2,16
Min.	-8,62	-15,22	-13,38	-16,75
Max.	8,62	15,68	5,88	15,42

As expected, the Z-score and the new ZL-score are highly correlated. The correlation between the Z-score and ZL-score of bankrupt firms and non-bankrupt firms is respectively 0,9833, and 0,9963. The correlation between the Z-score and the ZL-score of the total sample is 0,9961. The correlations imply that there is a difference in terms of linear relationship between the two scores, although they are highly linked to each other.

Table 11 reports the classified results of my training and test sample by using the ZL-score. The results of the training sample present that one statement prior to bankruptcy, the ZL-score predicts 81,54% of firms that go bankrupt next year correctly. Moreover, 74,96% of firms that continue their operations next year, are classified as so. Additionally, the results of the training sample present that two statements prior to bankruptcy, the ZL-score predicts 73,85% of firms that go bankrupt in two years correctly. 72,32% of firms that continue their operations in two years, is classified as so. The test sample, which is used to evaluate the obtained model, presents that 82,14% of bankrupt firms is classified correctly one year prior. Additionally, 73,49% of non-bankrupt firms is classified as non-bankrupt, which is correct. The classification accuracy for the test sample decreases for both bankrupt (78,57%) and non-bankrupt firms, compared to one statement prior to bankruptcy (70,34%). Note that the percentage wrong classifications decreases, compared to the Altman Z-score model for both bankrupt and non-bankrupt firms.

Table 11 Classification results ZL-score

Bankrupt firms training sample					
	# firms	*Correctly classified*	*Grey area*	*% correct*	*% incorrect*
One year prior	65	53	6	81,54	9,23
Two years prior	65	48	10	73,85	10,77
Non-bankrupt firms training sample					
	# firms	*Correctly classified*	*Grey area*	*% correct*	*% incorrect*
One year prior	1.777	1.332	188	74,96	14,46
Two years prior	1.777	1.285	236	72,32	14,41
Bankrupt firms test sample					
	# firms	*Correctly classified*	*Grey area*	*% correct*	*% incorrect*
One year prior	28	23	2	82,14	10,71
Two years prior	28	22	3	78,57	10,71
Non-bankrupt firms test sample					
	# firms	*Correctly classified*	*Grey area*	*% correct*	*% incorrect*
One year prior	762	560	104	73,49	12,86
Two years prior	762	536	111	70,34	11,16

Note: This table reports classification results using the ZL-score for the training and for the test sample. ZL-score <1,40 represents bankrupt firms, and ZL-score>2,58 represent financial soundness. The scores in between 1,40 and 2,58 are considered as the grey area, which is the uncertain area The training sample consists of 1.842 firms, of which 65 bankrupt firms. The test sample comprises the remaining firms: 762 non-bankrupt firms and 28 bankrupt firms.

To summarize, the empirical results present the highest classification accuracy for the ZL-score, compared to the Z-score and the LRA. Classification results of the ZL-score compared to the Z-score are more accurate for as well as the bankrupt firms as for non-bankrupt firms. Additionally, the percentage of wrong classified firms decreases for both bankrupt and non-bankrupt firms in the ZL-model compared to the Z-model. Comparing results of the LRA with the ZL-score, the LRA for the total sample predicts non-bankrupt firms more accurate, yet bankrupt firms are classified miserable. This implies that the LRA for the total sample may not be used for predicting bankruptcies for Dutch SMEs. The correctly classified results for the LRA for equal groups are less accurate for bankrupt firms, compared to the ZL-score, and approximately equal for non-bankrupt firms (~73%). To recapitulate, ZL-score model works better than the Altman Z-score model, and slightly better than the LRA.

5. Conclusions, discussion and recommendations for future research

5.1 Conclusion

Predicting financial distress will always be important for managers, investors, creditors, and other stakeholders. Even though researchers spend a long time on predicting bankruptcies, to my knowledge, the Altman Z-score model has not been investigated for Dutch SMEs in previous studies. The purpose of this study is firstly to examine whether the revised Altman Z-score model (1983) is valid for Dutch SMEs. The original Altman Z-score model (1968) is developed for publicly traded companies only. Due to the private nature of most Dutch SMEs, the revised model is examined, which substitutes market value by book value (Altman, 1983). Moreover, to develop a new model and for comparison reasons, the LRA is conducted. Secondly, to optimally predict bankruptcies for Dutch SMEs, my own ZL-score model is developed, based on the Altman Z-score and the LRA results.

The empirical analysis is conducted using a sample for the years 2010-2015 consisting of 2.632 firms of which 3,53% went bankrupt in the sample. The aim of this study is firstly to test whether the revised Altman Z-score model (1983) works for Dutch SMEs. The empirical findings indicate that the Altman Z-score model predicts correctly approximately 71% of bankruptcies one year prior. According to the Altman Z-score for the sample used in this paper, the accurate classification percentage decreases for both bankrupt and non-bankrupt firms two years prior to bankruptcy, compared to one statement prior to bankruptcy.

Furthermore, to develop a new model and for comparison reasons, the LRA is conducted. I draw conclusions based on two samples: the total sample and a randomly two equal groups (65 bankrupt and 65 non-bankrupt firms) sample. All individual ratios of the model significantly contribute to the prediction of bankruptcies for Dutch SMEs. Classification results of the LRA are not useful for the total sample, yet it is useful by splitting up the sample in equal groups. The total sample is highly tilted towards non-bankrupt firms in classifying correctly, and the accuracy results for the two equal groups are approximately equal for bankrupt and non-bankrupt firms (71%).

Moreover, inspired by the Altman model, I estimate a new bankruptcy model especially for Dutch SMEs. This new ZL-score model works better than the Altman Z-score model and the LRA as it predicts approximately 82% of bankrupt firms correctly one year prior. Besides that, according to the sample used in this paper, the percentage of wrong classified firms is the lowest for the ZL-score (9% for bankrupt firms one year prior), compared to the Z-score (14%) and the LRA (28%).

Good working bankruptcy models allow managers to take steps in order to prevent stakeholders against possible negative consequences of financial distress. Business cycles are patterns of economic fluctuations, resulting in expansions and recessions. Thus, new crises and bankruptcies come with certainty, and therefore studying financial distress is important. This research is especially interesting for Dutch SMEs, as the ZL-score model can be used to predict bankruptcies for SMEs in the Netherlands.

5.2 Discussion

Against different theories, the Z-score model and my own developed ZL-score model predict bankrupt firms more accurate than non-bankrupt firms. The theories state that an essential argument is related to earnings management in times firms face financial distress (Garcìa Lara, GarcíaOsma and Neophytou, 2009; Mselmi, Lahiani and Hamza, 2017). The aim of those firms is to hide the financial distress, in order to meet stakeholders' expectations, and eventually obtain financing (Garcìa Lara, GarcíaOsma and Neophytou, 2009; Mselmi, Lahiani and Hamza, 2017). Managers can adjust reported numbers on the financial statements, resulting in a lower estimate of bankruptcy probability. Indeed, Garcìa Lara, GarcíaOsma and Neophytou (2009) investigate that to hide financial distress, firms are culpable to manipulate statements beginning four years prior to bankruptcy. Those theories made me expect the models to predict non-bankrupt firms better. Nevertheless, empirical results of the Z-score model and the ZL-score model show that bankrupt firms are predicted with higher accuracy rates, compared to non-bankrupt firms. However, as expected, the LRA results present higher accuracy for non-bankrupt firms, compared to bankrupt firms.

Researchers discuss several classification results by predicting bankruptcies with the LRA. Bardos and Zhu (1997) correctly classify 85,5% of non-bankrupt firms, and 52% of bankrupt firms. The low accuracy for bankrupt firms may be explained by the imbalance in their data as the number of non-bankrupt firms included in the dataset is much larger than the number of bankrupt firms. These results correspond to my LRA results for the total sample, which presents an accuracy prediction highly tilted towards non-bankrupt firms. In contrast to my accuracy rates, Mselmi, Lahiani and Hamza (2017) report the highest accuracy for SMEs with the LRA. They investigate French SMEs, and correctly classify 82,85% of bankrupt firms and 95,71% of non-bankrupt firms. Those high accuracy rates can be explained by the fact that they did not restrict their financial ratios. However, results can be affected by including too many ratios. Every irrelevant included ratio reduces the accuracy of the estimated discriminant coefficients. The ratios included in all my models (Z-score, ZL-score and the LRA) are restricted to the five ratios used in the revised work of Altman (1983). It is possible that by including additional parameters, the models' accuracy increases. To conclude, researchers should be aware that results can be affected by including too many ratios. Every irrelevant included ratio reduces the accuracy of the estimated discriminant coefficients. Results can significantly deviate by choosing other ratios or sample sizes.

Currently, a commercial trend is going on within SMEs. This regards that crowdfunding is becoming more common. Crowdfunding results in freely negotiable debt, and therefore, in differences between market- and book value of the liabilities of enterprises, even though they are not listed. Additionally, this is interesting to consider since we are heading towards a world in which any type of investment through any type of platform becomes more common. To represent ratios I used book values, however, considering the trend, it is necessary to further deepen into this book value, since it may present a distorted view.

5.3 Recommendations for future research

This research can be approved in the following ways. First, future research should further investigate the accuracy of the ZL-score model for earlier years prior to bankruptcy. The main problem of bankruptcy models is the generalizability, since they are all based on specific datasets (e.g. Altman, et al., 2017; Almamy, Aston, & Ngwa, 2016; Balcaen, and Ooghe, 2006; Bellovary et al., 2007; Gerantonis, et al., 2009; Grice and Ingram, 2001). Due to data limitations, this research has a data restriction of one and two statements prior to bankruptcy. This is not in contrast to work of other researchers, as many studies that produce a model for predicting bankruptcy focus on one statement prior to bankruptcy (Altman, 1968; Lord, Weech-Maldonado, Davlaov, 2017; Mselmi, Lahiani, & Hamza, 2017). Due to the data restriction, this research focuses on generating a model that best achieves a late signaling of bankruptcy (Altman et al, 2017). However, researchers sometimes aim to produce bankruptcy models that warn as early as possible. Investigators point out that one year before bankruptcy, it is too late to undertake actions like the fulfillment of a turnaround plan (Pompe, and Bilderbeek, 2005). The obtained model is not tested for and based on statements published earlier years prior to bankruptcy. Therefore, I recommend future research to further deepen into the predictable value of the ZL-score model for earlier years prior to bankruptcy and to test with a bigger sample size, especially for the bankrupt group.

Moreover, I recommend future research to investigate the predictable value of the ZL-score model for other countries. The model is tested for Dutch SMEs, but possibly works for other countries.

Furthermore, this research firstly focusses on the predictable value of the Altman Z-score model, which restricts my dataset to the included financial ratios of that model. The original composition of ratios is developed in 1968, but the business economy changes constantly, and therefore the relevance of ratios may change. I recommend future research to focus on the composition of ratios and remove or add variables with the aim to increase the predictable value of the model. Superfluously, due to the data restriction, it is impossible to test for the accuracy prediction of other bankruptcy models. I chose to determine the predictable value of the Altman Z-score model for Dutch SMEs, since research presents that the model works for the previously investigated countries and firm sizes. Those results made me expect the model to work for my subject of interest, and therefore generate high accuracy rates for Dutch SMEs. Future research should consider other bankruptcy models or techniques and compare the accuracy results.

The ZL-score model provides in imitation of Altman (1983) a grey zone interval of 1,18. I recommend future research to reconsider the bandwidth of the grey interval, since that may result in increased accuracy rates and/or certainties of classifications. I expect the wrong classifications to increase when decreasing the grey zone, and I expect the wrong classifications to decrease when increasing the grey zone. By applying advanced statistical techniques, an optimal trade-off between increasing right classifications, decreasing wrong classifications, and certainty of classification can be provided by optimizing the grey zone.

Finally, future research should take a critical look at the limitations of the MDA. Both the Z-score and the ZL-score are based on that technique. I recommend further research to explore the consequence of the weak intuitive interpretation of calculating a score, the influence of macro-economic factors on the accuracy of the model, and the consequences of instable ratios over time.

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6. Appendix

Figure 6.1. Trade-off cut-off ZL-score

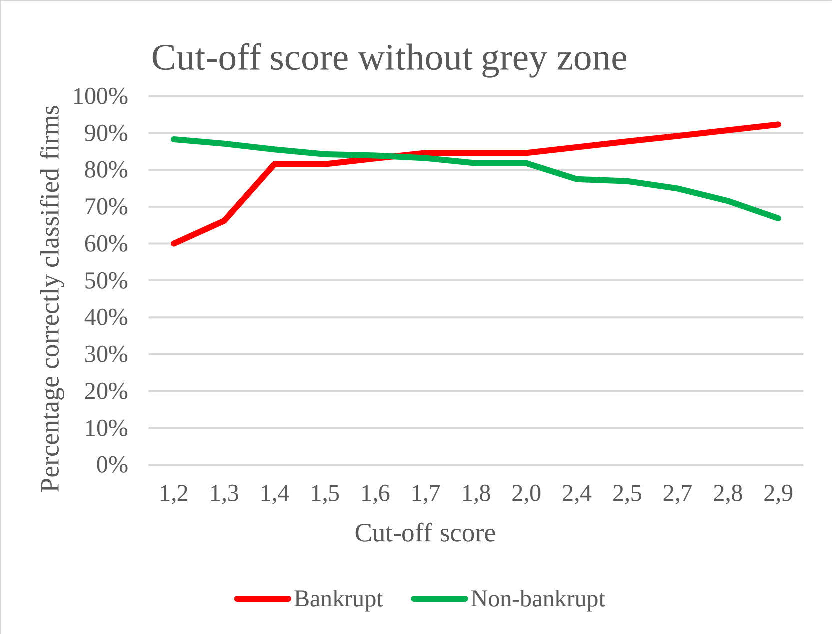
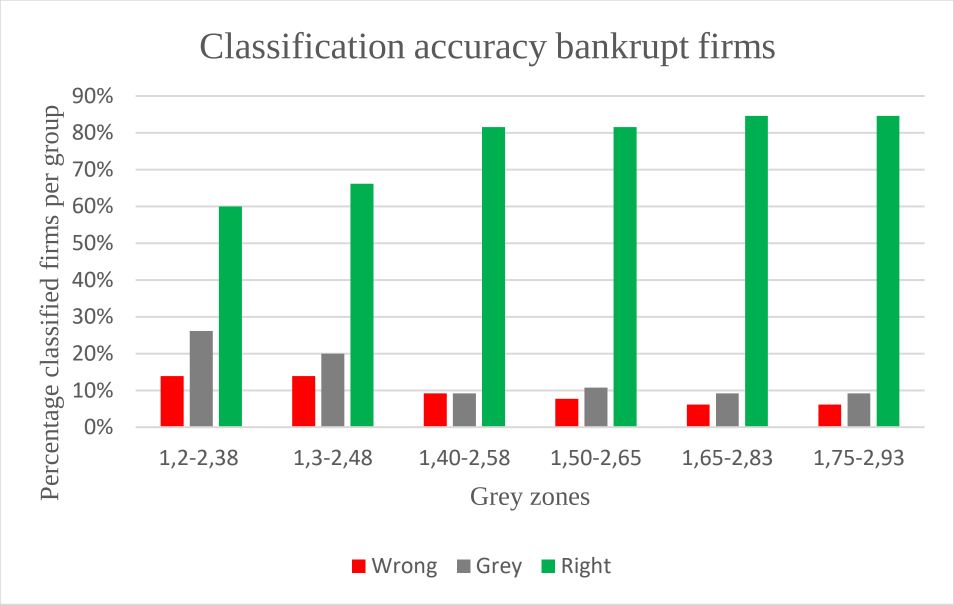


Figure 6.1 presents graphically that decreasing the cut-off score results in higher accuracy results for non-bankrupt firms, and lower accuracy results for bankrupt firms. Moreover, increasing the cut-off score results in higher accuracy rates for bankrupt firms and lower accuracy rates for non-bankrupt firms.

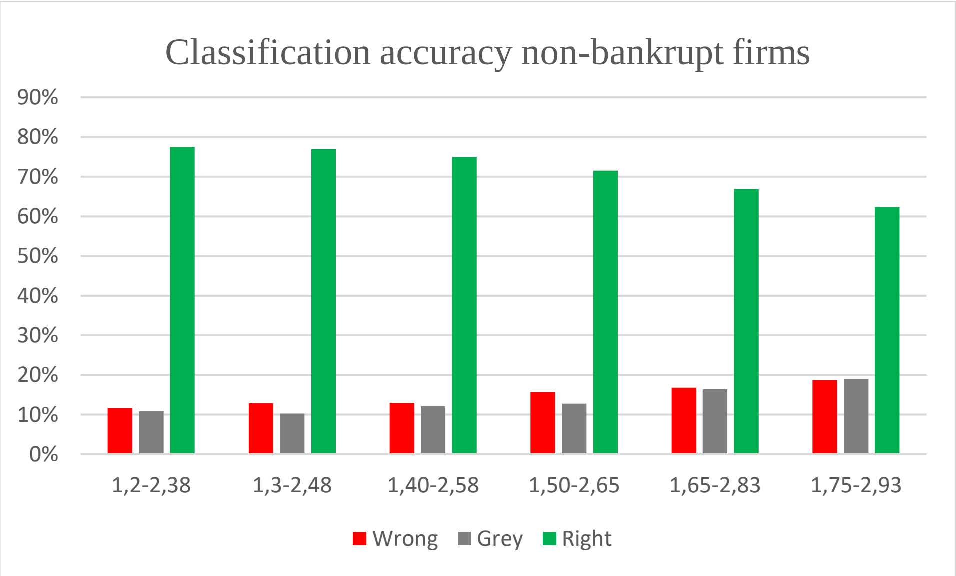
Results present that there is an equal trade-off between bankrupt and non-bankrupt firms at 1,63. At that score, the correctly classified percentage is approximately 83% for both bankrupt and non-bankrupt firms. Note that this accuracy rate is higher than the rates reported in Table 11, since this graph does not consider a grey zone.

Figure 6.2. Classification with grey zone ZL-score

6.2A



6.2B



6.2C

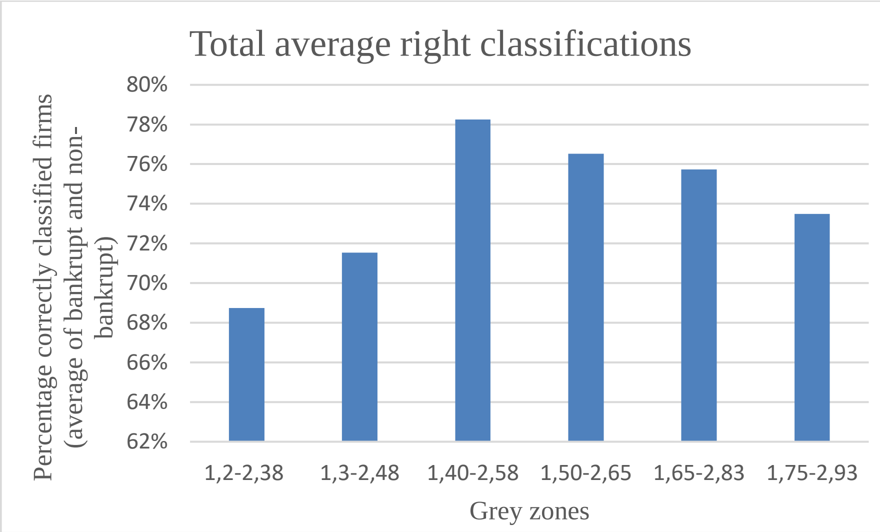


Figure 6.2A and 6.2B show the percentage right, wrong and grey classifications of the bankrupt and non-bankrupt test sample one year prior to bankruptcy for different grey zones. Firms with scores higher than the peak of the zone are rightly classified as non-bankrupt, firms with scores lower than the underside of the zone are rightly classified as bankrupt. Figure 6.2A clearly shows that increasing the values of the grey zone results in higher right accuracy rates for bankrupt firms, and 6.2B shows that increasing the values of the grey zone results in lower right accuracy rates for non-bankrupt firms.

Figure 6.2C presents the total average right classifications per grey zone. This is calculated by adding the percentages of accuracies of the bankrupt firms and non-bankrupt firms and dividing them by two. The figure presents that grey zone 1,40-2,58 provides the highest accuracy rates.

Figure 6.3. Wrong classifications ZL-score

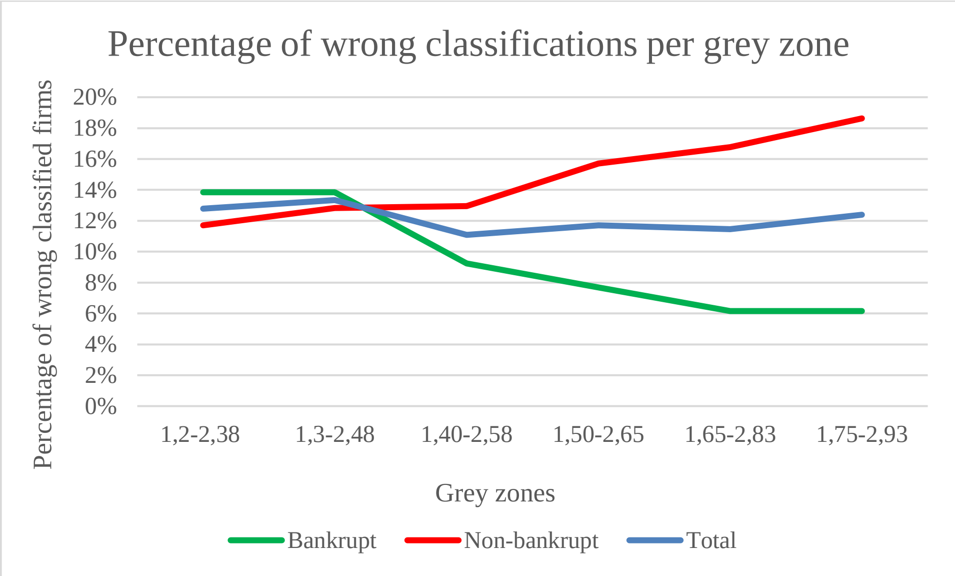


Figure 6.3 presents the percentage of wrongly classified firms[8] per grey interval for bankrupt firms, non-bankrupt firms and for the total sample. The line 'total' shows a weighted average of the wrongly classified firms in the bankrupt and non-bankrupt sample. The figure shows that the total wrong classifications are approximately equal for the intervals 1,20-2,38 to 1,75-2,93. The wrong classifications decrease from 1,75-2,93. The zone 2,00-3,18 wrongly classifies 28,71% of the total sample.

The ZL-score considers grey zone 1,40-2,58. That zone classifies 11,09 % of total firms, 9,23% of bankrupt firms, and 12,94% of non-bankrupt firms wrong.

[8] The total wrong classification is calculated by taking the average of the wrong classification results of the bankrupt and non-bankrupt firms. A classification is considered as wrong in case it is not labeled as right or grey: ((% wrong bankrupt) + (% wrong non-bankrupt))/2). E.g. cut-off scores between 1,20-2,38, a bankrupt firm is labeled as wrong in case the ZL-score is >2,38, and a non-bankrupt firm is labeled wrong in case the ZL-score is <1,20.

Figure 6.4. Random selection generator STATA for LRA

One statement prior to bankruptcy

Classified	Bankrupt	Non-bankrupt
Bankrupt	**49**	**16**
Non-bankrupt	**16**	**49**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**42**	**13**
Non-bankrupt	**23**	**52**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**47**	**21**
Non-bankrupt	**18**	**44**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**49**	**18**
Non-bankrupt	**16**	**47**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**47**	**17**
Non-bankrupt	**18**	**48**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**48**	**14**
Non-bankrupt	**17**	**51**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**45**	**19**
Non-bankrupt	**20**	**46**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**46**	**19**
Non-bankrupt	**19**	**46**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**50**	**18**
Non-bankrupt	**15**	**47**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**44**	**17**
Non-bankrupt	**17**	**48**
Total	65	65

Note that the numbers reported imply number of firms and the figure shows results one statement prior to bankruptcy. Random selection is repeated 10 times. In total 130 randomly selected firms are included per LRA, consisting of 65 bankrupt, and 65 non-bankrupt firms. The results show that one year prior to bankruptcy, on average, 71,85% of bankrupt firms is correctly classified and 73,54% of non-bankrupt firms.

Two statements prior to bankruptcy

Classified	Bankrupt	Non-bankrupt
Bankrupt	**48**	**16**
Non-bankrupt	**17**	**49**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**41**	**13**
Non-bankrupt	**24**	**52**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**47**	**20**
Non-bankrupt	**18**	**45**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**47**	**17**
Non-bankrupt	**18**	**48**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**45**	**18**
Non-bankrupt	**20**	**47**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**44**	**17**
Non-bankrupt	**17**	**48**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**46**	**17**
Non-bankrupt	**19**	**48**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**45**	**19**
Non-bankrupt	**20**	**46**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**49**	**16**
Non-bankrupt	**16**	**49**
Total	65	65

Classified	Bankrupt	Non-bankrupt
Bankrupt	**45**	**15**
Non-bankrupt	**20**	**50**
Total	65	65

Note that the numbers reported imply number of firms and the figure shows results two statements prior to bankruptcy. Random selection is repeated 10 times. In total 130 randomly selected firms are included per LRA, consisting of 65 bankrupt, and 65 non-bankrupt firms. The results show that two years prior to bankruptcy, on average, 70,31% of bankrupt firms is correctly classified and 74.15% of non-bankrupt firms.