Tilburg University

Master Thesis

Business and Pricing Strategies in Online Sharing Platforms

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Abstract

In this thesis we study different business and pricing strategies in the online sharing economy. We adapt the monopoly two-sided market model by Rochet and Tirole (2003) in two steps and then use the adjusted model to analyze different business strategies in the sharing economy.

First we treat the service price as exogenous and derive optimal pricing strategies that are closely related to the one from Rochet and Tirole (2003). Using these optimal pricing strategies we derive closed-form equilibrium results about the optimal transaction fees, profit and welfare for a non-profit and a for-profit sharing platform. Furthermore, we evaluate two different buyer-side demand function and find that the buyer's ability to shift utilities is beneficial for both the consumers and the platform.

In the second step the service price is treated as endogenous and the demand functions for strict and shifting utilities are combined. We evaluate two different business strategies for the non-profit and for-profit platform, one in which the platform and seller are both in control and one in which the platform is in full control. Here we define maximization problems for all strategies and analyze the numerical results on profit, welfare, transaction fees and service price.

Based on our results we claim that the business strategies in which both the platform and seller are in control establish the most benefits and have the least unwanted effects. When regulating the sharing economy we therefore state that the focus should not be on whether the platform uses a profit maximizing or welfare maximizing strategy, but instead on whether the platform lets the seller define its own service price or not.

Keywords: Online sharing economy, two-sided markets, pricing strategies

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1 Introduction

Recent research by TNO (2016) shows that 12 percent of the Dutch population earns money by offering services or products on online sharing platforms like Airbnb, Uber or Marktplaats. Moreover, 20 percent of this part of the population earns more than half of their income using these platforms. More and more jobs are offered as gigs, and this growing body of platform workers is changing the labor market as we know it. Platforms like Mturk and Uber do not have fixed staff employed, but offer jobs depending on demand and supply. This upcoming sharing economy raises a lot of questions and is frequently seen a disruptive. An often lined debate is about whether and how sharing platforms should be regulated. This debate is mostly focused on the potential treats for incumbent firms and the labor market. However, like Coyle (2016), we state that the debate should be focused on what business strategies will establish the most benefits and how the unwanted effects of the sharing economy can be avoided.

The online sharing economy follows the structure of two-sided markets, it is characterized by two distinct sides whose ultimate benefit stems from interacting trough a platform. Platform owners need to get both sides of the market on board to start operating. By charging different prices to each side, sharing platforms can treat one side of the market as profit neutral and the other side as a the profit maker. In other words the sharing platform owners can cross-subsidize between both sides which enables them to regulate the demand and supply offered on their platform.

This study focuses on the behavior of sharing platforms, by applying the well-know structure of two-sided markets we shed more light on their pricing structure and explain how prices and welfare are affected by different business strategies. We adapt the monopoly two-side market model of Rochet and Tirole (2003) such that it applicable to online sharing platforms by adding the service price and changing the demand functions. In order to implement all the adjustments we take a two-step approach. In the first step we assume an exogenous service price and keep the adjusted model closely related to the one in Rochet and Tirole (2003), such that we are able to define and compare closed-form equilibrium results. In the second step the service price becomes endogenous, here we define the profit and welfare maximization problem for different business strategies and present numerical results for each scenario.

The online sharing economy consists of many different types platforms, we therefore cannot capture all the characteristics of all sharing platforms. Our approach is to include a number of key ingredients and generalize our analysis such that it is relevant for various non-profit and for-profit peer-to-peer sharing platforms. We assume that the buyer and seller are charged by means of a per-transaction fee and that there is no subscription fee.

The following research questions are addressed in this research: (i) What are the optimal pricing strategies for non-profit and for-profit sharing platform under the assumption of strict and shifting utilities and how are they affected by an exogenous service price? (ii)

How does an endogenous service price set by the seller and an endogenous service price set by the platform affect the pricing strategies of non-profit and for-profit sharing platform? (iii) What do the results mean for the sharing economy in terms of possible regulation?

The outline of this thesis is as follows. In Chapter 2 we define the online sharing economy and explain what the difference is between a regular two-sided platform and a sharing platform. Chapter 3 provides a literature review on pricing strategies in two-sided markets that are relevant for our applications and on recent research about pricing strategies in the sharing economy. In Chapter 4 we derive the optimal pricing strategy for a non-profit and a for-profit monopoly sharing platform under the assumption that the service price is exogenous. We compare these pricing strategies with the ones from Rochet and Tirole (2003) and compare the non-profit and for-profit equilibrium results for different demand functions. In Chapter 5 the service price is endogenized and the profit and welfare maximization problems are defined while combining the demand functions. In Chapter 6 we discuss numerical results for the different business strategies and compare them based on several performance parameters. Finally the conclusions are summarized in Chapter 7 and Chapter 8 points out directions for further research.

2 Sharing economy platforms

There is large variety in type of activities offered by online sharing platforms, from professional work on platforms like Werkspot to accommodation on platforms like Airbnb. This wide range of activities makes it hard to define the sharing economy as a whole. Moreover, a lot of definitions and terminology dwell around in the literature making it even more difficult to determine what economic activity belongs to the sharing economy and how to regulate this upcoming economy. Based on existing literature we will first discuss the different definitions of the sharing economy and delimit the definitions for the purpose of this research. Second we focus on the general structure of online sharing platforms. Finally we discuss the different business strategies that a sharing platform can follow.

2.1 Definition

Stating one unambiguous definition of sharing economy platforms is not an easy task. This is partly due the fact that there is only a small difference between non-sharing platforms and sharing platforms, some even argue that this is not a 'real' difference but only a commercial one (Coyle, 2016). In the literature one can find different definitions of the sharing economy. For example, Hamari et al. (2015) define the online sharing economy as 'the peer-to-peer-based activity of obtaining, giving, or sharing the access to goods and services, coordinated through community-based online services'. Botsman (2015) defines the sharing economy as 'an economic system based on sharing underused assets or services, for free or for a fee, directly from individuals'. And Meelen and Frenken (2014) define the sharing economy as 'the phenomenon that allows consumers to use each other's underutilized goods, possibly for a fee'. These definitions all focus on three different parts. First they define between *whom* the sharing happens, which is between peers (i.e. individuals or consumers). Second they define *what* is shared between them, these are underused goods or services. And finally they define *how* the sharing takes place, here Hamari et. al. (2015) specifically define that sharing is coordinated through a platform the other definitions only specify that there is a possible fee to make the sharing happen. Combining these definition with the insights from Coyle (2016) we state that an online sharing economy platform can be characterized as a platform that (1) matches suppliers and consumers, (2) by letting suppliers use more intensively an asset they own, (3) through technologies that enable renting out the asset when it is not in use.

Based on these definitions it is clear that the sharing economy revolves around underused goods and idle capacity. But when are goods underused and when is capacity idle? Can you say that your house is underutilized when you are on vacation? That your lawn mower is underutilized when it is standing in you garage? It is not easy to answer these questions in an indisputable way. Throughout this research we assume that a good (or service) is underused if there is no lump sum or purchase amount involved in renting (or performing) the good (or service). This means that one already owns a lawn mower before one decides to rent it to someone else and one cannot increase his supply by purchasing more lawn mowers to rent out. If the latter is happening, an individual is purchasing lawn mowers with the purpose to earn money by renting them and is no longer participating in the sharing economy, but is in fact acting like a rental business. Platforms facilitating these kind of businesses are not sharing platforms but just two-sided rental platforms.

2.2 Structure

Like two-sided platforms, online sharing platforms are characterized by two distinct sides, a buyer-side and a seller-side, whose benefits come from interacting through the platform. Network externalities play an important role in these platforms, meaning that they can cross-subsidize between the different groups of end-users (Hamari et al., 2015). To get both sides on board, a platform can for example lower the price charged for one group of end users (e.g. the buyers) and increase the price charge to the other group (e.g. sellers), while keeping the total price the same. At first sight this results in an increase in the buyer-side demand and a decrease in the seller-side demand. However an increase in the buyer-side demand simultaneously increases the seller-side demand because there are more opportunities for transactions. This latter effect is typical for two-sided markets and is called the cross-externalities network effect (Rochet and Tirole, 2003). It causes the profit of a platform to not only depend on the total price charged to the end users, but also on the decomposition of transaction fees.

2.3 Business strategies

Schor (2014) states that the business model of a sharing platform are shaped by two dimensions: market orientation (for-profit vs. non-profit) and market structure (peer-to-peer vs. business-to-peer). Table 1 shows examples for each type.

		Type of provider	
		Peer to Peer	**Business to peer**
Platform orirentation	**For-profit**	Airbnb	Zipcar
	Non-profit	Food Swaps	Makerspaces

Table 1: Platform types. Adapted from "Debating the Sharing Economy" by J. Schor, 2014, *Great Transition Initiative*, p.4.

The intention of a platform to maximize profit or welfare influences the way sharing takes place and what part of the revenue ends up with the sellers and the platform. For-profit platforms, like Uber and Airbnb urge for profit maximization and growth. On the other hand are the non-profit platforms, like food swaps, who do not push for revenue but are designed to increase the welfare of their users. Even though the distinction between for-profit and non-profit sharing platforms is generally seen as the most important one, the division between P2P and B2P sharing platforms is considerable. On P2P platforms the sharing takes place directly between individuals and the platforms usually earns a commission per transaction. Hence P2P platforms often thrive to increase the number of transactions by attracting both sellers and buyers to their platform. On B2P platforms the transactions take place between the business and consumers. B2P platforms do no attract individual sellers, and therefore often aim to maximize the revenue per transaction instead of the number of transactions. (Schor (2014), Frenken et al. (2015))

As mentioned before, one important characterization of a sharing platform is the ability to cross-subsidize between the seller and buyer-side. A platform can adopt different pricing strategies but generally there is a profit-making side and a subsidized side. Table 2 provides some examples for the different pricing strategies where only the buyers are charged, only the sellers are charged or both are charged.

	Sharing platform	**Description of pricing**
Sellers are charged	Werkspot – platform for handymen and construction work	Subscription fee for workers
Buyers are charged	Thuisafgehaald – platform for meal-swaps	Transaction fee for meal buyers
Buyers and sellers are charged	Airbnb – Platform for private accommodations	Transaction fee for guests and hosts

Table 2: Pricing in P2P Sharing platforms. Adapted from "Competition in the Sharing Economy" by V. Demary, 2015, *IW policy paper*, p.13.

As can be seen in table 2, platforms also differ in how they charge consumers. We distinguish three types of charging; a subscription fee, a per-transaction fee or both a subscription and per-transaction fee. Because in the peer-to-peer sharing platforms a per-transaction fee is most common (Schor, 2014) and we want to stretch the platforms ability to cross-subsidize, we will focus on platforms that charge both buyers and sellers using a per-transaction commission fee.

In general a sharing platform sets the commission charged over the services price, which is split in a transaction fee charged to the buyer and a transaction fee charged to the seller and the seller sets the service price. However some platforms also influence or even set the price of the service that is provided by the seller. For example, Uber sets the price of the taxi ride and the transaction fee, were they charge a total commission of about 20-30% (Petropoulos, 2016). Airbnb on the other hand does not set the rental price, but they do recommend their sellers to charge a certain price depending on the size or location of an apartment. Airbnb charges a transaction fee of 3% fee to their hosts (the sellers) and a transaction fee of 6-12% to their guests (the buyers). On MTurk the price the requester (the seller) pays for a Human Intelligence Task ("HIT") consist of two components: an amount the requester pays to the workers (the buyers) and a fee payed to Mechanical Turk. Were the fee paid Mechanical Turk is depends on the amount paid to the Workers. In this research we distinguish two types of sharing platforms; (1) a sharing platform in which both platform owner and the seller are in control, and (2) a sharing platform that is in full control. Were in the first type the platform owner sets the transaction fees and the seller sets the service price. In the second type the platform owner sets both the transaction fees and service price.

2.4 Regulation

As stated in the introduction, online sharing platform raise a lot of questions with respect to regulations. The research about potential regulations in the sharing economy is very limited. A lot of questions are asked in the literature, but to our knowledge very little are answered. For example, Einav et al. (2015) state that the regulation must focus on protecting vulnerable consumers from a unfavorable market and point out that the timing of regulation is important. Koopman et al. (2015) claim however that even though regulation is justified to protect consumers, this does not mean that they accomplish those goals. But also their research raises more questions than answers towards the regulation of the sharing economy. By determining what business strategies establish the most benefits and what business strategies establish the most unwanted effects, we hope to give a good foundation to further research how to regulate the sharing economy.

3 Related literature

3.1 Pricing strategies in two-sided markets

There exists a lot of research about pricing strategies in two-sided platform markets. For a detailed discussion we refer to Rochet and Tirole (2006), here we only discuss the research that is most relevant to our application.

The initial work on two-sided markets by Rochet and Tirole (2003), and Armstrong (2006) focused on price theory in two-sided markets. Rochet and Tirole (2003) developed monopoly and duopoly models for two-sided markets with different governance structures (i.e. for-profit and not-for-profit). They use their models to shed more light on the logic behind the price allocation between the two sides of the market. Their basic models assume that fees are charged on a per transactions basis and that pricing is linear. The models developed in Armstrong (2006) assume that platforms charge a lump-sum fee instead of a per transaction fee. The transaction-based fee structure is representative for the payment card industry, but also for most online sharing platforms. Subscription to for example Airbnb is free, users are only charged when they make a transaction (i.e.

rent/lend an apartment). The lump-sum fee structure applies to for example the yellow pages and news papers.

The two-sided duopoly model by Rochet and Tirole (2003) allows on side of the platform to multi-home. They do this by capturing the dynamic aspects of multi-homing in the model. Let platform A offer a lower price to the seller than its concurrent platform B. Every seller has to make a decisions: join platform A, join platform B or join both platforms. Note that it is never optimal to only join the high-priced platform. Hence the seller's choice will be between joining both platforms or only joining the platform with the lowest costs. When the seller decides to join both platforms, there are more potential buyers available to transact with. But joining both platforms, will due to cross-network effects cause an increase in the buyer-side demand of the high-cost platform. With the result that the buyer-side demand on the low-cost platform will be lower when the seller joins both platforms as compared to when he only joins the low-cost platform. Rochet and Tirole (2003) mainly focused on platforms that only charge a per-transaction fee, but they also examine the case where there are lump-sum fees as well as per-transaction fees. This model boils down to the so called competitive bottleneck model by Armstrong (2006). In this research we only focus on monopoly sharing platforms and hence only use the theory derived for the monopoly two-sided markets.

Hyun (2016) also adopts the model of Rochet and Tirole. It is adopted such that it is applicable to the online two-sided platforms in which the sharing of content plays an important role. Note that Hyun (2016) talks about the sharing of contents on online website like Facebook or Twitters, he does not talk about the sharing of services. He argues that the option to share content allows someone to recommend (discourage) goods to your online friends leads to and increase (decrease) of demand, he refers to this effect as the direct network effect. Hyun (2016) builds a monopoly and duopoly platform model that incorporates both the cross-network effect among different groups and the direct network effects within a group. He finds that introducing direct network effect into one side of the market has two opposite effects on the pricing decision of platform; the demand-augmenting effect and the demand sensitizing effect. On one hand the demand-augmenting effect states that the when there are direct network effects on the buyer side, this increases marginal utility on the buyers side which increased the number of potential buyers and hence increases the buyer price. On the other hand the demand-sensitizing effect state then an increase in the buyer price decreases buyer side demand, moreover this increase is accelerated by the direct network effects. The decrease in buyer side demand in turn causes lower direct network effects resulting in lower marginal utility for any given price. The demand sensitizing effect provides incentive for a platform to lower buyer-side price because reducing price will attract more buyers.

Moreover, Huyn finds that the demand-augmenting effect predominates in the monopoly framework, whereas in the duopoly framework the demand-sensitizing effect overrules. This causes competing platform to discount the buyer-side price and increase the seller-side price (while lowering the sum of prices), which is a scenario that is regular for platforms under competitive conditions.

In this research we focus on online sharing platforms that charge only a per-transaction fee and no lump-sum fee. We choose to adopt the monopoly two-side market model by Rochet and Tirole (2003), because it most fitting to those applications. Furthermore, the monopoly model can relatively easy be extended to a duopoly model, which is helpful for future research. The changes made by Hyun (2016) to the monopoly model are also fitting to our application, unfortunately due to time limitations we are not able to implement them in our model.

3.2 Pricing strategies in sharing platforms

Benjaafar et al. (2015) developed an equilibrium model for the peer-to-peer sharing of products, where contrary to this research, they focus on whether individuals decide to own or not to own the product. The model is based on the idea that collaborative consumption has enabled product owners to get income from renting their products to non-product owners, while it enabled non-product owners to rent the product and get temporary access. They assume that the rental price and commission fee are set by the platform and take them as given to characterize equilibrium ownership and usage levels. They compare the equilibrium outcomes for several model setting and evaluating the impact on different problem parameters. Most importantly they compare models with and without collaborative consumption for profit maximizing and non-profit maximizing platforms. Their main finding is that consumers always benefit from sharing, where consumers that are neutral with respect to owning or not owning benefit the most.

As mentioned, Benjaafar et al. (2015) assume that the platform decides on the price and commission fees. Moreover, they focus on price as the primary decision variable of the platform owners and treat commission fees as being endogenously specified. They argue that platform-determined pricing is more likely when product or services are more homogeneous (like Uber) and seller-determined pricing is more likely when products or services are more heterogeneous (like Airbnb). Contrary to Benjaafar et al. (2015) we will put more focus on the commission fees that are set by the platform, in order to stretch the ability of sharing platforms to cross-subsidize between both user groups.

Another interesting and very recent research about the sharing economy is by Jiang and Tian (2016). They focus on the effects of sharing economy on the firm (i.e. the product manufacturer). The main question they address is how the sharing of products affects the manufacturer of those product. By doing so they delimit their research to the peer-to-peer sharing of products and exclude the offering of services. Were we investigate how the sharing platform and product owner should strategically set their transaction fees and service price in the sharing economy, they take another road and research how the product manufacturer should strategically set their product price in the sharing economy. Next to examining the effect of product sharing on the firms' profit they also look at how it affects consumer surplus and social welfare.

The research conducted by Fang et al. (2016) is perhaps most similar to our research. They develop a novel model of sharing platforms and derive equilibrium prices for both profit maximizing and non-profit maximizing platforms. Their results are validated by analysis using data from Didi Chuxing, a ridesharing platform in China. Contrary to our research, they include an extra opportunity for the product owners. They assume that the product owners experience a trade-off between using their resource directly or sharing it through a platform, allowing them to have two ways to derive benefit from the product. By defining a self-usage level and a sharing level that incorporate the fraction of time a seller usage the product and the fraction of time the seller rents the product, they include this trade-off in the model. Their main finding is that in practice the revenue of platforms can be limited due a relatively lower seller-side demand and hence platforms have incentives to increase seller-side demand by subsidizing that side of the market. We find similar results, namely that if the buyer-side utility is significantly higher than the seller-side utility (indicating a relatively low seller-side demand) the seller-side transaction fee becomes negative, meaning that the platform owner subsidizes that part of the market.

4 Monopoly Sharing Platform

In this chapter we derive the optimal pricing strategy for a non-profit and a for-profit monopoly sharing platform under the assumption of an exogenous service price. The monopoly two-sided market model from Rochet and Tirole (2003) is adjusted to determine closed-from equilibrium results in both scenarios. Section 4.1 states the modeling assumptions and the foundations of the model. In section 4.2 we derive the equilibrium prices for a profit maximizing sharing platform and in section 4.3 for a welfare maximizing sharing platform. To conclude we state the most important results in the final section of this chapter.

4.1 Modeling

In sharing platforms value is created by transactions between the buyer-side and the-seller side. A transaction is characterized by a buyer (superscript B) purchasing a service from a seller (superscript S) trough the platform. The seller charges a service price s to the buyer for delivering the service, and the platform charges a transaction fee (p^B) to the buyer and a transaction fee (p^S) to the seller for enabling the transaction (i.e. bringing together both sides, handling finances, etc.). Figure 1 gives an overview of the cash-flows between a buyer, the platform and a seller.

Figure 1: Cashflow structure platform. Where, $p^B_{tot} = s + sp^B$. is the total price paid by the buyer and $p^S_{tot} = s - sp^S$, is the total price received by the seller. Note that the platform can charge different buyer and seller transaction fees

The difference with the two-sided monopoly modeled by Rochet and Tirole (2003) is that here the service price paid to the seller is included in the model. In later research they do present a model in which the seller price is included (Rochet and Tirole, 2006), however not in a way that is representative for a sharing platform. The main difference lies in the different natures of a seller in a sharing platform and a seller in a regular platform. The seller in a sharing platform has limited idle capacity to offer and cannot increase his capacity by producing more goods, the seller in a regular platform can increase his capacity by producing more goods or acquiring more service. As a consequence, the capacity in a sharing platform can only be increased by acquiring more sellers that offer a good or service, while in a regular platform the capacity can be increased individually through the sellers and collectively by the platform. In Chapter 5 we elaborate more on the position of a seller associated with a sharing platform, for now we assume the seller's service price s is set exogenous and hence throughout this chapter the service price is treated as a fixed parameter.

There are different types of utilities associated with the platform and the service. Let b^B be the buyer's total utility related to using the platform for transacting, and let v^B be the buyer's total utility related to the service. This captures the idea that the buyer obtains benefits from using the platform (i.e. network benefits or social connections) that are independent from the benefits of the service (i.e. being able to mow his lawn). The seller's total utility related to the platform is b^S, which captures the benefits from using the platform, like reaching a larger group of potential buyers. Next to the benefits related to using the platform, the seller also has the benefit of a potential profit from renting his product or service. This benefit from the potential profit is not directly captured into the demand functions, instead it is assumed that the seller will only join the platform if the potential profits are at least zero.

4.2 For-profit monopoly

In this section we derive the equilibrium prices for a non-profit monopoly sharing platform while assuming that the service price is exogenous. First the quasi-demand functions are defined, where the first buyer-side quasi-demand function is applicable under the assumption of strict utilities and the second buyer-side quasi-demand function is applicable under the assumption of shifting utilities. The seller-side quasi-demand function is applicable under both assumptions. Second we derive a general solution for the optimal pricing decisions which is used to define closed-form equilibrium prices, platform profit and consumer welfare in both settings.

4.2.1 Quasi-demand functions

Following Hyun (2016) we assume that b^S is uniformly distributed on $[0, \bar{b}^S]$, b^B is uniformly distributed on $[0, \bar{b}^B]$ and v^B is uniformly distributed on $[0, \bar{v}^B]$. The distributions are independent of each other and independent of the price chosen by the platform and the seller. The buyer's demand depends on the transaction fee p^B charged by the platform and the service price s charged by the the seller. The seller's demand directly depends on the transaction fee p^S charged by the platform and indirectly on the service price. There are network externalities such that the per transaction net surplus of the seller (buyer) depends on the number of buyers (sellers) associated with the platform. However, like Rochet and Tirole (2003) we define buyer-side (seller-side) quasi-demand functions that are independent of the number of sellers (buyers).

4.2.1.a Buyer-side quasi-demand function 1

In this specification of the buyer-side demand we assume that the buyers evaluate the platform price and the service price separately. This means that (1) the buyers platform utility must be higher than the price paid to the platform and (2) the buyers service utility must be higher than the price paid for the service. Both statements need to hold in order to have any demand. The *buyer-side quasi-demand function 1* is then defined as follows:

$$D_1^B(p^B) = Pr(b^B \geq sp^B)Pr(v^B \geq s)$$

By using the uniform distributions and independence of utilities we obtain:

$$D_1^B(p^B) = \frac{(\bar{b}^B - sp^B)(\bar{v}^B - s)}{\bar{b}^B \bar{v}^B} \tag{1}$$

We choose to use this simple representation of the buyer-side demand so that we can derive intuitive results for the model and can compare our equilibrium results to those defined in Hyun (2016). The buyer-side quasi-demand has the following properties:

Remark 1. ***Properties buyer-side quasi-demand function 1***
Downward sloping and linear in p^B:

$$\frac{\partial D_1^B}{\partial p^B} = -\frac{s(\bar{v}^B - s)}{\bar{b}^B \bar{v}} < 0 \textit{ for all } \bar{b}^B \geq sp^B \textit{ and } \bar{v}^B \geq s; \textit{ else, } \frac{\partial D_1^B}{\partial p^B} = 0.$$
$$\frac{\partial^2 D_1^B}{\partial {p^B}^2} = 0.$$

Figure 2 illustrates how the buyer-side demand D^B is defined as a function of the platforms buyer-side transaction fee p^B. Point A in figure 2 corresponds to the condition that if $sp^B \geq \bar{b}^B$ then $D^B(p^B, s) = 0$. Usually one sees that when the price is zero, the demand reaches its maximum. However, in figure 2 the maximum demand is reached in point B where the value of the commission price becomes negative. In other words, the platform pays the buyer-side to join the platform. This happens because the buyer-side takes into account both the price paid to the platform and the price paid for the service. In this case, the service price is set such that only half of the buyer-side is expected to join the platform and hence (given a positive commission price) the platform will at most attract half of the potential buyer-side demand corresponding to point C. Because the platform has the ability to cross-subsidize between both sides of the platform, it in fact can be beneficial for the platform to set a negative buyer-side commission. As figure 2 clearly shows, a negative buyer-side commission increases the the buyer-side demand, which in turn increases the number of transaction and in some cases the platform's profit.

4.2.1.b Buyer-side demand function 2

In this demand specification we assume that the buyer can shift utilities. For example, when the buyer-side commission level is set such that the buyer has platform utility 'left-over' the buyer can shift this utility that is originally associated with the platform to the utility associated with the service price. The second buyer-side demand function allows

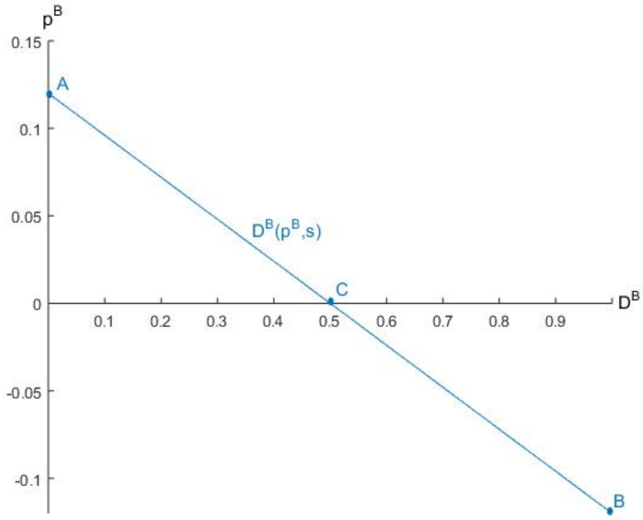

Figure 2: Buyer-side demand plotted against the buyer-side commission fee. Input parameters are $\bar{b}^B = 1.2$, $\bar{v} = 20$ and $s = 10$

for shifting in utilities and is defined as:

$$D_2^B(p^B) = Pr(b^B + v^B \geq sp^B + s)$$

Define $Z = v^B + b^B$ where v^B is uniformly distributed on $(0, \bar{v}^B)$ and b^B is uniformly distributed on $(0, \bar{b}^B)$. By using the convolution of two uniformly distributed variables the probability distribution function of Z is derived:

$$f_Z(z) = \begin{cases} \dfrac{z}{\bar{v}^B \bar{b}^B}, & \text{if } 0 \leq z \leq \bar{b}^B \\ \dfrac{1}{\bar{v}^B}, & \text{if } \bar{b}^B \leq z \leq \bar{v}^B \\ \dfrac{\bar{v}^B + \bar{b}^B - z}{\bar{v}^B \bar{b}^B}, & \text{if } \bar{v}^B \leq z \leq \bar{v}^B + \bar{b}^B \end{cases}$$

Using integration by parts the cumulative distribution function for the different intervals becomes:

$$F_Z(z) = \begin{cases} \dfrac{z^2}{2\bar{v}^B \bar{b}^B}, & \text{if } 0 \leq z \leq \bar{b}^B \\ \dfrac{2z - \bar{b}}{2\bar{v}^B}, & \text{if } \bar{b}^B \leq z \leq \bar{v}^B \\ \dfrac{(2\bar{v}^B + 2\bar{b})z - z^2 - (\bar{v}^B)^2 - (\bar{b}^B)^2}{2\bar{v}^B \bar{b}^B}, & \text{if } \bar{v}^B \leq z \leq \bar{v}^B + \bar{b}^B \\ 1 & \text{if } z \geq \bar{v}^B + \bar{b}^B \end{cases}$$

So for the demand function it is obtained that:

$$D_2^B(p^B) = \begin{cases} \dfrac{2\bar{v}^B\bar{b}^B - (sp^B+s)^2}{2\bar{v}^B\bar{b}^B}, & \text{if } 0 \leq (sp^B+s) \leq \bar{b}^B \\ \dfrac{2\bar{v}^B + \bar{b}^B - 2(sp^B+s)}{2\bar{v}^B}, & \text{if } \bar{b}^B \leq (sp^B+s) \leq \bar{v}^B \\ \dfrac{(\bar{v} + \bar{b}^B - s - sp^B)^2}{2\bar{v}\bar{b}^B}, & \text{if } \bar{v}^B \leq (sp^B+s) \leq \bar{v}^B + \bar{b}^B \end{cases}$$

Where the first case is not realistic, because the service price s will generally be larger than the upper limit of the benefit associated with the platform. The last case is also unlikely to occur, because when the total price paid by the buyer $(sp^B + s)$ is larger than the upper limit of the service utility $(\bar{v})$ the demand becomes very small. Therefore the second case is the most probable one, this case also has the largest interval. Hence, we assume that in the case of shifting utilities the buyer-side quasi-demand function is defined as:

$$D_2^B(p^B) = \frac{2\bar{v}^B + \bar{b}^B - 2sp^B - 2s}{2\bar{v}^B} \tag{2}$$

Again the most important properties for the buyer-side demand function 2 are stated in Remark 2, which also become clear from Figure 3.

Remark 2. ***Properties buyer-side quasi-demand function 2***
Downward sloping and linear in p^B:

$$\frac{\partial D_2^B}{\partial p^B} = -\frac{s}{\bar{v}^B} < 0 \textit{ for all } \bar{b}^B \geq sp^B \textit{ and } \bar{v}^B \geq s;\ \textit{else, } \frac{\partial D_2^B}{\partial p^S} = 0.$$

$$\frac{\partial^2 D_2^B}{\partial {p^B}^2} = 0$$

Figure 3 shows how the second buyer-side demand specification is defined as a function of the buyer-side transaction fee. Point A now corresponds to the conditions that if $sp^B + s \geq \bar{v}^B + 0.5\bar{b}^B$ the buyer-side demand becomes zero. This causes point A to be significantly higher and point B to be significantly lower than in Figure 2. At point C the buyer-side transaction fee is zero, because the 'left-over' buyer-side utility now can be used as extra service-utility the demand is higher than the demand at point C in Figure 2. Overall it holds that when the transaction fee is positive, the buyer-side demand is higher under the assumption of shifting utilities than under the assumption of strict utilities.

4.2.1.c Seller-side quasi-demand function

The seller-side quasi-demand function is defined as follows:

$$D^S(p^S) = Pr(b^S \geq sp^S) = \frac{\bar{b}^S - sp^S}{\bar{b}^S} \tag{3}$$

Again the demand depends on both the service price and the commission level, where we assume that service price is set such that after paying the fees the seller at least breaks

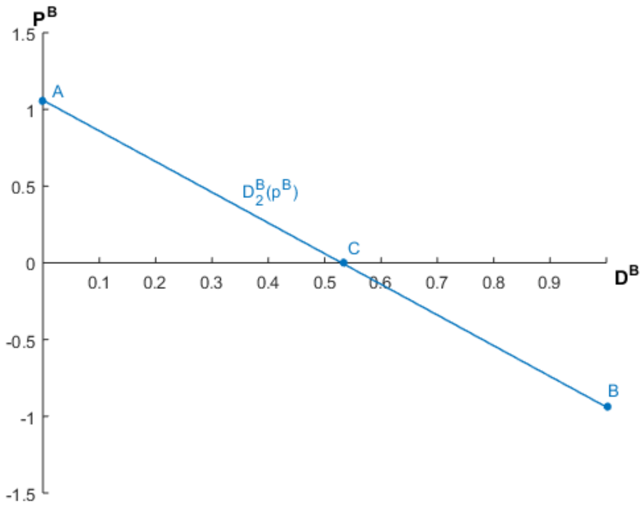

Figure 3: Buyer-side demand 2 plotted against the buyer-side transaction fee. Input parameters are $\bar{b}^B = 1.2$, $\bar{v} = 20$ and $s = 10$

even. Below we state the most important properties for the seller-side demand function:

Remark 3. ***Properties seller-side quasi-demand function***
Downward sloping and linear in p^S:

$$\frac{\partial D^S}{\partial p^S} = -\frac{s}{\bar{b}^S} < 0 \text{ for all } \bar{b}^S \geq p^S; \text{ else, } \frac{\partial D^S}{\partial p^S} = 0.$$

$$\frac{\partial^2 D^S}{\partial p^{S2}} = 0$$

Figure 4 shows the demand function for the seller side given different transaction fees. In point A the seller-side demand is zero, hence the platform will not choose buyer-side fee that is higher than the one in point A. This point corresponds to the condition that the seller-side commission fee times the service prices must be lower than the maximum seller-side utility. In point B the total transaction fee is zero and the seller-side demand is at its maximum.

Similar to Rochet and Tirole (2003) we take the matching process as given and focus on the proportion of matches that effectively result in a transaction. By assuming independence between b^B, v^B and b^S the proportion of transactions is equal to $D^B(p^B) * D^S(p^S)$. So, to reach the maximum number of transactions possible the platform must attract the upper bound on the number of buyers and the upper bound of the number of sellers. When the platform attracts all the possible sellers, but only a quarter of the possible buyers, they will only reach a quarter of the possible transactions. Hence it is important that the platform gets both sides of the market on board. Note that when deriving the pricing decision we only consider one buyer and one seller and hence talk about the probability of a transaction instead of the proportion of transactions.

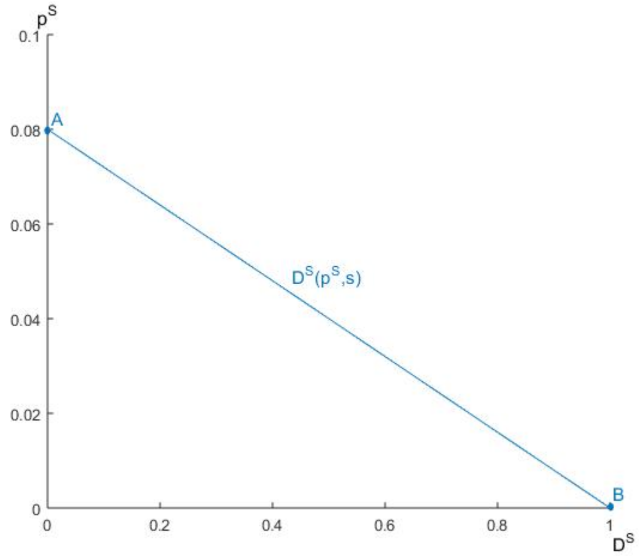

Figure 4: Seller-Side Demand plotted against the seller-side transaction fee. Input parameter are $\bar{b}^S = 0.8$ and $s = 10$.

4.2.2 Pricing decisions

In this section we derive the pricing decision for the for-profit monopoly platform. The per-transaction profit for the platform is given by $(sp^B + sp^S - c)$, where c is the platform's per transaction cost. As explained in the previous section the probability of an actual transactions is equal to $D^B * D^S$. Then the platform set the transactions fee to the maximizes the net profit:

$$\pi = (sp^B + sp^S - c)D^B(p^B, s)D^S(p^S, s) \tag{4}$$

Assuming that D^B and D^S are log concave, it is easily seen that π is also log concave. The maximization problem is then characterized by the following first-order conditions (FOCs):

$$\frac{\partial log(\pi)}{\partial p^B} = \frac{s}{sp^B + sp^S - c} + \frac{1}{D^B(p^B)}\frac{\partial D^B(p^B)}{\partial p^B} = 0 \tag{5a}$$

$$\frac{\partial log(\pi)}{\partial p^S} = \frac{s}{sp^B + sp^S - c} + \frac{1}{D^S(p^S)}\frac{\partial D^S(p^S)}{\partial p^S} = 0 \tag{5b}$$

From which the following conditions follows:

$$\frac{1}{D^B(p^B)}\frac{\partial D^B(p^B)}{\partial p^B} = \frac{1}{D^S(p^S)}\frac{\partial D^S(p^S)}{\partial p^S} \tag{6}$$

Price elasticities

The price elasticity of the quasi buyers and sellers demand are calculated using the definition: $e_d = |\frac{\partial Q}{\partial p}\frac{p}{Q}|$. Hence the price elasticity for the buyer side quasi-demand with respect to the buyer-side transaction feeis:

$$e_{D^B} = \left| \frac{p^B}{D^B(p^B)} \frac{\partial D^B(p^B)}{\partial p^B} \right| \rightarrow \frac{e_{D^B}}{p^B} = \left| \frac{1}{D^S(p^S)} \frac{\partial D^S(p^S)}{\partial p^S} \right|$$

And the price elasticity for the seller side quasi-demand with respect to the seller-side transaction fee is:

$$e_{D^S} = \left| \frac{p^S}{D^S(p^S)} \frac{\partial D^S(p^S)}{\partial p^S} \right| \rightarrow \frac{e_{D^S}}{p^S} = \left| \frac{1}{D^S(p^S)} \frac{\partial D^S(p^S)}{\partial p^S} \right|$$

By condition 6 we have that the following statement holds:

$$\frac{e_{D^B}}{p^B} = \frac{e_{D^S}}{p^S}$$

Combining the price elasticities with the first order conditions we obtain that:

$$\begin{aligned} -\frac{1}{D^B(p^B)} \frac{\partial D^B(p^B)}{\partial p^B} &= \frac{s}{sp^B + sp^S - c} \\ \frac{e_{D^B}}{p^B} &= \frac{s}{sp^B + sp^S - c} \end{aligned}$$

resulting in two conditions that resemble the Lerner formula for the buyer and seller-side demand respectively,

$$(sp^B + sp^S - c) = \frac{sp^B}{e_{D^B}} \tag{7}$$

$$(sp^B + sp^S - c) = \frac{sp^S}{e_{D^S}} \tag{8}$$

These results lead to a generalized version of Proposition 1 from Rochet and Tirole (2003) where the price characterization of the platform transaction fees looks like the classical Lerner formula. Were the only difference is the additional service price s that now plays a role in setting the transaction fees.

Proposition 4. *(a) The total commission fee of the monopoly sharing platform* $p = p^B + p^S$ *is given by the classical Lerner formula:*

$$sp - c = \frac{sp}{e}$$

(b) The fee structure is given by the ratio of elasticities that resembles the classical Lerner formula:

$$sp^B + sp^S - c = \frac{sp^B}{e_{D^B}} = \frac{sp^S}{e_{D^S}}$$

The Lerner formula suggests that there is some inverse relationship between markup over marginal cost and elasticity of demand. Market power is measured by the price elasticity of demand; more inelastic demand results in more market power and more elastic demand results in less market power. Intuitively this makes sense, when demand is more inelastic customers are less sensitive to price changes meaning that they will not change behavior much due a price change and this in turn makes it optimal for a firm to markup price more. On the other hand, when the price elasticity of demand is higher customers are more sensitive to a change in price an will change behavior a lot as response to a higher price.

In what follows the pricing decisions are derived for two different settings. The first setting is called: "Strict utilities". In this setting buyer-side quasi-demand function 1 is used. The second setting is called: "Shifting utilities", here buyer-side quasi-demand function 2 is used.

4.2.2.a Strict utilities

From remark 1 and 3 we obtain that D_1^B and D^S are log-concave. Hence we can substitute the demand functions and their derivatives in the maximization problem (eq. 2). The maximization problem is then characterized by using FOCs 5a and 5b:

$$sp^B + sp^S - c = \bar{b}^B - sp^B$$
$$sp^B + sp^S - c = \bar{b}^S - sp^S$$

This representation nicely shows that there is a trade-off when pricing both sides of the platform. The net benefits on both sides must be equal to obtain an optimal price.

Equilibrium prices

Using the price elasticities and the first order conditions we derive closed-form equilibrium prices for the platform.

$$\hat{p}^B_{fp_1} = \frac{1}{3s}[2\bar{b}^B - \bar{b}^S + c] \tag{9a}$$
$$\hat{p}^S_{fp_1} = \frac{1}{3s}[2\bar{b}^S - \bar{b}^B + c] \tag{9b}$$
$$\hat{p}_{fp_1} = \frac{1}{3s}[\bar{b}^B + \bar{b}^S + 2c] \tag{9c}$$

From these equations it is readily seen that an increase in the service price s, causes a decrease in both the buyer and seller-side transaction fee. This is logical, since we assumed that the buyer (seller) has a benefit b^B (b^S) associated with the platform that is independent of the service price. In other words, the buyer is not willing to pay more to the platform when the service becomes more expensive. Both transaction fees are ceteris paribus increasing in platform transaction costs, where the costs are equally divided over both sides of the platform.

The buyer-side transaction fee is ceteris paribus increasing in the buyer-side utility and decreasing in the seller-side utility. This trade-off captures the nature of two-sided markets; the platform aims to attract both sides of the market. When the seller has a higher utility than the buyer, the seller-side transaction fee will be higher than the buyer-side transaction fee in order to keep the net-utilities on both sides equal and attract the same amount of buyers and sellers. The equilibrium prices furthermore show that the transaction fees are not affected by the service utility, which stems from the assumption that the buyer evaluates the platform utility and service utility separately.

Compared to the equilibrium prices that can be derived from the for-profit monopoly two-sided market model in Rochet and Tirole (2003), the difference is the factor $\frac{1}{s}$. Where they make the assumption that the buyer and seller evaluate the transaction fee p, we included the service price and assumed that the buyer and seller evaluate the total amount paid to the platform sp.

Profit & Welfare

By plugging the closed-form equilibrium prices into the platforms profit function the optimal profit for a profit maximization platform under the assumption of strict utilities is derived:

$$\pi_{fp_1} = \frac{1}{27} \frac{(\bar{b}^B + \bar{b}^S - c)^3(\bar{v}^B - s)}{\bar{b}^B \bar{b}^S \bar{v}^B} \tag{10}$$

For the derivation of the welfare function W we refer to Section 4.3, here we merely state the welfare corresponding to the optimal profit.

$$W_{fp_1} = \frac{1}{27s} \frac{(\bar{b}^B + \bar{b}^S - c)^3(\bar{v}^B - s)}{\bar{b}^B \bar{b}^S \bar{v}^B} \tag{11}$$

Profit and welfare are both ceteris paribus increasing in buyer-side utility, seller-side utility and service-utility and decreasing in platform costs and service price. Where the welfare is a fraction s smaller than the the platform profit.

4.2.2.b Shifting utilities

From remark 3 it follows that that D_2^B is log-concave. Hence, the maximization problem in this case is also characterized by FOCs 5a and 5b resulting in:

$$\begin{aligned} sp^B + sp^S - c &= \bar{v}^B + 0.5\bar{b}^B - s - sp^B \\ sp^B + sp^S - c &= \bar{b}^S - sp^S \end{aligned}$$

These conditions already show that the trade-off in pricing both sides of the platforms is now also affected by $\bar{v}$.

Closed-form equilibrium prices

The following closed-form equilibrium prices are derived from the maximization problem:

$$\hat{p}^B_{fp_2} = \frac{1}{3s}\left(\bar{b}^B + 2\bar{v}^B - \bar{b}^S - 2s + c\right) \tag{12a}$$
$$\hat{p}^S_{fp_2} = \frac{1}{3s}\left(2\bar{b}^S - 0.5\bar{b}^B - \bar{v}^B + s + c\right) \tag{12b}$$
$$\hat{p}_{fp_2} = \frac{1}{3s}\left(\bar{b}^S + 0.5\bar{b}^B + \bar{v}^B - s + 2c\right) \tag{12c}$$

Comparing these equilibrium prices with the one derived for the first demand specification we see that, if $\bar{v} - s = 0.5\bar{b}^B$ the transaction fees are the same. Whenever there is enough utility left from the service price ($\bar{v} - s \geq 0.5\bar{b}^B$) the platform owner can use this to increase the buyer-side transaction fee. Even though the seller-side transaction fee becomes lower in that case, the platform's total transaction fee is still higher compared to for the strict utilities. Together with our finding that for a positive transaction fee the buyer-side demand function 2 is higher than buyer-side demand function 1, this indicates that the platform profit is higher in the case of shifting utilities. Furthermore for the applications that we have in mind it is unlikely that $(\hat{v} - s) \leq 0.5\bar{b}^B$, because the service utility is assumed to be significantly higher than platform utility. Therefore it is likely to be beneficial for the platform if buyers shift between the service utility and the platform utility.

Profit & Welfare

By plugging the optimal transaction fees into the platforms profit function the optimal profit for a profit maximization platform under the assumption of shifting utilities is derived:

$$\pi_{fp_2} = \frac{1}{27}\frac{(0.5\bar{b}^B + \bar{b}^S + \bar{v}^B - s - c)^3}{\bar{b}^S\bar{v}^B} \tag{13}$$

For the derivation of the welfare function W we again refer to Section 4.3, here we merely state the welfare corresponding to the optimal profit for this case:

$$W_{fp_2} = \frac{1}{27s}\frac{(0.5\bar{b}^B + \bar{b}^S + \bar{v}^B - s - c)^3}{\bar{b}^S\bar{v}^B} \tag{14}$$

Comparing the profit and welfare under the assumption of shifting utilities with the profit and welfare under the assumption of strict utilities we find that if,

$$(0.5\bar{b}^B + \bar{b}^S + \bar{v}^B - s - c)^3\bar{b}^B \geq (\bar{b}^B + \bar{b}^S - c)^3(\bar{v}^B - s)$$

the profit and welfare under the assumption of shifting utilities is higher than under the assumption of strict utilities. When plugging in $(\hat{v} - s) = 0.5\bar{b}^B$ (recall that under this assumption the transaction fee in both cases are equal), the profit and welfare under the assumption of utility shifting is $0.5\bar{b}^B$ higher than under the assumption of strict utilities. Not only confirming that it is indeed beneficial for the platform owner if users shift utilities, but also showing that it is beneficial for consumers to shift utilities.

4.3 Non-profit monopoly

For deriving the optimal pricing strategies of a non-profit monopoly platform Ramsey pricing is used. This means that we assume that a non-profit monopoly platform maximizes the welfare subject to the budget balance constraint. In this section we first define the total consumer welfare and then derive the net surplus for the different quasi-demand specifications defined in Section 4.2.1. Second a general formulation for the optimal pricing decisions is derived which is used to define the closed-from equilibrium prices, platform profit and consumer welfare under both the strict utilities and the shifting utilities assumption.

4.3.1 Welfare functions

When maximizing the welfare we assume that the consumer surplus is a valid measure of social welfare and define the welfare function as:

$$W = V^S(p^S)D^B(p^B) + V^B(p^B)D^S(p^S) \tag{15}$$

Where V^k is the net consumer surplus.

$$V^k(p^k) = \int_{sp^k}^{\infty} D^k(z)dz \tag{16}$$

The consumer net surplus can be viewed as the area under the demand function that lies left of the total price paid to the platform.

4.3.1.a Buyer-side welfare 1

Since we assumed that the consume surplus follows a uniform distribution on the interval $(0, \bar{b}^B)$, where $\bar{b}^B$ is the maximum benefit existing, the buyer-side demand is zero if $p^B \geq \bar{b}^B/s$. Hence, we can change the interval of integration into $\left[p^B, (b^B/s)\right]$. Using the buyer-side demand 1 the buyer-side net surplus for this case is then derived as follows:

$$\begin{aligned} V_1^B(p^B) &= \int_{p^B}^{\infty} D_1^B(z)dz = \int_{p^B}^{\bar{b}^B/s} \frac{(\bar{b} - sz)(\bar{v} - s)}{\bar{b}^B\bar{v}}dz \\ &= \left[-\frac{(\bar{v} - s)z(sz - 2\bar{b}^B}{2\bar{v}\bar{b}^B}\right]_{p^B}^{\bar{b}^B/s} \\ &= \frac{(sp^B - \bar{b}^B)^2(\bar{v} - s)}{2\bar{b}^B\bar{v}s} \end{aligned} \tag{17}$$

Where the buyer-side welfare 1 is then defined as: $W_1^B = V_1^B D^S$. And the properties of the buyer-side net surplus are:

Remark 5. ***Properties buyer-side net surplus 1***

Downward sloping and convex in p^B:

$$\frac{\partial V_1^B}{\partial p^B} = -\frac{(\bar{b}^B - sp^B)(\bar{v} - s)}{\bar{v}\bar{b}^B} = -D_1^B < 0$$
$$\frac{\partial^2 V_1^B}{\partial {p^B}^2} = \frac{s(\bar{v} - s)}{\bar{b}^B \bar{v}} > 0$$

4.3.1.b Buyer-side welfare 2

As motivated before only interval 2 for the second specification of the buyer-side demand is considered. Hence, we only define the net consumer surplus and welfare function on that specific interval. When $p^k \geq (\bar{v} - s + 0.5\bar{b})/s$, buyer-side demand function 2 becomes negative resulting in zero buyer-side demand. Therefore the interval of integration when deriving the net surplus for this case is $\left[p^k, (\bar{v} - s + 0.5\bar{b})/s\right]$.

$$\begin{aligned} V_2^B(p^B) &= \int_{p^B}^{(\bar{v}-s+0.5\bar{b}^B)/s} \frac{2\bar{v} + \bar{b} - 2zs - 2s}{2\bar{v}} dz \\ &= \left[-\frac{z(sz - 2\bar{v} + 2s - \bar{b}^B)}{2\bar{v}} \right]_{p^B}^{(\bar{v}-s+0.5\bar{b}^B)/s} \\ &= \frac{\left(2\bar{v} - 2s + \bar{b}^B - 2p^B s\right)^2}{8\bar{v}s} \end{aligned} \tag{18}$$

The buyer-side welfare 2 is then defined as: $W_2^B = V_2^B D^S$. The properties of the buyer-side net surplus are:

Remark 6. ***Properties buyer-side net surplus 2***
Downward sloping and convex in p^B:

$$\frac{\partial V_2^B}{\partial p^B} = -\frac{2\bar{v} + \bar{b}^B - 2sp^B - 2s}{2\bar{v}} = -D_2^B < 0 \quad \textit{Note that: } sp^B + s < \bar{v}$$
$$\frac{\partial^2 V_2^B}{\partial {p^B}^2} = \frac{s}{\bar{v}} > 0$$

4.3.1.c Seller-side welfare 1 & 2

Following the same motivation as for the buyer-side net surplus 1, the interval of integration for calculating the seller-side net surplus is $[p^S, b^S/s]$. The seller-side net surplus is then derived as follows:

$$\begin{aligned} V^S(p^S) &= \int_{p^S}^{\bar{b}^S/s} D^S(z)dz = \int_{p^S}^{\bar{b}^S/s} \left(\frac{\bar{b}^S - zs}{\bar{b}^S}\right)dz \\ &= \left[z - \frac{sz^2}{2\bar{b}^S} \right]_{p^S}^{\bar{b}^S/s} \\ &= \frac{(sp^S - \bar{b}^S)^2}{2\bar{b}^S s} \end{aligned} \tag{19}$$

The seller-side welfare 1 is defined as $W_1^S = V^S D_1^B$ and the seller-side welfare 2 is defined as $W_2^S = V^S D_2^B$. The properties of the seller-side net surplus are:

Remark 7. ***Properties seller-side net surplus***
Downward sloping and convex in p^S:

$$\begin{aligned}\frac{\partial V^S}{\partial p^S} &= -\frac{\bar{b}^B - sp^S}{\bar{b}^S} < 0 \\ \frac{\partial^2 V^S}{\partial p^{S2}} &= \frac{s}{\bar{b}^S} > 0\end{aligned}$$

Using these definitions we can derive the optimal pricing decision for the non-profit platform in both demand specifications. In the subsequent section we explicitly state the maximization problem and derive the equilibrium results.

4.3.2 Pricing decisions

The platform maximizes the welfare subject to the budget balance constraint:

$$\begin{aligned}&\max_{p^S,p^B} && W(p^S,p^B) = V^S(p^S)D^B(p^B) + V^B(p^B)D^S(p^S) \\ &\text{s.t.} && sp^B + sp^B = c\end{aligned}$$

From Remark 5, 6 and 7 we know that V^S and V^B are downward convex and by Remark 1, 2 and 3 we have that D^B and D^S are downward linear. Hence we have concave optimization problem subject to a linear constraint which can be solved using Lagrangian multipliers. The Lagrangian is defined as:

$$L(p^B, p^S, \lambda) = W(p^B, p^S) - \lambda(sp^B + sp^S - c)$$

resulting in the following system of equations to be solved:

$$\begin{aligned}\frac{\partial L}{\partial p^B} &= \frac{\partial W(p^B,p^S)}{\partial p^B} - \lambda s = 0 \\ \frac{\partial L}{\partial p^S} &= \frac{\partial W(p^B,p^S)}{\partial p^S} - \lambda s = 0 \\ \frac{\partial L}{\partial \lambda} &= sp^B + sp^S - c = 0\end{aligned}$$

It is easily seen that the optimization problem is characterized by two conditions:

$$\frac{\partial W}{\partial p^B} = \frac{\partial W}{\partial p^S} \tag{20a}$$

$$sp^B + sp^S = c \tag{20b}$$

where:

$$\frac{\partial W}{\partial p^B} = V^S(p^S)\frac{\partial D^B(p^B)}{\partial p^B} + D^S(p^S)\frac{\partial V(p^B)}{\partial p^B} = V^S(p^S)\frac{\partial D^B(p^B)}{\partial p^B} - D^S(p^S)D^B(p^B)$$
$$\frac{\partial W}{\partial p^S} = V^B(p^B)\frac{\partial D^S(p^S)}{\partial p^S} + D^B(p^B)\frac{\partial V(p^S)}{\partial p^S} = V^B(p^B)\frac{\partial D^S(p^S)}{\partial p^S} - D^B(p^B)D^S(p^S)$$

Using the quasi-demand function and price elasticities from section 4.2 we derive a generalization of Proposition 2 from Rochet and Tirole (2003):

Proposition 8. *Ramsey prices embody the average surpluses created on the others side of the market and are characterized by two conditions:*

$$sp^B + sp^S = c \textit{ (budget balance)}$$

and

$$\frac{sp^B}{e^B}\left[\frac{V^B}{D^B}\right] = \frac{sp^S}{e^S}\left[\frac{V^S}{D^S}\right]$$

Again the only difference with Rochet and Tirole (2003) is the additional parameter s that now plays a role in setting the transaction fees.

In the subsequent paragraphs we use the characterization of the optimization problem to derive the equilibrium prices for both demand specifications.

4.3.2.a Strict utilities

Under the assumptions of strict utilities the buyer and seller-side welfare are respectively characterized by W_1^B and W_1^S and the total consumer welfare is defined as $W_1 = W_1^B + W_1^S$. Using conditions 20a and 20b we state that the optimization problem is characterized by:

$$(\bar{b}^B - sp^B)^2 = (\bar{b}^S - sp^S)^2$$
$$sp^B + sp^S = c$$

These equations show that the optimal transaction fees for welfare maximization are set such that the net utilities on both sides of the market are equal. Note that we made a similar statement for the profit maximization sharing platform.

Equilibrium prices

The following closed-form equilibrium prices are derived:

$$\hat{p}^B_{np_1} = \frac{1}{2s}\left[\bar{b}^B - \bar{b}^S + c\right] \tag{21a}$$
$$\hat{p}^S_{np_1} = \frac{1}{2s}\left[\bar{b}^S - \bar{b}^B + c\right] \tag{21b}$$
$$\hat{p}_{np_1} = \frac{c}{s} \tag{21c}$$

The total transaction fee charged by the platform is set such that the budget balance constraint holds and hence does not depend on the utilities. The division of the total transaction fee over the buyer and seller-side depends on the difference in buyer and seller-side utility.

Compared to the optimal transaction fees for the profit maximizing platform (assuming strict utilities) we obtain that the total transaction fee is $\frac{1}{3s}[\bar{b}^B + \bar{b}^S - c]$ lower for the welfare maximizing platform. This difference in total transaction fee is equally divided over the buyer and seller-side transaction fee.

Profit & Welfare

When deriving the optimal non-profit transaction fee it is assumed that the platform breaks even resulting in zero platform profit:

$$\pi_{np_1} = 0 \tag{22}$$

The optimal welfare for a welfare maximizing sharing platform under the assumption of strict utilities is derived by plugging in the transaction fees into the total consumer welfare equation.

$$W_{np_1} = \frac{1}{8s}\frac{(\bar{b}^B + \bar{b}^S - c)^3(\bar{v}^B - s)}{\bar{b}^B\bar{b}^S\bar{v}^B} \tag{23}$$

Again the total consumer welfare is ceteris paribus increasing in buyer-side utility, seller-side utility and service utility and decreasing in platform costs and service price.

Compared to the consumer welfare in the for-profit sharing platform it is obtained that the consumer welfare is 3.375 times larger for the non-profit sharing platform (assuming strict utilities).

4.3.2.b Utility shifting

Under the assumptions of shifting utilities the total consumer welfare is defined as $W_2 = W_2^B + W_2^S$. Using conditions 20a and 20b we state that the optimization problem in this setting is characterized by:

$$(\bar{v} + 0.5\bar{b}^B - sp^B - s)^2 = (\bar{b}^S - sp^S)^2$$
$$sp^B + sp^S = c$$

Where again the total transaction fee divided such that the net-utilities on both sides of the platform are equal.

Equilibrium prices

In this setting the following closed-form equilibrium prices are derived:

$$\hat{p}^B_{w_2} = \frac{1}{2s}\left[0.5\bar{b}^B - \bar{b}^S + \bar{v} - s + c\right] \tag{24a}$$
$$\hat{p}^S_{w_2} = \frac{1}{2s}\left[-0.5\bar{b}^B + \bar{b}^S - \bar{v} + s + c\right] \tag{24b}$$
$$\hat{p}_{w_2} = \frac{c}{s} \tag{24c}$$

The total transaction fee is again defined such that the budget balance constraint holds and independent of the utilities. Equivalent to in the for-profit scenario, when $\bar{v} - s = 0.5\bar{b}^B$ the buyer and seller-side transaction fees under the assumption of shifting utilities are equal to those under the assumption of strict utilities.

Compared to the optimal transaction fees for the profit maximization platform we obtain that under the assumption of shifting utilities the total transaction fee is $\frac{1}{3s}[0.5\bar{b}^b + \bar{b}^s + \bar{v}^B - s - c]$ lower for the welfare maximizing platform. Again this difference in total transaction fees is equally divided over the buyer and seller-side.

Profit & Welfare

When deriving the non-profit equilibrium prices it is assumed that the platform breaks even resulting again in zero platform profit:

$$\pi_{np_2} = 0 \tag{25}$$

The maximum welfare when assuming shifting utilities is derived by plugging in the optimal transaction fees into the total welfare equation, resulting in:

$$W_{np_2} = \frac{1}{8s}\frac{(0.5\bar{b}^B + \bar{b}^S + \bar{v}^B - s - c)^3}{\bar{b}^S\bar{v}^B} \tag{26}$$

Comparing the welfare under the assumption of utility shifting with the welfare under the assumption of strict utilities we find a similar result as in the for-profit setting. Namely that if,

$$(0.5\bar{b}^B + \bar{b}^S + \bar{v}^B - s - c)^3\bar{b}^B \geq (\bar{b}^B + \bar{b}^S - c)^3(\bar{v}^B - s)$$

The welfare under the assumption of shifting utilities is higher than the welfare under the assumption of strict utilities. Indicating that in that case of a non-profit platform it also beneficial if the consumer can shift utilities.

Compared to the consumer welfare in the for-profit sharing platform the consumer welfare is 3.375 times larger for the non-profit sharing platform, which is the same under the assumption of strict utilities.

4.4 Conclusions

In this chapter we have derived and interpreted many results, therefore we give a short overview of the most important results so far before proceeding to the next chapter.

The following statements hold under both the strict and shifting utilities assumption:

- The buyer and seller-side transaction fees are strictly higher in the for-profit case than in the non-profit case.
- The profit in a non-profit sharing platform is zero and hence strictly lower than in a for-profit sharing platform.
- The consumer welfare is significantly lower in a for-profit sharing platform than in a non-profit sharing platform.

The following statements hold for both a non-profit and a for profit sharing platform:

- Under the assumption of shifting utilities the buyer-side transaction fee is higher, the seller-side transaction fee is lower and the total transaction fee is higher than under the assumption of strict utilities if $\bar{v} - s \geq 0.5\bar{b}^B$. Else, the opposite is true.
- The profit and total welfare are higher under the assumption of shifting utilities than under the assumption of strict utilities if $(0.5\bar{b}^B + \bar{b}^S + \bar{v}^B - s - c)^3\bar{b}^B \geq (\bar{b}^B + \bar{b}^S - c)^3(\bar{v}^B - s)$. Else, the opposite is true.
- Because it is most likely that $\bar{v} - s \geq 0.5\bar{b}^B$ and $(0.5\bar{b}^B + \bar{b}^S + \bar{v}^B - s - c)^3\bar{b}^B \geq (\bar{b}^B + \bar{b}^S - c)^3(\bar{v}^B - s)$, we claim that both the platform owner and the platform users benefit from the buyers ability to shift in utilities.

While keeping these statement in mind we elaborate on the model by endogenizing the service price in the next chapter.

5 Endogenizing the service price

In this chapter we extend the models in Chapter 4 by endogenizing the service price and combining the two buyer-side demand functions. Endogenizing the service price means that we include the service price as a decision parameter in the optimization problems. Two different cases of endogenizing the service price are considered. In the first case both the platform and seller are in control: the platform determines the transaction fees and the seller determines the service price. Here it is assumed that the platform is the first mover and the seller is the follower. In the second case only the platform is in control; the platform sets the transaction fees and the service price. Here the seller is a price-taker who joins the platform when the benefits outweighs the costs and the profit is positive. Before elaborating on these cases some preliminaries about the seller's role and the combined buyer-side demand function are stated. Second the maximization problems for both cases in the setting of a for-profit monopoly platform are derived and third the maximization problems in the setting of a non-profit monopoly platform. In Chapter 6 numerical results for all cases are discussed.

5.1 Preliminaries

Endogenizing the service price changes the role of the seller with respect to the monopoly sharing platform. The seller now has a more active role in either deciding or accepting the service price. Combining the buyer-side quasi-demand functions results in a more realistic demand specifications where there is a limit on the shifting between buyer-side platform and service utilities. First the role of the seller is described in more detail and second the combined demand function is evaluated.

The seller

Corresponding to the setting of a sharing platform, we assume that the service offered by the seller is a idle trait the seller posses. For example the seller owns a lawn mower, he uses the lawn mower now and then but most of the time it is just standing in the barn. The seller decides he wants to make some extra money by renting out his lawn mover and chooses to use an online platform for this. What price does the owner of the lawn mower then charges for this service?

First of all it is important to note that the owner of the lawn mower cannot increase his capacity as the demand for renting lawn mowers through the platform increases. Of course, in reality this is possible but then we are not talking about using idle capacity to make extra money and hence we are in the regular platform economy and not in the sharing platform economy. So when the demand for lawn mowers on the sharing platform increases, the sharing platform will try to increase the supply by attracting more sellers. The same reasoning holds for a platform like Airbnb, as demand for home renting increases Airbnb tries to attract more home-owners. Home-owners already associated with Airbnb will not buy more properties in order to increase the capacity on Airbnb (in reality this does happen, but that belongs to another discussion).

Based on this motivation we assume that when a seller sets the service price to maximize profits, quantities do not play a role in the maximization problem. The seller has fixed costs c^S, which embody all expenses he makes while delivering the service (e.g. they exclude sunk cost like the purchase cost of the lawn mower or purchase cost of the house). The net profit of the seller is obtained by the probability of a transaction times the profit per transaction. Define $g(s)$ as the *seller's net profit*, then:

$$g(s) = (s - sp^S - c^S)D^B(p^B, s) \tag{27}$$

Where we are from now on assuming that the seller-side quasi-demand is zero if $g(s) \leq 0$.

Buyer-side demand

Enabling the buyers to shift between the service and platform utility is a sensible assumption, however the practical implication of it are not always that realistic. When the service price is set endogenously, it can result in very high transaction fee and relatively low service prices. By means of an example it is easier to explain why this effect is unrealistic in practice. Imagine that you want to rent a room though a renting platform and you are considering two platforms:

1. Platform A. Here you pay 50 euros for the room to the seller and an extra transaction fee of 8%, equivalent to an amount of 4 euros, to the platform owner.

2. Platform B. Here you pay 25 euros for the room to the seller and an extra transaction fee of 116%, equivalent to 29 euros, to the platform owner.

In both situation the total price you (the buyer) pay is the same, it can however be argued whether someone is willing to pay a transaction fee of 116% to the platform. We argue that this is not realistic and propose a constraint that puts a cap on the utility shifting. By adding this constraint we capture the idea that a buyer is willing to pay a higher transaction fee when the service price is lower, but only up to a certain limit. The utility caps we propose are that in order to have any buyer-side demand, the transaction fee cannot be higher than the upper-bound of the platform utility and the service price cannot be higher than the upper-bound of the service utility. Figure 5 and 6 plot the buyer-side demand 1 and 2 for the transaction fee and service price respectively and show how the utility shifting constraints affect the buyer-side demand 2.

At point A in figure 5 the utility cap intersects with buyer-side demand 2, whenever the transaction fee is above point A the buyer-side demand is zero. At point B the buyer-side demand 1 intersects with the buyer-side demand 2, if the transaction fee lies between point A and B the buyer-side demand is higher in the second demand specification than in the first. When the transaction fee is lower than in point B the the buyer-side demand is higher in the first demand specification. Note that due the characteristics of the uniform distribution the latter is less likely to be optimal.

Figure 6 sketches a similar picture as figure 5. When the transaction fee is higher than in point A the utility shifting constraint makes the buyer-side demand zero. And when the transaction fee is between point A and B demand specification 2 ensures a higher demand than the first one.

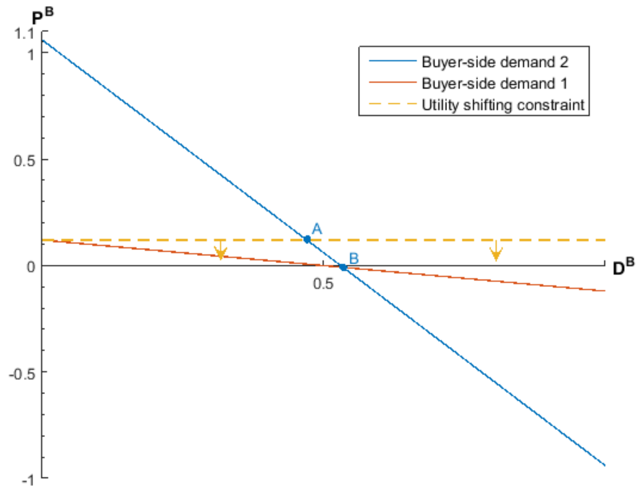

Figure 5: Buyer-side demand 1 and 2 plotted against buyer-side commission fee with the utility shifting constraint. Input parameters are $\bar{b}^B = 1.2$, $\bar{v} = 20$ and $s = 10$.

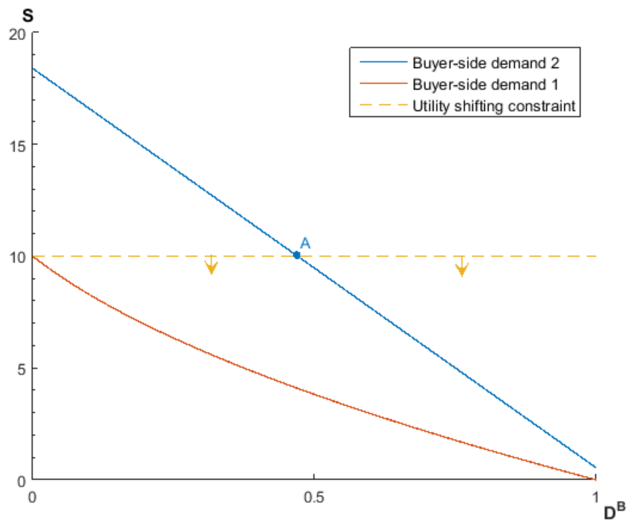

Figure 6: Buyer-side demand 1 and 2 plotted against service price with the utility shifting constraint. Input parameters are $\bar{b}^B = 1.2$, $\bar{v} = 20$ and $p^B = 0.12$.

5.2 For-profit monopoly

In general the maximization problem for a for-profit monopoly platform under the assumption of an endogenous service price extends to:

$$\begin{aligned}
\max_{p^S, p^B, s} \quad & \pi(p^B, p^S, s) = \left(sp^S + sp^B - c\right) D_2^B(p^B, s) D^S(p^S, s) \\
\text{s.t.} \quad & sp^B \leq \bar{b}^B \\
& s \leq \bar{v}^B
\end{aligned}$$

Where in case 1 the service price can be expressed as a function of the transaction fees and in case 2 the service price is simply a decision parameter.

5.2.1 Case 1: Platform and seller in control

In this case both the seller and the platform are in control. The platform sets the transaction fees and has the first mover advantage, the seller moves second and decides on the service price while taking into account the transaction fees. This specific case is assessed as a Stackelberg game, where the equilibrium can be solved by backward induction. First the follower's (i.e. the seller) problem must be solved to get the optimal response function to the leader's (i.e. the platform) decision. Then the platform's decision problem is solved while considering the optimal response of the seller. For every possible platform decision, the seller's optimal reaction can be determined by considering the platform's transaction fees as input parameters. Finally, the platform identifies the transaction fees that lead to the optimal payoff, assuming that the seller takes the optimal response to those transaction fees.

The seller's reaction function is derived by maximizing the seller's profit function:

$$\max_{s} \; g(s) = (s - sp^S - d) D_2^B(p^B, s)$$

By the first order condition we obtain that:

$$\frac{(1 - p^S)(2\bar{v}^B + \bar{b}^B - 4sp^B - 4s) + 2d(p^B + 1)}{2\bar{v}^B} = 0$$

and,

$$s = \frac{(p^S - 1)\bar{b}^B - 2(dp^B + d + \bar{v}^B) + 2p^S\bar{v}^B}{4(p^B + 1)(p^S - 1)}$$

Since we do not aim to find closed-form equilibrium prices and plugging in the closed-form expression for s complicates the maximization problem significantly, the optimal reaction function of the seller is introduced as a constraint in the maximization problem.

The maximization problem for a for-profit monopoly sharing platform where the platform and seller are in control is defined as:

$$\max_{p^S, p^B, s} \quad \pi(p^S, p^B, s) = \left(sp^S + sp^B - c\right) D_2^B(p^B, s) D^S(p^S, s)$$

s.t. Positive demand constraints:

$$D_2^B(p^B, s) \geq 0 \tag{28a}$$

$$D^S(p^S, s) \geq 0 \tag{28b}$$

$$g(s) \geq 0 \tag{28c}$$

Optimal response function:

$$g'(s) = 0 \tag{28d}$$

Utility shifting constraints:

$$sp^B \leq \bar{b}^B \tag{28e}$$

$$s \leq \bar{v}^B \tag{28f}$$

Commission fee constraints:

$$-1 \leq p^S \leq 1 \tag{28g}$$

$$-1 \leq p^B \leq 1 \tag{28h}$$

Where the positive demand constraints capture the assumption that there is no optimization problem when there is no demand. Constraints 28a and 28b are needless to explain, constraint 28c is equivalent to the assumption that the seller only joins the platform if the expected profit is positive. As explained above, the optimal response function is introduced as a constraint in 28d. The utility shifting constraint are represented in constraints 28e and 28f. Following the assumption that the transaction fees are a percentage of the service price, the transaction fees are bounded below by -1 and bounded above by 1 which is captured in constraints 28g and 28h.

5.2.2 Case 2: Platform in full control

In this case it is assumed that the platform determines both the service price and the transaction fees. Now the seller is no longer a profit maximizer, but the seller decides either to join the platform or not. It is assumed that the seller joins the platform when the seller-side demand and the seller's profit is positive, i.e. when $b^S \geq sp^S$ and $g(s) \geq 0$.

The maximization problem for a for-profit monopoly sharing platform where the platform is in full control is then defined as:

$$
\begin{aligned}
\max_{p^S,p^B,s} \quad & \pi(p^S,p^B,s) = \big(sp^S + sp^B - c\big)\, D^B(p^B,s)D^S(p^S,s) \\
\text{s.t.} \quad & \text{Positive demand constraints:} \\
& D_2^B(p^B,s) \geq 0 && \text{(29a)} \\
& D^S(p^S,s) \geq 0 && \text{(29b)} \\
& g(s) \geq 0 && \text{(29c)} \\
& \text{Utility shifting constraints:} \\
& sp^B \leq \bar{b}^B && \text{(29d)} \\
& s \leq \bar{v}^B && \text{(29e)} \\
& \text{Commission fee constraints:} \\
& -1 \leq p^S \leq 1 && \text{(29f)} \\
& -1 \leq p^B \leq 1 && \text{(29g)}
\end{aligned}
$$

This maximization problem is equivalent to the maximization problem when both the sharing platform and seller are in control. The only difference lies in the absence of the optimal response function (constraint 28d).

5.3 Non-profit monopoly

The maximization problem for a non-profit monopoly platform under the assumption of an endogenous service price in general extends to:

$$
\begin{aligned}
\max_{p^S,p^B,s} \quad & W(p^S,p^B,s) = V^S D^B + V^B D^S \\
\text{s.t.} \quad & \pi(p^S,p^B,s) = 0 \\
& sp^B \leq \bar{b}^B \\
& s \leq \bar{v}^B
\end{aligned}
$$

Where it is important to note that only the platform is assumed to be a non-profit organization, the seller acts individually and is still assumed to maximize profit.

In this setting it is assumed that the platform owner does not only determine the transaction fees but also determines (or affects) the service price. Therefore the net utility functions derived in the previous chapter have to be adjusted, the service price s becomes a decision parameter instead of an input parameter. The buyer-side net utility now depends on the total price paid by a buyer $sp^B + s$ and the total utility associated with the service and the platform $\bar{b}^B + \bar{v}^B$. The sellers utility depends on the price paid to the platform by a seller sp^S and the total utility with the platform $\bar{b}^S$. Using the second buyer-side demand specification the net-utility for the buyer-side becomes:

$$V_3^B(z) = \int_{sp^B+s}^{\bar{b}^B+\bar{v}^B} \frac{2\bar{v}^B + \bar{b}^B - 2(z)}{2\bar{v}^B} dz = \left[-\frac{z(z - 2\bar{v}^B - \bar{b}^B)}{2\bar{v}^B} \right]_{sp^B+s}^{\bar{b}^B+\bar{v}^B}$$
$$= \frac{(\bar{v}^B - sp^B - s)(\bar{v}^B - sp^B - s + \bar{b}^B)}{2\bar{v}^B}$$

and the net utility for the seller-side becomes:

$$V_2^S(z) = \int_{sp^S}^{\bar{b}^S} \frac{\bar{b}^S - z}{\bar{b}^S} dz = \left[z - \frac{z^2}{2\bar{b}^S} \right]_{sp^S}^{\bar{b}^S}$$
$$= \frac{(sp^S - \bar{b}^S)^2}{2\bar{b}^S}$$

In the remainder of this chapter these net-utilities are assumed when defining the maximization problem for case 1 and case 2.

5.3.1 Case 1: Platform and seller in control

For this scenario we make similar assumptions as in the for-profit setting. The differences are in objective function and the break-even constraint. The maximization problem for a non-profit monopoly sharing platform where both the platform and seller are in control is defined as:

$$\max_{p^S, p^B, s} \quad W(p^S, p^B, s) = V_2^S(p^S, s) D_2^B(p^B, s) + V_3^B(p^B, s) D^S(p^S, s)$$

s.t. Demand constraints:

$$D_2^B(p^B, s) \geq 0 \tag{30a}$$
$$D^S(p^S, s) \geq 0 \tag{30b}$$
$$g(s) \geq 0 \tag{30c}$$

Platform break-even constraint:

$$\pi(p^S, p^B, s) = 0 \tag{30d}$$

Seller profit maximization constraint:

$$g'(s) = 0 \tag{30e}$$

Utility shifting constraints:

$$sp^B \leq \bar{b}^B \tag{30f}$$
$$s \leq \bar{v}^B \tag{30g}$$

Commission fee constraints:

$$-1 \leq p^S \leq 1 \tag{30h}$$
$$-1 \leq p^B \leq 1 \tag{30i}$$

5.3.2 Case 2: Platform in full control

Also for this setting the assumptions are similar to the one in the for-profit setting, the difference is again the objective function and the break-even constraint. The maximization problem for a non-profit monopoly sharing platform where the platform is in full control is defined as:

$$\max_{p^S, p^B, s} \quad W(p^S, p^B, s) = V^S(p^S, s)D_2^B(p^B, s) + V^B(p^B, s)D^S(p^S, s)$$

$$\text{s.t.} \quad \text{Demand constraints:}$$

$$D_2^B(p^B, s) \geq 0 \tag{31a}$$

$$D^S(p^S, s) \geq 0 \tag{31b}$$

$$\text{Break-even constraints:}$$

$$g(s) = 0 \tag{31c}$$

$$\pi(p^S, p^B) = 0 \tag{31d}$$

$$\text{Utility shifting constraints:}$$

$$sp^B \leq \bar{b}^B \tag{31e}$$

$$s \leq \bar{v}^B \tag{31f}$$

$$\text{Commission fee constraints:}$$

$$-1 \leq p^S \leq 1 \tag{31g}$$

$$-1 \leq p^B \leq 1 \tag{31h}$$

In the next chapter numerical results for all the defined maximization problems are presented and discussed.

6 Numerical results

To obtain numerical results for the optimization problems we used the Matlab toolbox YALMIP. An example of the Matlab code is attached in Appendix A. In this section first the results for a for-profit monopoly sharing platform are discussed. Second we discuss the result for the non-profit monopoly sharing platform. Finally, we compare the numerical results in the for-profit scenario with those in the non-profit scenario. Note that at the beginning of a subsection when we state which parameter is treated as variable an which are treated as fixed, the statement holds for the remainder of that subsection.

6.1 For-profit

In this section numerical results for a for-profit monopoly sharing platform are presented. For each of the input parameters the effects on profit, welfare, transaction fees and service price are discussed. These performance indicators are compared for the setting where both the seller and platform are in control (C1) and the setting where the platform is in full control (C2).

6.1.1 Service utility

Figures 7 and 8 show the profit and welfare for an increasing service utility when the other input parameters are fixed [1].

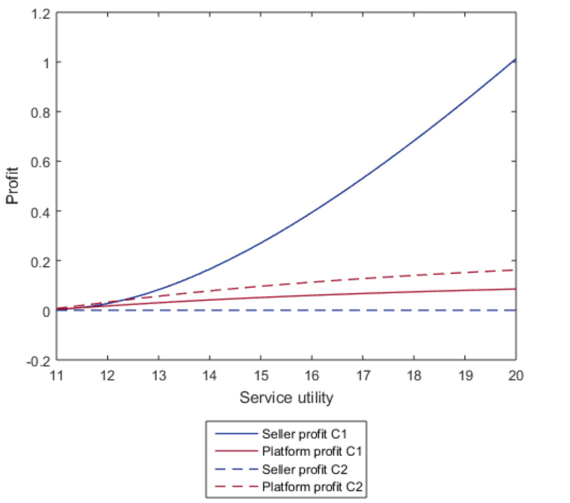

Figure 7: Platform and seller profit plotted against service utility for case 1 and 2. [1]

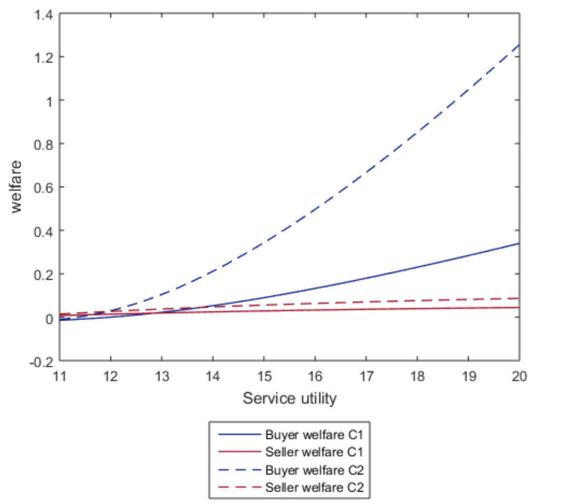

Figure 8: Platform and seller welfare plotted against service utility for case 1 and 2. [1]

[1]Input parameters are $\bar{b}^B = 1.2$, $\bar{b}^S = 1$, $c = 1$ and $d = 10$.

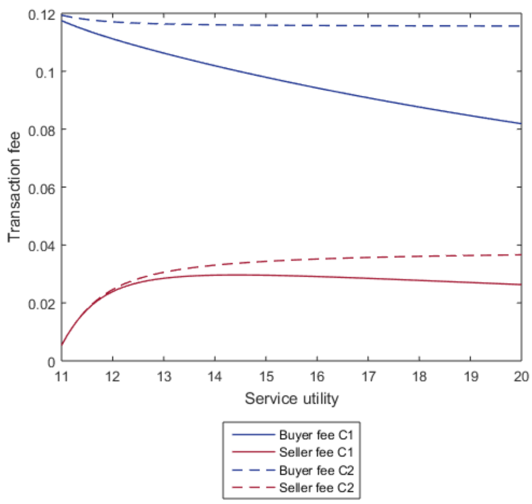

Figure 9: Buyer and seller-side fee plotted against service utility for case 1 and 2. [1]

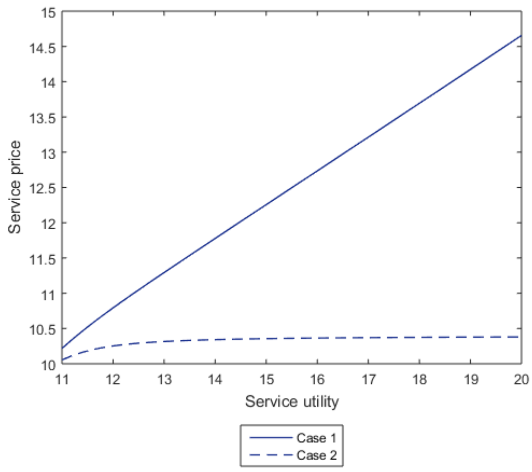

Figure 10: Service price plotted against service utility for case 1 and 2. [1]

When both the platform and seller are in control (C1) the seller's profit is significantly higher than the platform's profit. When the platform is in full control (C2) the seller's profit drops to zero while the platform's profit only slightly increases. The total profit is higher if the seller sets the service price (C1) than if the platform sets the service price (C2), which can be explained by the fact that the platform can only use a part of the 'left over' service utility to increase their transaction fees. This difference in total profit is also reflected in figure 8, where it is seen that if the platform is in partial control (C1) the total welfare per transaction is lower than when it is in full control (C2).

The seller-side welfare is not affected much by an increasing service utility. Recall that the service utility is part of the buyer-side welfare, which explains why the seller-side welfare is little affected by it.

The buyer-side welfare is increasing with the service utility. When the platform is in full control (C2) the service price is set such that the seller breaks even, causing the service price to be significantly lower (see fig. 10). As a consequence the platform owner can us part of the 'left over' service utility to increase the buyer-side transaction fee (see fig. 9). However, due to the utility shifting constraints a significant amount of the service utility cannot be used, which is reflected in a relatively high buyer-side welfare for case 2 (see fig. 8).

6.1.2 Buyer-side utility

In this section we evaluate the effect of an increasing buyer-side utility on the performance indicators while the other input parameters are fixed [2].

Figures 11 and 12 show the profit and welfare for an increasing buyer-side utility. Note that when the buyer-side utility is below 1, the seller's welfare is negative in both cases

[2]Input parameters are $\bar{v} = 15$, $\bar{b}^S = 1$, $c = 1$ and $d = 10$.

and there will be no seller engaged with the platform. Therefore, we only discuss the graphs on the interval where the buyer-side utility is larger than 1.

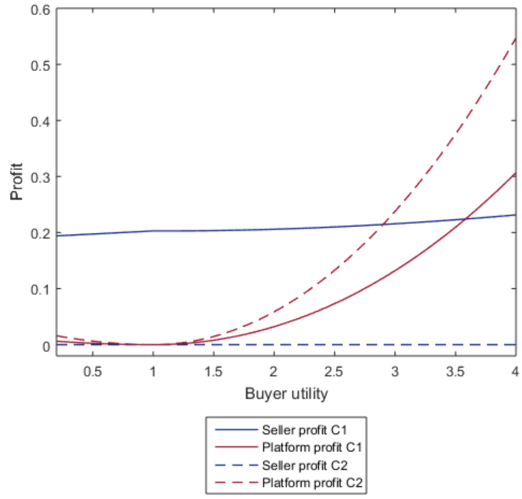

Figure 11: Platform and sellers profit plotted against buyer utility for case 1 and 2. [2]

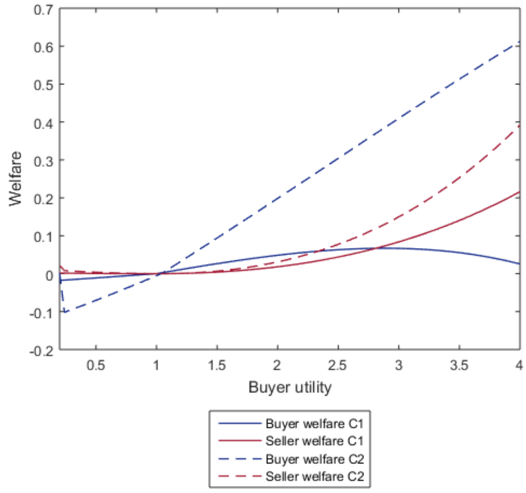

Figure 12: Platform and seller welfare plotted against buyer utility for case 1 and 2. [2]

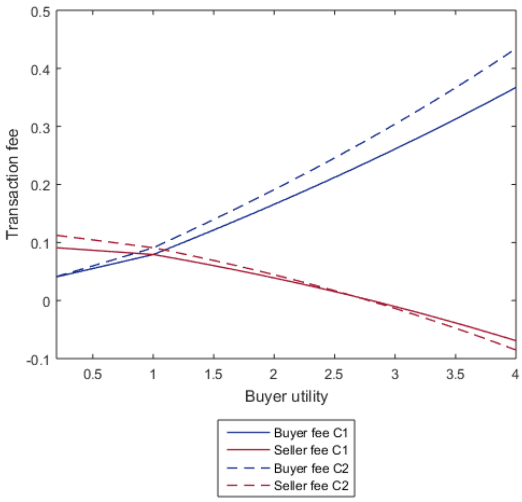

Figure 13: Buyer and seller-side fee plotted against buyer utility for case 1 and 2.[2]

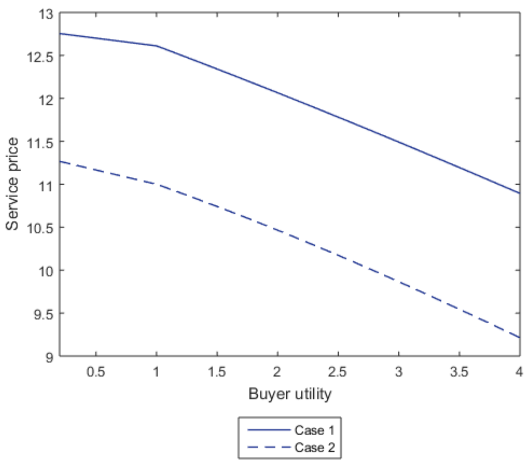

Figure 14: Service price plotted against buyer utility for case 1 and 2. [2]

The seller's profit does not increase much for an increasing buyer-side utility, while the platform's profit increases rapidly. A higher buyer-side utility directly increases the total transaction fee (see fig. 13) which in turn causes the platform profit to increase. The small increase in the seller profit is not due to an increase in the service price but due to a decreasing seller-side transaction fee (see fig. 13 and 14). Again we obtain that both the buyer and seller-side welfare are higher when the platform is in full control (C2) than when the platform and seller are in control (C1). The significantly higher buyer-side welfare is, as explained before, caused by the utility shifting constraints.

For both cases the transaction fees for both sides are the same when the buyer and seller-side utility are equal (see fig. 13). Moreover, if the buyer-side utility is significantly higher than the seller-side utility the seller-side transaction fee becomes negative, meaning that the platform owner subsidizes that part of the market. Recall that the buyer-side demand also depends on the service utility, so for the platform to benefit from the increase in buyer-side utility they have to make sure that the service price decreases (see fig. 14). If the platform is in full control (C2) the platform can lower the service price, however as soon as the seller's profit becomes negative the demand is lost. To keep the seller-side demand and profit positive, they therefore subsidize the seller-side. If both the platform and seller are in control (C1), the platform can use a similar tactic. By subsidizing the seller-side, they push the service price to decrease, resulting in more buyer-side demand. This intuition also explains the counter-intuitive result of an increasing seller's profit and a decreasing service price (C1), lowering the service price increases the buyer-side demand which in turn can increase the seller's profit.

6.1.3 Seller-side utility

In this section we let the service utility increase and treat the other input parameters as fixed [3].

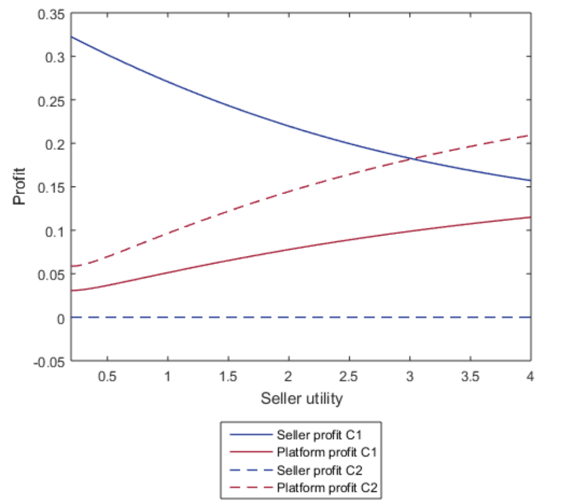

Figure 15: Platform and seller profit plotted against seller utility for case 1 and 2.[2]

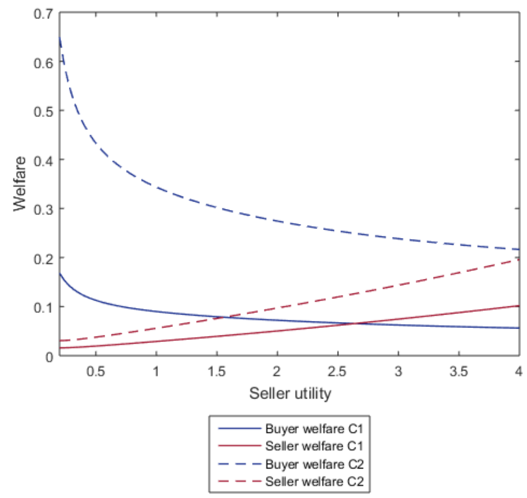

Figure 16: Platform and seller welfare plotted against seller utility for case 1 and 2.[2]

Figure 15 shows that when both the platform and seller are in control, the seller's profit is decreasing with the seller's utility while the platform's profit is increasing with the seller's utility. This reflects that the higher the seller's utility is, the more the platform can charge to the seller-side which lowers the seller's profit (see also fig. 17). When the platform is in full control (C2), obviously the seller's profit is zero and the platform's profit is increasing with the utility and strictly higher than when the platform is not

[3] Input parameters are $\bar{v} = 15$, $\bar{b}^B = 1.2$, $c = 1$ and $d = 10$.

in full control (C1). From figure 16 we see that for both cases the seller's welfare is increasing and the buyer's welfare is decreasing with an increasing seller's utility. For the same reasons as before the buyer-side and seller-side welfare are higher if the platform is in full control (C2).

As expected, the seller-side transaction fee and the service price are increasing with the seller's utility and the buyer-side transaction fee is decreasing with the seller-side utility. Overall, when the platform is in full control (C2) the service price is lower, which causes the transaction fees for that case to be higher. (see fig. 17, 18)

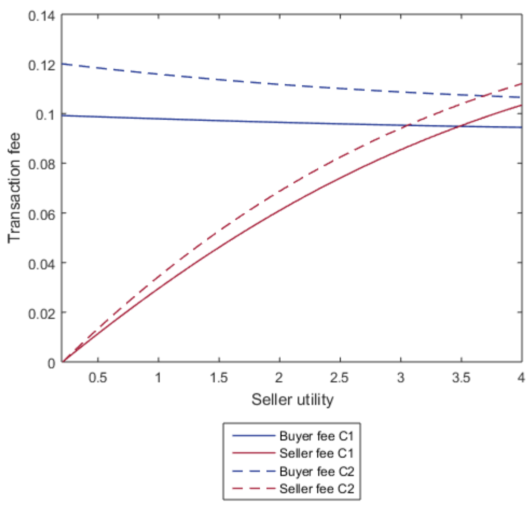

Figure 17: Buyer and seller-side fee plotted against seller utility for case 1 and 2.[2]

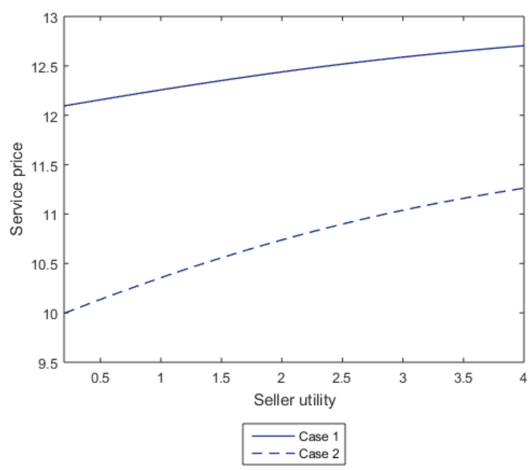

Figure 18: Service price plotted against seller utility for case 1 and 2.[2]

6.2 Non-profit

In this paragraph the results for the non-profit scenarios are presented. For each of the input parameters we again discuss the effects on profit, welfare, transaction fees and service price. The performance indicators are compared for the case where both the seller and platform are in control (C1) and the case where the platform is in full control (C2).

6.2.1 Service utility

In this section we let the service utility increase and treat the other input parameters as fixed [4].

In figure 19 we see that only if the seller sets the service price (C1) a profit is made. This profit is then increasing with the service utility. All other profits are forced to be zero by the budget balance constraints. The seller in case 1 still earns a profit because the assumption that the seller is a profit maximizer when setting the service price still holds. From figure 20 we see that letting the seller set the service price (C1) has a very strong

[4]Input parameters are $\bar{b}^B = 1.2$, $\bar{b}^S = 1$, $c = 1$ and $d = 10$.

negative effect on the buyer's welfare and a mediate negative effect on the seller's welfare. However, it can be clearly seen in figures 19 and 20 that a small increase in the seller's welfare is at the expense of a large decrease in the sellers profit. So, while a platform that is in full control (C2) is clearly more beneficial for the buyer, a situation in which both the platform and the seller are in control (C1) seems to be more beneficial for the seller.

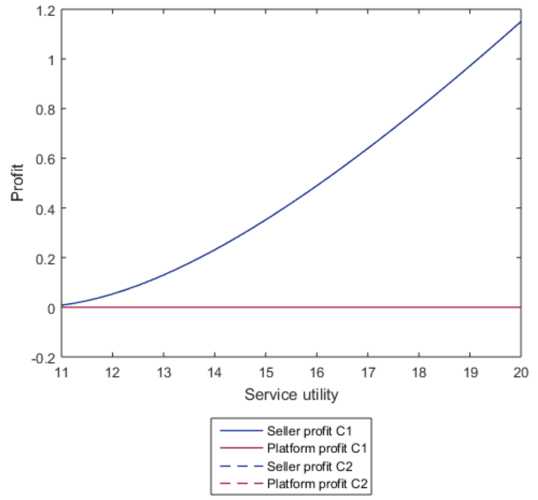

Figure 19: Platform and seller profit plotted against service utility for case 1 and 2.[4]

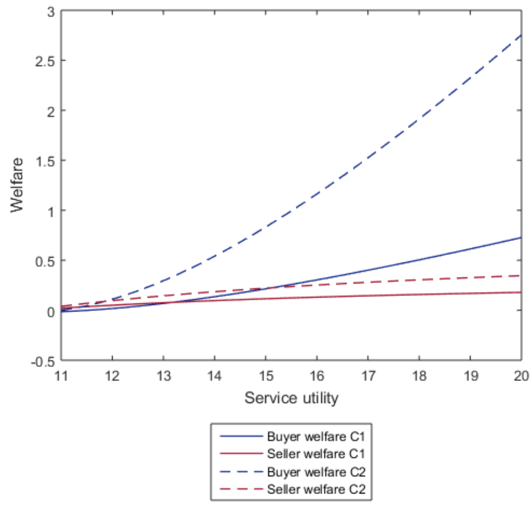

Figure 20: Platform and seller welfare plotted against service utility for case 1 and 2.[4]

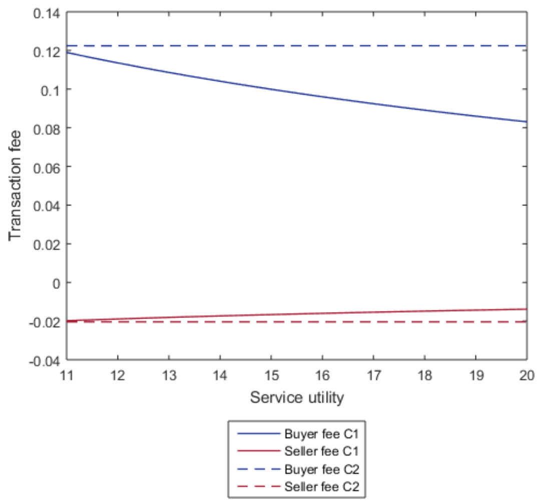

Figure 21: Buyer and seller-side fee plotted against service utility for case 1 and 2. [4]

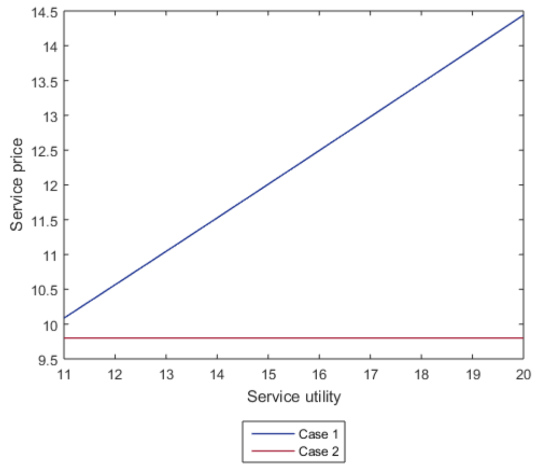

Figure 22: Service price plotted against service utility for case 1 and 2. [4]

Most importantly the figures 21 and 22 show that when the platform is in full control (C2) the transaction fees are fixed for an increasing service utility. Both the platform and the seller break-even in this case, which causes the service price to be fixed for an increasing service utility. A fixed service price and the utility shifting cap in turn cause the transaction fees to be fixed.

6.2.2 Buyer-side utility

In this section we let the buyer-side utility increase and treat the other input parameters as fixed [5].

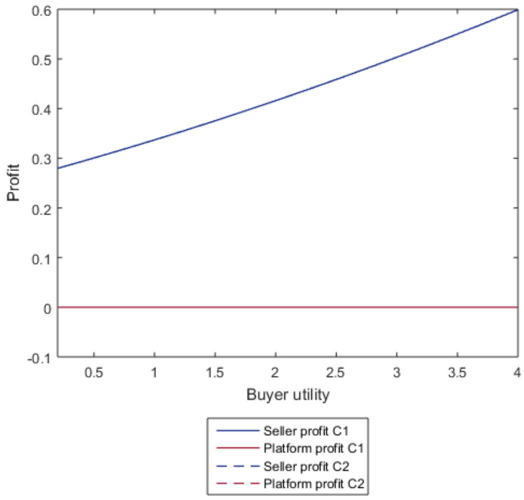

Figure 23: Platform and sellers profit plotted against buyer utility for case 1 and 2.[5]

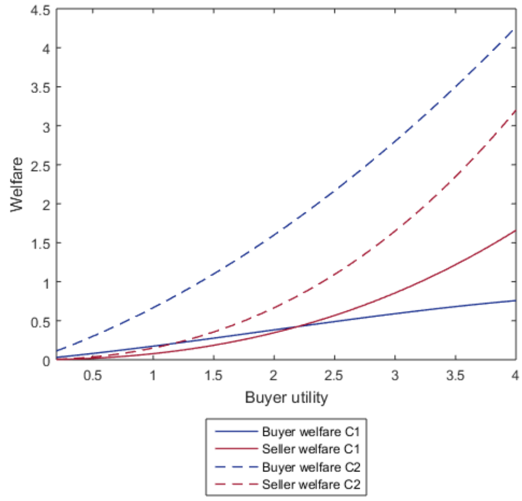

Figure 24: Platform and seller welfare plotted against buyer utility for case 1 and 2.[5]

The results in figures 23 and 24 are similar to those for an increasing service utility and do not need further explanation. Furthermore, we obtain that in both cases the buyer-side

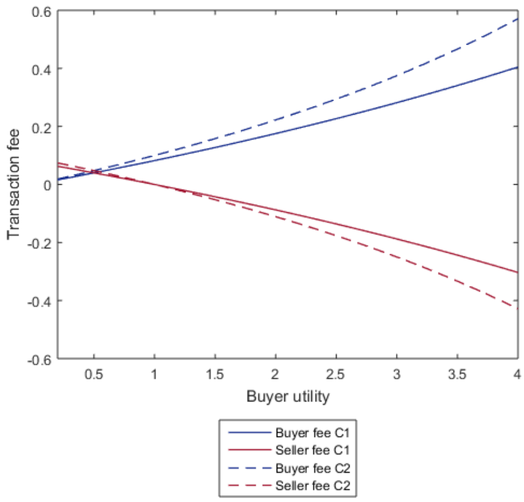

Figure 25: Buyer and seller-side commission fee plotted against buyer utility for case 1 and 2. [5]

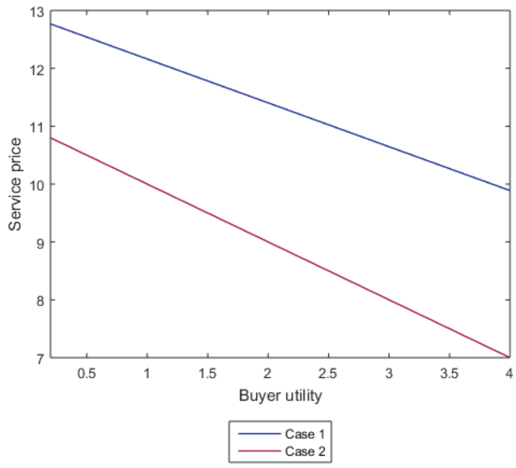

Figure 26: Service price plotted against buyer utility for case 1 and 2. [5]

fee is increasing and the seller-side fee and service price are decreasing in the buyer-side utility (fig. 25, 26). In both scenarios we obtain that if the buyer-side utility is higher than the seller-side utility the seller-side transaction fee becomes negative, meaning that the

[5]Input parameters are $\bar{v} = 15$, $\bar{b}^S = 1$, $c = 1$ and $d = 10$.

platform owner subsidizes that part of the market. Recall that in the for-profit scenario we obtained a similar result, however the subsidizing of the seller-side is significantly more extreme for the non-profit scenario. Using the same intuition as in the for-profit scenario, the intuition behind the decreasing service price can be explained.

6.2.3 Seller-side utility

In this section we let the buyer-side utility increase and treat the other input parameters as fixed [6].

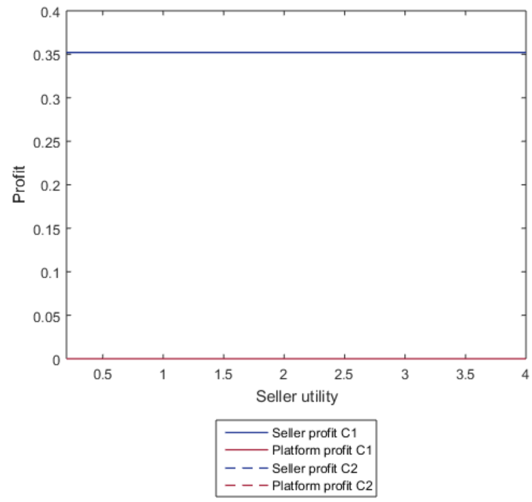

Figure 27: Platform and seller profit plotted against seller utility for case 1 and 2.[6]

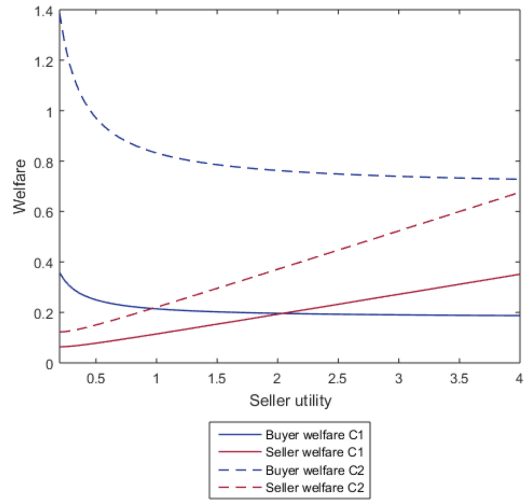

Figure 28: Platform and seller welfare plotted against seller utility for case 1 and 2.[6]

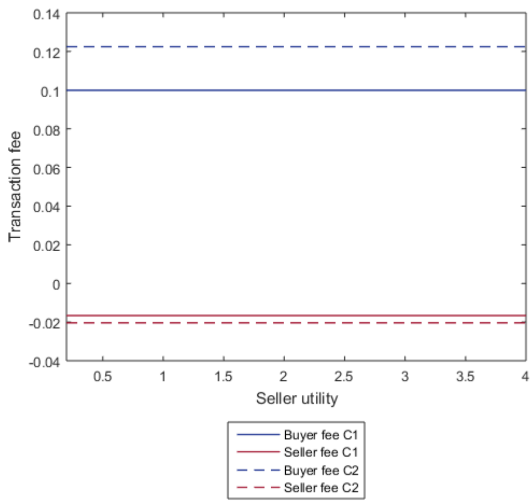

Figure 29: Buyer and seller-side commission fee plotted against seller utility for case 1 and 2. [6]

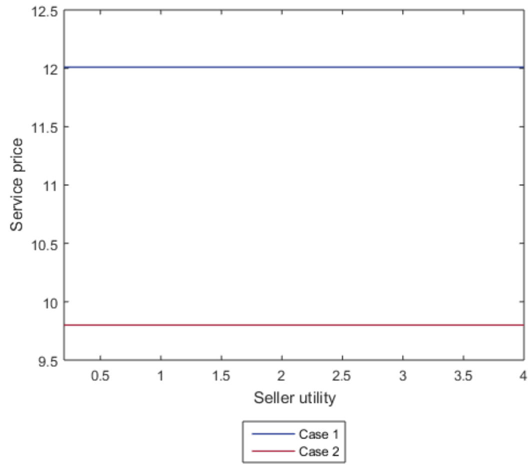

Figure 30: Service price plotted against seller utility for case 1 and 2. [6]

[6] Input parameters are $\bar{v} = 15$, $\bar{b}^B = 1.2$, $c = 1$ and $d = 10$.

When both platform and seller are in control (C2), the profit earned by the seller is positive and constant for an increasing seller utility. From figure 28 we obtain that the buyer-side welfare decreases and the seller-side welfare increases with the seller-side utility. Figures 29 and 30 show a picture we have not seen before. The transaction fees and service price are in both cases constant for an increasing service utility. The seller utility has no impact on both the service price and the transaction fees.

6.3 Conclusions

In this section we give a summary of the results for the non-profit and for-profit numerical results in both cases. We focus on how these different business strategies affect the seller's profit, the platform's profit, the buyer welfare and the seller welfare.

- When in a for-profit platform both the platform owner and seller are in control, we obtain that both the seller and platform profit are relatively high but the buyer and seller welfare are relatively low.
- If the for-profit platform is in full control, the seller's profit becomes zero and the platform owner has, compared to in the other strategies, the highest possible profit. The buyer welfare is relatively high in this scenario, the seller welfare is however low.
- When in a non-profit platform both the platform owner and seller are in control, the seller has, compared to in the other scenarios, the highest possible profit. The platform owner has zero profit and both the buyer and seller welfare are relatively high.
- If the non-profit platform is in full control, both the seller and platform have zero profit. For this business strategy the buyer and seller welfare are the highest.

Table 3 gives an overview of these results.

	For-profit	**Non-profit**
Platform & seller in control	✓ High seller profit ✓ High platform profit ✓ Medium buyer welfare ✓ Medium seller welfare	✓ Highest seller profit ✓ Zero platform profit ✓ High buyer welfare ✓ High seller welfare
Platform in control	✓ Zero seller profit ✓ Highest platform profit ✓ High buyer welfare ✓ Low seller welfare	✓ Zero seller profit ✓ Zero platform profit ✓ Highest buyer welfare ✓ Highest seller welfare

Table 3: Overview numerical results

Another important result that is not specific for one business strategy is that if the buyer-side utility is significantly higher than the seller-side utility the seller-side transaction fee becomes negative. In other words, when the buyer-side demand is significantly larger than the seller-side demand, the platform owner subsidizes the seller-side. This effect does not hold the other way around and is stronger in the non-profit scenarios than in the for-profit scenarios.

7 Conclusion

In this thesis we first defined different business and pricing strategies in the sharing economy. Then we adjusted the monopoly platform model by Rochet and Tirole (2003) in two steps such that it fitted the nature of a sharing platform.

As a first step we treated the service price as exogenous and evaluated the results under the assumptions of strict and shifting utilities. During this step we have defined results that are closely related to those in Rochet and Tirole (2003) and were able to define closed-form equilibrium results about transaction fees, profit and welfare. Furthermore we evaluated two different buyer-side demand function and found that the buyer's ability to shift utilities is beneficial for both the consumers and the platform.

In the second step the service price was treated endogenous and the demand functions for strict and shifting utilities were combined. Here we evaluated two different business strategies, one were the platform and seller are both in control and one were the platform is in full control. Unfortunately, we could not define closed-form equilibrium results in this step. Instead we defined the profit and welfare maximization problem for both strategies and presented numerical results on profit, welfare, transaction fees and service price for each scenario.

This research is one of the first to analyze the effects of different business and pricing strategies in the sharing economy. Whilst the closed-form equilibrium and numerical results are both theoretical they give a lot of insight in how the different business and pricing strategies affect the platform and seller profits, the consumer welfare and the prices in the sharing economy. We will briefly highlight our most important findings:

- Both the sharing platform owner and the sharing platform users benefit from the users ability to shift utilities.
- When the platform is in full control the buyer and seller welfare increases at the expense of the seller's profit. Which is beneficial for the buyer but harmful for the seller.
- Consumer welfare is significantly higher in a non-profit platform than in a for-profit platform.
- When defining the seller's profit as part of the seller's welfare, the consumer welfare becomes significantly higher for the case where the platform and seller are both in control.
- When the buyer-side demand is significantly larger than the seller-side demand, the platform owner subsidizes the seller-side.

Based on these conclusions we state that the business strategies that establishes the most benefits are the ones in which both the platform and the seller are in control and it less important if that is a non-profit or for-profit platform. Even though that in the non-profit scenario the buyer and seller welfare are the highest, we find that they increase at the expense of the seller's profit. The business strategy that causes the most unwanted effects is the for-profit platform that is in full control, because in this scenario the seller has zero profit and a low welfare.

Hence we claim that when regulating the sharing economy the focus should not be on whether the sharing platform is for-profit or non-profit, but on whether the platform is in full control or not. This is especially important as more and more seller's becomes dependent on the online sharing market for part of their income.

8 Limitations & Further research

This research provides interesting insights in how different business strategies in the sharing economy affect the pricing, profit and consumer welfare. However there are some limitations and directions for future research.

First of all, we did not derive closed-form equilibrium results for our model with an endogenous service price. Due to time limitations we chose to present numerical results instead. By showing numerical results for the three most important input parameters (buyer-side utility, seller-side utility and service utility) and providing intuition that explains the results we tried to ensure that our numerical results are robust. However deriving closed-form equilibrium and additional robustness checks is definitely an addition and should be considered in further research.

Another important direction for further research is to extend on the model we have developed. For example by including the include the direct network effects that are proposed in Hyun (2016). Or by including competition between sharing platforms and competition between sharing platform and regular platform into the model. Based on the literature on duopoly two-sided market models from Rochet and Tirole (2003) this is sensible next step.

Furthermore, during this research we have considered static pricing while dynamic pricing is also common in online sharing platforms. The literature on dynamic pricing in the sharing economy is still in its infancy and therefore any contribution on this part is welcome.

Finally, we have assumed that the demand is uniformly distributed and did not check our results for other demand specifications. It would be interesting to investigate the results from our model under different and perhaps more realistic demand functions. Of course, ultimately it would be best if we can validate our results by using real data from the sharing economy.

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APPENDICES

Appendix A. Matlab code

A.1 For-profit scenarios

```
%FOR PROFIT
%% Case 1. Platform and buyer in control: include that g'(s)=0
clear all
%define paramaters
r=1; %b^S
c=2; %c1
d=10; %c2
v=15; %v^B
q=1.2; %b^B

%define decisions variables
x = sdpvar(3,1)
%define constraints and objective
Constraints=[   x(1)*x(3)+ x(3) <= v+0.5*q, %buyerside demand positive
                x(2)*x(3) <= r, %sellerside demand positive
                x(3)-x(3)*x(2)>=d, %profit seller positive
                x(1)*x(3)+x(2)*x(3)-c >=0 %profit platform positive
                x(1)*x(3)<=q, %utility constraint 1
                x(3)<=v, %utility constraint 2
                ((1-x(2))*(2*v+q-4*x(1)*x(3)-4*x(3))+d*(2*x(1)+2))==0,
                %g'(s) positive
                -1<=x(1)<=1,
                -1<=x(2)<=1,
                0<=x(3)<=100];
Objective=-((x(1)*x(3)+x(2)*x(3)-c)*((2*v+q-2*x(3)*x(1)-2*x(3))/(2*v))*((r-
    x(2)*x(3))/(r)));

sol=optimize(Constraints,Objective);

solution=value(x);
p1=solution(1)
p2=solution(2)
s=solution(3)

%platform profit:
pi=(p1*s+p2*s-c)*((2*v+q-2*s*p1-2*s)/(2*v))*((r-p2*s)/(r));
%seller profit:
g=(s-s*p2-d)*((2*v+q-2*s*p1-2*s)/(2*v));
%seller welfare:
ws=(((s*p2-r)^2)/(2*r))*((2*v+q-2*s*p1-2*s)/(2*v));
%buyer welfare:
wb=(((v-p1*s-s)*(v-s*p1-s+q))/(2*v))*((r-p2*s)/(r));

%% Case 2: platform in full control -> no g'(s) in constraints
```

```
%define paramaters
q=1.2; %b^B
r=1; %b^S
c=2; %c1
d=10; %c2
v=15; %v^B

%define decisions variables
x = sdpvar(3,1)
%define constraints and objective
Constraints=[   x(1)*x(3)+ x(3) <= v+0.5*q, %buyer side demand positive
                x(2)*x(3) <= r, %seller side demand positive
                x(3)-x(3)*x(2)== d, %profit seller positive
                x(1)*x(3)+x(2)*x(3)-c >=0 %profit platform positive
                x(1)*x(3)<=q, %utility constraint 1
                x(3)<=v, %utility constraint 2
                -1<=x(1)<=1,
                -1<=x(2)<=1];
Objective=-((x(1)*x(3)+x(2)*x(3)-c)*((2*v+q-2*x(3)*x(1)-2*x(3))/(2*v))*((r-
    x(2)*x(3))/(r)));

sol=optimize(Constraints,Objective);
solution=value(x)
p1=solution(1)
p2=solution(2)
s=solution(3)

%platform profit:
pi=(p1*s+p2*s-c)*((2*v+q-2*s*p1-2*s)/(2*v))*((r-p2*s)/(r));
%seller profit:
g=(s-s*p2-d)*((2*v+q-2*s*p1-2*s)/(2*v));
%seller welfare:
ws=(((s*p2-r)^2)/(2*r))*((2*v+q-2*s*p1-2*s)/(2*v));
%buyer welfare:
wb=(((v-p1*s-s)*(v-s*p1-s+q))/(2*v))*((r-p2*s)/(r));
```

A.2 Non-profit scenarios

```
%% Case 1. Platform and buyer in control: include that g'(s)=0
%x(1)=p^B, x(2)=p^S, x(3)=s
clear all
%define parameters
q=1.2; %b^B
c=1; %c1
d=10; %c2
v=15; %v^B
r=1.2; %b^S

%define decisions variables
x = sdpvar(3,1);

%define constraints and objective
Constraints=[   x(1)*x(3)+ x(3) <= v+0.5*q, %buyerside demand positive
                x(2)*x(3) <= r, %sellerside demand positive
                x(3)*x(2)+x(3)*x(1)-c==0, %profit platform positive
                x(3)-x(3)*x(2)-d>=0, %profit seller positive
                x(1)*x(3)<=q, %utility constraint 1
                x(3)<=v, %utility constraint 2
                ((1-x(2))*(2*v+q-4*x(1)*x(3)-4*x(3))+d*(2*x(1)+2))/(2*v)==0,
    %g'(s)=0 seller profit maximization constraint
                -1<=x(1)<=1,
                -1<=x(2)<=1];
Objective=-((((x(2)*x(3)-r)^2)/(2*r))*((2*v+q-2*x(3)*x(1)-2*x(3))/(2*v))+(((v-x(1)*x(3)-x(3))*(v-x(3)*x(1)-x(3)+q))/(2*v))*((r-x(2)*x(3))/(r)));

sol=optimize(Constraints,Objective);

solution=value(x);
p1=solution(1)
p2=solution(2)
s=solution(3)

%platform profit:
pi=(p1*s+p2*s-c)*((2*v+q-2*s*p1-2*s)/(2*v))*((r-p2*s)/(r));
%seller profit:
g=(s-s*p2-d)*((2*v+q-2*s*p1-2*s)/(2*v));
%seller welfare:
ws=(((s*p2-r)^2)/(2*r))*((2*v+q-2*s*p1-2*s)/(2*v));
%buyer welfare:
wb=(((v-p1*s-s)*(v-s*p1-s+q))/(2*v))*((r-p2*s)/(r));


%% Case 2: platform in full control -> no g'(s) in constraints
%define parameters
q=1.2; %b^B
```

```
c=1; %c1
d=10; %c2
v=15; %v
r=1.2; %b^S

%define decisions variables
x = sdpvar(3,1)

%define constraints and objective
Constraints=[    x(1)*x(3)+ x(3) <= v+0.5*q, %buyerside demand positive
                 x(2)*x(3) <= r, %sellerside demand positive
                 x(3)*x(2)+x(3)*x(1)-c==0, %profit platform positive
                 x(3)-x(3)*x(2)-d==0, %profit seller positive
                 x(1)*x(3)<=q, %utility constraint 1
                 x(3)<=v, %utility constraint 2
                 -1<=x(1)<=1,
                 -1<=x(2)<=1];
Objective=-((((x(2)*x(3)-r)^2)/(2*r))*((2*v+q-2*x(3)*x(1)-2*x(3))/(2*v))
    +(((v-x(1)*x(3)-x(3))*(v-x(3)*x(1)-x(3)+q))/(2*v))*((r-x(2)*x(3))/(r)));

sol=optimize(Constraints,Objective);
solution=value(x)
p1=solution(1)
p2=solution(2)
s=solution(3)

%platform profit:
pi=(p1*s+p2*s-c)*((2*v+q-2*s*p1-2*s)/(2*v))*((r-p2*s)/(r));
%seller profit:
g=(s-s*p2-d)*((2*v+q-2*s*p1-2*s)/(2*v));
%seller welfare:
ws=(((s*p2-r)^2)/(2*r))*((2*v+q-2*s*p1-2*s)/(2*v));
%buyer welfare:
wb=(((v-p1*s-s)*(v-s*p1-s+q))/(2*v))*((r-p2*s)/(r));
```