Master Thesis

Bitcoin: a diversifier, hedge or safe haven

An academic research concerning the risk management properties of bitcoin

November 2017

Author: R.W.J. Hazewinkel
ANR: 841963
MSc. Finance – Tilburg University, school of Economics and Management

Supervisor: Dr. L.T.M. Baele
Co-reader: Dr. F. Castiglionesi

Preface and Acknowledgements

By writing this thesis I will finalize the Master of Finance at Tilburg University School of Economics and Management. The recent developments for cryptocurrencies and bitcoin in particular have stimulated me to incorporate this topic in my thesis. Throughout the process of writing I have gained insights in cryptocurrencies and portfolio management, which have proven to be highly interesting. Hopefully I will get the opportunity to incorporate this knowledge in my career.

Several people have supported me throughout the process of writing my master thesis, whom I would like to thank.

First and foremost, I would like to thank my supervisor Dr. L.T.M. Baele for guiding me in writing my master thesis. His feedback has always been invaluable, from the beginning to the end. Second, I would like to thank Esteban Willemse for the countless hours of checking my writing and tackling several obstacles. I have always appreciated our talks and discussions, which have contributed to my work.

R.W.J. Hazewinkel

Tilburg, November 2017

Abstract

Over the past seven years the cryptocurrency bitcoin has been a topic of interest for researchers, financial analysts and consumers. The uncertainty surrounding the nature of bitcoin and its recognition as a financial asset have yielded numerous papers focusing on these topics. By combining the regression analysis of Baur & Lucey (2010) and the Modern Portfolio Theory (Markowitz, 1952, 1959) in one study, this thesis closes a gap in the literature concerning the risk management properties of bitcoin. This thesis aimed to investigate the diversification, hedge and safe haven properties of bitcoin for a US portfolio including nine assets. The regression results suggest that bitcoin can mainly acts as a diversifier. Despite the high volatility of bitcoin, the low correlation of bitcoin yields diversification benefits for the US portfolio. The results of mean-variance spanning test (Chen et al., 2010) show that inclusion of bitcoin is beneficial for the risk-return trade-off for the US portfolio. Last of all, the application of the Black-Litterman model expresses doubt towards the results of the Modern Portfolio Theory. Nevertheless, additional testing with the Black-Litterman model suggests a small inclusion of bitcoin is beneficial for the US portfolio, these finding are in line with the results of the Modern Portfolio Theory. By summarizing the results above, it can be concluded that the inclusion of bitcoin can improve the performance of a US portfolio. Therefore, this thesis advocates bitcoin should be taken seriously by investors as a financial asset, however investors should stay vigilant of bitcoin's risks.

Keywords: bitcoin, diversifier, hedge, safe haven, OLS regression analysis, Modern Portfolio Theory, diversification, mean-variance spanning test, Black-Litterman model.

Table of Content

Chapter 1: Literary Review

This chapter will provide a general view on bitcoin. Section 1 of this chapter will explain the basics of bitcoin. The second section will discuss the technology behind bitcoin called ‘Blockchain’. The third section discusses the market for bitcoin, including the intermediaries involved.

After a background is established concerning the market and technology behind bitcoin, section 1.4 discusses whether or not bitcoin can be considered a true currency. Section 1.5 will summarize the current literature regarding the determinants of the value of bitcoin. Altogether, the first five sections provide a basic understanding of bitcoin, this will allow section 1.6 to summarize the literature regarding the risk management properties of bitcoin –the central topic of this thesis.

1.1 What is Bitcoin?

The digital currency bitcoin was first introduced by Nakamoto (2008). In 2009, it became the first cryptocurrency to be fully decentralized. This means bitcoin is not controlled by a central bank or other entity such as a government. Instead, bitcoin’s ability as a medium of exchange is built on a peer-to-peer network called ‘the Blockchain’. This network validates the transactions and controls the creation of new bitcoins (Lo & Wang, 2014). According to its creator —Nakamoto, which is a pseudonym for an individual or group of programmers, the goal of bitcoin is to serve as an alternative option to existing payment systems. Bitcoin facilitates transactions across different currencies and national borders while avoiding the intervention of central banks and sovereign entities (Lo & Wang, 2014). According to the creator, transactions with bitcoin should be free of any exploitation by traditional financial intermediaries. Last of all, bitcoin is pseudo-anonymous, which means the identity of a buyer or seller with bitcoin is unknown (Polasik, Piotrowska, Wisniewski, Kotkowski, & Lightfoot, 2014).

The upcoming section will provide a brief explanation of the Blockchain and its mechanisms. Subsequently, the bitcoin market will be discussed.

1.2 The Blockchain

In order to explain the Blockchain, the basics will be discussed first. Afterwards, the creation of bitcoin will be clarified.

1.2.1 The Peer-to-Peer Network

The network behind bitcoin which is called Blockchain records two types of data, objects and blocks. The former refers to one single trade between two parties and the latter refers to a record of approved transactions that took place in the past (Peng, 2013). Summarizing, a block consists of several objects that have been validated. Each transaction is unique and for every transaction a unique address is used by both the receiving and sending party. For example, a party could make multiple transactions, but regardless of sending or receiving a bitcoin for goods or services, the address is unique and will only be used for the specific transaction. Each bitcoin has its own chain of transactions, these chains are published on a public transaction ledger. What follows is verification of this public transaction ledger with its included transactions. After the verification process has been completed, the data is added to the Blockchain as the second type of object: a block (Peng, 2013).

Before a transaction is approved and added to the Blockchain it needs to be verified. This is done by so called 'miners'— individuals who use their computers to verify the public ledger by solving a computational problem. The incentive for miners to verify the ledger comes from the fact that in return for verifying the ledgers, the first miner to solve the problem is rewarded with a number of bitcoins – this process is called mining (Lo & Wang, 2014). Nevertheless, for miners to solve this problem, powerful computers are required. The computers necessary for mining bitcoin require a lot of computational capacity which demands high-tech equipment. Since this computer hardware consumes substantially more energy than an average computer, electricity cost and the price of the equipment play an important role in the profitability of a miner (Peng, 2013).

1.2.2 Miners and Mining Bitcoin

As central banks print currency bills to facilitate the creation and supply of money, the principle of mining bitcoin forms the creation of money or supply of bitcoin. Thus, every time the miners solve the computational problem bitcoins are created as a reward for the miner's effort. In addition, an algorithm adjusts the computational problem –making it more difficult to solve the next problem for miners (Peng, 2013). Figure 1 illustrates the difficulty of the problem over the lifespan of bitcoin. The difficulty of this computational problem can be measured with the Hash rate, the higher this rate the more difficult it is to solve the problem. Besides adjusting the computational problem, the algorithm monitors the total number of bitcoins which is fixed at an amount of 21 million (Lo & Wang, 2014). Figure 2 shows the number of bitcoins in circulation over time.

Miners can work individually or as part of a mining pool (Peng, 2013). The reason for miners to work together in a mining pool is to increase their computational power in order to solve the problem in a smaller time span. Since the problem becomes more difficult to solve as the algorithm increases its complexity with every bitcoin mined, Lo & Wang (2014) expect the mining of bitcoin will become more concentrated throughout the lifespan of bitcoin. This is due to the fact that individual miners will realize their own efforts are no longer profitable due to the increased efficiency offered by mining pools (Peng, 2013).

To summarize, the public ledger is available to every user on the peer-to-peer network. Every new transaction is checked by miners to ensure coins are not being re-used, which eliminates the possibility of fraud. When the transaction is approved, it is added to the Blockchain. This way the Blockchain acts as a neutral entity, while regulating the supply and quantity of bitcoin (Peng, 2013).

1.3 The Bitcoin Market

This section provides a closer look at the bitcoin market. Before discussing the development of the market, the upcoming paragraphs will elaborate on the parties involved in the market.

1.3.1 Intermediaries in the Bitcoin Market

Lo & Wang (2014) recognize three types intermediaries or parties that play a role in the bitcoin market: exchanges, miners, and payment providers. The first type of intermediaries discussed by Lo & Wang (2014) are the exchanges where buyers and sellers of bitcoin trade on the Blockchain. Lo & Wang (2014) point out that there were five big exchanges which accounted for 95% of the total bitcoin trading volume between March 2012 and March 2014 (see Table 1). Reviewing this table based on the last two years of data, the five biggest exchanges are: Okcoin, Huobi, Btcchina, Bitfinex, Lakebtc. Together these five exchanges account for 95% of the bitcoin trading volume (see Table 2), this suggests the market is still concentrated. However, Figure 3 implies the exchange market for bitcoin is less concentrated than before. This graph ranks each exchange as based on the order books, eliminating the volume differences created by exchanges with and without transaction fees, allowing for a better approximation of the bitcoin market share for each exchange (Cieśla, 2017). The reason Mt. Gox is absent in Table 2 is due to its bankruptcy filed on February 28, 2014 –this event will be discussed later in section 1.3.3.

Besides the bitcoin exchanges, the other two intermediaries discussed by Lo & Wang (2014) are miners and payment providers. Since the former is discussed in the previous section, this paragraph will focus on the payment providers. Due to the novelty and uncertainty surrounding bitcoin, a lot of companies did not recognize the cryptocurrency as a valid means of payment. Especially in the early years, the acceptance of bitcoin among companies was low (Polasik et al., 2014). This problem is tackled by payment providers, who are specialized in servicing bitcoin transactions on behalf of companies. In addition to processing the transactions, payment providers enable companies to keep a bitcoin balance and to exchange these bitcoin between regular currencies. Last of all, payment providers offer the service to manage the verification process on behalf of account holders, which means transactions are faster and more secure for companies who decide to accept bitcoin as a means of payment (Lo & Wang 2014). As stated by Polasik et al., (2014), payment providers such as BitPay and Coinbase play a crucial role in the number of companies that accept bitcoin as a means of payment. This effect is compared to the role of Paypal in driving online shopping.

Despite the fact that Lo & Wang (2014) recognize three intermediaries for bitcoin, there is yet another party involved in bitcoin —the users or wallet owners of bitcoin. The upcoming section will focus on this group.

1.3.2 Bitcoin Users

The bitcoin market is international and pseudo-anonymous, which makes it difficult to get a precise estimate of the total number of users (Segendorf, 2014). One way to analyze the usage of bitcoin throughout the years is to estimate the number of wallets. A bitcoin wallet is a software program where bitcoins can be stored. Nevertheless, this number is likely to over-estimate the total amount of users, since one user can have multiple wallets (Polasik et al., 2014). Despite this estimation error, Polasik et al., (2014) uses the quarterly reports of the news platform called Coindesk to estimate the developments of the bitcoin market. Coindesk is a news website focussing on the developments of bitcoin and other cryptocurrencies such as Ethereum. More information regarding Coindesk will be provided in chapter three. According to Coindesk (2017), for the second quarter of 2013 the total number of wallets is estimated at 765,039, this amount increased to 5.4 million in 2014. By the end of the second quarter of 2015, Coindesk estimates a total of 9.3 million. In 2016 the growth of the wallets sustained, estimating 13.5 million wallets in quarter one and 17.8 million by the end of 2016.

To conclude this section, by estimating the number of bitcoin wallets, one can get an implication of the development of bitcoin. However, other measures must be employed to provide additional insights. Looking back to the previous section, the Blockchain is built upon two objects: transactions and blocks. Since every transaction is recorded once it is verified, this measure could be a better way to illustrate the usage and development of bitcoin over time. Therefore, Figure 4 shows the total number of bitcoin transactions recorded in the Blockchain, here the trend shows an increase of the number of transactions over time. Since the changes in the number of transactions do not provide any insights in the size and growth of the bitcoin market, the upcoming section will elaborate on this.

1.3.3 Market Development of Bitcoin

This section will discuss the growth of the bitcoin market by elaborating on several events in the history of bitcoin.

The first description of bitcoin appeared in the paper of Nakamoto (2008). The creator of this proposal mined the first 50 bitcoin in 2009 as a demonstration to a group of online observers. First off, the circulation of bitcoin was among enthusiasts and volunteers from around the world. In the same year, the first purchase of goods and services was made by using bitcoin (Wallace, 2011). Two pizzas were purchased at a cost of 10,000 bitcoin, using the Bitcoin Price Index (Coindesk, 2017) this amount would be equivalent to 10.9 million US dollar on April the 1st 2017 for an average price of $1,089.51 US dollar per bitcoin. The increase of the bitcoin price is illustrated in Figure 5, in this graph the movement of the Bitcoin Price Index (XBP) is regressed over time. The XBP reached its all-time high on June 11th 2017 at a price of $3,018.54 US dollar –more information concerning this index will be provided in section 3.1.

In 2010 Mt. Gox, started trading bitcoins, the first day 20 bitcoins were exchanged at a price of 4.951 US cents. In 2011, a website selling illegal drugs called The Silk Road accepted only bitcoins as payment. The association of The Silk Road with bitcoin generated a reputation of lawlessness. However, this association did not hurt bitcoin—even the arrest of The Silk Road operator by the United States authorities in October 2013 generated publicity making bitcoin even more popular (Yermack, 2013).

In 2013, Mt. Gox and other exchanges reported rapid growth in bitcoin trading. By the end of 2013, bitcoin reached a price of $752.97 US dollar compared to $13.51 US dollar in 2012 on the same day. After the peak in 2013, the price dropped back to $315.09 US dollar by the end of 2014. Figure 6 captures the growth of bitcoin by presenting the market capitalization of bitcoin over time. The increase in trade and popularity of the digital currency in 2013 gave rise to several hacking attacks targeting bitcoin exchanges. In February 2014, Mt. Gox filed for bankruptcy after a value of almost $500 million US dollar worth of bitcoin vanished (Badev & Chen, 2014). Since the incident with Mt. Gox several exchanges took precautions in order to prevent events like this from happening again.

Section 1.1 to 1.3 of this chapter have provided a basic understanding concerning bitcoin's: technology and market development. With this knowledge, the upcoming sections will focus on the academic discussion involving bitcoin. Over the past seven years, there has been a discussion whether bitcoin is an alternative to traditional currencies (European Central Bank, 2012). Therefore, section 1.4 will summarize the literature regarding the question if bitcoin is a currency. Second of all, in order to provide a deeper understanding of bitcoin as a currency and its value, section 1.5 will elaborate on the value determinants of bitcoin. The last section of this chapter will focus on the risk management properties of bitcoin.

1.4 Bitcoin as a Currency

By aiming to provide clarity on bitcoin, European Central Bank (2012) does not only explain the technology behind bitcoin, furthermore it attempts to answer the question whether bitcoin can be seen as a currency. Over the past seven years academics debated whether bitcoin can be considered a currency or an alternative asset. This section provides a summary of the academic literature concerning this discussion. The information regarding the nature of bitcoin as a currency is useful for the assessment of bitcoin as a potential risk management tool for investors—the main topic of this paper. Therefore, this section is vital in serving as a source of background knowledge for section 1.6 of this thesis.

As argued by the European Central Bank (2012), money is traditionally defined as having three functions: first of all, it acts as a medium of exchange, secondly it is a unit of account and last of all, a store of value. The academic literature surrounding bitcoin as a currency mainly focusses on these three requirements.

According to Yermack (2013) bitcoin does not behave like a currency. By using the requirements mentioned above, Yermack (2013) concludes the following. First of all, bitcoin can be seen as a medium of exchange due to the growing number of merchants that accept bitcoin. However, the global commercial use of bitcoin is modest. Second of all, bitcoin fails as a unit of account for the following three reasons. Merchants are required to quote the prices of goods and services in at least four decimal numbers, which is likely to confuse both buyers and seller. In addition, bitcoin is traded for a different price among its exchanges without the chance for an arbitrage opportunity. Last of all, the bitcoin price exhibits relatively high volatility. These three characteristics make it inappropriate to use bitcoin as a unit of account. Third of all, as a store of value, bitcoin faces several problems. Besides the high volatility, the most significant issue is the fact that bitcoin cannot be deposited in a bank. Instead it is stored in digital wallets which in turn are subject to several security problems such as hacks and theft (Yermack, 2013).

Badev & Chen (2014) investigate if bitcoin is a medium of exchange, the empirical results from this research are in line with Yermack (2013). By examining publicly available transaction-level data of bitcoin, Badev & Chen (2014) aim to answer the question if bitcoin is an investment or a payment system. Analysis suggests more than 50% of the bitcoins in circulation have not been used in transactions for the past three months. One third of the bitcoins in circulation have not been used in a transaction for one year prior to the analysis. Of all the gathered transactions, 50% involved an amount of less than $100 US dollar. Badev & Chen (2014) estimate the number of daily bitcoin users has doubled every eight months. However even with this growth, bitcoin's daily transaction volume is relatively small in comparison with the domestic volume of US payment systems (Badev & Chen, 2014). Summarizing these observations indicate bitcoin is hardly used as a payment system.

In addition to the results of Badev & Chen (2014). Woo, Gordon, & Iaralov (2013) show that bitcoin can serve as a store of value. Over 2013, the price of bitcoin rose while the number of commercial transactions decreased. Woo et al., (2013) suggest the appreciation in price is due to bitcoin being a store of value, rather than a medium of exchange.

Opposite to Yermack (2013), there is academic literature stating bitcoin meets the third requirement— a store of value. Van Alstyne (2014) provides four arguments why bitcoin has value. First of all, bitcoin has a technological value, which comes from preventing a coin being spent twice. Second of all, the low transaction fees of bitcoin in comparison to credit card companies make it an attractive way to trade. Third of all, since each transaction requires authentication on the public ledger, bitcoin is better at detecting fraud than credit cards. Last of all, Van Alstyne (2014) advocates bitcoin has value simply because people accept it, indicating this is similar to any other form of money. More precisely, money can be anything varying from paper such as bills to rocks like gold or silver. Van Alstyne (2014) states that as long as it is used in trade, kept as wealth and measured in prices, bitcoin can be a currency.

Jenssen (2014) adopts the three attributes of a currency with an alternative approach. Bitcoin can be accepted as a medium of exchange if there is a demand for bitcoin to be used in trade. Consequently, this demand originates from bitcoin's advantages in terms of transaction costs which are substantially lower in comparison to other payment systems. In turn, if bitcoin is accepted as a medium of exchange, it must also be a store of value (Ostroy & Starr, 1990, as cited in Jenssen, 2014).

By recognizing bitcoin as a store of value, Jenssen (2014) advocates two theories about how the value of bitcoin is determined. The first theory states the value of bitcoin is determined by its demand, since the supply of bitcoin is relatively fixed at a total number of 21 million bitcoin that will eventually be in circulation. More precise, the demand for bitcoin arises from traders aiming to minimize their transaction cost. The second theory demonstrates mining bitcoin is resource intensive, which implies a computer-labor power source of value, Jenssen (2014) refers to this as the "proof-of-work" feature of the mining protocol. Opposing the first theory of Jenssen (2014), Huhtinen (2014) indicates the intrinsic value of bitcoin can be zero. The main reason states that the value of bitcoin is freely defined by the demand of the market without the interference of central banking policies, and for this reason bitcoin does not possess any value.

The last two papers of this section provided a more detailed discussion regarding bitcoin as a store of value. The next section will elaborate on this by focusing on the value determinants of bitcoin.

1.5 The Value Drivers of Bitcoin

Buchholz, Delanney, Warren, & Parker (2012) is one of the first academic papers to investigate the value drivers of bitcoin. By using an Autoregressive Conditional Heteroscedasticity (ARCH) and a Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, this paper aims to explain the effects of volatility on the bitcoin price. The results suggest that the interaction between the supply and demand of bitcoin plays a role in determining its price. To be more precise, the supply of bitcoin determines its availability and bitcoins in circulation and therefore the scarcity of bitcoin on the market. In turn, the demand for bitcoin is primarily determined by its demand as a medium of exchange. Buchholz et al., (2012) conclude by stating that the bitcoin price is influenced by the interaction between its supply and demand.

In contradiction to Buchholz et al., (2012), Kristoufek (2013) believe it is not possible to explain the price formation of bitcoin with standard economic theories. Since bitcoin is free from any regulation by a central bank or government, there are no supply and demand fundamentals present in the market of bitcoin making it isolated from the real economy. As an alternative, results indicate the bitcoin price is driven by bitcoin's attractiveness for investors, which is measured with search queries on Google and Wikipedia (Kristoufek, 2013). Next to Buchholz et al., (2012) and Kristoufek (2013), Van Wijk (2013) attempts to explain the value of bitcoin. By employing stock equity indices, exchange rates and commodity indices, results suggests global financial development has a significant impact on the value of bitcoin in the long run (Van Wijk, 2013).

Ciaian, Rajcaniova, & Kanes, (2014) remark that the research above investigates the impact of factor separately, neglecting the possibility of interaction between the variables. Attempting to close this research gap, Ciaian et al., (2014) incorporate the three price factors of bitcoin discussed in the previous papers: supply-demand fundamentals, attractiveness for investors and global financial indicators in order to find answers for the value determinants of bitcoin. The results of this paper indicate the supply and demand fundamentals along with bitcoin's attractiveness for investors are significant determinants for bitcoin's value. The outcome of this paper does not support previous findings by Van Wijk (2013) which states financial development is a driver for the bitcoin value. Similar to the outcome of Ciaian et al., (2014) are the results of Polasik et al., (2014), which shows the price of bitcoin is primarily determined by its attractiveness for investors and demand as a medium of exchange. Both papers use Google search trends as a measurement of bitcoin's attractiveness.

Kristoufek (2015) utilizes a continuous wavelets method to provide a different perspective for identifying potential drivers of bitcoin's value. The factors investigated are divided into three sources: fundamental, speculative and technical. First of all, Kristoufek (2015) concludes that bitcoin's usage in trade, money supply and price level play a role in bitcoin's value over a long-term time frame. Second of all, the increasing bitcoin price motivates miners. However, since the difficulty and technological necessities for mining have risen sharply, this effect disappears over time. Third of all, the interest of investors in bitcoin is a driver in the long run; in times of rapid price increases it pushes prices up, while in times of rapid decline it pushes the price down. The interest of investors is measured by gathering data from Google searches and Wikipedia searches for bitcoin. Last of all, the Chinese market is identified to be a main driver for bitcoin price. Reason for investigating this relationship is due to events in China such as: bitcoin restrictions by the Chinese government, and acceptance of bitcoin as a payment system by Baidu, a big Chinese player in online shopping (Kristoufek, 2015).

Kristoufek (2015) is not the only one providing evidence of long-term drivers for bitcoin. Bouoiyour, Selmi, Tiwari, & Olayeni, (2016) use a technique called Empirical Mode Decomposition (EMD) to create frequency components for the bitcoin price. These components enable the distinction between short-term and long-term effect on bitcoin price variation. The results of this test suggest that the fluctuations in the bitcoin price are due to long-term effects rather than short-term—long-term effects are considered to be one year or longer (Bouoiyour et al., 2016)

Despite the outcome of Kristoufek (2013) that suggest the amount of Google searches is a driver for the bitcoin value in the long-term, Huhtinen (2014) was not able to find estimates which support this result. Nevertheless, using a multivariate regression other results were found. Huhtinen (2014) finds two drivers with a positive effect on the bitcoin price, the number of bitcoin transactions and the Hash Rate. In addition, the multivariate regression results are tested with a Granger test. The outcome from this test indicate the causality of the regression analysis remains questionable, the results support the view that the bitcoin price itself drives Google searches, transactions, and network attendance (Huhtinen, 2014).

Bouoiyour & Selmi (2015) believe previous research only included a limited number of explanatory variables, which may lead to outcomes subject to noise. Therefore, Bouoiyour & Selmi (2015) employ an Autoregressive Distributed Lag (ARDL) method to investigate the short-term and long-term effects of the following seven variables: investor attractiveness, exchange trade volume, monetary bitcoin velocity, estimated output volume, Hash rate, gold price and the Shanghai stock market. The latter was included in order to test the findings of Kristoufek (2015) which state the Chinese market is a main driver for the bitcoin price. The results of Bouoiyour & Selmi (2015) vary in terms of time domain. In the short run, investor attractiveness, exchange trade ratio, estimated output volume, and the Shanghai stock market are positive drivers for the bitcoin price. As a long-term driver, the exchange trade ratio becomes less significant. In addition, the Hash rate shows to be a positive driver in the long-term.

To summarize this section, several academic papers provide results with regards to the value determinants of bitcoin. In addition, across these papers, aligning conclusions do exist. First of all, as shown above, estimates suggest the supply and demand of bitcoin are factors influencing its value (Buckholz et al., 2012; Ciaian et al., 2014; Polasik et al., 2014; Kristoufek 2015). Secondly, several papers point out that the attractiveness of bitcoin for investors shows to be a determinant for the value of bitcoin (Kristoufek 2013; Ciaian et al., 2014; Polasik et al., 2014; Kristoufek 2015; Bouoiyour & Selmi 2015). Thirdly, both Kristoufek (2015) and Bouoiyour & Selmi (2015) show that the Chinese economy is a value determinant for bitcoin. Last of all, results from Huhtinen (2014) and Bouoiyour & Selmi (2015) suggest that the Hash rate is a potential driver for the value of bitcoin. Despite the similarities found in the results, there are conclusions that do not align. Van Wijk (2013) finds that global financial development is a determinant of the bitcoin value, while results from Ciaian et al., (2014) do not support this outcome.

Section 1.4 summarized the academic literature regarding bitcoin as a currency. In addition, section 1.5 provided a focus on the literature studying the value determinants of bitcoin. The information from these two sections will aid in explaining the nature of the risk management properties of bitcoin, which will be discussed in the last section of this chapter.

1.6 Bitcoin as a Tool for Risk Management

The last two sections provided a summary regarding the academic literature concerning the nature of bitcoin as a currency and the potential factors determining the value of bitcoin. These two sections will aid in understanding the risk management properties of bitcoin, which will be discussed in this section.

In addition to discussing whether bitcoin should be viewed as a currency, Yermack (2013) investigates the correlation of bitcoin against gold and currencies such as the euro, yen, Swiss franc and British pound. By denoting the price of bitcoin in US dollar, Yermack (2013) shows that the pairwise correlations with bitcoin are close to zero. This means bitcoin’s value is completely unrelated to that of gold and the other currencies. Yermack (2013) concludes bitcoin is useless as a tool for risk management against other assets, stating it is a poor hedge and safe haven.

Despite the fact that Yermack (2013) concludes bitcoin is useless as a tool for risk management, other research presents alternative findings. Therefore, the upcoming paragraphs will provide a summary concerning the academic literature involving the diversification, hedge and safe haven properties of bitcoin.

In light of the discussion whether bitcoin is a currency, Ennis (2013) compares bitcoin with a commodity instead of a currency. Since the supply of bitcoin looks similar to the supply of gold, Ennis (2013) compares bitcoin to gold. Focusing on gold's role as a hedge and safe haven for equity, bonds and currencies, Ennis (2013) investigates if bitcoin could play a similar role. By using a GARCH analysis, Ennis (2013) finds bitcoin can be a possible hedge for the euro, this is not the case for the dollar. Secondly, estimates suggest bitcoin can take the role of a hedge against movement of European equity markets. In addition, despite the fact that estimates indicate bitcoin is a diversifier for US equity markets, it can neither act as a hedge nor a safe haven for these markets. Thirdly, bitcoin is a potential hedge and safe haven for both the European and US bond market.

Ennis (2013) was not the only research comparing bitcoin to gold as a tool for risk management. Dyhrberg (2016a) employs an asymmetric GARCH method to investigate the financial asset properties of bitcoin in comparison to both gold as a commodity and the US dollar as a currency. The results prove opposite to the conclusion of Yermack (2013). Dyhrberg (2016a) indicates bitcoin is similar to gold in being a store of value and in a way similar to the US dollar as a medium of exchange. The overall conclusion suggests the hedging capabilities of bitcoin are between gold and the US dollar.

Complementary to the literature above, Dyhrberg (2016b) continues the research with regards to the risk management properties of bitcoin. First of all, results suggest bitcoin can be used as a hedge against stocks listed in the Financial Times Stock Exchange Index –this index contains the 100 largest companies listed on the London stock exchange. Second of all, bitcoin can be used as hedge against the US dollar. The overall conclusion states bitcoin shows similar hedging capabilities in comparison to gold, making it a tool for hedging market specific risk (Dyhrberg, 2016b).

As discussed in the precedent research, bitcoin shows similarities to gold in terms of being a hedge and a safe haven. Focusing on the latter, Bouri, Azzi, & Dyhrberg, (2016a) investigate the safe haven property of bitcoin around the bitcoin price crash of 2013. By using an asymmetric GARCH, Bouri et al., (2016a) study the safe haven effect of bitcoin against the six currencies, bitcoin is most traded for: the US dollar, the Australian dollar, the Canadian dollar, the British pound, euro and the Japanese yen. The results indicate bitcoin had a safe haven property similar to gold, prior to the crash in 2013. After the bitcoin price crash in 2013, the safe haven effect disappeared. Secondly, estimates show that adding bitcoin to a US equity portfolio may lead to an effective reduction of the overall portfolio risk. However, results indicate that the effect of adding bitcoin to a portfolio prior to the crash is larger than afterwards.

Bouri, Gupta, Tiwari, & Roubaud, (2016b) analyze whether bitcoin can act as a hedge against global uncertainty. By merging the volatility indices (VIX) of 14 equity markets, Bouri et al., (2016b) create a variable for global uncertainty called World VIX. First of all, results from the OLS regressions indicate that global uncertainty has a negative effect on the return of bitcoin. Secondly, quantile regression estimates suggest bitcoin can act as a hedge in times of global uncertainty, especially over shorter investment horizons. To summarize, temperary investments in bitcoin can aid investors in hedging the uncertainty for the global equity market (Bouri et al., 2016b)

In addition to investigating the safe haven properties of bitcoin around the crash in 2013, the paper Bouri et al., (2016a) tests for the inclusion of bitcoin to a portfolio. A number of papers focus on the inclusion of bitcoin in an investment portfolio, the upcoming paragraphs will elaborate on this field.

Brière, Oosterlinck, & Szafarz, (2013) use weekly data over the period 2010 to 2013, in order to analyze a bitcoin investment for a US investor using Modern Portfolio Theory (MPT) (Markowitz, 1952,1959). The portfolio already includes, equity, bonds, currencies and investments such as currencies, real estate and hedge funds. By employing the mean-variance spanning (MVS) tests of Huberman & Kandel, (1987) and Ferson, Foerster, & Keim (1993), results from both tests suggest the inclusion of bitcoin to a US portfolio offers diversification advantages. Adding a bitcoin allocation of 3% to a diversified portfolio significantly improves the risk-return trade-off. To conclude, bitcoin might have a high volatility accompanied with a high return. However, this high risk is compensated in diversified portfolios due to the low correlations with the other assets (Brière et al., 2013).

In addition, the paper from Eisl, Gasser and Weinmayer (2015) attempts to answer the question whether ‘bitcoin can improve portfolio diversification.’ The results from the Value-at-Risk framework suggest bitcoin should be including in optimal portfolios. Despite the fact that adding bitcoin will increase the overal risk of the portfolio, the additional risk is compensated by an increase in the portfolio return, leading to a better risk-return ratio.

Bouri, Molnár, Azzi, Roubaud and Hagfors (2016c) investigate the three risk management abilities, diversification, hedge and safe haven for bitcoin. This paper is the first to study these three risk management properties of bitcoin in one overview by following the regression model of Baur & Lucey (2010) and Ratner & Chiu (2013) who differentiate between a diversifier, hedge and a safe haven. More precise, Bouri et al., (2016c) employ a dynamic conditional correlations (DCC) model (Engle, 2002) to estimate the correlations for the regressions. By including MSCI stock indices, bonds, commodities, and the US dollar index, the results of this study suggest that bitcoin is not a strong hedge and can only function as a diversifier. Moreover, bitcoin can only be a safe haven in the case of market turmoil for Asian equity markets.

To summarize, previous literature suggests the addition of bitcoin to a portfolio leads to potential benefits for investors. Nevertheless, the number of papers focussing on this topic is limited. Furthermore, several papers study the diversification, hedge and safe haven abilities of bitcoin against assets and indices. However, the papers discussed above are fragmented. To be more precise, not all three risk management properties are studied together. Bouri et al., (2016c) aim to close a gap in the literature by providing results concerning the diversification, hedge and safe haven advantages of bitcoin in one overview. Nevertheless, despite the fact that results indicate bitcoin as a diversifier, Bouri et al., (2016c) do not elaborate on this by applying their estimates to the inclusion of bitcoin to an investment portfolio such as MPT. This thesis will address this gap in the literature.

Similar to Bouri et al., (2016c), this thesis will follow the approach of Baur & Lucey (2010) in order to evaluate the diversification, hedge and safe haven properties of bitcoin. From the perspective of a US investor, this thesis will test the risk management abilities of bitcoin against nine other assets. The assets consist of five equity indices from Morgan Stanley Capital International (MSCI) , two indices for gold and commodities, a bond index and a US dollar index. After analyzing the diversification, hedge and safe haven advantages of bitcoin, this thesis will use the Modern Portfolio Theory of Markowitz (1952, 1959) to analyse the performance for a portfolio that includes bitcoin. Last of all, this thesis will test whether the inclusion of bitcoin improves the portfolio risk-return trade-off by doing a mean-variance spanning test.

The previous paragraphs presented the gap in the literature relevant for this thesis. The next chapter will discuss the research questions and theoretical framework.

Chapter 2: Research Questions

With regards to the current state of the literature involving the diversification, hedge and safe haven advantages of bitcoin, the main research question for this thesis can be formulated as follows:

- *What are the diversification, hedge and safe haven advantages for a US investor holding a portfolio including bitcoin?*

2.1 Secondary Research Questions

In order to answer the main research question in a constructive manner, this thesis knows three secondary research questions. Since this paper is structured in three parts, each part will answer one of the secondary research questions in the order presented below.

By using the regression model of Baur & Lucey (2010), the first part of this thesis will answer the question illustrated below.

2.1.1: Does bitcoin possess any diversification, hedge and safe haven properties against the assets in the US portfolio?

With the Modern Portfolio Theory (Markowitz, 1952, 1959), part two will answer the question stated below.

2.1.2: How does the proportion of bitcoin change with the risk and return of the portfolio?

The Last part of this thesis will employ the mean-variance spanning test discussed in Chen, Chung, Ho, & Hsu (2010) to answer the question presented below.

2.1.3: Is the inclusion of bitcoin to the US portfolio improving the risk and return trade-off?

Chapter three will discuss the methodology surrounding the secondary research questions. The next section will focus on the definitions of a diversifier, hedge and safe haven.

2.2 Theoretical Framework

Before discussing the data & methodology for this paper, it is essential to create a clear separation of the three risk management properties —diversifier, hedge and safe haven. This thesis employs the definitions as provided by Baur & Lucey (2010) —who differentiate between a diversifier, hedge and safe haven. The definitions below are vital to the regression model discussed in section 3.2.1.

Diversifier

“A diversifier is defined as an asset that is positively (but not perfectly) correlated with another asset or portfolio on average.”

Hedge

“A hedge is defined as an asset that is uncorrelated or negatively correlated with another asset or portfolio on average.”

In times of market turmoil, a hedge could show a positive correlation, and a negative correlation in average market conditions, this is the difference between a hedge and safe haven.

Safe haven

“A safe haven is defined as an asset that is uncorrelated or negatively correlated with another asset or portfolio in times of market turmoil.”

A safe haven has the distinguishing characteristic to be negatively correlated with another asset in times of market stress. This characteristic does not mandate the correlation to be positive or negative on average, only to be zero or negative in periods of market turmoil. The mechanism of a safe haven asset works as follows; in times of extreme market conditions the negative correlation with the other asset, makes the price of the safe haven asset rise while the price of the other asset drops, compensating the investor for the losses in bad times (Baur & Lucey, 2010).

Chapter 3: Data & Methodology

The previous chapter provided the research questions for this thesis. This chapter will discuss the data and methodology used to answer the research questions. The first section of this chapter will discuss the data used for this paper, the second section will provide the methodology for each of the three steps of this thesis. Since there are several variables used in this paper, Table 3 provides an overview of the variables of each equation.

3.1 Data

The dataset for this thesis consists of the daily return in US dollar for bitcoin and nine other assets part of the US portfolio. These assets consist of five MSCI indices, a bond index, a currency index, and commodity indices. The data spans from Monday July 26th 2010 to Friday June 16th 2017. Because bitcoin is traded continuously, the observations for holidays, Saturdays and Sundays are eliminated. Eventually, this yields 1799 daily return observations, the descriptive statistics for the daily return are displayed in Table 4. Datastream is used to acquire the daily returns of the financial assets, the daily return for bitcoin is obtained from the Bitcoin Price Index (Coindesk, 2017). Since this thesis will evaluate the results from the viewpoint of a US investor, the returns of bitcoin and the assets are denominated in US dollar.

The Coindesk Bitcoin Price Index represents the average bitcoin price across leading global bitcoin exchanges (Coindesk, 2017). The index is intended to serve as an industry basis, useful for participants and accounting professionals. Currently, the XBP represent the average price of Bitstamp, Coinbase, itBit, and Bitfinex. To be included exchanges need to meet a set of criteria, exchanges are added and deleted when criteria are met or violated. (Coindesk, 2017).

The MSCI indexes included in this thesis are, World, US, Europe, Pacific, and Emerging Markets (EM). The MSCI World represents the international stocks, MSCI US and MSCI Europe represent the stocks in the United States and Europe. MSCI Pacific is a proxy for Asia Pacific stocks. The MSCI EM index represents the stocks for 24 emerging market countries (MSCI Inc., 2017). For the US bond market, the US Benchmark 10 Year government index is used as a proxy. The commodity and gold index of Standard & Poor's Goldman Sachs is used as a proxy for the commodity and gold market. Last of all, the US dollar Index represents the performance of the US dollar.

3.2 Methodology

Given the problem statement for this thesis, the methodology is structured to answer every secondary research question in the order in which it is stated. First of all, by combining the definitions from the theoretical framework with the econometric regression of Baur & Lucey (2010), step one will test if bitcoin can act as a diversifier, hedge or safe haven against movement of the assets listed in Table 4. Second of all, in step two this paper will use Modern Portfolio Theory of Markowitz (1952, 1959), to test the performance of a US portfolio including bitcoin. Last of all, step three will extend on the MPT by investigating whether the addition of bitcoin to a portfolio improves the portfolio risk-return trade-off, this will be done with a mean-variance spanning test.

3.2.1 Identifying the Diversification, Hedge and Safe Haven Properties

By combining the definitions of the three risk management properties from the theoretical framework with Equation 3.1, this model will be used to answer research question 2.1.1. The regression of Equation 3.1 is based on the approach of Baur & Lucey (2010) and tests for the diversification, hedge and safe haven abilities of bitcoin against the other assets. One should note that this model is not identical to the regression method of Bouri et al., (2016c), since the regression below does not involve the dynamic conditional correlation model mentioned in section 1.6.

Equation 3.1: Regression model for the three risk management properties

$$R_{bitcoin,t} = a + b_1 R_{asset,t} + b_2 R_{asset,t\,(q5\%)} + b_3 R_{asset,t\,(q2.5\%)} + b_4 R_{asset,t\,(q1\%)} + e_t$$

First of all, for this equation $R_{bitcoin,t}$ and $R_{asset,t}$ are the daily returns for bitcoin and one of the assets in the portfolio. Second of all, $R_{asset,t\,(q\ldots\%)}$ is the term which accounts for the negative shocks of the asset, allowing focus on falling returns as a result of market turmoil. Therefore, this term includes the return of the asset that are in the q% lowest quantile. Similar to Baur & Lucey (2010) this thesis uses the 5%, 2.5% and 1% quantile. If the return of the asset exceeds the lowest quantiles the value of $R_{asset,t\,(q)}$ will be zero. Last of all, b_1 indicates the average effect between the returns of bitcoin and the assets, while b_2, b_3, and b_4 resembles the effect between the returns of bitcoin and the return of the asset in times of market stress. The betas for this regression are standardized coefficient estimates, which are equal to the correlation coefficient between bitcoin and the variable for each beta.

Combining the definitions from the theoretical framework with Equation 3.1, the two can be related with the following hypothesis (Baur & Lucey, 2010).

- *Hypothesis 1: If b_1 is positive but not equal to 1, this suggests bitcoin is a diversifier against movements of the selected asset on average.*

- *Hypothesis 2: If b_1 is zero or negative, this would suggest bitcoin could act as a hedge against movements of the selected asset on average.*

- *Hypothesis 3: If the sum of b_1, plus b_2, b_3, and b_4 is negative or equal to zero in times of market turmoil for the selected asset, this implies bitcoin serves as a safe haven asset.*

For Hypothesis 3, summing b_1 and b_2 would result in the estimate between the return of bitcoin and the asset return in the 5% lower quantiles. Therefore, in order to calculate the estimate between the return of bitcoin and the 1% lower quantile of the asset, one needs to sum up b_1, b_2, b_3, and b_4.

3.2.2 Bitcoin & Modern Portfolio Theory

Step two of this thesis will focus on the portfolio implications of adding bitcoin to a portfolio, and will therefore answer research question 2.1.2. This will be done by following the Modern Portfolio Theory of Markowitz (1952, 1959) as the underlying framework. The theory presents an approach where adding assets to an investment portfolio decreases the level of risk without sacrificing the return of the portfolio. More precisely, this theory is an econometric model illustrating the benefits of diversification. For this theory, return is measured using the probability distribution of the expected return. Risk can be measured as the variability or standard deviation around the expected value of the probability distribution of the return (Bodie, Kane, & Marcus, 2014).

Each asset has its own return and standard deviation, combining a group of assets forming a portfolio offers diversification benefits. The expected return for the portfolio is equal to the weighted average of the assets included in the portfolio. However, this is not the case for the risk of the portfolio. Equation 3.2 and Equation 3.3 present the formulae for the portfolio expected return and risk (Bodie et al., 2014).

Equation 3.2: Portfolio expected return for n assets

$$E(r_p) = \sum_{i=1}^{n} w_i \, E(r_i)$$

Equation 3.3: Portfolio variance for n assets

$$Var(r_p) = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j p_{ij} \sigma_i \sigma_j$$

For equation 3.2, $\sum_{i=1}^{n} w_i$ = 1, n is the total number of assets in the portfolio. The weight of the funds invested in the assets *i*, is given as w_i. The expected return of asset *i* and the portfolio are respectively $E(r_i)$ and $E(r_p)$.

In equation 3.3, p_{ij} is the correlation coefficient between the returns of asset *i* and asset *j*. The standard deviation of the return (r_i, r_j) for asset *i* and asset *j* are σ_i and σ_j. Since the variance of a portfolio is not the weighted average of the assets in the portfolio but involves the correlation coefficient, there can be benefits from diversification. The correlation coefficient has a value between -1 and 1. In the situation of a correlation coefficient with the value of -1, this means perfect negative correlation where the benefits of diversification are the most. While a correlation coefficient of zero means no correlation exists at all, a correlation coefficient of 1 indicates a perfect positive correlation meaning the assets move in the same direction (Chen et al., 2010).

Looking back at equation 3.2 and 3.3, the return and risk of the portfolio can shift by changing the weights of the assets included in the portfolio. The possibility of creating portfolios by changing the asset proportions allows for different portfolio returns and risk while having the same asset types in each portfolio. The portfolios with the highest return to risk ratio or Sharpe ratio can be found on a line called the efficient frontier. Were for a given level of risk and return an efficient portfolio can be found, the minimum-variance portfolio offers the lowest portfolio risk on the efficiency frontier. The best portfolio choice depends on the investor's tradeoff between risk and return (Bodie et al., 2014).

By finding the minimum-variance portfolio the efficiency frontier can be drawn, from there the proportion of bitcoin can be monitored for a given level of risk. In order to stick to realistic investment possibilities, short positions are excluded and the weights of the assets sum to 1. With the equation below, one can estimate the asset weights for a minimum-variance portfolio containing n assets (Chen et al., 2010).

Equation 3.4: Minimum-variance portfolio

$$Min\ \sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i\, w_j \rho_{ij}\, \sigma_i \sigma_j$$

Subject to $w_1 + w_2 + \ldots + w_n = 1$

3.2.3 Mean-Variance Spanning Test

This section will discuss the methodology for the mean-variance spanning test, which will be used to answer research question 2.1.3. The MVS test was first introduced by Huberman and Kandel (1987), adopting the MPT as the underlying framework (Chen et al., 2010). This method enables to test whether adding a new asset to a benchmark portfolio leads to any improvement of the mean-variance frontier Kan & Zhou (2008). More precise, this test measures the statistical difference between the efficiency frontier of the benchmark portfolio and extended portfolio (Maroney & Naka, 2006). In the case that the benchmark portfolio and extended portfolio correspond, there is spanning. Meaning, investors will not benefit from adding the new asset to the benchmark portfolio. When the results suggest spanning does not exists, this means the inclusion of the new asset benefits from diversification advantages which could not be created with the assets in the benchmark portfolio (Maroney & Naka, 2006). The effect of adding a new asset to the benchmark portfolio can be tested with an ordinary least squares (OLS) regression, presented in Equation 3.5 below (Chen et al., 2010).

Equation 3.5: Spanning test OLS regression

$$R_{ext} = a + \beta R_{bench} + e$$

For this equation, R_{ext} and R_{bench} are the returns for the extended portfolio and benchmark portfolio on the efficiency frontiers of each portfolio at a level of risk. The null hypothesis for this test states that spanning between the portfolios exist, meaning the addition of the new asset does not improve the portfolio performance, as a result of diversification benefits. Equation 3.6 illustrates the conditions for the null hypothesis.

Equation 3.6: Null hypothesis for spanning

$$H_0 : a = 0 , \ \beta = 1$$

The one hypothesis for this test states that when alpha is not equal to zero and beta does not equal 1, meaning the addition of the new asset to the benchmark portfolio creates new diversification benefits, improving the performance of the portfolio (Chen et al., 2010).

This chapter discussed the methodology necessary to answer the secondary research questions for each part of this thesis. The upcoming chapter will present the results.

Chapter 4: Results

The results for this thesis will be presented in the same order as the sections in the methodology. First of all, the regression output of Equation 3.1 will be discussed, this will allow to answer research question 2.1.1. Second of all, the outcome for the portfolio including bitcoin will be presented. Last of all, the results of the mean-variance spanning test will be discussed. However, before presenting the MVS test results the performance for the portfolio excluding bitcoin will be discussed.

4.1 Regression Output for the Risk Properties of Bitcoin

Table 5 provides the results for the regression in Equation 3.1, together with the hypothesis in section 3.2.1, the diversification, hedge and safe haven properties for bitcoin can be analyzed.

The b_1 coefficients for the regressions are all statistically different from 1. For the MSCI World, MSCI Europe and commodity index, the b_1 estimates are positive and relatively close to zero. This would suggest bitcoin can act as a potential diversifier against average movement of international stocks, European stock market and commodities. The fact that these estimates are positive and close to zero would suggest the diversification effect is relatively strong. The remaining b_1 estimates show a negative coefficient for the average effect. Combining this with the hypothesis from section 3.2.1 would suggest bitcoin can act as a potential hedge against average movement of the MSCI USA, MSCI Pacific, MSCI EM, bond index, gold index and US dollar index. By taking a closer look at the value of all the estimates, it can be concluded that all are close to zero. This would mean that the average hedge effect of bitcoin against these assets is relatively weak.

The previous paragraph evaluated the diversification and hedge capabilities of bitcoin against the average movement of each asset, now the safe haven property of bitcoin will be discussed. According to hypothesis 3 in section 3.2.1 a safe haven effect arises when the sum of the betas is negative or equal to zero. Looking at the results in Table 5, the sum of the b_1, b_2 and b_3 for the commodity index meets the criteria. The coefficient estimate of -0.067 would suggest bitcoin can act as a weak safe haven against negative market returns for commodities, this effect only exist for market returns which are part of the 2.5% quantile.

This section provides the answers for the question whether bitcoin possess any diversification, hedge and safe haven properties against the assets in the US portfolio. The estimates show that bitcoin can mainly act as a diversifier. The negative coefficient estimates for the MSCI USA, MSCI Pacific, MSCI EM, bond index, gold index and US dollar index are close to zero, indicating a weak hedging effect. According to the results bitcoin can act as a safe haven for bad commodity returns in the 2.5% quantile. Since this section answered the first research question for this thesis, the upcoming section will explain the results vital to research question 2.1.2.

4.2 Performance for the Bitcoin Portfolio

This section will start by introducing the input for the portfolio, followed by evaluating the efficiency frontier. The last paragraph will provide the answers for research question 2.1.2 by focusing on how the portfolio changes for a given level of risk, return and bitcoin weight.

By using the daily returns from part one of this thesis, the monthly returns contain 89 observations (Table 6). The expected return for bitcoin and the nine assets are estimated by taking an equally weighted average, the statistics are presented in Table 7. The standard deviations in this table are from the variance-covariance matrix shown in Table 8, which in turn is based on the monthly historical returns. Last of all, Table 9 presents the correlations between the assets of the portfolio, from this table one can note that the correlation of bitcoin with the other assets is relatively low, implying bitcoin can offer diversification benefits for the portfolio.

In order to create the efficient frontier for the portfolio including bitcoin, first the minimum-variance portfolio is estimated –this is the starting point for the efficiency frontier. Table 10 presents the implications for the minimum-variance portfolio, for the sake of illustration the portfolio with equal weights is added along with the MVP allowing short sales. From Table 10 the difference between the two MVP's is clear: the portfolio with short sales offers a better Sharpe ratio and lower standard deviation. Both minimum-variance portfolios include a relatively small proportion of bitcoin, for the MVP with short sales the proportion of bitcoin is a short position. The biggest difference between the two minimum-variance portfolios is the proportions in MSCI World, MSCI Pacific, and MSCI EM, which are zero for the portfolio without short sales but not for the MVP with short sales. Summarizing, the efficiency frontier for the portfolio including bitcoin can be drawn in Figure 7. In addition to the efficiency frontier, both minimum-variance portfolios are included as are the expected return and risk for the individual assets.

Focusing on a portfolio where short sales are not allowed, the portfolios on the efficiency frontier can be analyzed as follows. Table 11 presents the relative changes of the efficiency frontier and asset weights for a given portfolio risk –starting with the MVP as the first observation. Despite the fact that bitcoin is the riskiest asset in the portfolio, there is a small allocation for bitcoin in the minimum-variance portfolio. Looking back at Table 9, this can be due to the low correlation of bitcoin. From the MVP, any increase in the portfolio risk means the asset weight of bitcoin increases, until a maximum of 100% at a portfolio expected return of 22.99% and risk of 64.55%. As the risk for the portfolio increases, the weight for the other assets decrease to zero in the following order, Gold index, Commodity index, MSCI EU, US dollar, bond index, and MSCI USA.

In addition, Table 12 illustrates a similar shift, here the asset proportions are illustrated for a given level of portfolio return. Last of all, Table 13 shows how the portfolio return and risk change for a given proportion of bitcoin, at a 9% allocation to bitcoin, the remaining asset allocation is assigned to MSCI

USA (84%) and the Bond Index (7%). This table shows that for a bitcoin allocation between 1 to 10 percent, can be combined with the other assets in the portfolio. Since the efficiency frontier for the portfolio including bitcoin is defined, the upcoming section will answer the question whether adding bitcoin to a US portfolio is actually beneficial for the risk-return trade-off.

This section provided the answers necessary for research question 2.1.2. The upcoming section will discuss the benchmark portfolio and the results of the mean-variance spanning test.

4.3 Results for the Mean-Variance Spanning Test

Before discussing the results for the mean-variance spanning test, the portfolio implications will be interpreted for the benchmark portfolio –the portfolio excluding bitcoin. Table 14 presents the equally weighted portfolio and minimum-variance portfolio with and without short sales. When comparing the equally weighted portfolios of the benchmark and extended portfolio, the difference in the portfolio return and risk can be explained by the lack of bitcoin in the benchmark portfolio.

Despite the fact that bitcoin is a risky asset, the minimum-variance portfolio for the extended portfolio shows a slightly lower risk and higher return, when not allowing for short sales. This would suggest that the diversification benefits of bitcoin compensate for the addition risk. Figure 8 presents the efficiency frontier for the benchmark portfolio, including the minimum-variance portfolio and individual assets.

Figure 9 illustrates the efficiency frontier of the extended and benchmark portfolio, here the frontier for the extended portfolio lies above the frontier of the benchmark portfolio, this indicates that for the same level of risk, the portfolio including bitcoin has a higher expected return. Table 15 confirms this remark with regards to both efficiency frontiers by presenting the returns for both portfolios for the same level of risk. The upcoming paragraph will expand on the results from Figure 9 and Table 15 by presenting the results of the mean-variance spanning test.

Section 3.2.3 introduced the mean-variance spanning test (Chen et al., 2010), the results for the OLS regression are illustrated in Table 16. For alpha the coefficient estimate is -0.005 and for beta 1.938, both estimates are statistically significant at the 1% level. Looking back at the hypothesis from section 3.2.3 the H_0 states spanning exists when alpha is equal to zero and beta equals 1. With the results from the regression H_0 can be rejected. This suggest that the inclusion of bitcoin leads to diversification advantages, beneficial for the US portfolio. Therefore, research question 2.1.3 can be answered by concluding spanning does not exist and that adding bitcoin to the US portfolio is beneficial for the risk-return trade-off.

4.4 Black-Litterman and the Portfolio including Bitcoin

Part two and three of this thesis applied the Modern Portfolio Theory (Markowitz, 1952, 1959) to the US portfolio including bitcoin. Part two shows that for a bitcoin allocation of 1 to 10 percent, bitcoin can be combined with the other assets in the portfolio. The results of the spanning test in part three indicate the inclusion of bitcoin leads to diversification advantages, which is beneficial for the performance of the US portfolio. However, as mentioned by Black & Litterman (1991) the Modern Portfolio Theory has two major limitations. First of all, the expected return for the individual assets is very hard to predict, furthermore when based on the historical returns this can yield inaccurate estimates for the future. Secondly, the efficiency frontier is very sensitive to the assumptions of the expected return for the individual assets. As an alternative, the Black-Litterman model is developed. This model allows investors to combine equilibrium returns with a set of views that investors have on certain assets (Black & Litterman, 1991).

In order to test the results from part two, this section will apply the Black-Litterman model to the case of bitcoin. More precise, this model attempts to answer the question whether a portfolio allocation of 5% for bitcoin is feasible given the expected return used in part two. After introducing the methodology and assumptions of the model, the results will be discussed.

4.4.1 Methodology for the Black-Litterman model

The first step for the model is determining the equilibrium returns. These returns are used as a starting point, set by the market. Using reverse optimization, the implied equilibrium excess returns (IER) can be calculated using the formula of Equation 3.7 below (Idzorek, 2005).

Equation 3.7: Implied Equilibrium Excess Returns

$$\mu^{IER} = \gamma \sum w^*$$

For the equation above, μ^{IER} is the vector for the implied equilibrium excess return. Gamma (γ) is the risk aversion coefficient, Sigma ($\sum$) is the annual variance-covariance matrix of the excess returns and w^* the equilibrium weights. By annualizing the daily variance of the assets in the US portfolio, Table 17 presents the variance-covariance matrix Sigma – assuming 252 trading days in a year.

For the equilibrium weights of the market, 60% is allocated to the MSCI World and 40% to the bond index, all other asset weights are equal to zero. Last of all, the annual market risk premium for equities (MSCI World) is assumed to be 5.5%, this yields a gamma of 3.14. The IER resulting from Equation 3.7 can be viewed as the equilibrium portfolio, from the IER views can be added for the assets in the portfolio. By adding views investors can deviate from the equilibrium portfolio and adjust the returns for the assets with views from the IER, investors will hold the equilibrium portfolio when no views are introduced (Idzorek, 2005).

The views can be combined with the IER using the Black-Litterman formula stated in Equation 3.8 (Idzorek, 2005), this combination yields the Black-Litterman expected excess returns (BL ER).

Equation 3.8: The Black-Litterman Formula

$$BL\,[R] = [\,(\tau\sum)^{-1} \; + \; P'\Omega^{-1}P]^{-1}\,[(\tau\sum)^{-1}\,\mu^{IER} + \; P'\Omega^{-1}Q]$$

When combining the IER and the views, investors can assign a level of certainty/uncertainty towards the views and a level of precision for the IER, these factors can both be found in the formula above. Omega (Ω) is a confidence matrix, where the diagonal elements contain the variance of each view, this matrix represents the uncertainty in the views. Tau (τ) is a scalar for the precision of the IER. The matrix P, selects the assets involved in each view, Q is a matrix that contains the magnitude of the views on the selected assets (Idzorek, 2005). BL [R] represents the Black-Litterman expected excess return vector, which is a result of the IER adjusted in the direction of the views, assets not incorporated in the view are affected by the view due to the correlation with the asset part of the view (Drobetz, 2001). Last of all, The Black-Litterman optimal weights can be found by using Equation 3.7, solving for w. When there are no constraints applied to the model, the weights for the BL ER will only change for the assets included in the view (Idzorek, 2005), this change in the included asset will be financed by a short position on the risk-free asset.

When referring back to the start of this section, the Black-Litterman model will test the feasibility of a bitcoin allocation of 5%, given the expected return of bitcoin used in part two. In order to do so, one view will be introduced. The view states that bitcoin will have an expected excess return based on its historical return. Looking back at part two, the monthly return for bitcoin is expected to be 22.99%. On an annual basis this means the expected excess return for bitcoin is 274.98% assuming a risk-free rate of 1%, this will be the view.

In order to test this view, assumptions need to be made regarding Omega and Tau. According to Walters (2013), Tau (τ) is one of the most confusing parts of the model. Discussing there is no correct method for determining its value. Academic literature chooses different values for Tau, while others ignore the value (Walters, 2013). For instance, Lee (2000) sets the value of Tau between 0.01 and 0.05 (Idzorek, 2005), while He & Litterman (1999) use a value of 0.025 (Walters, 2013). Last of all, Idzorek (2005) states the easiest way to align the Black-Litterman model is by assuming Tau. Therefore, this thesis assumes Tau equals 0.025. Last of all, by varying Omega (Ω) –the uncertainty of the view, the feasibility of a 5% allocation can be tested. The upcoming section will calculate the Black-Litterman expected excess returns for this view, along with the corresponding optimal weights.

4.4.2 Results for the Black-Litterman model

This section will start by presenting the results for the IER and Black-Litterman expected excess returns. Followed by presenting the results for the Black-Litterman expected excess returns and optimal weights for a changing Omega.

When implementing the assumptions stated in the previous section, the equilibrium expected excess returns can be calculated for the assets in the US portfolio using Equation 3.7. Table 18 presents the IER along with the optimal weights, which are identical to the equilibrium weights. In addition to the IER, this table provides the Black-Litterman expected excess returns, for a standard deviation of 2% in the view, corresponding to an Omega of 0.04%. By looking at the difference between the IER and BL ER, the return for bitcoin changed in the direction of the view. For the Black-Litterman weights, both the allocation with short sales and without short sales are presented in the table.

By using the Black-Litterman expected excess returns and weights for a standard deviation of 2% as a starting point, Table 19 provides the returns and weights for a changing Omega, not allowing for short sales. This table shows that when increasing the uncertainty surrounding the view, the return and weight for bitcoin decrease. At a standard deviation of 62.5%, the estimated allocation for bitcoin is 5% with an estimated return of 18.6%. With the relative high uncertainty in the view, it can be concluded that a bitcoin allocation of 5% for the Black-Litterman model is not feasible, given the expected return of bitcoin used in part two of this thesis.

The result from the previous paragraph expresses doubt towards the input of part two of this thesis, which can be explained by the first limitations of the MPT named by Black & Litterman (1991).

Nevertheless, this does not mean a bitcoin allocation in the US portfolio is futile. By fixing the standard deviation of the view at 2% and decreasing the magnitude (Q) of the view, Table 20 shows that for a lower expected excess return of bitcoin, an allocation of 1 to 10 percent is possible. When Q is equal to 1% and its return is lower than that of the MSCI World, bitcoin is still allocated in the portfolio. This would suggest that despite bitcoin's low expected excess return, its diversification advantages make it worth to include bitcoin in the US portfolio.

To summarize this section, despite the fact that the first test for a changing Omega expresses doubt towards the results of part two, the Black-Litterman model shows that by decreasing the magnitude of the view a small bitcoin allocation in the portfolio is still possible, this is in line with the results from part two of this thesis.

Since this chapter discussed the results for each part of this thesis including the Black-Litterman tests. The next chapter will focus on the conclusions of the research, discussion and the recommendations and limitations.

Chapter 5: Conclusion

This study aimed to investigate the diversification, hedge and safe haven properties of bitcoin for a US portfolio. The results indicate bitcoin mainly offers diversification advantages for the US portfolio. By discussing the conclusion for each part of this thesis, the first section will answer the main research question: *What are the diversification, hedge and safe haven advantages for a US investor holding a portfolio including bitcoin?*

After presenting the conclusions of the research, this chapter will provide a discussion by comparing the results to other literature. Furthermore, the last section will present the limitations and recommendations for future research.

5.1 Conclusions of Research

By using the regression from Baur & Lucey (2010), the first part of this thesis presents the findings necessary for answering the research question whether bitcoin possesses diversification, hedge and safe haven properties against the assets in the US portfolio. The results show bitcoin can mainly act as a diversifier against the other assets. In addition, estimates indicate bitcoin can act as a hedge, however all coefficients are close to zero resulting in a very weak hedging effect. Last of all, bitcoin can only act as a weak safe haven against down movements for commodities.

In order to answer research question 2.1.2, the second part of this thesis applies the US portfolio to the Modern Portfolio Theory (Markowitz, 1952, 1959). Despite the high volatility of bitcoin, the efficiency frontier indicates that for the minimum-variance portfolio a small proportion of the total weight is allocated to bitcoin. By increasing the risk for the efficiency frontier, the proportion of bitcoin increases. The results suggest that for an allocation between 1 to 10 percent, bitcoin can be combined with the other assets in the US portfolio, due to the low correlation of bitcoin with other assets this creates diversification benefits.

Part three applies the mean-variance spanning test (Chen et al., 2010) to answer the question whether the inclusion of bitcoin is improving the risk-return trade-off for the US portfolio. The results from the regression analysis provide evidence against the hypothesis for spanning. More precise, the inclusion of bitcoin leads to diversification advantages for the portfolio, which are beneficial for the risk and return trade-off for the US portfolio. This suggests investors should include bitcoin in their portfolio.

Section 4.4 tests the feasibility of a bitcoin allocation of 5% in the US portfolio, assuming the expected return of bitcoin in part two. By using the Black-Litterman model (Black & Litterman, 1991), the outcome of the first test suggests that a small allocation of bitcoin is not as feasible, given the high level on uncertainty for the expected return of bitcoin in part two. Despite the doubt towards the expected return of bitcoin used in part two, the Black-Litterman model does not rule out an allocation of bitcoin to the US portfolio. By lowering the magnitude of the expected excess return, an allocation of 1 to 10 percent is still feasible, this conclusion is in line with the result from part two.

With the conclusions presented in the paragraphs above, one can state that the results of the research suggests bitcoin offers diversification advantages for the US portfolio. Including a small proportion of bitcoin is beneficial for the risk-return trade-off.

5.2 Discussion

The previous section provided the answers for the research questions. This section will focus on the discussion by comparing the results of this thesis with academic literature performing similar tests.

The initial proposal for this thesis was to extend upon the work of Bouri et al., (2016c), who use a method partially based on the regression model of Baur & Lucey (2010). When comparing the results of Bouri et al., (2016c) with the results from section 4.1, one can note the similarities in the outcome with respect to the diversification benefits of bitcoin. Nevertheless, the same can not be said with regards to the outcome for the safe haven effect of bitcoin. This difference might be explained by the fact that Bouri et al., (2016c) employ a DCC model to estimate the correlations for the regressions.

Looking back at section 1.6, Brière et al., (2013) studied the investment of bitcoin for a US investor holding a diversified portfolio by applying the Modern Portfolio Theory (Markowitz, 1952,1959) and conducting two spanning tests. The results of Brière et al., 2013 suggest bitcoin offers diversification benefits due to its low correlation. Both spanning tests indicate inclusion of bitcoin can improve the risk-return trade-off for a diversified portfolio. Despite the similarities, the findings of this thesis are complementary since the dataspan of this thesis is significantly larger than that of Brière et al., (2013).

5.3 Recommendations & Limitations

This section will elaborate on the recommendations and limitations for this thesis. The data will be discussed first, followed by the methods and theory as presented in the order of the methodology.

For this thesis bitcoin is tested against nine other assets. With the selection of these assets, one can remark that this thesis provides a general view regarding the risk management properties of bitcoin. Future research can provide new insights by expanding the portfolio with new assets such as inflation linked bonds and alternative assets like real estate. In addition, this thesis adopts the viewpoint of a US investor. Future research can explore the risk diversification, hedge and safe haven properties of bitcoin from a different perspective.

As noted by Bouri et al., (2016c), during the sample period of their study the bitcoin prices experienced high volatility, this can suggest that the risk management effect for bitcoin is not constant over time. For this reason, Bouri et al., (2016c) indicate that this creates the possibility for future studies focusing on the time-varying risk management effect of bitcoin. It should be noted that the data span for this thesis includes around 63% of the same dates as Bouri et al., (2016c), making this suggestion for future research applicable to the situation of this thesis.

The daily return for bitcoin is obtained from the Bitcoin Price Index (Coindesk, 2017). However, when exchanges fail to meet the index criteria, this can lead to temporary or permanent removal. This might generated noise in the price estimate of the index. Despite these situations, the XBP remains a strong measure for the bitcoin price, since it captures the global price of bitcoin across different exchanges. Alternative sources used by academic literature only consider the price from single exchanges instead of an average across global exchanges.

As mentioned in the discussion regarding the first part of this thesis, Bouri et al., (2016c) employ a DCC model in order to eliminate the heteroscedasticity in the return series. Due to a limited time availability the regression model from section 3.2.1 does not utilize a model to take these errors into account. In addition, for the regressions of Baur & Lucey (2010) only daily returns are used as the input for the regressions. Future research can extend on this thesis by changing the frequency of the input.

The second and third part of this thesis is built on the methodology of Modern Portfolio Theory (Markowitz, 1952,1959), where the third part utilizes a spanning test (Chen et al., 2010). Black & Litterman (1991) develop an alternative for the MPT, discussing the MPT consists of two major limitations. First of all, expected returns are difficult to estimate, besides when based on historical return this provides inaccurate estimates for the future. Secondly, the efficiency frontier is very sensitive to the input for the individual assets. Despite the limitations, MPT is a fundamental tool for financial theory and practice. Besides, throughout its existence the MPT proved to be the base for other portfolio theories (Mangram, 2013). One of these is the mean-variance spanning test used in part three. This thesis utilizes a spanning test where short sales and transaction costs are abscent, however, these factors do contribute to the decision of the investor in real life (Nijman, de Roon, & Werker, 2001). Future research can expand on this thesis by employing a spanning test including these factors.

Section 4.4 tests the results of part two and part three by applying a Black-litterman model. The first test shows that the expected return for bitcoin in part two does not yield a feasible allocation of 5% for bitcoin, which can be explained by the first limitation named by Black & Litterman (1991). Despite this shortcoming, the results for part two and part three are not purposeless. This thesis attempted to close a literary gap by combining the regression of Baur & Lucey (2010) and Modern Portfolio Theory (Markowitz, 1952, 1959). In addition, as explained in the discussion, the results for part two and three are complementary to the results of Brière et al., (2013).

The Black-Litterman model of section 4.4 is used as a test for part two and part three. Due to the limited time availability, the equilibrium weights are standardized. A suggestion for future research would be to focus on the application of bitcoin to the Black-Litterman model, especially since there is little to no academic literature regarding bitcoin and the Black-Litterman model in comparison to bitcoin and the MPT.

Last of all, in light of the discussion in section 1.4 surrounding the nature of bitcoin, the low correlation of bitcoin might suggest that cryptocurrencies should be considered as an alternative asset, due to the potential diversification benefits. In order to test this hypothesis, future research can perform similar tests to this thesis by incorporating alternative cryptocurrencies.

Summarizing, this thesis shows that despite the high volatility and return for bitcoin, its low correlation with other assets offers diversification benefits. Inclusion of bitcoin to the US portfolio can improve the risk-return trade-off for the portfolio. This thesis advocates bitcoin should be taken seriously by investors as a financial asset, however investors should remain vigilant to bitcoin’s risks.

Bibliography

Badev, A., & Chen, M. (2014). Bitcoin: Technical Background and Data Analysis. *Finance and Economics Discussion Series Divisions of Research & Statistics and Monetary Affairs Federal Reserve Board, Washington, D.C.*

Baur, D. (2012). Asymmetric volatility in the gold market. *Journal of Alternative Investments 14(4): 26–38.*

Baur, D. G., & Lucey, B. M. (2010). Is gold a hedge or a safe haven? An analysis of stocks, bonds and gold. *The Financial Review, 217-229*.

Black, F., & Litterman, R. (1991, October). Global Asset Allocation with Equities, Bonds, and Currencies. *Goldman, Sachs & Company*.

Bodie, Z., Kane, A., & Marcus, A. J. (2014). Portfolio Theory and Practice. In Bodie, Kane, & Marcus, *Investments* (10 ed., pp. 117-255). Berkshire: Mc Graw Hill Education.

Bouoiyour, J., & Selmi, R. (2015). What Does Bitcoin Look Like? *Annals of Economics and Finance 16-2*, 449-492.

Bouoiyour, J., Selmi, R., Tiwari, A. K., & Olayeni, O. (2016). What drives Bitcoin price? *Economics Bulletin Volume 36, Issue 2*.

Bouri, E., Azzi, G., & Dyhrberg, A. H. (2016a). On the return-volatility relationship in the Bitcoin market around the price crash of 2013. *Economics Discussion Papers, No. 2016-41*.

Bouri, E., Gupta, R., Tiwari, A. K., & Roubaud, D. (2016b). Does bitcoin hedge global uncertainty? Evidence from Wavelet-based Quantile-in-Quantile regressions.

Bouri, E., Molnár, P., Azzi, G., Roubaud, D., & Hagfors , L. I. (2016c). On the hedge and safe haven properties of Bitcoin: Is it really more than a diversifier? *Financial Research Letters, 20, 192-198*.

Brière, M., Oosterlinck, K., & Szafarz, A. (2013). Virtual Currency, Tangible Return: Portfolio Diversification with Bitcoins.

Buchholz, M., Delanney, J., Warren, J., & Parker , J. (2012). Bits and Bets, Information, Price Volatility, and Demand for Bitcoin. *Economics 312*.

Chen, W., Chung, H., Ho, K., & Hsu, T. (2010). Portfolio Optimization Models and Mean-Variance Spanning Tests. In C. F. Lee, A. C. Lee, & J. Lee, *Handbook of Quantitative Finance and Risk Management* (pp. 165-184). Springer.

Ciaian, P., Rajcaniova, M., & Kanes, D. (2014). The economics of Bitcoin Price formation.

Cieśla, K. (2017). *Ranking System*. Retrieved from Bitcoinity.org: http://data.bitcoinity.org/markets/rank_explanation

Coindesk. (2017). *Coindesk Bitcoin Price Index*. Retrieved from Coindesk: https://www.coindesk.com/price/bitcoin-price-index/

Drobetz, W. (2001). How to avoid the pitfalls in portfolio optimization? Putting the Black-Litterman approach at work. *Swiss Society for Financial Market Research*, 59-75.

Dyhrberg, A. H. (2016a). Bitcoin, gold and the dollar – A GARCH volatility analysis. *Financial Research Letters 16 (2016) 85-92*.

Dyhrberg, A. H. (2016b). Hedging capabilities of bitcoin. Is it the virtual gold? *Financial Research Letters 16 (2016) 139-144*.

Eisl, A., Gasser, S. M., & Weinmayer, K. (2015). Does Bitcoin Improve Portfolio Diversification?

Engle, R. F. (2002). Dynamic condition correlation - A simple class of multivariate GARCH models. *J. Bus. Econ. Statist*, 339-350.

Ennis, C. (2013). Is Bitcoin like Gold? An Examination of the Hedging and Safe Haven Properties of the Virtual Currency. *Master of Science of Finance*.

European Central Bank. (2012). Virtual Currency Schemes.

Ferson, W. E., Foerster, S. R., & Keim, D. B. (1993). General Tests of Latent Variable Models and Mean-Variance Spanning. *Journal of Finance 48*, 131-156.

Hayes, A. (2014). What factors give cryptocurrencies their value: An empirical analysis.

Hayes, A. (2016). Cryptocurrency Value Formation: An Emperical study leading to a cost of production model for valuing Bitcoin. *Telematics and Informatics*.

He, & Litterman, R. (1999). The Intuition Behind Black-Litterman Model Portfolios. *Goldman Sachs Asset Management Working paper*.

Huberman, G., & Kandel, S. (1987). Mean-Variance Spanning. *The Journal of Finance*, 873-888.

Huhtinen, T. P. (2014). Bitcoin as a monetary system: Examining attention and attendance.

Idzorek, T. M. (2005). A Step-By-Step Guide To The Black-Litterman Model. Incorporating user-specified confidence levels.

Iwamura, M., Kitamura, Y., Matsumoto, T., & Saito, K. (2014). Can We Stabilize the Price of a cryptocurrency?: Understanding the Design of Bitcoin and Its Potential to Compete with Central Bank Money.

Jenssen, T. B. (2014). Why Bitcoins Have Value, and Why Governments Are Sceptical. *Master's thesis - Department of Economics - University of Oslo*.

Kan, R., & Zhou, G. (2008). Tests of mean-variance spanning. *Annals of Economics and Finance*, 145-193.

Kristoufek, L. (2013). Bitcoin meets Google Trends and Wikipedia: Quantifying the relationship between phenomena of the Internet era. *Scientific Reports 3 (3415)*, 1-7.

Kristoufek, L. (2015). What are the Main Drivers of the Bitcoin Price? Evidence from Wavelet Coherence Analysis.

Lee, W. (2000). Advanced Theory and Methodology of Tactical Asset Allocation. *New York: John Wiley & Sons*.

Lo, S., & Wang, C. (2014). Bitcoin as Money? *Current Policy Perspectives - Federal Reserve Bank of Boston*.

Mangram, M. E. (2013). A simplified perspective of the Markowitz Portfolio Theory. *Global Journal of Business Research*, 59-70.

Markowitz. (1952). Portfolio selection. *Journal of Finance*(7), 77-91.

Markowitz. (1959). Portfolio selection: efficient diversification of investments.

Maroney, N., & Naka, A. (2006). Diversification Benefits of Japanse Real Estate of the Last Four Decades. *J Real Estate Finan Econ 33*, 259-274.

MSCI Inc. (2017, September 29). *MSCI Emerging Markets Index*. Retrieved from MSCI: https://www.msci.com/documents/10199/c0db0a48-01f2-4ba9-ad01-226fd5678111

Nakamoto, S. (2008). Bitcoin: A peer-to-peer electronic cash system. .

Nijman, T. E., de Roon, F. A., & Werker, B. J. (2001). Testing for Mean-Variance spanning with short sales constraints and transaction costs: The case of emerging markets. *Journal of Finance, 56(2)*, 721-742.

Peng, S. (2013). Bitcoin: Cryptography, Economics, and the Future. *Senior Capstone Thesis - School of Engineering and Applied Science*.

Polasik, M., Piotrowska, A., Wisniewski, T. P., Kotkowski, R., & Lightfoot, G. (2014). Price Fluctuations and the Use of Bitcoin: An Emprical Inquiry.

Ratner, M., & Chiu, C. C. (2013). Hedging stock sector risk with credit default swaps. *International Review of Financial Analysis, 18-25*.

Segendorf, B. (2014). Have virtual currencies affected the retail payments market? *Sveriges Riksbank Economics Commentaries, (2)*.

Van Alstyne, M. (2014). Why Bitcoin Has Value. *Communications of the ACM*.

Van Wijk, D. (2013). What can be expected from the Bitcoin? *Working Paper No. 345986, Erasmus Rotterdam Universiteit*.

Wallace, B. (2011). *The Rise and Fall of Bitcoin*. Retrieved from Wired: http://www.wired.com/magazine/2011/11/mf_bitcoin.

Walters, J. (2013). *blacklitterman*. Retrieved from blacklitterman: www.blacklitterman.org

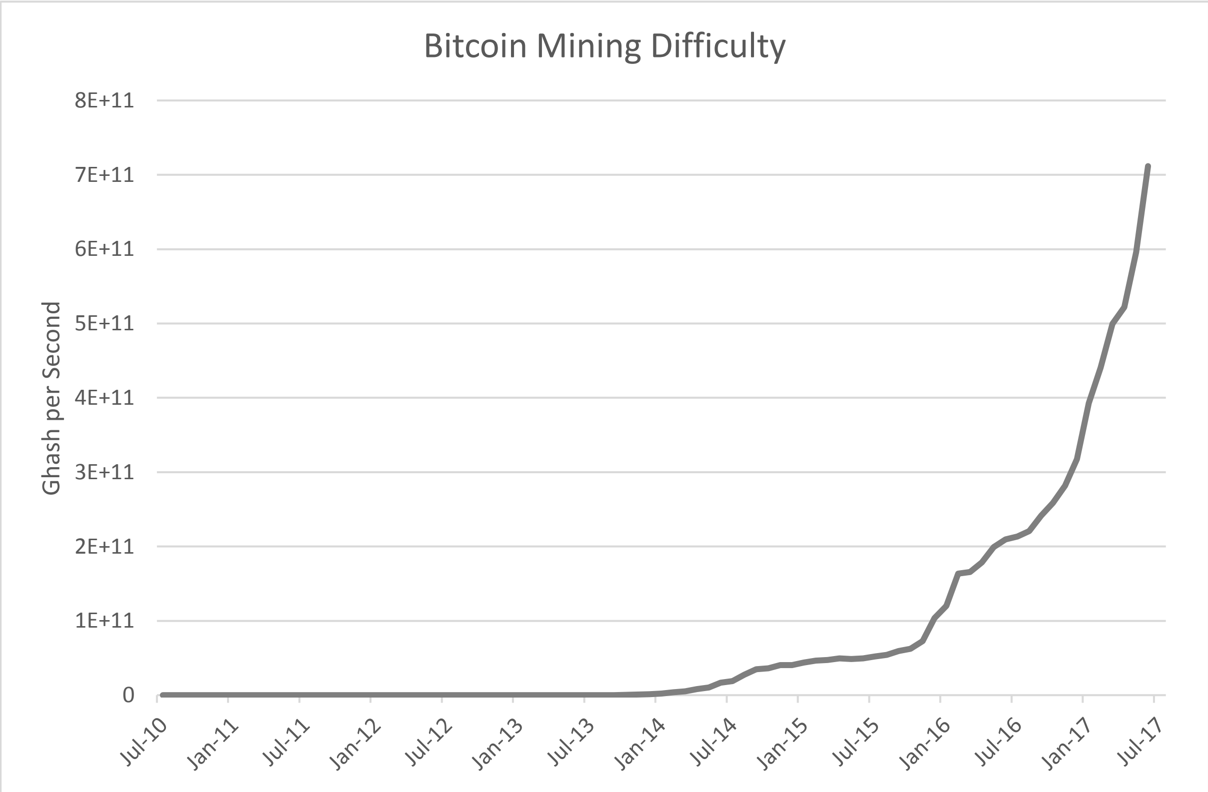
Woo, D., Gordon, I., & Iaralov, V. (2013). Bitcoin: a first assessment. *FX and rates*.

Yermack, D. (2013). Is Bitcoin a real currency? An economic appraisal. *NBER Working Paper Series*.

Appendix

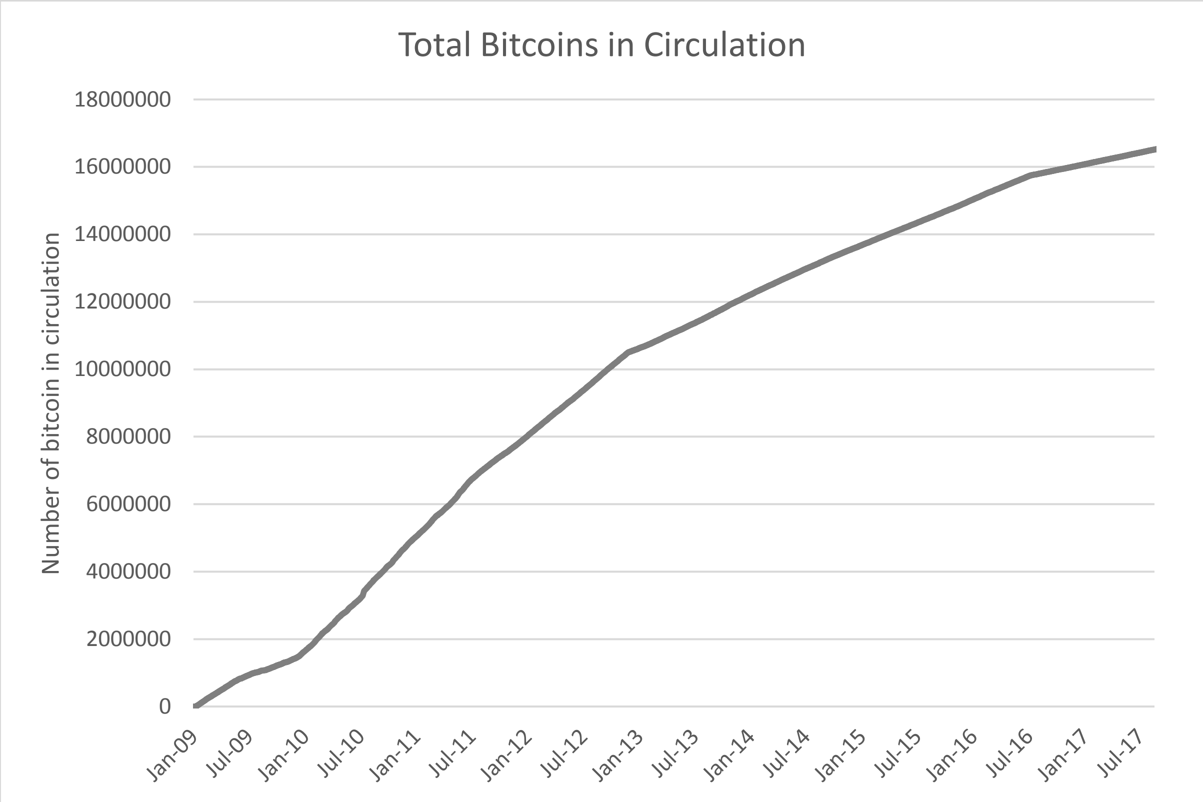
A.1 Figures

Figure 1. The Difficulty of solving the Computational Problem for Miners, regressed on time.



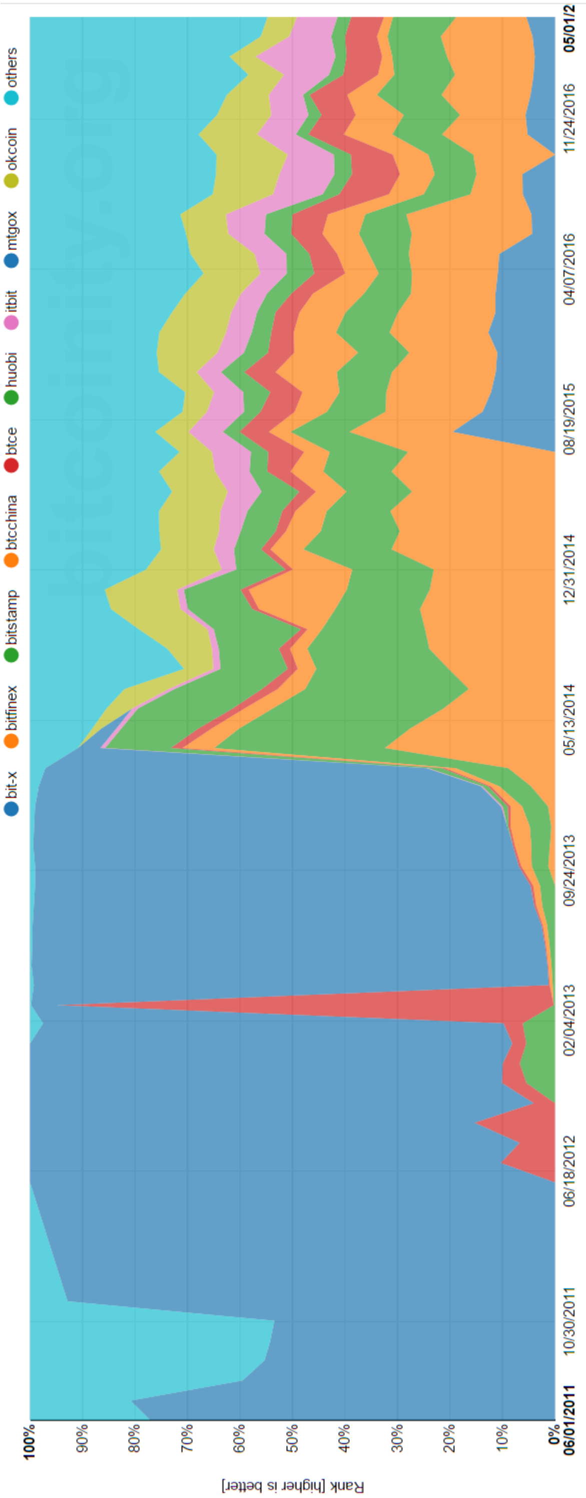
Note: Using monthly observations, this graph plots the difficulty for miners in solving the computational problem over the period July 2010 to June 2017. On the y-axis, the difficulty as measured in Ghash per second, which is a billion hash per second –the concept of hash is explained in section 1.2.2. Source: http://bitcoinity.org/.

Figure 2. The number of bitcoins in circulation, regressed over time.



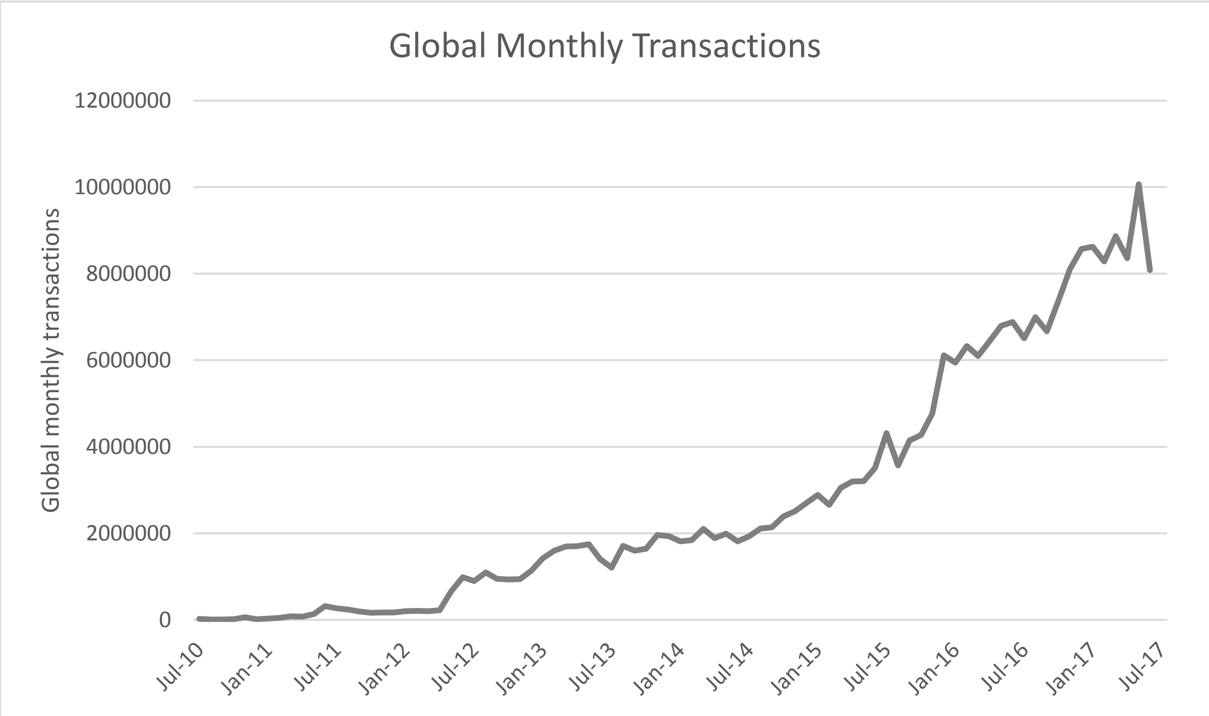
Note: With monthly observations from January 2009 to June 2017, this graph presents to total amount of bitcoin in circulation. Source: quandl.com

Figure 3. Market share of bitcoin exchanges throughout the lifespan of the cryptocurrency.



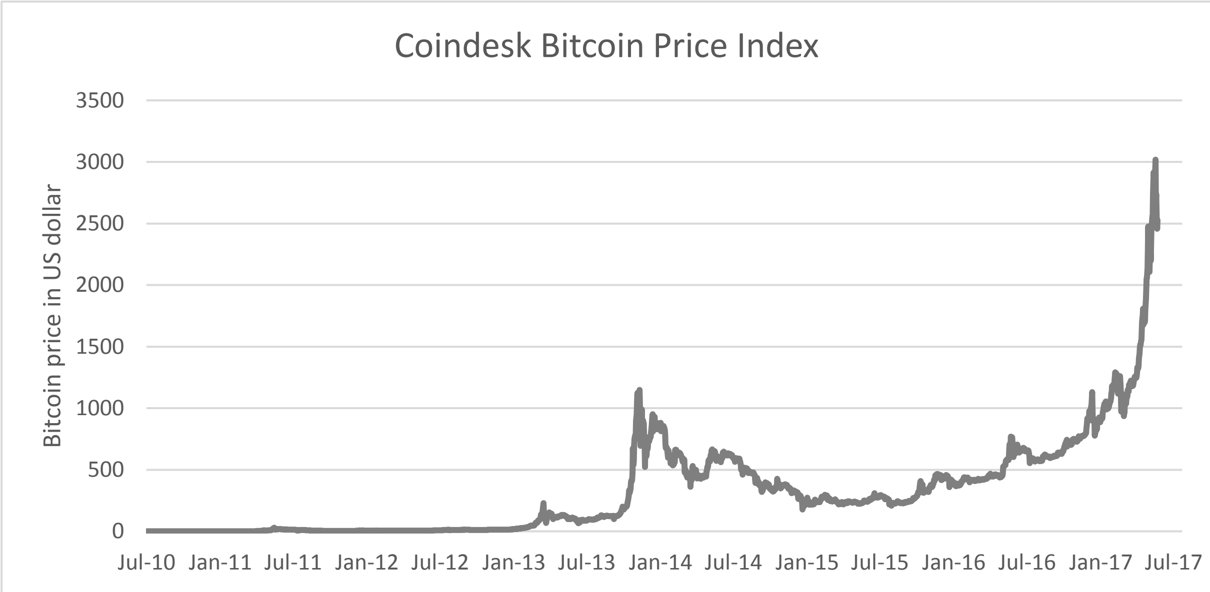
Note: The illustration above shows the market share for exchanges trading bitcoin for all available currencies in percentages. Using monthly observations this image regresses the developments for the market share over time. The data includes the period of June 2011 to May 2017 – data before this period displays only Mt. Gox as the available exchange. The market share for this graph is not base on the trading volume but on the rank, which is based on the order book of each exchange. Source: http://bitcoinity.org/

Figure 4. The total number of transactions recorded in the Blockchain.



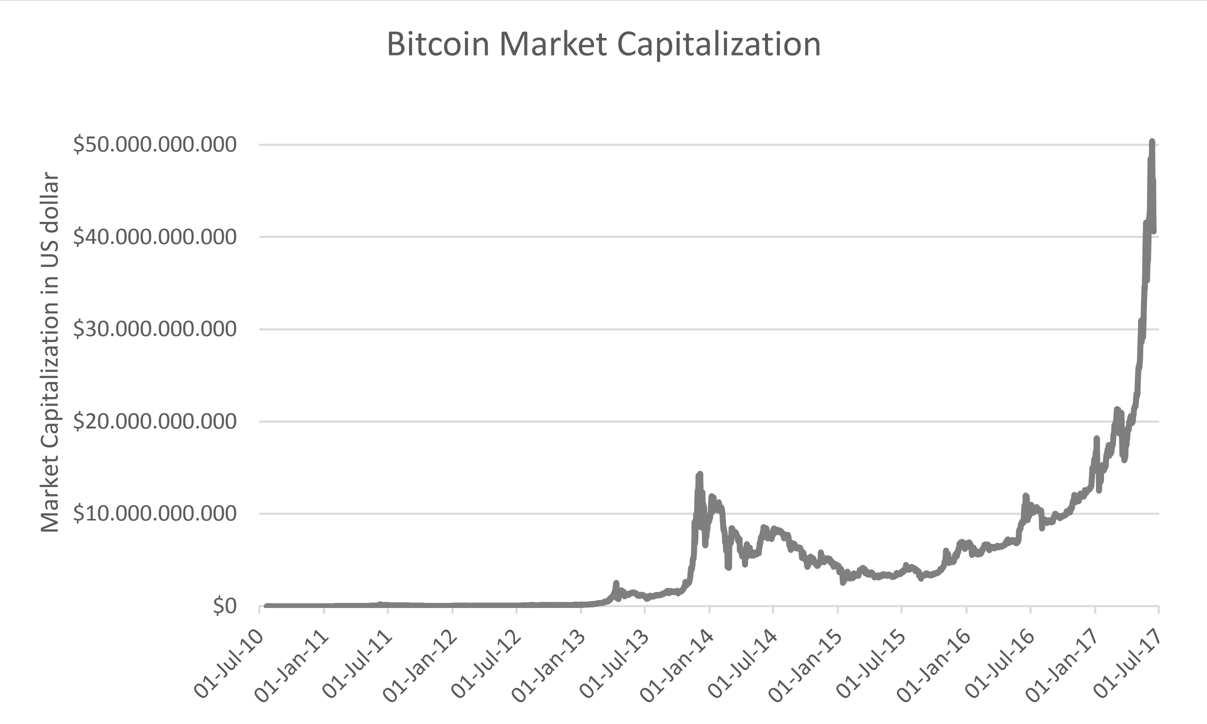
Note: The figure above illustrates the total transactions in bitcoins for each month over the period July 2010 – June 2017. Source: http://bitcoinity.org/

Figure 5. XBP in US dollars regressed over time.



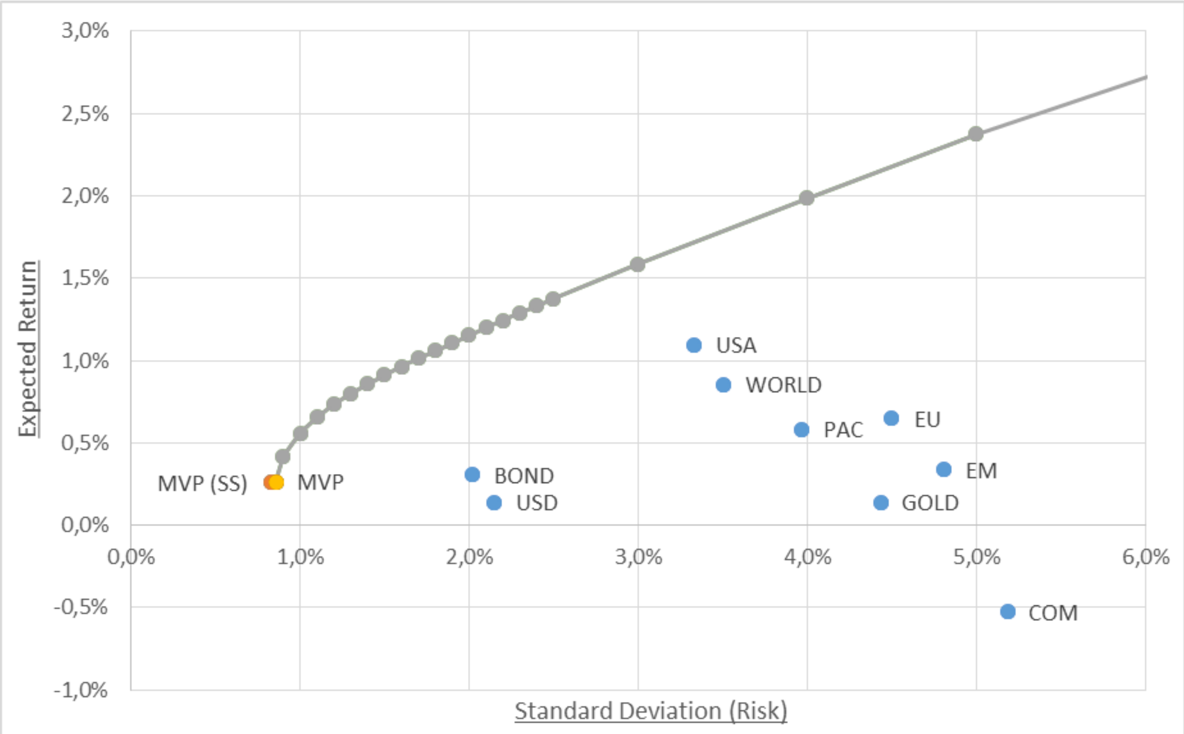
Note: This graph plots the daily closing price for the Bitcoin Price Index of Coindesk in US dollars. Source: https://www.coindesk.com/price/

Figure 6. Market capitalization of bitcoin



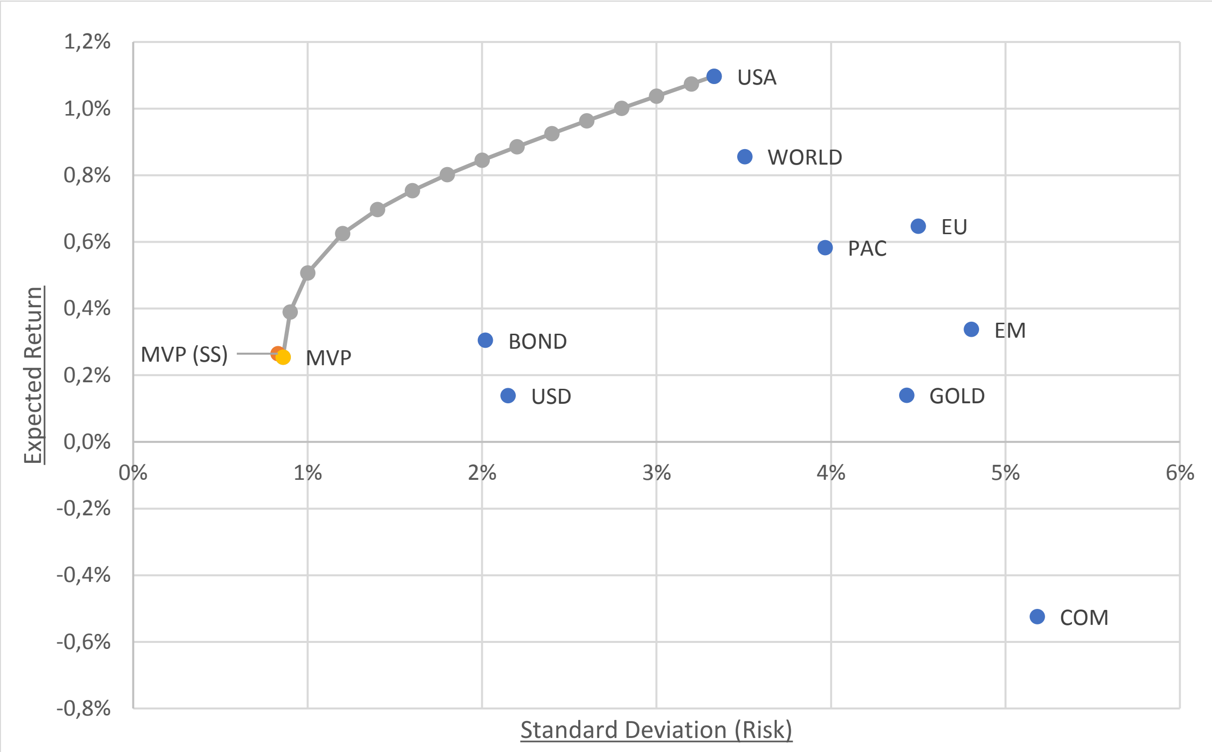
Note: The graph above shows the market capitalization of bitcoin in US dollars for the period July 2010 to June 2017, using daily observations. Source: http://bitcoinity.org/

Figure 7. Efficiency Frontier & Individual Assets for the portfolio including bitcoin (extended portfolio)



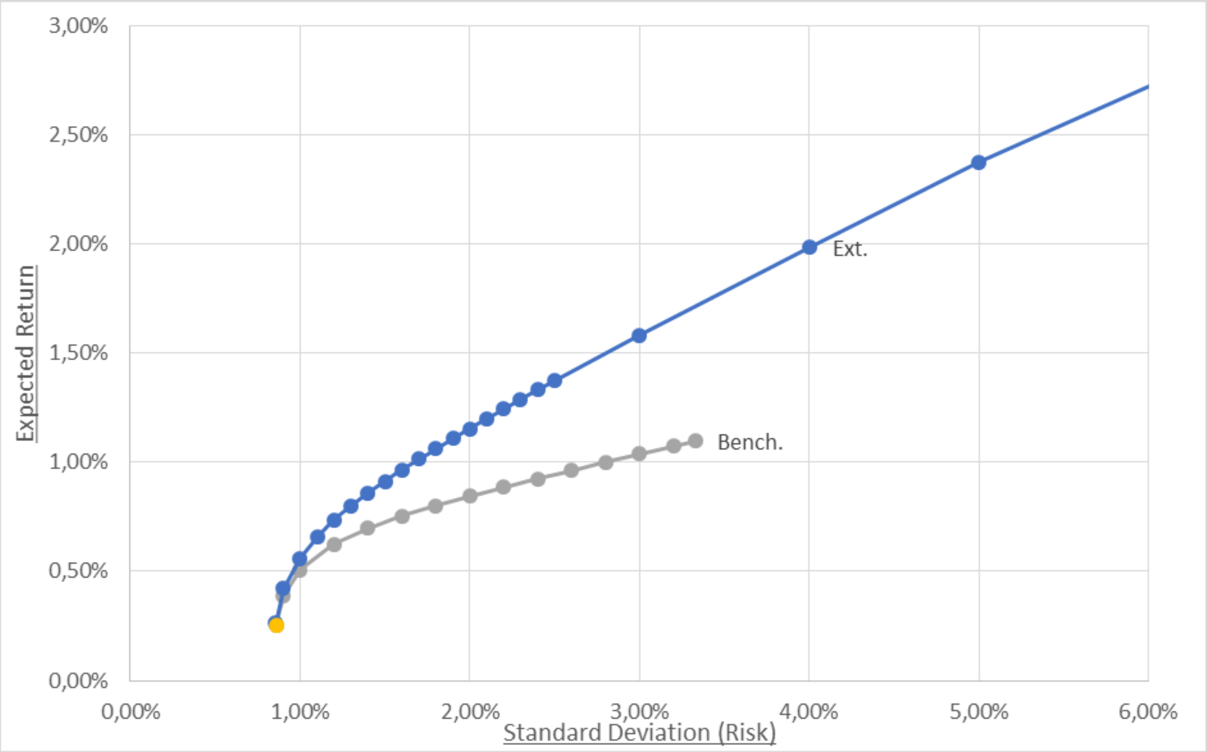
Note: The efficiency frontier for the portfolio including bitcoin ends with the portfolio where 100% of the total asset weights is allocated to bitcoin, the corresponding portfolio risk is 64.55% with an expected return of 22.99%. The end of the efficiency frontier is removed from this figure in order to keep a clear overview of the portfolio performance.

Figure 8. Efficiency Frontier & Individual Assets for the portfolio excluding bitcoin (benchmark portfolio)



Note: The figure above illustrates the efficiency frontier for the portfolio excluding bitcoin.

Figure 9. Efficiency Frontier for the benchmark and extended portfolios



Note: The figure above illustrates the efficiency frontier for the extended portfolio and benchmark portfolio.

A.2 Tables

Table 1. Market share of bitcoin exchanges (March 2012 – March 2014).

	Volume (MLN)	MKT Share
Mt Gox	41,6	51,7%
Btcchina	9,88	12,3%
Huobi	9,16	11,4%
Bitstamp	8,59	10,7%
Btce	6,86	8,5%
Bitfinex	1,99	2,5%
Bitcoin24	0,81	1,0%
Campbx	0,64	0,8%
Localbitcoins	0,49	0,6%
Other	0,4	0,5%

Note: This table illustrates the total market share of each exchange, including all currencies, (Lo & Wang, 2014). The volume is the sum of bitcoins traded over the selected period. Source: http://bitcoinity.org/.

Table 2. Market share of bitcoin exchanges (November 2015 – November 2017).

	Volume (MLN)	MKT Share
Okcoin	520	39,5%
Huobi	491	37,3%
Btcchina	214	16,2%
Others	27,3	2,1%
Bitfinex	17,8	1,4%
Lakebtc	12,9	1,0%
Bitflyer	10,9	0,8%
Kraken	8,48	0,6%
Bitstamp	7,58	0,6%
Coinbase	7,55	0,6%

Note: This table illustrates the total market share of each exchange, including all currencies. The volume is the sum of bitcoins traded over the selected period. Source: http://bitcoinity.org/.

Table 3. Variable list

$R_{bitcoin,t}$	Return of bitcoin at time t
a	Constant for the simple linear regression
b_1	Estimated beta coefficient for the average effect
b_2	Estimated beta coefficient for 5% quantile
b_3	Estimated beta coefficient for 2.5% quantile
b_4	Estimated beta coefficient for 1% quantile
$R_{asset,t}$	Return of asset at time t
$R_{asset,t\,(q)}$	Return of asset at times of market turmoil (q)
e_t	Error term
$E(r_p)$	Expected portfolio return
w_i	Weight of asset i
$E(r_i)$	Expected return of asset i
$Var(r_p)$	Portfolio variance
p_{ij}	Correlation coefficient between asset i and j
σ_i	Standard deviation of asset i
R_{ext}	Return of the extended portfolio
R_{bench}	Return of the benchmark portfolio
μ^{IER}	Implied equilibrium excess return
γ	Risk aversion coefficient
Σ	Variance-Covariance matrix
w^*	Equilibirum weights
$BL\ [R]$	Black-Litterman Expected excess return
τ	scalar Tau for the precision in IER
P	selection matrix for the views
Ω	confindence matrix
Q	A matrix containing the magnitude of each view

Note: each of these variable is used for the equations of this thesis.

Table 4. Descriptive statistics, daily returns, 26 July 2010 – 16 June 2017

	Mean	Std. dev.	Min.	Median	Max.	Skewness	Kurtosis
Bitcoin	0,8035%	6,5439%	-35,81%	0,25%	64,82%	1,30	12,64
MSCI World	0,0432%	0,8339%	-5,11%	0,06%	4,20%	-0,48	4,59
MSCI EU	0,0344%	1,2134%	-8,77%	0,03%	5,94%	-0,35	4,10
MSCI USA	0,0557%	0,9065%	-6,71%	0,04%	4,80%	-0,40	5,04
MSCI PAC	0,0320%	1,0137%	-6,38%	0,05%	5,22%	-0,30	3,03
MSCI EM	0,0161%	0,9715%	-6,31%	0,05%	4,93%	-0,31	2,87
BOND Index	0,0157%	0,4400%	-1,90%	0,02%	1,87%	-0,16	1,14
COM Index	-0,0305%	1,1995%	-6,52%	0%	5,63%	-0,10	2,50
GOLD Index	0,0066%	1,0626%	-9,34%	0%	4,72%	-0,65	6,31
USD Index	0,0071%	0,4783%	-2,39%	0%	2,19%	0,09	1,68

Note: the descriptive statistics is based on 1799 daily observations in US dollar.

Table 5. Results for the risk management properties of Bitcoin

MSCI WORLD			MSCI Europe		
Bitcoin	Coeff. Est.	t-stat	Bitcoin	Coeff. Est.	t-stat
b_1	0,013	-3,84***	b_1	0,053	-4,54***
$b_1 + b_2$	0,012	0,13	$b_1 + b_2$	-0,037	-1,34
$b_1 + b_2 + b_3$	0,063	1,39	$b_1 + b_2 + b_3$	0,050	0,88
$b_1 + b_2 + b_3 + b_4$	0,064	2,04**	$b_1 + b_2 + b_3 + b_4$	0,043	0,86
MSCI USA			**MSCI Pacific**		
Bitcoin	Coeff. Est.	t-stat	Bitcoin	Coeff. Est.	t-stat
b_1	-0,003	-4,78***	b_1	-0,028	-6,35***
$b_1 + b_2$	0,020	0,47	$b_1 + b_2$	0,003	0,35
$b_1 + b_2 + b_3$	0,078	1,97**	$b_1 + b_2 + b_3$	-0,041	-0,80
$b_1 + b_2 + b_3 + b_4$	0,081	2,75***	$b_1 + b_2 + b_3 + b_4$	0,004	0,85
MSCI Emerging Markets			**Bond Index**		
Bitcoin	Coeff. Est.	t-stat	Bitcoin	Coeff. Est.	t-stat
b_1	-0,020	-5,87***	b_1	-0,022	-3,19***
$b_1 + b_2$	-0,002	0,17	$b_1 + b_2$	-0,015	-0,09
$b_1 + b_2 + b_3$	-0,030	-0,57	$b_1 + b_2 + b_3$	-0,031	-0,54
$b_1 + b_2 + b_3 + b_4$	0,016	1,24	$b_1 + b_2 + b_3 + b_4$	-0,044	-1,17
Commodity Index			**Gold Index**		
Bitcoin	Coeff. Est.	t-stat	Bitcoin	Coeff. Est.	t-stat
b_1	0,006	-6,27***	b_1	-0,006	-5,68***
$b_1 + b_2$	0,089	2,02**	$b_1 + b_2$	-0,025	-0,47
$b_1 + b_2 + b_3$	-0,067	2,53*	$b_1 + b_2 + b_3$	-0,007	-0,05
$b_1 + b_2 + b_3 + b_4$	-0,134	0,30	$b_1 + b_2 + b_3 + b_4$	0,056	2,58*
US dollar Index					
Bitcoin	Coeff. Est.	t-stat			
b_1	-0,036	-3,93***			
$b_1 + b_2$	-0,030	-0,28			
$b_1 + b_2 + b_3$	0,003	0,65			
$b_1 + b_2 + b_3 + b_4$	-0,009	0,24			

*Note: *** , ** , * indicate statistical significance at the 0.01, 0.05, 0.10 level.*

Table 6. Descriptive statistics, monthly returns, 26 July 2010 – 16 June 2017

	Mean	Std. dev.	Min.	Median	Max.	Skewness	Kurtosis
Bitcoin	22,998%	64,550%	-59,55%	4,96%	360,28%	2,91	10,15
MSCI World	0,856%	3,506%	-11,19%	0,78%	9,19%	-0,54	1,38
MSCI EU	0,647%	4,500%	-13,45%	0,73%	12,15%	-0,31	1,15
MSCI USA	1,097%	3,329%	-9,67%	0,95%	9,58%	-0,44	1,05
MSCI PAC	0,583%	3,965%	-12,45%	0,95%	8,92%	-0,46	0,54
MSCI EM	0,338%	4,804%	-14,60%	0,29%	11,92%	-0,36	0,55
BOND Index	0,305%	2,019%	-5,22%	0,28%	6,72%	0,16	0,60
COM Index	-0,524%	5,182%	-13,94%	-0,67%	10,18%	-0,27	-0,16
GOLD Index	0,140%	4,433%	-11,64%	0,15%	8,83%	-0,30	-0,11
USD Index	0,139%	2,148%	-4,03%	0,03%	5,53%	0,22	-0,45

Note: the descriptive statistics for the monthly return is based on 89 observations.

Table 7. Individual assets, monthly returns

	Exp. ret.	Std. dev.	Sharpe ratio
Bitcoin	22,998%	64,550%	0,36
MSCI World	0,856%	3,506%	0,24
MSCI EU	0,647%	4,500%	0,14
MSCI USA	1,097%	3,329%	0,33
MSCI PAC	0,583%	3,965%	0,15
MSCI EM	0,338%	4,804%	0,07
BOND Index	0,305%	2,019%	0,15
COM Index	-0,524%	5,182%	-0,10
GOLD Index	0,140%	4,433%	0,03
USD Index	0,139%	2,148%	0,06

Note: The Sharpe ratio is the Expected return divided by the standard deviation.

Table 8. Variance-Covariance Matrix

	Bitcoin	MSCI WORLD	MSCI EU	MSCI USA	MSCI PAC	MSCI EM	BOND INDEX	COM INDEX	GOLD INDEX	USD INDEX
Bitcoin	0,41667	0,00236	0,00296	0,00212	0,00294	-0,00271	-0,00011	0,00037	-0,00006	-0,00159
MSCI WORLD	0,00236	0,00123	0,00145	0,00112	0,00119	0,00129	-0,00035	0,00090	0,00007	-0,00023
MSCI EU	0,00296	0,00145	0,00202	0,00119	0,00135	0,00160	-0,00040	0,00114	0,00024	-0,00049
MSCI USA	0,00212	0,00112	0,00119	0,00111	0,00100	0,00108	-0,00036	0,00076	-0,00004	-0,00010
MSCI PAC	0,00294	0,00119	0,00135	0,00100	0,00157	0,00143	-0,00023	0,00085	0,00009	-0,00024
MSCI EM	-0,00271	0,00129	0,00160	0,00108	0,00143	0,00231	-0,00020	0,00129	0,00056	-0,00037
BOND INDEX	-0,00011	-0,00035	-0,00040	-0,00036	-0,00023	-0,00020	0,00041	-0,00033	0,00034	0,00001
COM INDEX	0,00037	0,00090	0,00114	0,00076	0,00085	0,00129	-0,00033	0,00269	0,00044	-0,00039
GOLD INDEX	-0,00006	0,00007	0,00024	-0,00004	0,00009	0,00056	0,00034	0,00044	0,00197	-0,00035
USD INDEX	-0,00159	-0,00023	-0,00049	-0,00010	-0,00024	-0,00037	0,00001	-0,00039	-0,00035	0,00046

Note: this VCV-matrix is based on the monthly data.

Table 9. Correlation matrix

	Bitcoin	MSCI WORLD	MSCI EU	MSCI USA	MSCI PAC	MSCI EM	BOND INDEX	COM INDEX	GOLD INDEX	USD INDEX
Bitcoin	1	0,10	0,10	0,10	0,11	-0,09	-0,01	0,01	0,00	-0,11
MSCI WORLD	0,10	1	0,92	0,96	0,85	0,77	-0,50	0,50	0,05	-0,31
MSCI EU	0,10	0,92	1	0,80	0,76	0,74	-0,44	0,49	0,12	-0,51
MSCI USA	0,10	0,96	0,80	1	0,76	0,68	-0,53	0,44	-0,03	-0,14
MSCI PAC	0,11	0,85	0,76	0,76	1	0,75	-0,28	0,41	0,05	-0,29
MSCI EM	-0,09	0,77	0,74	0,68	0,75	1	-0,20	0,52	0,26	-0,36
BOND INDEX	-0,01	-0,50	-0,44	-0,53	-0,28	-0,20	1	-0,31	0,38	0,03
COM INDEX	0,01	0,50	0,49	0,44	0,41	0,52	-0,31	1	0,19	-0,35
GOLD INDEX	0,00	0,05	0,12	-0,03	0,05	0,26	0,38	0,19	1	-0,37
USD INDEX	-0,11	-0,31	-0,51	-0,14	-0,29	-0,36	0,03	-0,35	-0,37	1

Note: The correlation matrix above is based on the monthly data.

Table 10. Portfolio performance and asset weights (extended portfolio)

	Equal Wgt.	Min. var. portfolio	
		With SS	No SS
Bitcoin	10%	-0,03%	0,05%
MSCI WORLD	10%	51,82%	0,00%
MSCI EU	10%	6,79%	17,11%
MSCI USA	10%	-25,77%	0,08%
MSCI PAC	10%	-9,60%	0,00%
MSCI EM	10%	-7,09%	0,00%
BOND INDEX	10%	39,93%	37,78%
COM INDEX	10%	6,13%	5,60%
GOLD INDEX	10%	0,95%	0,73%
USD INDEX	10%	36,88%	38,64%
total weight	100%	100%	100%
Portfolio Exp. ret.	2,6578%	0,2601%	0,2632%
Portfolio Std. dev.	6,8818%	0,8290%	0,8574%
Portfolio variance	0,4736%	0,0069%	0,0074%
Sharpe ratio	0,386	0,314	0,307

Note: The table above is based on the monthly data, showing the weights for the minimum-variance portfolio with and without short sales.

Table 11. Asset weights for a given Portfolio Standard Deviation (extended portfolio)

		Asset weights for a given Portfolio Standard Deviation									
Portfolio			MSCI					INDEX			
Std. dev.	Exp. ret.	Bitcoin	WORLD	EU	USA	PAC	EM	BOND	COM	GOLD	USD
0,857%	0,263%	0,05%	0%	17,11%	0,08%	0%	0%	37,78%	5,60%	0,73%	38,64%
0,90%	0,42%	0,26%	0%	11,72%	11,58%	0%	0%	41,65%	2,81%	0,07%	31,91%
1,00%	0,56%	0,46%	0%	6,93%	21,69%	0%	0%	44,47%	0,24%	0%	26,22%
1,10%	0,66%	0,64%	0%	1,89%	29,69%	0%	0%	46,33%	0%	0%	21,44%
1,20%	0,74%	0,85%	0%	0%	33,81%	0%	0%	46,96%	0%	0%	18,39%
1,30%	0,80%	1,06%	0%	0%	35,59%	0%	0%	46,90%	0%	0%	16,45%
1,40%	0,86%	1,25%	0%	0%	37,19%	0%	0%	46,85%	0%	0%	14,71%
1,50%	0,91%	1,43%	0%	0%	38,67%	0%	0%	46,80%	0%	0%	13,11%
1,60%	0,96%	1,59%	0%	0%	40,07%	0%	0%	46,76%	0%	0%	11,58%
1,70%	1,01%	1,76%	0%	0%	41,41%	0%	0%	46,71%	0%	0%	10,12%
1,80%	1,06%	1,91%	0%	0%	42,72%	0%	0%	46,67%	0%	0%	8,70%
1,90%	1,11%	2,06%	0%	0%	43,99%	0%	0%	46,63%	0%	0%	7,32%
2,00%	1,15%	2,21%	0%	0%	45,24%	0%	0%	46,59%	0%	0%	5,96%
2,10%	1,20%	2,36%	0%	0%	46,47%	0%	0%	46,55%	0%	0%	4,63%
2,20%	1,24%	2,50%	0%	0%	47,68%	0%	0%	46,51%	0%	0%	3,31%
2,30%	1,29%	2,64%	0%	0%	48,87%	0%	0%	46,47%	0%	0%	2,01%
2,40%	1,33%	2,78%	0%	0%	50,06%	0%	0%	46,43%	0%	0%	0,72%
2,50%	1,37%	2,93%	0%	0%	51,10%	0%	0%	45,97%	0%	0%	0%
3%	1,58%	3,71%	0%	0%	55,31%	0%	0%	40,98%	0%	0%	0%
4%	1,98%	5,19%	0%	0%	63,38%	0%	0%	31,43%	0%	0%	0%
5%	2,37%	6,63%	0%	0%	71,23%	0%	0%	22,13%	0%	0%	0%
6%	2,76%	8,06%	0%	0%	78,99%	0%	0%	12,95%	0%	0%	0%
7%	3,14%	9,48%	0%	0%	86,69%	0%	0%	3,83%	0%	0%	0%
8%	3,52%	11,07%	0%	0%	88,93%	0%	0%	0%	0%	0%	0%
10%	4,26%	14,42%	0%	0%	85,58%	0%	0%	0%	0%	0%	0%
20%	7,76%	30,42%	0%	0%	69,58%	0%	0%	0%	0%	0%	0%
30%	11,20%	46,12%	0%	0%	53,88%	0%	0%	0%	0%	0%	0%
40%	14,62%	61,74%	0%	0%	38,26%	0%	0%	0%	0%	0%	0%
50%	18,03%	77,34%	0%	0%	22,66%	0%	0%	0%	0%	0%	0%
60%	21,45%	92,91%	0%	0%	7,09%	0%	0%	0%	0%	0%	0%
64,55%	22,998%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%

Note: All asset weights sum up to 100%, each row represents a portfolio situated on the efficient frontier. The top observation of this table represents the minimum-variance portfolio without short sales.

Table 12. Asset weights for a given Portfolio Expected Return (extended portfolio)

		Asset weights for a given Portfolio return									
Portfolio			MSCI					INDEX			
Exp. ret.	Std. dev.	Bitcoin	WORLD	EU	USA	PAC	EM	BOND	COM	GOLD	USD
0,263%	0,857%	0,05%	0,00%	17,11%	0,08%	0%	0%	37,78%	5,60%	0,73%	38,64%
0,3%	0,86%	0,10%	0%	15,85%	2,77%	0%	0%	38,69%	4,95%	0,57%	37,07%
0,4%	0,89%	0,24%	0%	12,43%	10,08%	0%	0%	41,15%	3,17%	0,16%	32,78%
0,5%	0,95%	0,38%	0%	8,97%	17,38%	0%	0%	43,30%	1,34%	0%	28,63%
0,6%	1,04%	0,53%	0%	4,95%	24,98%	0%	0%	45,26%	0%	0%	24,28%
0,7%	1,15%	0,74%	0%	0%	32,85%	0%	0%	46,99%	0%	0%	19,43%
0,8%	1,30%	1,06%	0%	0%	35,57%	0%	0%	46,90%	0%	0%	16,47%
0,9%	1,47%	1,38%	0%	0%	38,30%	0%	0%	46,81%	0%	0%	13,51%
1,0%	1,67%	1,71%	0%	0%	41,02%	0%	0%	46,72%	0%	0%	10,54%
1,1%	1,88%	2,03%	0%	0%	43,75%	0%	0%	46,64%	0%	0%	7,58%
1,2%	2,10%	2,36%	0%	0%	46,47%	0%	0%	46,55%	0%	0%	4,62%
1,3%	2,33%	2,68%	0%	0%	49,20%	0%	0%	46,46%	0%	0%	1,66%
1,4%	2,56%	3,02%	0%	0%	51,61%	0%	0%	45,37%	0%	0%	0,0%
1,5%	2,80%	3,39%	0%	0%	53,62%	0%	0%	42,98%	0%	0%	0,0%
2,0%	4,04%	5,25%	0%	0%	63,69%	0%	0%	31,07%	0%	0%	0,0%
2,5%	5,32%	7,10%	0%	0%	73,75%	0%	0%	19,15%	0%	0%	0,0%
3,0%	6,63%	8,95%	0%	0%	83,82%	0%	0%	7,23%	0%	0%	0,0%
3,5%	7,94%	10,97%	0%	0%	89,03%	0%	0%	0%	0%	0%	0,0%
4,0%	9,30%	13,25%	0%	0%	86,75%	0%	0%	0%	0%	0%	0,0%
5%	12,08%	17,82%	0%	0%	82,18%	0%	0%	0%	0%	0%	0,0%
6%	14,93%	22,39%	0%	0%	77,61%	0%	0%	0%	0%	0%	0,0%
7%	17,80%	26,95%	0%	0%	73,05%	0%	0%	0%	0%	0%	0,0%
8%	20,69%	31,52%	0%	0%	68,48%	0%	0%	0%	0%	0%	0,0%
9%	23,60%	36,08%	0%	0%	63,92%	0%	0%	0%	0%	0%	0,0%
10%	26,51%	40,65%	0%	0%	59,35%	0%	0%	0%	0%	0%	0,0%
11%	29,42%	45,22%	0%	0%	54,78%	0%	0%	0%	0%	0%	0,0%
12%	32,34%	49,78%	0%	0%	50,22%	0%	0%	0%	0%	0%	0,0%
13%	35,26%	54,35%	0%	0%	45,65%	0%	0%	0%	0%	0%	0,0%
14%	38,19%	58,91%	0%	0%	41,09%	0%	0%	0%	0%	0%	0,0%
15%	41,11%	63,48%	0%	0%	36,52%	0%	0%	0%	0%	0%	0,0%
16%	44,04%	68,05%	0%	0%	31,95%	0%	0%	0%	0%	0%	0,0%
17%	46,97%	72,61%	0%	0%	27,39%	0%	0%	0%	0%	0%	0,0%
18%	49,90%	77,18%	0%	0%	22,82%	0%	0%	0%	0%	0%	0,0%
19%	52,83%	81,74%	0%	0%	18,26%	0%	0%	0%	0%	0%	0,0%
20%	55,76%	86,31%	0%	0%	13,69%	0%	0%	0%	0%	0%	0,0%
21%	58,69%	90,88%	0%	0%	9,12%	0%	0%	0%	0%	0%	0,0%
22%	61,62%	95,44%	0%	0%	4,56%	0%	0%	0%	0%	0%	0,0%
22,998%	64,55%	100,00%	0%	0%	0%	0%	0%	0%	0%	0%	0,0%

Note: All asset weights sum up to 100%, each row represents a portfolio situated on the efficient frontier. The top observation of this table represents the minimum-variance portfolio without short sales.

Table 13. Portfolio Expected Return and Standard Deviation for a given bitcoin weight (extended portfolio)

	Asset weights for a given Bitcoin weight										
	MSCI					INDEX				Portfolio	
Bitcoin	WORLD	EU	USA	PAC	EM	BOND	COM	GOLD	USD	Std. dev.	Exp. Ret.
0,05%	0,00%	17,11%	0,08%	0,00%	0,00%	37,78%	5,60%	0,73%	38,64%	0,857%	0,263%
1%	0%	0%	35%	0%	0%	47%	0%	0%	17%	1,27%	0,78%
2%	0%	0%	43%	0%	0%	47%	0%	0%	8%	1,86%	1,09%
3%	0%	0%	51%	0%	0%	46%	0%	0%	0%	2,54%	1,39%
4%	0%	0%	57%	0%	0%	39%	0%	0%	0%	3,19%	1,66%
5%	0%	0%	62%	0%	0%	33%	0%	0%	0%	3,87%	1,93%
6%	0%	0%	68%	0%	0%	26%	0%	0%	0%	4,56%	2,20%
7%	0%	0%	73%	0%	0%	20%	0%	0%	0%	5,25%	2,47%
8%	0%	0%	79%	0%	0%	13%	0%	0%	0%	5,96%	2,74%
9%	0%	0%	84%	0%	0%	7%	0%	0%	0%	6,66%	3,01%
10%	0%	0%	90%	0%	0%	0%	0%	0%	0%	7,37%	3,28%
20%	0%	0%	80%	0%	0%	0%	0%	0%	0%	13,43%	5,48%
30%	0%	0%	70%	0%	0%	0%	0%	0%	0%	19,73%	7,67%
40%	0%	0%	60%	0%	0%	0%	0%	0%	0%	26,09%	9,86%
50%	0%	0%	50%	0%	0%	0%	0%	0%	0%	32,48%	12,05%
60%	0%	0%	40%	0%	0%	0%	0%	0%	0%	38,88%	14,24%
70%	0%	0%	30%	0%	0%	0%	0%	0%	0%	45,29%	16,43%
80%	0%	0%	20%	0%	0%	0%	0%	0%	0%	51,71%	18,62%
90%	0%	0%	10%	0%	0%	0%	0%	0%	0%	58,13%	20,81%
100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	64,55%	22,998%

Note: All asset weights sum up to 100%, each row represents a portfolio situated on the efficient frontier. The top observation of this table represents the minimum-variance portfolio without short sales.

Table 14. Portfolio performance for the portfolio excluding bitcoin (benchmark portfolio)

	Equal Wgt.	Min. var. portfolio	
		With SS	No SS
MSCI WORLD	11%	53,22%	0%
MSCI EU	11%	6,40%	17,09%
MSCI USA	11%	-26,73%	0,27%
MSCI PAC	11%	-9,91%	0,00%
MSCI EM	11%	-6,91%	0,00%
BOND INDEX	11%	39,85%	37,95%
COM INDEX	11%	6,12%	5,57%
GOLD INDEX	11%	0,93%	0,68%
USD INDEX	11%	37,03%	38,45%
total weight	100%	100%	100%
Portfolio Exp. ret.	0,3978%	0,2647%	0,2544%
Portfolio Std. dev.	2,3409%	0,8291%	0,8580%
Portfolio variance	0,0548%	0,0069%	0,0074%
Sharpe ratio	0,170	0,319	0,296

Note: The table above is based on the monthly data, showing the weights for the minimum-variance portfolio with and without short sales.

Table 15. Portfolio returns for the Benchmark and Extended Portfolio given the same level of risk

std. dev.	Ret. Bench. Port.	Ret. Ext. Port.
1,00%	0,51%	0,56%
1,20%	0,63%	0,74%
1,40%	0,70%	0,86%
1,60%	0,75%	0,96%
1,80%	0,80%	1,06%
2,00%	0,84%	1,15%
2,20%	0,89%	1,24%
2,40%	0,93%	1,33%
2,60%	0,96%	1,42%
2,80%	1,00%	1,50%
3,00%	1,04%	1,58%

Note: This table captures the return of the portfolios for the same level of risk.

Table 16. Results for the Mean-Variance Spanning Test

	Coeff. Est.	Std. err.	t-stat
α	-0,005	0,000	0,000***
β	1,938	0,004	0,000***

*Note: ***, **, * indicate statistical significance at the 0.01, 0.05, 0.10 level.*

Table 17. Variance-Covariance Matrix (annualized)

	Bitcoin	MSCI WORLD	MSCI EU	MSCI USA	MSCI PAC	MSCI EM	BOND INDEX	COM INDEX	GOLD INDEX	USD INDEX
Bitcoin	1,079	0,005	0,008	0,006	-0,002	-0,001	-0,002	0,007	0,003	-0,002
MSCI WORLD	0,005	0,018	0,022	0,017	0,009	0,014	-0,004	0,012	0,002	-0,004
MSCI EU	0,008	0,022	0,037	0,018	0,011	0,020	-0,005	0,016	0,004	-0,006
MSCI USA	0,006	0,017	0,018	0,021	0,004	0,011	-0,004	0,011	0,000	-0,002
MSCI PAC	-0,002	0,009	0,011	0,004	0,026	0,015	0,000	0,005	0,005	-0,002
MSCI EM	-0,001	0,014	0,020	0,011	0,015	0,024	-0,003	0,011	0,004	-0,003
BOND INDEX	-0,002	-0,004	-0,005	-0,004	0,000	-0,003	0,005	-0,004	0,002	0,000
COM INDEX	0,007	0,012	0,016	0,011	0,005	0,011	-0,004	0,036	0,008	-0,004
GOLD INDEX	0,003	0,002	0,004	-0,0002	0,005	0,004	0,002	0,008	0,028	-0,004
USD INDEX	-0,002	-0,004	-0,006	-0,002	-0,002	-0,003	0,000	-0,004	-0,004	0,006

Note: This table represents the annual variance-covariance matrix, used as input for the Black-Litterman model.

Table 18. Returns and Optimal Weights for the IER and BL ER

	Expected excess returns		Optimal weights		
	Implied ER	BL ER	Implied W	BL W (SS)	BL W (no SS)
Bitcoin	0,77%	270,97%	0%	79,79%	44,3%
MSCI WORLD	2,80%	4,18%	60,00%	60,00%	27,6%
MSCI EUROPE	3,56%	5,56%	0%	0%	1,6%
MSCI USA	2,71%	4,30%	0%	0%	3,3%
MSCI PACIFIC	1,74%	1,12%	0%	0%	0,9%
MSCI EM	2,31%	2,08%	0%	0%	0,0%
BOND INDEX	-0,13%	-0,66%	40,00%	40,00%	22,3%
COM INDEX	1,79%	3,67%	0%	0%	0,1%
GOLD INDEX	0,65%	1,46%	0%	0%	0%
USD INDEX	-0,67%	-1,11%	0%	0%	0%
		Risk-free	0%	-79,79%	0%

Note: The output above is based on annual returns, for the Black-Litterman Expected Excess Returns the optimal weights with (SS) and without (no SS) short sales are presented.

Table 19. Black-Litterman Expected Excess Returns & Optimal Weights for a changing Omega

Std. dev. View	2%	10%	20%	30%	40%	50%	60%	62,25%
Omega (Ω)	0,04%	1%	4%	9%	16%	25%	36%	38,75%
BL ER								
Bitcoin	271,0%	200,8%	111,2%	64,0%	40,3%	27,5%	19,9%	18,6%
MSCI WORLD	4,2%	3,8%	3,4%	3,1%	3,0%	2,9%	2,9%	2,9%
MSCI EUROPE	5,6%	5,0%	4,4%	4,0%	3,9%	3,8%	3,7%	3,7%
MSCI USA	4,3%	3,9%	3,4%	3,1%	2,9%	2,9%	2,8%	2,8%
MSCI PACIFIC	1,1%	1,3%	1,5%	1,6%	1,6%	1,7%	1,7%	1,7%
MSCI EM	2,1%	2,1%	2,2%	2,3%	2,3%	2,3%	2,3%	2,3%
BOND INDEX	-0,7%	-0,5%	-0,3%	-0,3%	-0,2%	-0,2%	-0,2%	-0,2%
COM INDEX	3,7%	3,2%	2,6%	2,2%	2,1%	2,0%	1,9%	1,9%
GOLD INDEX	1,5%	1,3%	1,0%	0,8%	0,8%	0,7%	0,7%	0,7%
USD INDEX	-1,1%	-1,0%	-0,9%	-0,8%	-0,7%	-0,7%	-0,7%	-0,7%
Optimal Weights								
Bitcoin	44,3%	36,4%	23,8%	15,7%	10,4%	7,3%	5,3%	5,0%
MSCI WORLD	27,6%	11,2%	13,3%	15,7%	17,3%	25,3%	36,8%	56,4%
MSCI EUROPE	1,6%	7,3%	8,6%	9,4%	9,9%	8,2%	5,4%	0,2%
MSCI USA	3,3%	14,8%	17,6%	19,7%	20,7%	17,0%	11,2%	0,3%
MSCI PACIFIC	0,9%	3,8%	4,7%	4,9%	5,2%	4,2%	2,8%	0,0%
MSCI EM	0,0%	0%	0%	0,4%	0,3%	0,3%	0,2%	0,0%
BOND INDEX	22,3%	24,3%	29,0%	33,5%	35,5%	36,8%	37,8%	38,0%
COM INDEX	0,1%	0,3%	0,0%	0,3%	0,2%	0,3%	0,2%	0%
GOLD INDEX	0%	0,4%	0,9%	0,3%	0,3%	0,3%	0,2%	0%
USD INDEX	0%	1,5%	2,1%	0,0%	0,1%	0,2%	0,1%	0,1%
BL Portfolio								
Tot. Weight	100%	100%	100%	100%	100%	100%	100%	100%
Return	121,41%	74,40%	27,87%	11,54%	5,74%	3,56%	2,64%	2,52%
Risk (SD)	46,37%	38,22%	25,47%	17,59%	12,79%	10,22%	8,90%	8,72%
Sharpe ratio	2,62	1,95	1,09	0,66	0,45	0,35	0,30	0,29

Note: This table presents the Black-Litterman expected excess returns and weights for a changing Omega, the scalar Tau is constant at 0.025 and Q is constant at 274.98%. Short sales are not allowed.

Table 20. Black-Litterman Expected Excess Returns & Optimal weights for a changing Q

Magnitude (Q)	10%	5%	4%	3%	2%	1%
BL ER						
Bitcoin	9,87%	4,94%	3,95%	2,97%	1,98%	0,99%
MSCI WORLD	2,85%	2,83%	2,82%	2,82%	2,81%	2,81%
MSCI EUROPE	3,63%	3,59%	3,59%	3,58%	3,57%	3,56%
MSCI USA	2,76%	2,73%	2,73%	2,72%	2,71%	2,71%
MSCI PACIFIC	1,72%	1,73%	1,73%	1,73%	1,73%	1,74%
MSCI EM	2,30%	2,30%	2,30%	2,31%	2,31%	2,31%
BOND INDEX	-0,15%	-0,14%	-0,14%	-0,14%	-0,13%	-0,13%
COM INDEX	1,85%	1,82%	1,81%	1,81%	1,80%	1,79%
GOLD INDEX	0,68%	0,66%	0,66%	0,66%	0,66%	0,65%
USD INDEX	-0,69%	-0,68%	-0,68%	-0,68%	-0,68%	-0,67%
Optimal Weights						
Bitcoin	2,61%	1,21%	0,92%	0,64%	0,36%	0,07%
MSCI WORLD	19,15%	18,92%	17,90%	40,18%	40,59%	24,76%
MSCI EUROPE	10,75%	11,08%	11,35%	5,37%	5,47%	9,57%
MSCI USA	22,18%	22,64%	23,47%	10,15%	10,86%	19,25%
MSCI PACIFIC	5,50%	5,73%	5,92%	2,61%	2,77%	4,92%
MSCI EM	0,37%	0,41%	0,29%	0,02%	0,03%	0,19%
BOND INDEX	38,80%	39,34%	39,46%	39,17%	39,50%	39,33%
COM INDEX	0,31%	0,39%	0,39%	0,27%	0,02%	0,31%
GOLD INDEX	0,32%	0,29%	0,29%	0,28%	0,39%	0,47%
USD INDEX	0,01%	0,01%	0,00%	1,31%	0,02%	1,14%
BL Portfolio						
Tot. Weight	100%	100%	100%	100%	100%	100%
Return	1,86%	1,67%	1,65%	1,61%	1,64%	1,60%
Risk (SD)	7,58%	7,25%	7,21%	7,06%	7,21%	7,06%
Sharpe ratio	0,25	0,23	0,23	0,23	0,23	0,23

Note: This table presents the Black-Litterman Expected Excess Returns & Optimal Weights for a changing magnitude of the view on bitcoin. Tau is constant at 0.025, the standard deviation for the view is fixed at 2%. Short sales are not allowed.