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The Battle for Parity: Risk Allocation Matters Too!

Author:
R.J.M. Uijting BSc.

Supervisor:
prof. dr. H.M. Prast

Abstract

In the past years, risk budgeting has gained increased interest from academics and practitioners in the asset management industry. The question arose how to create diversified portfolios could be protected from adverse economic regimes. Risk Parity is a strategy that equalizes risk contribution among assets, thereby diversifying risk rather than capital. This thesis presents: 1) an overview of the Risk Parity strategy, 2) four multi-regional backtests comparing the performance of Risk Parity with 60/40, 1/N and GMV strategies in a consistent manner, and 3) the effect of asset inclusion/substitution on portfolio performance. The performance is measured using total returns, risk-adjusted returns maximum drawdowns and Gini coefficients. Using a 1997-2017 sample, we find that unlevered RP cannot produce higher total return than the 60/40 and 1/N strategies. In terms of risk adjusted returns, it outperformed all peers in 3 out of the 4 backtests. The robustness show that the choice of assets and asset classes had an affect on the outcome of the backtests. Furthermore, The origin of strategy's performance is examined in every backtest.

Student Name	Ruben Joseph Maria Uijting
ANR	216301
SNR	U1254243
Phone Number	
E-mail	

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Chapter 1

Introduction

1.1 Motivation

Over time, individuals and institutions have accumulated excess wealth. When put on a savings account, it returns at most the interest rate and is likely to reduce purchasing power if the Consumer Price Index (CPI) exceeds the interest rate. The individuals and institutions with excess wealth therefore had incentives to seek methods to maintain or increase the value of the excess wealth, all in order to maximize and smoothen consumption over their life cycles. Hence, investors emerged with the goal to maximize the value of the capital while minimizing the downside risk. Investing is the exchange of liquidity for an asset which is believed to return superior relative to a benchmark. Investments are made to: (1) maximize quantity of capital or (2) ensure that present cash maintains equivalent real future value. The question that investors need to answer is: What mix of assets has the best chance of delivering good returns over time through all economic environments?

The art of investing is not a exact science. The study of finance tries to price assets on their risk-level and expected rate of return. Hence, theories were developed to explain current prices, but also to estimate the expected returns of assets. These theories are used by investors to create portfolios that fulfill a previously set goal. One of the building blocks of modern portfolio theory is the modern portfolio theory (MPT) by Markowitz (1952). MPT considers how an investor should choose a portfolio with a good trade-off between risk and return. It shows how an investor can create a combination of available assets with the maximum amount of return per unit of risk: the tangency portfolio. The theory had a tremendous impact in the finance world and is still frequently used these days. Recent literature about the MPT suggests that the theory has downsides: (1) the tangency portfolio tends to concentrate on specific asset weights and these are extremely sensitives to changes in input, and (2) this portfolio construction approach requires the estimation of expected returns of assets, which is extremely challenging in practice. (Merton, 1980)

Sharpe (1964) developed the Capital Asset Pricing Model (CAPM) as an addition to the MPT. It presumes that all investors invest as MPT investors, so that the tangency portfolio must be the market portfolio. The outcome provided by CAPM is that all investors should hold the market portfolio, levered according to each investor's risk preference. Historically, the market portfolio has brought about significantly lower Sharpe ratios than the tangency portfolio. There are three reasons for this: (1) the market weights of stocks relative to bonds have varied over time in such a way that the risk-return characteristics of the market are inside the hyperbola of a mean-variance diagram[1], (2) the market portfolio allocates a much larger fraction of its capital to stock than what has been optimal historically, and (3) the risk-adjusted returns are not balanced across assets.

The two theories discussed above illustrate several issues: concentrated assets in the portfolio, the correctness of estimated expected returns are of huge importance to portfolio quality, and theoretical portfolios perform poorly in the real world. Another problem that academics and practitioners have tried to tackle is diversification. Typical 60/40 and 1/N portfolios diversify their funds over different asset classes, thereby reducing risk. Although the funds are diversified, risk is still concentrated in certain asset classes, most predominantly high-beta assets. For example, a 60/40 portfolio has 60% of its funds invested in equity, but more than 90% of the risk is allocated to equity (Kazemi, 2012). The financial crisis of 2007-2009 showed that adverse equity shocks destroyed much of the value of portfolios. Although correlations tend to rise in times of crisis (Koestrich, 2015), investors suffered severe losses due to their high risk allocation in single asset classes. So did capital diversification truly reduce risk? The financial crisis kick-started the search for theories and models that would prevent such extreme losses, shedding new light on the practice of risk-budgeting.

Risk Parity (RP) is an approach to portfolio construction which focuses on the diversification of risk among factors and asset classes rather than the allocation of capital. The concept was first discussed by Booth and Fama (1992) and continues to attract more attention from both academics and practitioners. Investment funds such as Bridgewater Associates[2], the Wisconsin State Investment Board[3] and The Pennsylvania Public Schools Employees' Retirement System[4] have already created portfolios using this approach. The portfolio construction method requires no estimation of expected returns, and some academics suggest the method is empirically superior to other general portfolio theories, making it respectively robust and attractive.

RP offers a simple solution to the aforementioned problems: diversify the portfolio, not through capital allocation, but through risk allocation. This means an equal amount of risk contribution of each asset class, thus the strategy invests more capital

[1] Also known as Markowitz' bullet. It shows the combination of assets which have the highest return for a given amount of risk.

[2] The Bridgewater adopted a RP framework with their All Weather Fund. Since inception, it realized a 9.3% total annualized return.

[3] This fund allocated 600 million to a RP strategy.

[4] This pension fund uses a RP strategy as an asset class in their 53.5 billion dollar AUM.

in low-beta assets (i.e. bonds) rather than high-beta assets (i.e. equity). Due to substantial allocation to 'safe' assets, RP strategies are less aggressive than traditional allocation strategies (such as the 60/40 strategy), but can produce a higher Sharpe Ratio. RP investors can overcome this problem by using leverage. This allows them to lever up the risk-balanced portfolio to desired levels of expected return and risk. The result is risk-balanced portfolio which allows so-claimed 'true' diversification and preference-customization [5].

Besides the theoretical justification, empirical studies suggest that a RP portfolio is superior to other traditional portfolios. The historical outperformance of RP is quite robust. Several academics (Asness, Frazzini, and Pedersen (2012), Kaya and Lee (2012) and Chaves et al. (2011)) suggest that RP strategies were historically superior. The popularity of RP is the result of (1) the intuitive reasoning of balancing risk rather than invested capital, (2) the historical evidence for this approach over traditional approaches and, (3) the boost of interest in the subject after the financial crisis of 2007-2009.

Although the empirical literature concerning RP provides promising results, the arguments above miss justification. First of all, the intuition that a RP portfolio is superior to equity-dominated portfolio relies on the implicit expectation of asset returns. An investor happily invests in an equity dominated portfolio if the equity risk premium versus the bond risk premium is high enough. Hence, the intuition that 'a high-beta strategy entails too much risk' is only correct if the return of high-beta assets relative to low-beta assets is not high enough to compensate for the additional risk. The RP intuition that risk should be equalized across assets is only entirely correct if the risk-adjusted returns are unequal across assets.

One cannot merely assume that equal risk contribution across asset classes is superior because it is better diversified. An investor, however, should believe that the returns of equity are not sufficiently high to compensate for the additional risk. This means that RP is not only a method about equally spreading risk in the portfolio, but also an implicit belief on risk-adjusted returns of assets. As Asness, Frazzini, and Pedersen (2012) note: "A RP investor should not say: *equal risk is always the best regardless of expected returns*. Instead, they should say: *we do not believe expected returns are high enough on equities to make them a disproportionate part of our risk budget*." Hence, RP investors need to explain why high-beta assets offer lower risk-adjusted returns compared to low-beta assets in order to justify a higher allocations to bonds. This thesis delves deeper into the RP theory and aims to answer the following question: (Why) does a RP portfolio perform better than traditional portfolios?

[5] Although leverage introduces risks and other practical concerns

1.2 Problem Description

The goal of this thesis is to create an overview of the RP theory and complement theoretical properties with an empirical study. It extends the literature by combining (1) studies about RP to present an overview of its properties, and (2) empirical tests of RP. It will examine RP on both a regional scale and global scale. Financial literature predominantly focuses on the USA as a region for back-testing portfolio theories. Therefore, there has been little to no research done on the European markets regarding RP. This does not imply that outcomes differ between regions, yet it provides an opportunity to fill a gap in the literature. For this reason, the regional sample will consist of European indices. The following subjects will be covered regarding RP:

i) How has Risk Parity performed compared to traditional heuristic portfolios and where does Risk Parity return come from?

ii) How does asset selection and inclusion affect Risk Parity portfolios?

iii) What does leverage mean for the performance of Risk Parity portfolios?

iv) Why would an investor choose a Risk Parity strategy?

The remainder of this thesis is structured as follows. Chapter 2 discusses the literature of RP and general portfolio management knowledge . In Chapter 3, the analysis framework and methodology is explained. Chapter 4 discusses the data used. Chapter 5 presents the results and discusses the robustness. The discussion of the results and the applied methodology is held in the Chapter 6. Finally, Chapter 7 draws conclusions.

Chapter 2

Literature and Design

This chapter discusses the relevant literature and the empirical design for this thesis. The chapter starts with the basics of portfolio construction in section 2.1. It serves as a backbone for the empirical section of this thesis by providing the tools and knowledge for the analysis. The subsequent section 2.2 provides literature about traditional allocation strategies and discusses their characteristics. The chapter ends with section 2.3 which discusses the literature about RP, how the strategy works and how a RP portfolio can be constructed.

2.1 Definitions Portfolios

This section discusses the theory of asset allocation strategies that will be used in the empirical part of the thesis. The chosen strategies were subject to one constraint: estimated returns should not be an input variable to construct the portfolio weights. This allows for the most robust estimation, comparable strategies and less reliance on external factors. Therefore, the chosen strategies can be constructed *ex ante*. In order to elaborate on RP, we first discuss the fundamentals of portfolio construction.

In portfolio construction, the goal is to search for feasible weights w_i to minimize the risk σ_p to reach a target return $E(R_p)$ for portfolio p. Commonly in the literature, the risk-free rate is referred as r_f. This figure is used to calculate the excess returns of asset or portfolios. Usually, the risk-free rate is represented by a short-term bond rate that does not face liquidity nor default risk. The number of assets used in portfolio construction are denominated by N. The vector of all weights w_i for all assets i is $N \times 1$-dimensional and contains all assets in the portfolio. Each asset has a risk σ_i, and a correlation with other assets $\rho_{i,j}$. From the combination of assets, the $N \times N$ variance-covariance matrix Σ can be constructed. This matrix has the individual assets' variances σ_i^2 on the diagonal axis and the covariance between assets $\sigma_{i,j} = \rho_{i,j}\sigma_i\sigma_j$ on the off-diagonal elements. A universal optimization technique is defined as follows

$$\begin{aligned} & \min w'\Sigma w \\ w.r.t. \quad & \mu' w \geq E(R_p) \\ s.t. \quad & 0 \leq w_i \leq 1, \\ & \sum_{i=1}^{N} w_i = 1 \end{aligned} \tag{2.1}$$

where μ' stands for the transpose of the expected return vector. In this simple technique, the variance of a portfolio is minimized for a target return. The weights of individual assets are restricted to positive values only and have to sum up to 100%, which also implies that going short is not allowed.

2.1.1 Risk

Risk is a subject that is frequently investigated by academics. The most used risk measure in finance is the standard deviation (SD). It is a measure that indicates how observations are dispersed around the mean value of the sample. The SD is calculated as the square root of the variance. The variance is calculated as the average from the squared deviations of the observations from their mean

$$SD = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2} \tag{2.2}$$

The SD assumes a normal distribution and stationary data. In finance, data series are rarely stationary, therefore the data must be modified to use the SD properly. This is usually done by using returns rather than prices since returns usually remain stationary around 0. The normal distributions assumption is also unlikely to be valid. Finance data usually has a form of skewness, kurtosis or long tails. For example, when 'bad' news enters the markets, stock prices tend to drop dramatically, this causes stock price data to contain many extreme values. As a reaction to the shortcoming of the SD, academics developed new risk measures to compute risk more correctly. This thesis recognizes the downsides of using the SD as risk measure, but it is outside of the scope of this thesis to investigate the impact of other risk measures. It is, however, interesting to investigate the implications of other risk measures in a RP framework. For now, the SD is considered as the measure for risk. The SD, also known as the volatility, of the portfolio can be calculated as follows

$$\sigma_p = \sqrt{w'\Sigma w} \tag{2.3}$$

Individual assets in a portfolio contribute their part of volatility to the total portfolio volatility. This comprises of the total contribution of risk (TRC) and the marginal contribution of risk (MCR) of asset i. The MCR is the derivative of portfolio risk with respect to the individual asset's weight. For interpretation purposes, it is the total volatility that would be added to the portfolio if its weight would increase from 0% to 100% *ceteris paribus*.

$$\begin{aligned} MCR_i \equiv \frac{\partial \sigma_p}{\partial w_i} = \frac{\Sigma w}{\sigma_p} \\ = \frac{w_i \sigma_i + \sum_{j \neq i} w_j \sigma_{i,j}}{\sqrt{w' \Sigma w}} \end{aligned} \tag{2.4}$$

The TRC of an asset in a portfolio is its MCR times the asset's weight. It represents the percentage point volatility that is added to the portfolio volatility by asset i.

$$\begin{aligned} TRC_i \equiv w_i \times \frac{\partial \sigma_p}{\partial w_i} = w_i \times \frac{w \Sigma}{\sigma_p} \\ = \frac{w_i^2 \sigma_i + \sum_{j \neq i} w_j w_i \sigma_{i,j}}{\sqrt{w' \Sigma w}} \end{aligned} \tag{2.5}$$

Now we know the asset's individual contribution to risk, we can define the portfolio's risk σ_p as parts of the assets it contains.

$$\sigma_p \equiv \sqrt{w' \Sigma w} = \sum_{i=1}^{N} TRC_i \tag{2.6}$$

2.1.2 Return

The return of an asset is the difference in value of an asset between different points in time, thereby assuming that no dividend is paid and all earnings are retained. In this thesis, the following definition of returns is used:

$$R_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} \tag{2.7}$$

where $R_{i,t}$ stands for the return of asset i, and $P_{i,t}$ represents the price of asset i at time t. Asset prices tend to by more volatile when the time window becomes smaller. As the time windows become larger, the prices and returns are more smoothed out. Also, the more frequent the data are, the more robust estimations become. To calculate the return of a portfolio, the following definition is used:

$$\begin{aligned} E(R_p) = \sum_{i=1}^{N} w_i \left(\frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}\right) \\ = \sum_{i=1}^{N} w_i R_i = w' R \end{aligned} \tag{2.8}$$

where $E(R_p)$ stands for the return of a portfolio, w stands for the vector of weights, and R stands for the vector of returns. There are two methods for calculating average returns. The first one is the arithmetic average. This method returns the average of individual events by simply dividing the sum of observations by the number of observations. This resembles an investor who adds and withdraws capital from a strategy. In this thesis, the used return series are not independent. For example, a negative return in a previous month has an effect on wealth levels in the current month. Also, the capital allocated to a strategy will not be withdrawn nor supplemented in this thesis. Therefore the geometric average, which accounts for reinvestment of capital, is more appropriate. This method is defined as follows:

$$\bar{R}_{i,t} = [(1 + R_{i,t}) + (1 + R_{i,t-1}) + \ldots + (1 + R_{i,t-T})]^{1/T} - 1 \tag{2.9}$$

where $\bar{R}_{i,t}$ is the average return of asset i at time t, $R_{i,t}$ the return of asset i at time t, and time T denotes the amount of periods in the past. By approximation, the formula for the geometric average can also be written as

$$\bar{R}_{i,t} = R^*_{i,t} - 0.5 \times \sigma^2_{i,t} \tag{2.10}$$

where $R^*_{i,t}$ stands for the arithmetic return of a series and $\sigma_{i,t}$ stands for the volatility of asset i at time t.

2.1.3 Risk-Adjusted Return

In the literature, performances of assets and/or portfolio's are commonly compared to each other. There is, however, no uniform measure that the entire industry uses. To overcome this problem, risk-adjusted returns are used. It represents the risk-return trade-off of an asset or portfolio. The most famous risk-adjusted return measure is the Sharpe Ratio (SR) by Sharpe (1964). It represents the average return excess of the risk-free rate, divided by its corresponding volatility. The definition is as follows:

$$SR = \frac{R_p - R_f}{\sigma_p} \tag{2.11}$$

The measure presents several drawbacks. First, it assumes a normal distribution of returns, when in reality that is not always the case. Second, there are difficulties with the interpretation of the measure. For example, when the SR is negative, the interpretation is not intuitive: a higher volatility would result in a lower return. Other measures have been developed to compensate for these drawbacks. The Modigliani-Modigliani Measure (Modigliani and Modigliani, 1997), better known as M^2, is such a measure. It measures an asset's/portfolio's return as if it has the same volatility as a chosen benchmark by using the SR. The definitions is as follows:

$$\begin{aligned} M^2 &= (\frac{\sigma_b}{\sigma_p})(R_p - r_f) + r_f \\ &= \frac{R_p - r_f}{\sigma_p} * \sigma_b + r_f = SR_p * \sigma_b + r_f \end{aligned} \tag{2.12}$$

where σ_b stands for the volatility of the chosen benchmark. The interpretation is more intuitive and the outcome is presented in percentage points. For example, if we compare a portfolio with a $M^2 = 5\%$ and the benchmark with a return of 4%, we can conclude that the difference of 1 percent point is the risk adjusted return with adjusted volatility. Although this measure does not meet the normality assumption, the interpretation allows for easier comparison between portfolios.

2.2 Traditional Allocation Strategies

This section presents the traditional allocation strategies that are commonly used in the literature. We define the selected strategies and discuss why they are used.

2.2.1 Equally Weighted Portfolio

The Equal Weight Strategy (1/N) is a portfolio construction method where the capital allocation among the asset classes is equalized. It is the most robust method of asset allocations since it does not require any inputs like volatility or expected return. Chaves et al. (2011) discussed the strategy and called it the most naive portfolio heuristic. The portfolio weights are constructed as $w_i = 1/N$ where w_i corresponds to the weight of asset i, and N corresponds to the number of assets in the portfolio. By definition, the weights of the assets cannot be negative, nor can the sum of the weight be higher or lower than 100%. Consequently, the portfolio's weights can be defined as follows:

$$w_i = w_{j \neq i} = \frac{1}{N} \tag{2.13}$$

Besides the ease of construction, the inclusion of this strategy in this thesis is otherwise motivated. DeMiguel, Garlappi, and Uppal (2007) explain that 1/N strategies 1) are easy to compute, and 2) besides the advances made in estimating parameters of sophisticated models, investors still use simple rules for wealth allocation. They advocate that the strategy is not a good asset-allocation strategy, but a good benchmark to compare other portfolio methods. The portfolio strategy can be considered to be anti-cyclical since all assets have positive weight regardless of their correlation or past returns. Therefore an exposure to assets in different economic regimes is likely. This is, however, strongly dependent on the tactical asset selection. As explained before, the portfolio SD $\sigma_{1/N}$ is calculated as:

$$\sigma_{1/N} = \sqrt{w' \Sigma w} \tag{2.14}$$

w and w' represents the weight vector and transpose weight vector respectively for each chosen asset. The variance-covariance matrix is represented by Σ.

2.2.2 Global Minimum Variance Portfolio

In 1952, Markowitz (1952) invented and formalized the mean-variance framework of asset allocation. This method aims to minimize the variance of an asset allocation for a target return. The method is also used to find combinations of assets that minimizes the variance of a portfolio. In a mean-variance graph, as shown in figure 2.2, the portfolio's position is the most left on the so-called Markowitz' bullet, which represents portfolios that minimize the volatility for a given amount of return. In 1962, Sharpe (1964) complemented the theory by formalizing the tangency portfolio and the capital allocation line. The tangency portfolio is the portfolio that maximizes the risk-return award, namely the SR. It represents the average return in excess of the risk-free rate per unit of risk, in this case the SD. The capital allocation line represents the combination between a risk-free asset and the tangency portfolio. By finding the portfolio with the maximum SR, one can theoretically lever it along the capital allocation line. Thus, the maximum return for the risk tolerance of an investor is realized.

However, as regularly pointed out by academics (Merton (1980)), future returns are hard to estimate, and often significantly deviate from realized returns. Using this strategy as a benchmark comes with disadvantages, leading to a widely used global minimum variance portfolio (GMV). The GMV does not require estimated returns but does require estimates of the covariance matrix. The paper by Merton points out that covariances can be estimated with higher accuracy than the returns, resulting in more robust weight estimates. To reach a portfolio allocation with minimum variance, the weights are defined as

$$
\begin{aligned}
w_{GMV} &= arg \min_{w} w' \Sigma w \\
s.t. \sum_{i=1}^{N} w_i &= 1 \\
0 \leq w_i &\leq 1
\end{aligned}
\tag{2.15}
$$

Maillard, Roncalli, and Teïletche (2010) and Chaves et al. (2012) point out that the GMV equalizes risk contribution (RC), similar to the RP portfolio. The difference, however, is that the GMV equalizes on a marginal basis, whereas the RP portfolio equalizes the TRC. The portfolio volatility can be calculated as$\sqrt{w' \Sigma w}$. The portfolio variance is minimized, so by definition the volatility of a GMV is smaller than other portfolio heuristics, as shown by Maillard, Roncalli, and Teïletche (2010)

$$
\sigma_{GMV} < \sigma_{1/N} \tag{2.16}
$$

2.2.3 60/40 portfolio

The 60/40 portfolio is a portfolio that allocates 60% of the funds to high-beta assets (e.g. equity), and the remaining 40% to low-beta assets (e.g. bonds). One reason for using such a portfolio is to profit from high-yielding assets while protecting the funds from negative shocks by investing in safer assets with a low correlation with the risky assets. The strategy gained much attention in the previous century, but lately has lost its attractiveness (Maillard, Roncalli, and Teïletche, 2010) due to the recognition that the portfolio strategy is a bad diversifier, and the low return of bonds has deterred investment managers. Formally, the weight of the risky assets can be written as

$$w_{risky} = 0.6 \tag{2.17}$$

Consequently, the weights of the safe assets in the portfolio can be written as

$$w_{safe} = 1 - w_{risky} = 0.4 \tag{2.18}$$

Like the 1/N strategy and GMV strategy, the variance of the portfolio equals $w\Sigma w$. The portfolio variance is largely dependent on the choice of assets, but typically, one can state that that

$$\sigma_{GMV} < \sigma_{1/N} < \sigma_{60/40} \tag{2.19}$$

2.3 Risk Parity

This section explains the theory of RP. The idea of RP is not new, Booth and Fama (1992) were some of the first to document the RC of an asset in the context of a multi-class allocation. Qian (2005) formalized the theory and called it RP. Since the economic crisis of 2007-2009, more focus has been placed on controlling risk in portfolio. Practitioners eventually developed the RP strategy, which has become increasingly popular ever since. RP has some attractive properties. It requires no expected return input, it is less reliant on growth in the economic cycle due to higher investment in low-beta assets, it less volatile compared to peers in theory, it can be levered up to realize target returns and there is no intended focus on certain asset classes. There also are, a few downsides for this strategy. The use of leverage is costly and therefore erodes profits; different risk measures result in different weights making the result highly dependent on the used measure; the variance-covariance matrix is assumed to be constant which is not always the case; and there is a timing problem regarding acquiring assets and/or leverage on the right time.

Besides informing the reader about RP, this section identifies gaps in the literature. It starts by delving deeper into the findings of the literature by presenting the advantages and pitfalls of the heuristic method. Subsequently, it presents the definition of RP and the mathematical construction. Finally, an example of how RP exactly works with real data is presented.

2.3.1 Overview

Performance Portfolios

The following section presents a summary of academics' review of RP portfolios and the most meaningful bodies of work regarding performance are discussed. The first study (Maillard, Roncalli, and Teïletche, 2010), compares three strategies (1/N, GMV, and the RP) in both a theoretical and real-life setup. In their theoretical setup, they find that the 1/N has the most balanced weights, but highest variance and concentrated risk in individual assets. The GMV portfolio has the lowest variance per definition, but the most extreme weights and RCs. The RP portfolio has more balanced weights and RCs. In their empirical sample, the authors use three datasets, each consisting of four assets: US Equity, Agricultural Commodities, and the Global Diversified Portfolio. The authors backtest from 1995 to 2008 using monthly rebalancing. In the equity portfolio, the 1/N and RP portfolio perform similarly due to high asset correlations, the difference lies in the contribution or risk, which is higher for the 1/N portfolio, while the RC weights are more equal for the RP portfolio. The GMV produces the highest SR but is most imbalanced in asset weights and RC. As the authors demonstrate in their theoretical background, with low correlations, the RP portfolio tends to result in weights that are proportional to their inverse volatility. The 1/N is dominated by the RP portfolio on all measures. The GMV dominates the RP portfolio in terms of volatility and returns, but has much larger drawdowns and concentrations of risk and weights. RP presents the advantage of having lower concentration in assets, and therefore less exposure to idiosyncratic risk. The last backtest is performed using a global portfolio consisting of major indices covering all demographic regions. The correlation of assets are much more spread than the previous backtests. The RP portfolio dominates when comparing drawdowns, return, volatility and SRs. Only the 1/N has better weight concentrations. The authors findings suggest that RP seems to perform better as correlation coefficients are more dispersed in the portfolio. This seems intuitive as heterogeneity in volatilities and correlations are linked to concentration measures of a RP portfolio.

Asness, Frazzini, and Pedersen (2012) find that RP outperforms the value-weighted and 60/40 portfolio in the very long run. The authors do not use the exact RP approach, rather, they use the inverse-volatility weighted method as a proxy for RP. Although the method results in a different portfolios compared to RP, the inverse-volatility strategy and the RP strategy tend to have similar results. They find that over a period of 1923 - 2010 a levered RP portfolio consisting only of a stock and bond index outperforms a 60/40 portfolio. They expanded the portfolio by including (global) asset classes and found similar results. Asness, Frazzini, and Pedersen provide a reason for the outperformance of RP: the security market line is too flat. As they explain: investors have an appetite for high return but are constrained by the amount of leverage they can take on. For this reason, they invest in higher yielding, though more volatile, assets. The authors suggest that most investors do so, resulting in lower return potential for high-beta assets. It implies that low-beta assets have higher risk-return characteristics. Therefore, the authors state that a strategy focusing on low-beta assets is prone to outperform high-beta strategies in terms of risk-adjusted returns. The authors do not explain why RP outperformed in certain periods of their 87-years sample. Ruban and Melas (2011) investigate the return differences between RP and a 60/40 strategy while focusing on specific periods. Their RP weights were a function of the inverse volatilities of included assets. In the two asset class backtests, which lasted from 1976 to 2009, they found that in economic growth regimes the 60/40 strategy outperformed the RP strategy. Although their setup is simple, the outcome suggests that RP can underperform under specific market conditions.

Chaves et al. (2011) find that RP does not consistently produce a higher SR than a 1/N and 60/40 strategy, but it does outperform minimum variance and mean-variance efficient strategies. Their sample, which ranged from 1980 to 2010, included nine asset classes. RP had the most balanced risk allocations and least volatile performance statistics. The authors perform a sensitivity analysis where the number of assets varies for each portfolio. Their results are ambiguous. For two of the included asset classes, the result is most optimal for all portfolios. The returns deteriorate when more asset classes are included, up until six asset classes. From that point onward, the performance improves again. The authors emphasize that the result is highly dependent on the investment universe, underlining that asset (class) selection is critical. They state that more research is necessary to draw good conclusions on the effect of asset class inclusion on RP performance. Finally, Peters (2011) investigates the source of RP's empirical outperformance. The author compares a RP portfolio with a static 50% equity, 25% bonds and 25% alternatives and a liability hedging portfolio using the Citi Liability Index in the period 1995 - 2010. He finds that RP outperforms its peers by dynamically rebalancing more weight to high-beta assets when volatilities are low, and vice versa. RP therefore adds value since low-beta assets (e.g. bonds) tend to outperform high-beta assets (e.g. equity) in periods of high volatility.

Overall, the literature indicates that RP tends to outperform traditional strategies. It is, however, subject to the time period used in the framework. Peters (2011) finds that RP outperforms while Chaves et al. (2011) don't find RP superiority. The latter use a time period before 2000 while the former is mostly after the 2000s. In addition, Chaves et al.'s framework allocates lower weights to pro-cyclical high-beta assets compared to Peters's. The performance of these assets was different given that two different time periods were used (Thiagarajan and Schachter, 2011). In all of these studies, the choice of assets was different. For example, Chaves et al. use the BarCap Aggregate Bond index, which produced an astonishing 0.82 SR over the past 30 years, which Peters' did not. Including specific assets can alter the results. Furthermore, the studies by Maillard, Roncalli, and Teïletche (2010) and Chaves et al. (2011) mainly use USA based portfolios for their backtests, although world portfolios are not as often constructed. The choice of assets and time periods can affect the outcome of constructed portfolios. Thiagarajan and Schachter (2011) rightly state that more work on the robustness of different sample universes and sample intervals is needed.

The Investment Universe is Infinite

A RP portfolio is restricted to only positive weights in the chosen asset classes. Most investors cannot have short positions, therefore this strategy is applicable to many investors in the world. An investor also does not have to include assets in which he wants to go short in, only those he wants to include in his portfolio. Imposing limits on asset weights has a downside, namely that it reduces the diversification benefits. A pitfall when using RP is the timing of acquiring assets. RP is a method that doesn't include analysis of returns nor views on market movements. By not analyzing this, one can miss the opportunity to invest or withdraw funds at the right moment[1]. Several studies[2] underline that the choice of asset classes included in the portfolio is of great importance to its performance. Chaves et al. (2011) show that RP performance drastically changes when different assets of the same class are substituted. Every asset has different characteristics and is weighted differently due to different co-dependency on other assets. Normally, the RP literature uses indices to assess RP's performance. Since every index is constructed differently, outcomes are greatly dependent on how indices are constructed, regardless of seemingly 'equal' characteristics or coverage. The choice of assets within classes is not the only issue, as Inker (2011) points out. He questions whether asset classes such as government bonds and commodities have a risk premium in the long run. Stocks and bonds have positive risk premia due to claim on cash streams of business operations. He states that commodity securities have a buyer and seller, so in the end it is a zero-sum game. The same holds for government bonds where the timing of acquisition is of great importance due to changing yields and opportunity costs. All in all, critical thinking on assets to include is critical for portfolio performance.

RP is said to have less reliance on growth in the economic cycle to generate the required rate of return. Kunz (2011) argues that RP portfolios are designed to generate equity-like returns while minimizing the variance. He explains that if inflation is

[1] A real-life investor might incorporate views in a RP portfolio by, for example, include a Black-Litterman framework.

[2] See Inker (2011), Kunz (2011) and Chaves et al. (2011).

high and a central bank is increasing interest rates, a RP strategy with less exposure to growing markets (e.g. equity markets) reduces the maximum draw-down of the portfolio. When considering two main drivers of financial markets, namely growth and inflation, we can state the following: bonds tend to react favorably to disinflation and deflation which increases them in value; commodities are favorable asset classes in times of rising inflation; stocks normally increase in value during times of economic prosperity; and low growth is favorable for long-term fixed assets. A good portfolio should contain assets that do well in every economic regime. The RP strategy gives a positive weight to every asset class included, so only when the right asset classes are included, the portfolio tends to smooth earnings over time. Maillard, Roncalli, and Teïletche (2010) find empirical evidence for the idea that RP portfolios are more stable over time. In their multinational sample that ranges from 1995 to 2009, they find that an RP portfolio is superior to equity dominated portfolios in terms of maximum drawdown of all possible intervals.

The previous section made clear that the performance of any portfolio is hugely dependent on the asset (classes) included. For RP in particular, the literature is not entirely clear on how robust RP performance is to changing assets. This will be tested in this thesis. Moreover, RP performance is more smooth over time relative to peers. An article by ReSolve Asset Management[3] indicates that RP performs well in the different economic regimes, yet it does not provide a solid conclusion on how growth and inflation affect the performance. This will be discussed in this thesis.

The Riskiness of Risk

Risk-Parity (RP) is an approach to investment portfolio management which focuses on the diversification of risk among factors and asset classes rather than the allocation of capital. This is achieved by making the contribution of risk equal by asset class. This is at odds with other heuristic portfolio theories, for example the 60/40 portfolio where 60% of the funds are invested in risky assets (equity), and the remainder in fixed income (bonds). Since bonds are less risky than equity, the asset class equity contributes to more than 90% of the total risk of the portfolio, according to Kazemi (2012) and the example in section 2.1. These findings suggest that simple allocation strategies are susceptible to shocks in the equity market since the risk is not diversified well. RP overcomes this problem by equally weighting the RC of each included asset (class). It allows investors to equalize RC across assets and thereby diversify on riskiness of assets rather than through asset classes or location wise. This means that less-volatile assets will be over-weighted compared to riskier assets.

One of the main strengths of RP strategies is the independence of mean estimations based on historical asset return. That is, RP is not affected by errors in estimation

[3] Management (2015)

of expected returns, nor human behavioral biases such as the look-ahead bias. Furthermore there is no intended focus on a specific asset class so that no asset can be favored relative to others. Additionally, the strategy incorporates the low correlations between classical and alternative asset classes (e.g. commodities) (Qian, 2005), resulting in lower concentration of weights in certain assets. The RP strategy requires a risk measure of assets as input, for which the literature commonly uses the SD. Although estimation errors might occur with estimating risk measures, these are more stable over time and therefore more reliable than estimating returns. The problem, as with almost any portfolio construction strategy, is that existing risk measures fall short in certain situations. For example, when dealing with return series of assets, the SD tends to be low during bull markets and high during bear markets (Danielsson, Valenzuela, and Zer, 2016). This poses a problem for a RP portfolio. In the bull market just before a crisis, the SD was low. Therefore, relatively more capital is allocated to stocks rather than other asset classes. The SD implies low risk, however, it does not include the risk of extremes. Therefore, this measure is flawed when used to serve as a complete assessment of risk, thus clearly posing a threat to performance

The RP strategy is said to perform stably across different economic states (Bilan, 2016). Allen (2010) finds that RP overperforms during bear markets and underperforms during bull markets. These are attractive characteristics, but Kunz (2011) points out that with time and different states of the economy, the correlations between assets change. The general form of RP assumes these correlations to be consistent over time and the probability of crises happening to be equal across states. This, however, is not always true, as the crisis of 2007-2009 pointed out (Rankin and Idil, 2014). An estimation of the variance-covariance matrix is important to incorporate the state of the economy. The literature acknowledges the issue with the SD, but provide no clear solution to this problem. The Value at Risk is a widely adopted risk measure in the finance industry, yet this measure suffers is sensitive to extreme events, invites excessive risk taking and suffers from interpretation issues which have severe adverse effects (Einhorn and Brown, 2008). Alankar, DePalma, and Scholes (2013) discuss a RP strategy where tail risk is incorporated in weight construction. As they argue, standard risk measures inadequately incorporate extreme events, resulting in extreme losses when a crisis hits. In their empirical analysis, the authors find that Tail RP reduces losses by 50%. Though appealing, their measure requires estimating the probability of a crisis through implied volatility via the option-market. The strategy is time consuming and dependent on the universe of assets, as their conclusion is based on recent data and a two-asset class portfolio[4].

[4]The authors use a bond index and equity index in the period 2003-2013. The bond index Barclays Capital US Aggregate Index is used, which is known for its extraordinary performance throughout the years. Therefore, the outcome might be biased because of the authors' selectiveness.

Leverage propels performance

A RP strategy can experience increased efficiency through the use of leverage (Kunz, 2011). Similar to levering the tangency portfolio in the CAPM framework, an investor can apply this to a RP portfolio. Instead of increasing expected return by investing more in equities, levering the RP portfolio results in higher returns compared to peers (Asness, Frazzini, and Pedersen (2012), Chaves et al. (2011), Qian (2011) etc.). However, it needs to have a superior SR, otherwise levering would result in an inferior portfolio. Asness, Frazzini, and Pedersen (2012) suggest leverage aversion as a reason why RP portfolios empirically outperformed. Frazzini and Pedersen (2014) argue that the tangency portfolio overweights safer assets. Some investors choose to invest in riskier portfolios since they are leverage constraint. Overweighing risky assets drives the price up, similarly reducing the expected return. Therefore safer assets have higher risk-adjusted returns. So, investors who are able to apply leverage can benefit from the mismatch of risk-adjusted returns across assets, thereby being rewarded in higher returns. In line with their theory, Black (1972) shows that if investors have a leverage constraint, the risk-adjusted returns of low-beta assets are higher compared to high-beta assets. This is visualized by a security market line which is too flat. Along similar lines, Jensen, Black, and Scholes (1972) and Frazzini and Pedersen (2014) find evidence for leverage aversion in every asset class. In addition, they find that low-beta assets have higher risk-adjusted returns than high-beta assets. But why not invest in the tangency portfolio? The tangency portfolio is a portfolio which is constructed *ex post*, the implications of which have been explained in the previous paragraphs. RP portfolio construction relies on *ex ante* information. Also, RP investing suggests allocating capital according leverage aversion theory, which is more in safer assets relative to riskier assets so that RC are equalized. This method is, as mentioned previously, justified by Black (1972) and Frazzini and Pedersen (2014) in the past.

The use of leverage is accompanied by several pitfalls. First, not every investor can use leverage. Pension funds, for example, have a fiduciary duty. It is difficult for those institutions to use leverage since it would impose extra credit risk, which pension claimants are not likely to approve. For other institutions it might be challenging to find credit for investment, as a typical leveraged portfolio requires a substantial amount of funds (Kazemi, 2012)[5]. This raises practical concerns that fall outside the scope of this thesis, but remain fruitful for later research. Second, leverage has a cost. The empirical outperformance of RP is subjective to borrowing rates. Last, the timing of leverage issuance is important. As borrowing costs become cheaper in periods of low distress - since confidence is high and liquidity costs are low - it is easier for investors to borrow funds. However, when money is borrowed in favorable times it may result in adverse effects when the economic environment worsens and

[5]In his study, he finds that for every dollar invested, 54 cents need to be borrowed to lever up the portfolio to match the volatility of a 60/40 portfolio

interest yields rise. Given the above-mentioned reasons, the use of leverage is not without costs, and investors should act wisely by taking these into consideration.

2.3.2 Fundamentals

Risk-Parity (RP) equalizes the TRC among assets in the portfolio. This is achieved by making the contribution of risk equal[6] among asset(s classes). There is no exact formula for the weight calculation of a RP portfolio. Asset weights are found by running an optimization algorithm that equalizes the TRC and finds the individual weights in a heuristic fashion. For equal RC to be achieved, the following optimization problem needs to be solved:

$$\begin{aligned} \min \sum_i \sum_{i \neq j} (w_i MRC_i - w_j MRC_j)^2 \\ = \min \sum_i \sum_{i \neq j} (TRC_i - TRC_j)^2 \end{aligned} \tag{2.20}$$

The optimization problem minimizes the differences between the TRC of all assets. In other words, if the distance between all is effectively reduced to 0, all assets contribute the same amount of risk to the portfolio. This results in:

$$TRC_i = TRC_{j \neq i} = \frac{1}{N} \tag{2.21}$$

As shown in section 2.1, the sum of all TRC's results in the portfolio's volatility. In the RP case, the portfolio volatility is also equal to $N \times TRC_i \in TRC$ since every asset contributes the same amount of volatility and we have N assets. When comparing the volatilities between different portfolio construction methods, Maillard, Roncalli, and Teïletche (2010) find that the variance of a RP portfolio is (by definition) equal or higher than a minimum variance portfolio, but lower than a 1/N and 60/40 portfolio.

$$\sigma_{GMV} < \sigma_{RP} < \sigma_{1/N} < \sigma_{60/40} \tag{2.22}$$

The risk of a RP portfolio is usually lower than its peers, as the previous formula suggests. Qian (2005) states that due to the higher low-beta exposure of RP, it will not be able to outperform a 60/40 portfolio in the long run. He therefore stresses that investors should use leverage to create equity-like return. In general, the amount of leverage used in a RP strategy is the ratio between the SDs of a benchmark portfolio and the RP portfolio, but any ratio or leverage level can be selected. The general form for the volatility looks as follows.

$$k = \frac{\sigma_b}{\sigma_{RP}} \tag{2.23}$$

[6] This implies $TRC_i = TRC_{i \neq j}$

where σ_b is the volatility of the benchmark portfolio, and k denominates the multiplier for the weights of a leveraged portfolio. Note that the multiplier k equalizes the risk of a levered portfolio and the benchmark portfolio. The vector of weights of RP portfolio is then calculated as follows

$$w_{levered} = k \times w_{unlevered} \tag{2.24}$$

2.3.3 Theoretical Example

Let's consider an example with real data and no-rebalancing. The investment universe consists of two assets: assets A (e.g. equity) and B (e.g. bonds). Asset A is represented by the MSCI World Equity index and has an annual volatility of 14.65%. Asset B is represented by the Bank of America Merrill Lynch Global Bond index and has an annual volatility of 5.37%. The time frame spans from January 1997 to July 2017. The correlation between the two assets is $\rho = 0.04$[7] and for this example, we assume that both assets have a SR of 0.3. We construct a 60/40, 1/N, GMV and a RP portfolio for comparison reasons since we do not have to estimate the returns. The two portfolio strategies that allocate the most capital to high-beta assets (1/N and 60/40) have the highest SDs. The RC[8] per asset to the portfolio as shown in table 2.1, we see that the 60/40 portfolio and the 1/N portfolio have a great amount of their risk allocated to the risky asset, although the capital allocation is rather equally spread. The GMV portfolio is the combination of assets that minimizes the portfolio's variance. We see that minimum variance is achieved when 93% of the capital allocation is invested in the Asset B. This also means that 96% of the risk allocation is allocated to asset B, resulting in the safest, though most concentrated, portfolio. The RP strategy also overweighs Asset B, but the RC per asset class is equal, namely 50%. By equalizing the RC, the capital allocation is more balanced relative to the GMV, and the portfolio's SD is much lower than the 60/40 and 1/N portfolios.

[7]This number fluctuates over the years, however, the yearly mean and median observations suggest this is a reasonable estimate.

[8]The calculations can be found in Appendix A.1

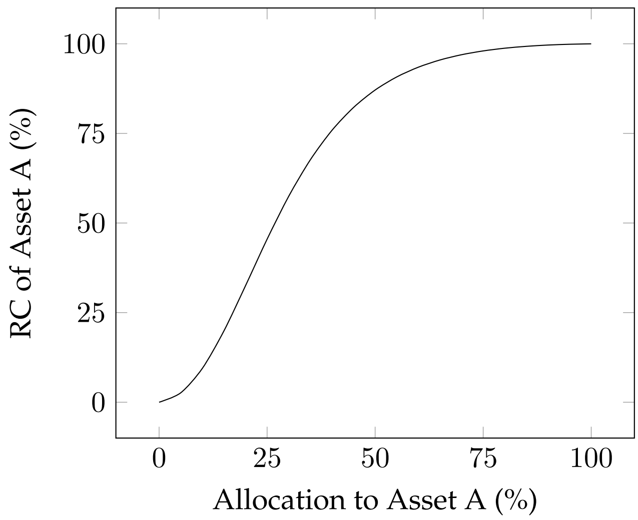
TABLE 2.1: Hypothetical portfolio weights and RC

Portfolio		Weights	Risk Contr.	Portfolio SD
60/40	Asset A	60%	93.48%	9.14%
	Asset B	40%	6.52%	
1/N	Asset A	50%	87.11%	7.91%
	Asset B	50%	12.89%	
GMV	Asset A	7%	4.37%	5.15%
	Asset B	93%	95.93%	
RP	Asset A	27%	50.00%	5.68%
	Asset B	73%	50.00%	

* Asset weights were restricted to positive weights only and had to add up to 100%

Figure 2.1 further illustrates how risky assets can concentrate risk allocations within a portfolio. We again use assets A and B and their characteristics for this example. We see that the marginal RC of Asset A is substantial as more capital is allocated into it. This stresses how important diversification across assets is.

FIGURE 2.1: RC of Asset A at various allocation levels



What about returns? Let's assume that there is a risk-free asset that returns 1%, and as assumed previously, both Asset A and Asset B have a similar risk-adjusted return, namely a $SR = 0.3$. This means that Asset A has a implied total return of 5.4%[9] and Asset B has an implied total return of 2.61%[10]. Now we know the returns of the assets, we can calculate the return of the portfolio. Table 2.2 shows the return characteristics of each portfolio. In addition, the figure introduces a leveraged RP portfolio.

[9] 0.3 * 15% + 1% = 5.4%
[10] 0.3 * 5% + 1% = 2.61%

TABLE 2.2: Hypothetical portfolio weights and RC

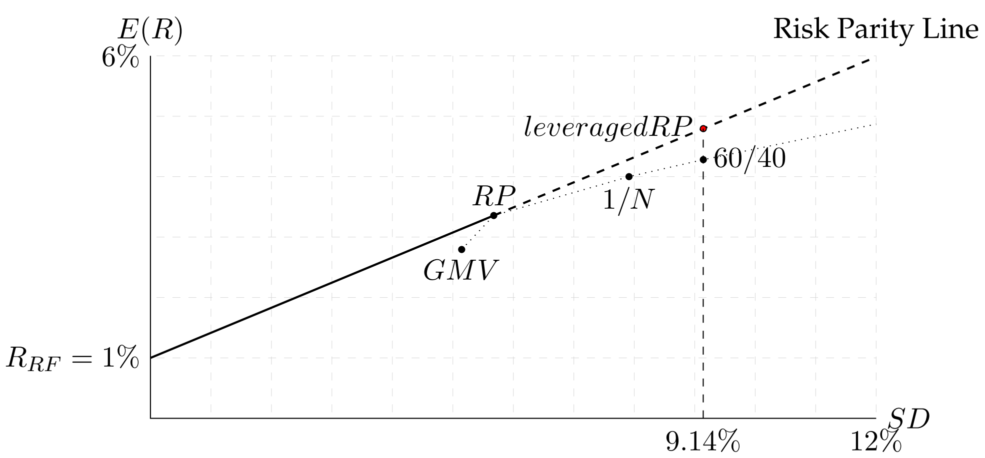
Portfolio		Weights	SR	SD	Return
60/40	Asset A	60%	0.36	9.14%	4.28%
	Asset B	40%			
1/N	Asset A	50%	0.38	7.91%	4.00%
	Asset B	50%			
GMV	Asset A	7%	0.35	5.15%	2.79%
	Asset B	93%			
RP	Asset A	27%	0.42	5.68%	3.36%
	Asset B	73%			
Leveraged RP	Asset A	43%	0.42	9.14%	4.80%*
	Asset B	118%			

Asset weights were restricted to positive weights only and had to sum up to 100%

* $43\% \times (5.40\% - 1\%) + 118\% \times (2.61\% - 1\%) + 1\% = 4.80\%$

When we lever up the RP portfolio to the risk of the portfolio with the highest return, we achieve a higher return for the same amount of risk. This implies that we have to use leverage, which obviously has its own restrictions as pointed out in section 2.3. But in this two-asset example, we see that a higher return can be achieved using a RP strategy with leverage. Due to the imperfect correlation and and overweighting safer assets with same risk-adjusted return characteristics, a RP portfolio was able to achieve a higher SR than other strategies. So, by (de-)levering up the portfolio's weights proportional to the ratio of the the SDs, we can achieve an allocation that suits individual risk preference while still being (sub-)optimal. Figure 2.2 presents a mean-variance diagram with the portfolios plotted inside. It helps to visualize the return/risk characteristics between the portfolios. Also, it helps in understanding how levering a RP portfolio can result in a superior portfolio.

FIGURE 2.2: Mean-variance diagram



The Risk Parity Line in figure 2.2 is the line that originates at the risk-free rate with a slope that is equal to the SR of the RP portfolio. It represents the combination of the RP portfolio with the risk-free asset that has the same SR. The line below the RP portfolio is a positive investment in the RP portfolio and the risk-free asset; the dashed line above the RP portfolio is a positive investment above 100% in the RP portfolio while shorting the risk-free asset; and the RP dot is is a 100% investment in the RP portfolio. In this example, the RP portfolio had the highest SR, so one can create superior portfolios and customize their portfolios to their own risk tolerance. The portfolio shows a superior performance relative to its peers. We can decompose the return in two ways. First, the return consists of the risk free rate plus the weight times the excess return of individual assets[11]. In this way, we can observe the source of excess return at asset class level (Qian, 2011). Second, the return consists of the total return of assets minus the funding costs of leverage[12] It implies that we have borrowed 61 basis points at the short-term risk-free asset return to invest in the leveraged RP portfolio at a cost of 1%.

In this example, we have seen that RP has the potential to outperform traditional strategies. The portfolio diversifies at risk level, thereby being less risky than other portfolios. The risk-adjusted return of the RP portfolio is the highest and the weights are fairly balanced. Following the example of the CAPM, an investor can use leverage to customize the RP portfolio to their risk tolerance level. This makes RP more attractive than the other used methods. In this example, the cost of leverage has not been taken into account, since it merely serves the purpose of pointing out the mechanism of how the RP portfolio can be superior to peers.

[11] $4.80\% = 1\% + 43\% \times 4.40\% + 118\% \times 4.61\%$
[12] $4.80\% = 43\% \times 5.40\% + 118\% \times 2.61\% - 1\% \times (118\% + 43\% - 100\%)$

Chapter 3

Methodology

This chapter introduces the methodology used to analyze the performance of the RP strategy. The allocation strategies mentioned in chapter 2.2 will be used as comparison to evaluate the performance of RP. All returns of the portfolios are excess of the risk-free rate. Metrics used to investigate performances are returns, volatility, drawdowns, RCs, and diversification. The chapter starts with discussing the data. Sequentially, it continues with explaining how portfolios are constructed and maintained over time. Afterwards, the methodology for evaluation performances is explained.

3.1 Data

The performance of RP is evaluated using a sample that ranges from 6 January 1995 until 30 June 2017. Six asset classes are included in the portfolios, namely: equity, bonds, commodities, real estate, inflation linked assets, and private equity[1]. According to a survey of Towers Watson, the typical portfolio of large institutional funds consisted of these classes[2]. Since this thesis is reviewing performance with market representative portfolios, this allocation is considered to be the norm for this thesis. The chosen asset classes will be represented by indices for several reasons. First, creating own-made indices is susceptible to mistakes in calculations and very time consuming. Second, it can be assumed that issuers of indices have better information resources due to their specialization and size. Therefore, one can assume that their indices are of good quality and accurate. Third, the use of indices allows for more robust estimation of the variance-covariance matrix due to a lower number of assets. One should take into account that the variance-covariance matrix reflects the movement of the index and the specified basket of investments it represents, not the movements of individual assets. Fourth, financial firms that create indices issue shares. Investors can buy these shares to have exposure to the specified basket of underlying investments. These shares usually trade at large exchanges and appear to be quite liquid due to their stock exchange availability (IA, 2017). This is an advantage for this thesis since liquidity issues of asset classes such as real estate and private

[1]The chosen assets offer returns in the four economic scenarios mentioned in the literature review

[2]See https://www.towerswatson.com/en/Insights/Newsletters/Americas/insider/2015/10/2014-asset-allocations-in-fortune-1000-pension-plans for more details.

equity can be regarded as non-existent[3]. Last, these indices have long time-spans, making them ideal for research purposes. There are, however, some disadvantages to using indices. Choosing indices is subject to the selection bias (Heckman, 1977). Also, an index might not reflect the true return characteristics of an asset class, although this is a general concern with asset selection. Furthermore, while choosing indices that have a long lifespan might be ideal for research purposes, the survivorship bias (Brown et al., 1992) could play a role. Individual indices remain available as long as investors are interested in them, meaning they offer better risk-return statistics than competitors that are terminated due to inferiority. All in all, it is assumed that the benefits of using indices outweigh the disadvantages.

The analysis will start with a chapter that discusses the descriptive statistics of the data used. This includes the statistics of the chosen asset classes, and the way that portfolios are built up. This thesis investigates the effect of asset selection in a RP portfolio. For this reason, data of comparable indices were used to investigate if substitution assets have an effect on performance. Furthermore, the distributions are checked and altered if necessary. In addition, the correlations between assets and portfolios are investigated, given that section 2.3 underlined that a RP portfolio performs better when the correlation among assets is more dispersed. Furthermore, Koestrich (2015) finds that the correlations of assets in times of crisis are closer to 1. Investigating the correlations over time allows to check whether correlations are non-constant and how correlations affect performance.

3.2 Portfolio Construction

I consider four portfolio strategies for backtesting: 1/N, GMV, 60/40 and the RP strategy. Table 3.1 presents the used portfolios and their strategies respectively.

TABLE 3.1: Portfolio summary

	Strategy	**Definitions**
$1/N$	Equalizes w_i	$w_i = w_{j \neq i} = \frac{1}{N}$
60/40	60% equity, 40% bonds	$w_{equity} = 1 - w_{bonds}$
GMV	Minimizes σ_p	$arg\min w'\Sigma w$
RP	Equalizes TRC	$TRC_i = TRC_{j \neq i} = \frac{1}{N}$

The benchmark strategies are chosen for the comparison of heuristic methods that are common in the literature, but every portfolio also serves as a comparison for additional reasons. First, the 1/N strategy equalizes the weights equally across asset classes. RP equalizes the RC of asset classes. It is therefore interesting to explore the

[3] This does not mean that liquidity will be ignored, it means that liquidity constraints are not used in the analysis.

diversification differences between the two strategies. Second, the 60/40 strategy is the high-beta strategy. Comparing it with a RP strategy helps in creating a contrast between high-beta portfolios and risk-balanced portfolios. Lastly, the GMV strategy always results in a portfolio with the lowest volatility. The comparison between RP and GMV helps to see where RP lays on the mean-variance area. Low volatility does not mean low risk; using the GMV strategy helps to uncover what reduces risk in portfolios. All strategies can be constructed *ex ante* so that the look ahead bias (Carhart et al., 2002) cannot play a role. The portfolios are constructed as they were defined in section 2.2 and 2.3. The 60/40 and 1/N strategies have fixed weights, so they do not require any steps for the calculation of asset weights. The construction of the GMV and the RP strategy's weights require several steps. These steps are as follows.

GMV steps

Step 1 The sample variance-covariance matrices are estimated using a 36-month period.

Step 2 The GMV algorithm finds the combination of assets that minimizes the total volatility of the portfolio. The algorithm has two constraints: 1) no negative weights, and 2) the sum of all asset weights should equal 100%.

Step 3 After the weights are set, the algorithm moves to $t + 1$ and starts over with step 1. This continues until the algorithm has iterated over every week of data.

RP steps

Step 1 In period t, the sample variance-covariance matrices are estimated using a 36-month period.

Step 2 The RP algorithm uses the variance-covariance matrix to calculate the marginal RC of each asset class, which it then transforms into the percentage of RC of the portfolio's total volatility. The algorithm then finds the asset weights so that the percentage of RC equals $1/N\%$, where N stands for the number of asset classes The algorithm has two constraints: 1) no negative weights, and 2) the sum of all asset weights should equal 100%.

Step 3 After the weights are set, the algorithm moves to $t + 1$ and starts over with step 1. This continues until the algorithm has iterated over every week of data.

Asset weights can only be calculated when enough data is available to estimate the VCV matrix. So, asset classes will be omitted if the data are insufficient. After the weights are determined, they are used to calculate annualized returns for all periods. These returns are excess of the risk-free rate. From those excess returns, the volatility and other statistics can be calculated.

In the initial setup, the portfolios are rebalanced monthly. In the sensitivity analysis, it will be repeated in a quarterly and yearly frequency. Returns tend to be volatile when the time frame is small. Taking a larger time period makes the data less noisy. Therefore the variance-covariance matrix will initially be estimated using a rolling average of 36 months of data.[4] This allows for proper estimation of the variance-covariance matrix, since the amount of time periods is greater that the number of assets. Although the choice of estimation period is in line with other backtest studies, varying the period might affect the outcome. To ensure robustness, the analysis will be repeated using a 12-month and 60-month VCV estimation period.

The literature commonly uses a shrinkage technique for the estimation of the variance-covariance matrix. As has been shown, this leads to a reduction in the tracking error, as Ledoit and Wolf (2004) and Lee (2011) demonstrate. Here, the shrinkage estimator is not used since the number of securities ($N = 8$) per portfolio is low compared to number of periods ($t = 1022$). That is, the shrinkage estimator only results in better results when $N > t$[5], which is not the case with this thesis' data[6]. The returns are calculated *ex post*. Because of *ex post* return calculation, there is no look-ahead bias. Since capital will be 'rolled-on' to the next period, the geometric average will be used to calculate the annual returns, the formula can be found in section 2.1.

3.3 Performance Evaluation

3.3.1 Risk Parity Performance

The RP strategy allocates much capital to low-volatility assets. Therefore, it is not likely that the return of a 0% leveraged RP portfolio will have a higher total return than a high-beta strategy such as the 60/40 strategy. Levering up the portfolio is therefore preferable when investing RP style. In order for RP to be attractive, the risk-return trade-off of the RP strategy should be superior to the benchmark portfolio. For this reason, the main focus when assessing the performances of the strategies will be placed on observing the produced volatility-return trade-offs.

Besides returns, the volatility and concentration characteristics are equally important to investigate in this thesis. Volatility will, as explained previously, be expressed in SDs. Additionally, maximum drawdowns (MD) will be included to draw conclusion on both riskiness and smoothness of returns over time. Volatilities do not always reflect the true riskiness of a portfolio. The MD shows how risky a portfolio is in terms of capital loss. It is important for an investor to know what the continuity of the invested capital is. For example, a GMV portfolio has the lowest possible volatility, but the strategy is likely to extremely overweight low-risk assets. These assets, however, can experience severe shocks, and when the asset weights are concentrated, the portfolio is susceptible to shocks, so large capital drops can occur. Low volatility portfolios are not equal to safe portfolios, therefore the MD is a good

[4] Academics usually take a long period to estimate the VCV matrix: Bilan (2016) use a 50-day window with daily return data, Chaves et al. (2012) use a 5-year window using monthly data, Kaya and Lee (2012) use a 40-month period using monthly data, and Marra (2016) use a 12-month window using weekly data.

[5] A general rule is that $\frac{t}{N} \geq 1$ for a variance-covariance matrix to be singular and non-invertible.

[6] With $N = 8$, only 8 variance estimates and $\frac{8*(8-1)}{2} = 28$ will be estimated.

metric to assess a specific risk.

When evaluating asset class weight and risk concentration, we make use of the so called Gini-coefficient. This measure is widely used by economist measure the inequality of a population. The Gini coefficient ranges between one (perfect equality) and zero (perfect inequality). Maillard, Roncalli, and Teïletche (2010) and Chaves et al. (2012) applied the Gini-coefficient as a measure of portfolio concentration by computing RC inequality (or RC concentration). In the literature, we find that the Herfindahl measure of portfolio concentration is generally used, but the Gini-coefficient presents the advantage of having a value always situated between zero and one, which allows for clearer interpretation and better comparison between portfolios. Furthermore, we check the correlation of the constructed portfolios with the asset classes. This helps to understand where return characteristics of portfolios are coming from. The calculation of the Gini coefficient is given by the formula below. x_i denotes the weight/RC of asset class i and there are n asset classes.

$$\begin{aligned} G &= \frac{1}{n}\left(n+1-2\left(\frac{\sum\limits_{i=1}^{n}(n+1-i)y_i}{\sum\limits_{i=1}^{n} y_i}\right)\right) \\ &= \frac{2\sum_{i=1}^{n} i y_i}{n\sum_{i=1}^{n} y_i} - \frac{n+1}{n} \end{aligned} \tag{3.1}$$

As pointed out in the introduction and the literature review, Asness, Frazzini, and Pedersen (2012) and Chaves et al. (2011) point out that when all assets have similar risk-return characteristics, adding more alternatives to a portfolio makes it more diversified. However, the former authors provide reasons and proof of why this is not the case. In this thesis, it is therefore important to test how the asset classes are positioned along the security market line over time. According to the theory of Asness, Frazzini, and Pedersen, the security market line should be more flat and low-beta assets should yield higher risk-return characteristics than high-beta assets. To test this, we not only plot the SML line over the entire dataset, but also over subperiods to show how the SML has been historically. From the graph, we can derive in which periods a RP should be able to outperform.

3.3.2 Sensitivity to Leverage

The previous section assumed that the RP strategy was not levered. In order to understand what leverage does with performance, the RP portfolios will be levered up 10%, 25%, 50% and 100% of their initial capital investment. The same performance statistics will be assessed as was proposed in section 3.3.1. With the use of leverage, costs will be imposed. So, actual leverage costs will be used, which are represented by the 5-year borrowing rate of European securities[7]. Although a 5-year borrowing rate seems appropriate as leverage costs, the literature frequently uses short term rates as a proxy for the costs. Short term rates have a few downsides. For example, by constantly reissuing debt, transactions costs rise, the susceptibility to short term yields rises and a insolvency risk arises when no new debt can be attracted. This thesis makes use of a short term bond as a proxy for costs. Bond yield curves show that short term bonds have lower rates since there is less insecurity due to the shorter time frame. Including short term bonds as a proxy for leverage costs allows to see how attractive a leveraged RP strategy is when the previously mentioned downsides are non-existent. For both cases of leverage costs, we look whether the marginal benefit of using leverage is higher than the marginal costs. Similarly, the realized annualized returns will be compared to the peer strategies.

3.3.3 Asset Choice

The performance of portfolios is hugely dependent on the choice of assets. For this reason, the strategy's portfolios will be constructed using four different asset mixes: two European oriented mixes and two global oriented mixes. As mentioned before, financial literature rarely uses Europe as a geographic to conduct backtests. Therefore this thesis includes it as a separate regional focus. The global oriented mix is included because investors have fast and wide access to the whole world due to improving (information) infrastructure. To conclude whether changing assets changes outcomes, at least two portfolios of each regional focus need to be constructed. These will be used to create comparable portfolio and test them with each other. The performances will be evaluated by using the performance method described in section 3.3.1. In addition, by adding or dropping asset classes in a RP portfolio, the result tends to be ambiguous (Chaves et al., 2012). Therefore, the effect of adding different asset classes and assets will be examined in the robustness section of the results chapter. This will be done by dropping random asset classes from all portfolios and assess the performance statistics.

In order to analyze the performance, we will construct four portfolios. As discussed before, this thesis has a focus on both Europe and the World. The first two portfolios consist of world indices but differ in the indices included. This way we can draw conclusions on the performance of a world portfolio all the while limiting the effect of individual indices having an effect on the outcome. Note that the chance that an individual index still affects the outcome is not eliminated, it's merely reduced. The same procedure holds for the European portfolios, but these portfolios consist mostly of European indices. Hence, the analysis will be done with four portfolios differing in geographical orientation and assets included.

[7]Nasdaq (2017) states that 20%/30% can be considered as a buy-and-hold strategy, meaning that a portfolio can be completely turned-over after 5 years, therefore a 5-year interest rate is used.

Chapter 4

Data

In this chapter, the data used in this thesis is presented, the reasons for including certain asset (classes) is explained, and the limitations of the data are explored. The chapter first discusses general information regarding the data, followed by the discussion of each asset class' statistics. Finally, the division of assets among portfolios is presented and discussed.

4.1 Asset classes

As mentioned in section 2.3, portfolio performance is strongly dependent on choice of asset classes. This section discusses the assets and asset classes included in the performance evaluation. For the remainder of this thesis, the risk-free rate is represented by the European 90-day bond rate. The risk-free rate has been chosen because it does not face liquidity risk in the short term, nor is the asset assumed to face default risk. The choice for an European rate is based on the regional focus of thesis. In the descriptions of the asset classes, we do not elaborate on the liquidity. As explained previously, the use of indices allows for more liquidity than actual direct investments. Of course, this remains a point of discussion, but this will be addressed in chapter 6. The starting date of each portfolio was the 5th December 1997. Since most indices existed three years from that point on, volatility statistics could be calculated. Furthermore, the correctness of data is important for the analysis. Therefore the skewness, kurtosis and normality are reviewed when discussing the asset classes. This thesis frequently uses the SD which assumes normality, therefore this is consistently checked. We know that return data suffers from statistical moments and outliers since news and market conditions heavily affect security prices, thereby violating the normality assumption. Therefore the normality tests are performed on the aggregate of all data. However, one should not forget that the return data suffers from these statistical moments. Moreover, the data is not checked on autocorrelation since previous returns have little causality when explaining future returns. The returns are stationary and news/market driven, finding positive autocorrelation statistics would likely be caused by new political decisions, market news or sentiment where the previous day price movement is the result rather than the cause.

4.1.1 Equities

Table 4.1 presents the summary statistics of chosen equity indices. Due to the generally high volatility and movement with the market, equity is considered to be a high-beta asset. Both Europe and the World are represented in order to differentiate between a Global and an European portfolio. Each demographic class has two broad comparable indices which are used to construct two similar portfolios for robustness checks.

TABLE 4.1: Summary statistics equity indices

	Ann. Return(%)	**SD(%)**	**Skewness**	**Kurtosis**	**Start date**
MSCI World	6.35	15.98	-0.603	3.851	06-01-1995
MSCI Emerging Market	5.62	21.69	-0.691	5.168	06-01-1995
FTSE All World	6.49	16.18	-0.637	4.024	06-01-1995
MSCI Europe	6.44	19.69	-0.481	2.905	06-01-1995
STOXX Europe	6.65	18.44	-0.335	3.064	06-01-1995

We see that the the world indices have lower volatilities than the European indices, but the returns seem roughly equal. This is most likely due to diversification benefits. The world indices are less susceptible to regional shocks, thereby claiming the so-called "free lunch". This tells us only little about the attractiveness of an index. It is hard to determine sources of idiosyncratic risk of any index since there are many assets included, so this information washes away. Hence, the idiosyncratic risk of the index can be considered to be the susceptibility to the market, therefore we can call it the market risk of equity. Equity markets around the world are intertwined, so when shocks happen, other regions and markets are affected. This may be a disadvantage regarding diversification. A portfolio containing both a European and a global index can be beneficial if the correlation between the two is less than 1. Table B.1 presents the correlation of all indices used in this thesis. For the equity indices, we see that the correlations between assets are quite clustered, averaging around 0.9. The emerging market index is less correlated to the others, possibly because the markets are different (advanced vs. developing) in the sense that the technology is less advanced. Therefore, these markets could be less intertwined with advanced countries, and shocks in either country would not influence the other as much. The correlations are unequal to 1, so there exists a potential diversification benefit. Figure B.3 shows the frequency distribution of returns of the equity indices on aggregate. Table 4.1 adds to this with the statistics on kurtosis and skewness. We see that the distributions are shifted more towards the right, implying higher right tail values. The kurtosis statistics show that the frequency distribution can be considered normal[1]. Finally, the distributions of the individual indices and the aggregate is tested

[1]The outcomes cluster around the value 3, which is generally considered to be a normal distribution.

on normality, table B.4 shows the results. For every index, we see that the assumption of normality is met, so the data can be used for the analysis without further adjustments.

4.1.2 Bonds

Table 4.2 presents the summary statistics of chosen bond indices. Both Europe and the World are represented in order to differentiate between a globally diversified and a European diversified portfolio. In addition, each demographic class has two broad comparable indices that are used to construct two seemingly similar portfolios for robustness checks. The classes consist of an overall bond type that include mainly corporates and some governments, and a government type that only consists of government bonds. Bonds appear to have much lower volatilities and returns relative to equity. In general, they are considered to be low-beta assets

TABLE 4.2: Summary statistics bond indices

	Ann. Return(%)	**SD(%)**	**Skewness**	**Kurtosis**	**Start date**
JPM Global All	0.81	3.18	-0.264	1.214	06-01-1995
BofA ML Global Broad	4.67	5.36	-0.015	1.116	06-01-1995
BofA ML Global Broad Gov.	4.53	6.58	0.048	1.192	06-01-1995
Barclays EU Agg*	0.20	3.37	-0.263	1.986	06-01-1995
IBOXX EU Overall*	0.34	3.46	-0.352	2.180	06-01-1995
BofA ML EU Gov.*	5.28	4.04	-0.449	2.605	06-01-1995

We see that in both demographic classes the government indices are superior. This, however, says nothing about the overall attractiveness of bonds. The correlation of bonds with the rest of the indices is low, as can be seen in table B.1. This makes it a good asset class to include in a portfolio to make it more diversified. We observe a big difference in annual return between the BofA indices and all others. An explanation could be the superior managing of the indices by BofA, since demographic spread and asset inclusion is comparable with other indices. Since multiple portfolios with different included assets are constructed, this poses no problem for the analysis. Overall, the indices seem to have less volatility and lower returns relative to the equity indices. This makes sense since bond returns are fixed yields and/or coupons. Equity returns dividend or the earnings of a company with theoretical unlimited potential, therefore usually equity returns are higher. Furthermore, bond holders have the first claim in case of bankruptcy, while equity holders have residual claim. Therefore, bonds are safer than equity. The distribution of bond returns is tilted to the right, as can be seen in the skewness statistic. Regarding kurtosis, the values are rather low. The possible reason for this is that the difference in return is high among the bond indices, resulting in the distribution of returns to be wider than those of the equity asset class, where the returns among indices are more clustered together. Table B.4 shows that the normality tests, they also reject the hypothesis that the distributions are non-normally distribution.

4.1.3 Commodities

Two global commodity indices have been chosen in order to make comparable portfolios. They are comparable in such a way that they are globally oriented, include the same variety of commodities and weigh them roughly in the same manner. On aggregate, the returns lie between those of bonds and equities. The volatility tends to be higher than the bond asset class because of low liquidity, climate and natural disasters, politics and technology. The indices have a low correlation with all the other indices, as can be seen in table B.1. Adding a commodity index in a portfolio can create diversification benefits.

TABLE 4.3: Summary statistics commodity indices

	Ann. Return(%)	**SD(%)**	**Skewness**	**Kurtosis**	**Start date**
Thompson Reuters Equal Weight	2.43	10.97	-0.237	4.683	06-01-1995
S&P GSCI Commodity	6.48	18.82	-0.462	3.311	06-01-1995

The skewness and kurtosis statistics of the commodity indices are comparable to those of the equity indices, having a negative skewness and high kurtosis. In other words, the return distribution is tilted towards right tail values and the returns are clustered according to a normal distribution. The normality test in table B.4 show that the return distributions of the commodity indices can be regarded as normally distributed.

4.1.4 Real Estate

The two real estate indices each represent a different region. Given that MSCI constructed and managed both indices, both are selected, resulting in fewer differences in the indices aside from the regional focus. The European index is more volatile, but it has a significantly higher return, possibly because of the booming housing prices in Europe as a result of the low interest rates. Also, Europe is more developed than the rest of the world as a whole, making access to finance easier, so it is easier to invest in real estate, thus boosting the return of the index. The world index is more regionally diversified, therefore regional booms and busts are canceled out, possibly explaining the lower annual return of the index.

TABLE 4.4: Summary statistics real estate indices

	Ann. Return(%)	SD(%)	Skewness	Kurtosis	Start date
MSCI World Real Estate	3.12	19.66	-0.405	5.802	06-01-1995
MSCI EU Real Estate	6.40	20.29	-0.902	6.952	06-01-1995

We see that the correlations (table B.1) are high with equities. This makes sense given that real estate has become an investment vehicle over the past decades. Given the nature of susceptibility to crises and consumer demand, the indices are likely to move closely to equities. The indices are also moderately correlated with commodities, which makes sense since it is a primary good that people need, similar to most of the commodities. The other correlations are quite low. Regarding the the return distributions of the real estate indices, we see that the skewness is low and the kurtosis quite high. In other words, the return frequencies are shifted to right tail values and the returns are are more peaked when compared to a standard normal distribution. The normality tests, shown in figure B.4, show that these indices can be considered as normally distributed.

4.1.5 Private Equity

The private equity indices have high volatilities because the business of private equity is more risky in nature than the other asset classes. The two indices were arbitrarily selected. The TR Private Equity is an index which compensates generously for the additional risk of private equity, but the S&P index does not. Private equity funds have been criticized by academics and practitioners for performing poorly relative to peers. Two indices cannot provide a complete picture of the attractiveness of private equity, but investors are not eager to invest in a risky index such as the S&P index when it returns relatively little. The source of private equity returns are similar to those of equity, but private equity faces more risk since the invested companies are usually smaller and less liquid. The investor, however, is compensated for this as a result of the high growth potential of these companies.

TABLE 4.5: Summary statistics private equity indices

	Ann. Return(%)	SD(%)	Skewness	Kurtosis	Start date
S&P Listed Private Equity	2.75	25.70	-0.602	9.350	01-11-2003
TR Private Equity Buyout	15.22	28.28	0.262	20.729	06-01-1995

The correlations of private equity with the rest of the indices in table B.1 are around 0.5 with all classes except for bonds and inflation-linked assets. Private equity is pro-cyclical which RE and equity also are. Also, private equity firms can have their business model intertwined with other asset classes. Bonds and inflation-linked assets are non-correlated since they are not a product which are produced by any firm, they are simply finance vehicles. Similarly to all the above-mentioned asset classes, the private equity indices are considered to be normally distributed, as shown in figure B.4. Notably, both indices have a high degree of peakedness. Also, the skewness statistics have the opposite signs: the S&P index is more skewed to the right while the TR index is skewed more to the left. This, however, poses no problem for normality.

4.1.6 Inflation-linked Bonds

The final asset class is inflation-linked bonds. Similar to real estate, these are selected based on their demographic region. The global index has performed better in terms of return and volatility, probably because it is more diversified. We see typical lower returns than equity-like indices, but the lower volatility compensates for this.

TABLE 4.6: Summary statistics Inflation indices

	Ann. Return(%)	SD(%)	Skewness	Kurtosis	Start date
BofA ML Global Infl.-Linked Gov	3.75	4.87	-0.128	3.534	01-01-1998
BofA ML Euro Infl.-Linked Gov	2.30	5.14	-0.212	3.278	01-11-1998

The inflation-linked assets have very low correlations with every asset classes, as the correlation table B.1 indicates. This is because of their inflation-protected nature. Therefore, the general economic regime plays no role and the value of the bond rises equally with the CPI rate. The inflation-linked bonds have a low correlation with 'simple' bonds because bonds are long-term and have fixed rate, so that their return is independent on state of the economy, only the solvency of the issuer is a factor. For the inflation-linked bonds, the movements of the coupons are different due to the level of CPI. The skewness and kurtosis are relatively normal, both indices are skewed to right tail values and their peakedness is fairly similar to that of a standard normal distribution. The normality tests in figure B.4 show that the return distributions can all be considered as normally distributed.

4.2 Asset Mixes

As explained in the methodology, this thesis will assess the performance of several portfolios to examine the robustness of RP. Table 4.7 presents the asset mixes that are used to examine RP performance. An 'x' indicates the inclusion of an index in the portfolio. All portfolios cover the same period, namely: 5 December 1997 - 30 June 2017. Each portfolio consists of a fixed selection of assets that represent an asset class. For example, portfolio 1 includes the MSCI World and MSCI EM as representatives for the equity asset class. The return of an asset class is the simple average of included asset which represent that asset class. Each asset mix includes the MSCI EM index since this is a common asset in portfolios and usually classifies as equity.

Emerging market indices in Europe arose too late to be included in this analysis, therefore the MSCI EM was included in all asset mixes. The world asset mixes each consist of a different normal equity index accompanying the PE index. The same holds for the European asset mixes. A similar division holds for bond assets: global bond indices are varied within each regional asset mix, and each regional asset mix has a fixed government bond. Not all assets are varied since it is outside of the scope of this thesis. Finally, the four remaining asset classes each have two indices that are included in one of the two asset mixes of each region.

TABLE 4.7: Asset Mixes

	Asset mix 1	**Asset mix 2**	**Asset mix 3**	**Asset mix 4**
MSCI World	x			
FTSE All World		x		
MSCI Europe			x	
STOXX Europe				x
MSCI Emerging Market	x	x	x	x
JPM Global All	x			
BofA ML Global Broad		x		
BofA ML Global Broad Government	x	x		
Barclays EU Agg			x	
IBOXX EU Overall				x
BofA ML EU Government			x	x
Thompson Reuters Equal Weight	x		x	
S&P GSCI Commodity		x		x
MSCI World Real Estate	x		x	
MSCI EU Real Estate		x		x
S&P Listed Private Equity	x		x	
TR Private Equity Buyout		x		x
BofA ML Global Infl.-Linked Gov	x		x	
BofA ML Euro Infl.-Linked Gov		x		x

Chapter 5

Results

This chapter discusses the results of proposed methodology. The structure of this chapter is as follows: in part I, the performance of all constructed portfolios will be discussed. In part II, the robustness and sensitivity of the portfolios will be tested.

5.1 Part I - Performance

This part discusses the performance of the constructed portfolios. It focuses on the return, volatility/risk, connectedness, return origin and the effect of leverage on the portfolio. It sequentially discusses the portfolios in the following order: first, the two global portfolios will be discussed, followed by the two European portfolios.

5.1.1 Global portfolios - Portfolio 1

Portfolio Statistics

First of all, the assets that are included in portfolio 1 can be found in table 4.7 and the asset weights can be found in both B.2 and the asset weights over time can be found in figure B.5. The returns are excess of the risk free rate. Note that all portfolios have the same start and end date, namely: 5 December 1997 - 30 June 2017. In order to analyze the portfolio, we start by observing the general portfolio characteristics presented in table 5.1 at face value. We note that the equity dominated 60/40 strategy has the highest return, but also the highest volatility, followed by the 1/N, the RP strategy and finally the GMV strategy. Each strategy's return is positively related with volatility, the average risk/return trade-off over the almost 20 year sample is presented by the SR. We see that the the RP strategy scores the best with a SR of 32.66%, which is approximately 10 percentage points higher than the benchmark strategies. Meaning, for every unit of volatility RP returns around 0.1% more return than its benchmarks. This is also reflected by the M^2 measure introduced in section 2.1. The M^2 shows the return of a benchmark strategy as if it would have the same volatility as the RP strategy but with its own SR. Now it is easier to see what the higher SR of the RP strategy means and how they compare: the benchmark strategies all have lower returns and the RP strategy seems superior based on the previously discussed statistics.

TABLE 5.1: Portfolio 1 Annualized Statistics

	60/40	**1/N**	**GMV**	**RP**	**Lev. RP**
Excess Return (%)	2.86	2.34	0.84	1.47	3.16
SD Full (%)	11.20	9.73	3.33	4.50	6.75
SR (%)	25.53	24.06	25.26	32.66	46.77
M^2	1.15	1.08	1.14	1.47	2.10
MD Full (%)	-61.21	-60.90	-72.05	-50.57	-64.73
MD 5 year (%)	-55.28	-49.90	-59.95	-34.88	-47.23
MD 1 year (%)	-32.27	-37.23	-34.09	-19.42	-27.56
GINI weights	0.300	1.000	0.217	0.547	0.547
GINI RC	0.008	0.500	0.2224	1.000	1.000

Not every investor is the same. Investors differ in preferences and goals. For instance, for an investor that would set a higher benchmark than the risk-free rate, he would care less for Share Ratios or other risk-adjusted returns, he wants a higher total return. That investor, if he does not have the possibility to use leverage, would not choose a GMV or RP strategy since the yield is too low. When he does have the possibility, levering up the portfolio would make sense. He could lever-up his portfolio to the level of the desired return and/or riskiness. For now, we assume that no constraints or risk are imposed with the use of leverage, this will be discussed later in this section. 5.1 shows us the return characteristics of a leveraged RP strategy with a leverage ratio of 1.5. This ratio is chosen for illustration purposes. We see that with 'only' 50% extra capital invested, we can obtain a superior return with lower volatility than the 60/40 and 1/N strategy. Figure 5.1 shows how 1000 euro invested according to each strategy develops over time. It is interesting to see that the SR of a leveraged RP increases as leverage increases. This has a simple reason: the full benefits of return are absorbed while the volatility of the strategy's portfolio did not grow at the same rate. We see that the M^2 measure is higher than the RP strategy since it has a higher SR, meaning it outperforms unlevered RP as well. For now, the costs of leverage have not been taken into account. Overall, in these portfolio of assets the RP and levered RP have performed better than their peers.

FIGURE 5.1: Wealth plot Portfolio 1

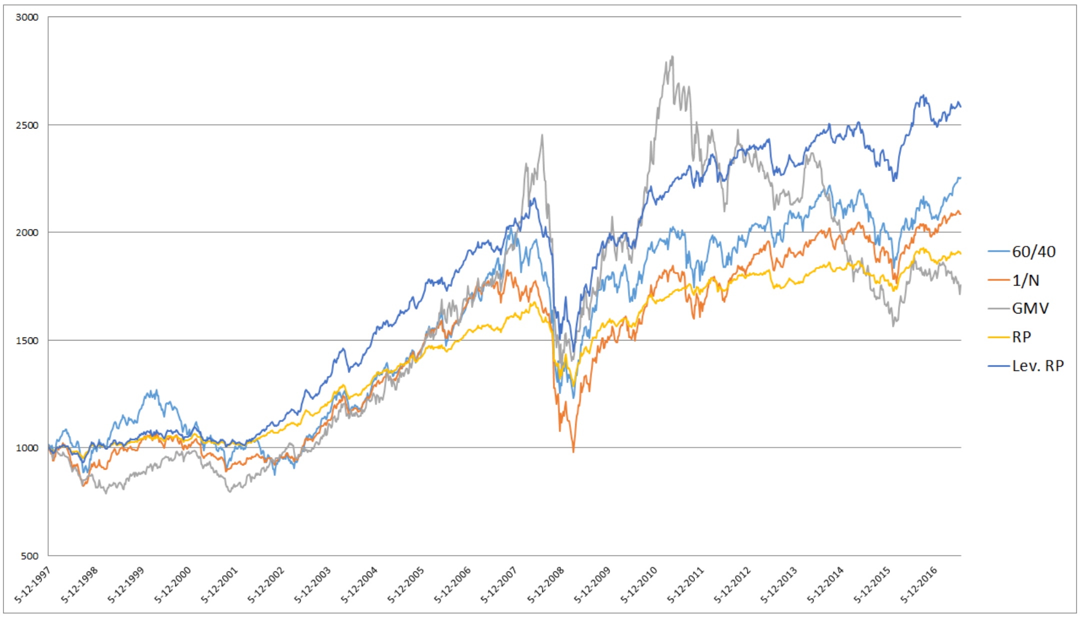


Table 5.1 shows the MDs and Gini coefficients of all strategies. The MD statistic is the cumulative loss of a specified period. For each portfolio, three periods have been defined to stress how risky a strategy is in terms of downsides and continuity: whole sample, 5-year rolling period and 1-year rolling period. Over the whole sample, we see that the GMV strategy has lost the most value (72.05%) in the whole sample period and the 5-year rolling period. In the short run (1-year), the MD is lower than the 1/N, but comparable. This is probably due to the concentration of asset weights, as shown by its Gini coefficient of 0.217. The high concentrations lead to higher idiosyncratic asset class risk because of the lower diversification. This means that adverse shocks within or across asset classes can lead to high losses. A well diversified portfolio in capital terms does not mean it does not face this risk. The 1/N portfolio has a Gini coefficient of 1 since capital is divided equally over all asset classes, but the MD over the whole period is still quite high: -60.90%. This gives reason to believe that the asset classes are highly correlated in terms of crisis, which the literature also suggests (Koestrich, 2015). In the period of the financial crisis (2007-2009), we see a large decrease in portfolio value for all strategies. To test whether the correlations in that time were indeed higher, we use a T-test. The correlations of all assets in the period 2006-2010 are subtracted to the correlations of the whole sample. Only left diagonal values are used since the right side values are similar, which implies that including them would dilute the outcome. We test whether the result differs from 0, since we then know that the correlations were higher in the period including the global financial crisis. These results are found in B.2. The T-test indeed confirms that the correlations in the period 2006-2010 were different than the correlations of the whole sample at at least a 1-percent level. This also tells us that it is logical that all strategies decrease sharply in value in that period, regardless of tactical asset weights, as is shown in figure 5.1. The 60/40 strategy has similar drawdown statistics to the 1/N strategy, most likely because of the high

capital investment in high-beta assets in combination with a high concentration of assets (Gini coefficient of 0.300). The RP portfolio concentrates its assets seemingly well with a Gini coefficient of 0.547. This is partly due to the strategy that RP always invest something in the chosen asset classes. The MD statistics are the lowest of all strategies, most likely because the strategy equalizes risk across asset classes, which minimizes the potential downfall of value. The levered portfolio does equally well in terms of the Gini coefficients, but has higher MD statistics since more capital is invested, so there is more sensitivity to individual and overall asset class risk when no other assets are included. The MD statistics are quite high for the entire sample, but decrease rapidly when the time period is shortened. In this portfolio, the RP portfolios perform promisingly in terms of asset concentration and drawdown of capital in the short-term at least, dependent on the level of leverage.

The Gini coefficients of the RCs is the last metric discussed in this paragraph. Obviously, the (levered) RP strategies have a RC Gini coefficient of 1 since the strategy equalizes RCs. The 60/40 portfolio performs the worst of all strategies with a RC Gini coefficient of 0.008, meaning that the source of risk is not diversified and most of it comes from equity. Although the GMV minimizes risk, its RC Gini coefficient is 0.221 since it invests most capital in bonds, which is the source of the risk. The 1/N portfolio has a RC Gini coefficient of 0.500 which is fairly diversified. This is likely due to the equal allocation to each asset class, meaning that is has exposure to every asset class. Not every asset class has the same risk characteristics; the 1/N portfolio takes the average of every asset class' RC leading to a fair value. This is, however, not reflected in the return characteristics (lowest SR, M^2 and MD 1-year).

Return Origin

The previous findings seem to suggest that the RP strategy has performed better than the 60/40, 1/N and GMV strategy, but it does not explain the cause. Therefore, the question remains: where do the returns come from and why do some outperform others? To answer this question, we will have to consider how the portfolio is correlated with the asset classes' return and how asset classes performed in the sample period. Table 5.2 presents the correlations of the four strategies with their asset class components. In general, a high correlation tells us where return comes from and how sensitive a change in the return of an asset classes is to the return of a strategy. Preferably, a portfolio's return should not have extremely high correlations with asset classes to protect itself from adverse shocks in asset class returns.

For the 60/40 portfolio, we clearly see that the return is highly, nearly perfectly (99.19%), correlated with equity returns. This means that the returns are mostly explained by equity returns. Bond returns have a much lower correlation, so the 60/40 strategy relies more equity than bonds for returns. This strategy included bonds to diversify in order to protect against shocks. We see that most of the risk is incorporated by the equity, suggesting that this is poor diversification.

TABLE 5.2: Correlation Portfolio 1 with Asset Classes

	Equity(%)	**Bonds(%)**	**Comm.(%)**	**RE(%)**	**PE(%)**	**Infl.(%)**
60/40	99.19	12.27				
1/N	74.73	1.78	56.76	86.62	72.48	0.14
GMV	22.15	91.32	26.23	20.44		
RP	61.09	36.04	55.46	70.90	60.26	18.79

The 1/N strategy's return has less extreme correlations with equity, but they remain on the high side (around 74.73%). Bonds have a lower correlation (1.78% on average) than the 60/40 portfolio. The lower correlation is explained by the relative lower weights to the asset classes, 1/N allocates 16.67% while the 60/40 strategy allocates 40%. Furthermore, real estate and private equity both have high correlations (86.62 and 74.48%) with the 1/N strategy. These are both high-beta asset classes. This is followed by commodities with a correlation of 56.76% and finally inflation linked bonds with a correlation close to zero (0.14%). By dividing capital equally across asset classes, we see that the correlations are higher for the high-beta classes, and lower for the low-beta classes. Dividing capital equally over classes might not be the ideal strategy when an investor wants to protect himself from shocks, regardless of the asset class.

The GMV strategy strongly overweights low-beta classes with respect to the high-beta classes. We see that equity, commodities and real estate all have much lower correlations with the returns of the GMV portfolio relative to previously discussed strategies. The correlations of inflation linked bonds and private equity with GMV returns are not discussed since the weights are zero or close to zero. The weights of the asset classes can be found in table 3.1. Bonds have a high capital allocation, which is why the bond asset class is highly correlated with the strategy's return. The strategy minimizes the variance, yet it concentrates risk and dependency in one asset class, making it sensitive to idiosyncratic shocks.

Lastly, we consider the RP strategy. Asset weights are determined by the volatility of an asset class and the main objective is to equalize RCs among them. This results in seemingly equal correlations for all strategies. Of course, when equalizing RCs using volatilities, this results in more equal correlations since correlations are a product of volatilities, but these correlations show that there are no extreme dependencies on certain asset classes. It is interesting to see that the high-beta asset classes still have a higher correlation with returns than the low-beta asset classes. Most likely, this is due to the higher volatility of these assets and positive correlations between asset classes. The main takeaway here is that the RP strategy has more balanced correlations with asset classes and, therefore, the portfolio is more diversified than its benchmarks in this portfolio.

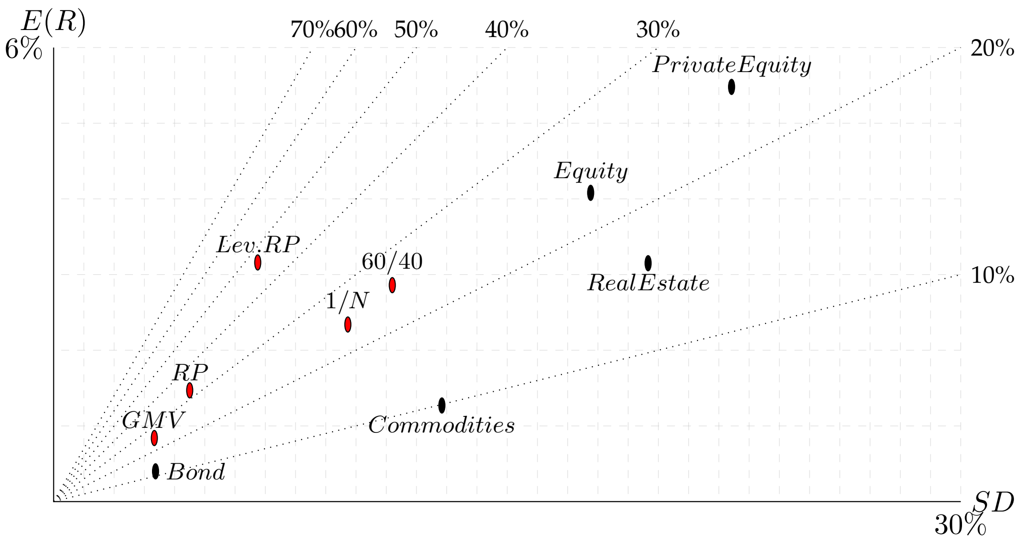
We have identified how the asset classes were correlated with the strategies. This, however, does not tell us why the strategy has the return it has. We use figure 5.2 which shows the position of asset classes in a mean-variance framework and table 5.3 which shows the statistics of the asset classes in portfolio 1. According to the CAPM theory, markets are efficient and all asset classes should have the same risk-return trade-off. The mean-variance diagram shows that this assumption does not hold. Inflation-linked bonds, equity and private equity have performed better than bonds, commodities and real estate.

TABLE 5.3: Asset Class Statistics Portfolio 1

	Equity(%)	**Bonds(%)**	**Comm.(%)**	**RE(%)**	**PE(%)**	**Infl.(%)**
Excess Annual Return	4.08	0.40	1.27	3.15	5.48	1.45
Volatility	17.76	3.37	12.84	19.66	22.42	4.54
SR	22.98	12.00	9.92	16.02	24.45	31.95

Equity and private equity, both high-beta assets, outperform. This suggests that strategies that invest more capital in these classes are prone to outperforming those who do not. For example, we see that the 60/40 strategy has performed better than the 1/N portfolio, most likely because of the higher exposure to the higher yielding equity class, while 1/N has the same capital exposure to every class. The GMV strategy has performed similar to the 60/40 and 1/N strategy despite the large dollar allocation to the lower yielding bond and commodity asset class. The correlations between the commodities and bonds are negative (as can be seen in table B.1, which likely caused diversification benefits, leading to a higher SR. The RP portfolio outperformed in terms of SR when compared to its peers. Although it has a large capital allocation to bonds and commodities, the higher SR is likely caused by a large allocation to the high yielding inflation linked bonds and an exposure to equity and private equity. Note that this analysis only holds for this portfolio, this is not a reflection of all asset classes in the world, since those have other (cor)relations with each other.

FIGURE 5.2: Mean-Variance Diagram Portfolio 1



The excess return-volatility characteristics of the strategies and asset classes are plotted in this mean-variance graph. The dotted lines depict the different levels of SR.

Leverage

The use of leverage is not free. Until now, we have assumed that the use of leverage has no cost, and we saw that the levered RP portfolio performed very well by obtaining a higher SR and higher returns that are significantly better than benchmarks. The use of borrowed capital did cause the MD to rise as was explained in the return characteristics section. Although the current financial climate allows for negative interest rates, borrowing capital is usually linked to a positive interest rate, just like households that acquire a house using the bank must pay mortgage interest. We have argued earlier in the section 3 that a turnover ratio of 5 years would be considered normal. For this reason, we assume that a 5-year bond yield is an appropriate cost for the use of leverage. The asset is the "Euro area 5-year Government Benchmark bond yield". The data stretches from January 1995 to June 2017. For comparison, the 3-month LIBOR rate is also used as a proxy for leverage. If a fund is able to refinance it debts with short term bonds, the short-term LIBOR rate serves as a fine proxy. The 3-month rate is chosen for the similar maturity as the risk-free rate. The difference between the risk-free asset and the 3-month LIBOR rate reflects the perceived riskiness of the economy since its priced according to how market players view each others solvency/default risk. The spread is presented by figure B.10. This makes the LIBOR rate suitable for cost of short term financing. We assume that the cost of leverage, in this case the total return of the bond, is yearly imposed on the return of the portfolio. The return of the asset is subtracted from the return of a leveraged RP portfolio according the the level of leverage used. Table 5.4 shows the characteristics of a levered RP portfolio with different levels of leverage.

TABLE 5.4: Portfolio 1 Leveraged Portfolio

	No lev.%	10%	25%	50%	100%
Excess annual Return (%)	1.47	1.81	2.31	3.16	4.84
SD (%)	4.50	4.95	5.62	6.75	9.00
SR(%)	32.66	36.51	41.13	46.77	53.82
MD Full (%)	-50.57	-53.82	-58.29	-64.73	-74.58
MD 5-year (%)	-34.88	-37.59	-41.45	-47.23	-57.34
MD 1-year (%)	-19.42	-21.12	-23.61	-27.56	-34.81
Cost leverage - 5-year (%)	0.00	0.43	0.98	1.94	3.93
Cost leverage - LIBOR (%)	0.00	0.32	0.71	1.38	2.78
Net Excess Return 5-year(%)	1.47	1.38	1.33	1.22	0.93
Net Excess Return LIBOR(%)	1.47	1.49	1.60	1.78	2.06

We clearly see that the source of leverage is important for the outcome. The use of a 5-year bond as a proxy for leverage cost leads to lower return as the level of level of leverage increases. In other words, the marginal cost of leverage is higher than the marginal benefit of the use of leverage. When the real costs of leverage equal a 5-year European bond, the use of leverage is not beneficial. The opposite holds when the 3-month LIBOR rate is used as proxy for leverage costs. The net excess return increases as the level of leverage increases. Furthermore, we see that the level of leverage positively affects the SR. The leverage causes the return to disproportionally rise relative to the rise in volatility. Furthermore, the MDs increase as leverage increase. This makes sense since more capital invested in the similar proportions means that the portfolio is more sensitive to (idiosyncratic) shocks. The MDs levels increase a fair amount. To illustrate this, the 100% leveraged portfolio has an excess after leverage cost return of 2.06% in the optimistic case. This is lower than benchmark strategies such as the 60/40 and 1/N. Their MD is also lower. So, even though the 100% portfolio has the highest SR, it cannot beat its peers in terms of real excess returns and downside risk.

5.1.2 Global portfolios - Portfolio 2

Portfolio Statistics

The assets that are included in portfolio 2 can be found in table 4.7, the asset weights can be found in both B.2 and the asset weights over time can be found in figure B.6. Similar to the previous section, we start the analysis by observing the general portfolio characteristics presented in table 5.5. We see that the 60/40 portfolio has the highest total return, followed by the RP portfolio, the 1/N portfolio, and finally the GMV portfolio. As the allocation to high-beta assets increases, the SRs decrease with this combination of assets. The GMV portfolio produces the highest volatility-return trade-off, this is also reflected by the M^2, where the portfolio is treated as if it would have the same volatility as the RP strategy. The GMV stands out with the highest value, followed by the RP strategy, and finally the 1/N portfolio and the 60/40. In terms of volatility-return trade-off, the GMV performs the best, however it has the lowest total return.

TABLE 5.5: Portfolio 2 Annualized Statistics

	60/40	**1/N**	**GMV**	**RP**	**Lev. RP**
Excess Return (%)	3.80	3.56	2.84	3.61	4.16
SD (%)	11.46	9.89	4.07	5.63	6.19
SR (%)	33.16	35.98	69.70	64.05	67.12
M^2	1.87	2.03	3.92	3.61	3.78
MD Full (%)	-66.88	-71.54	-62.06	-67.76	-71.12
MD 5 year (%)	-58.82	-53.94	-37.43	-49.32	-52.60
MD 1 year (%)	-34.90	-34.48	-17.18	-24.39	-26.45
GINI weights	0.300	1.000	0.209	0.629	0.629
GINI RC	0.001	0.377	0.221	1.000	1.000

If an investor would lever up a portfolio to achieve a higher return, the GMV portfolio would be the best option in this setting. It has a higher reward for volatility than its peers. Even a 50% levered RP strategy cannot achieve a higher SR than the GMV portfolio, so levering up other portfolio's makes little sense given the information up to now. Figure 5.3 shows how 1000 euro invested according to each strategy develops over time. We see that the leveraged RP portfolio clearly outperforms the other strategies. This is due to 50% extra capital invested, no leverage costs are imposed, and the RP strategy relatively performs well. This is, however, not the optimal solution. If the GMV portfolio would have been levered, the total return would be higher. When considering volatility-return characteristics with these combination of assets, the GMV has performed better than its peers.

FIGURE 5.3: Wealth plot Portfolio 2

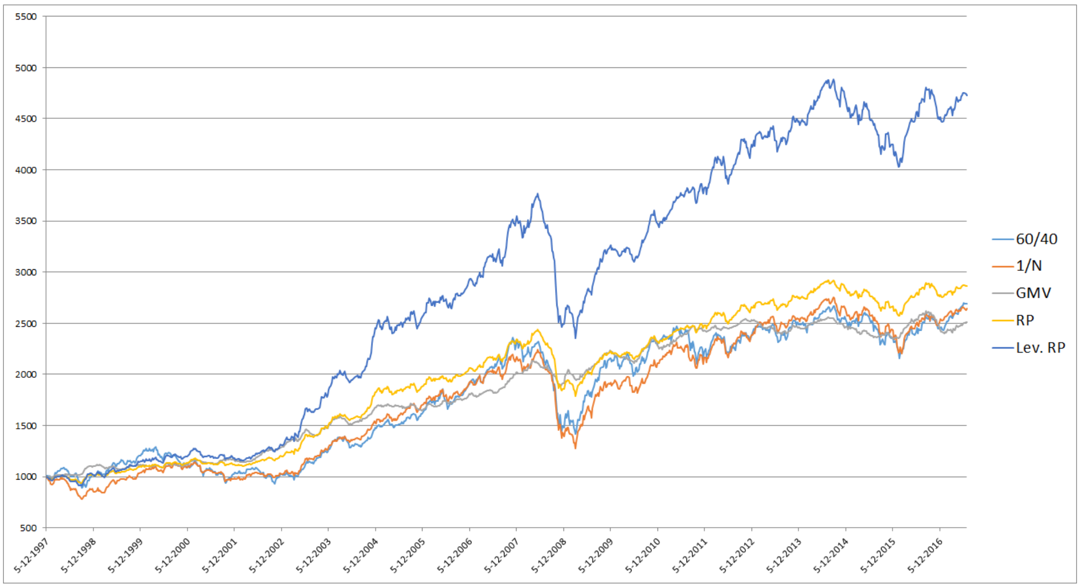


Table 5.5 shows the MDs and Gini coefficients of all strategies. Over the whole sample, we see that the 1/N strategy has lost the most value (71.54%). The 60/40 portfolio performed the worst in the 5-year and 1-year rolling sample. Similar to the volatility-return trade-off discussed in the previous paragraphs, a higher allocation to high-beta assets results in worse characteristics. In the discussion op portfolio 1's results, we partly explained the high MDs by the concentrations of capital and risk. The 60/40 strategy has a Gini coefficient of 0.300. The high concentrations leads to higher idiosyncratic asset class risk because of the lower diversification. This means that adverse shocks within or across asset classes can lead to high losses. The 1/N portfolio has a Gini coefficient of 1.000 since capital is divided equally over all asset classes, but the MD over the whole period is the highest of all. The 60/40 strategy has similar drawdown statistics as the 1/N strategy, most likely because of the high capital investment in high-beta assets in combination with a high concentration of assets. The RP portfolio concentrates its assets seemingly well with a Gini coefficient of 0.629. This is partly due to the strategy that RP always invest a positive weight in the chosen asset classes. The MD statistics are the second-lowest of all strategies, most likely because the strategy equalizes risk across asset classes, thereby minimizing the potential downfall of value. The levered portfolio does equally well in terms of the Gini coefficients, but has higher MD statistics since more capital is invested, so there is more sensitivity to individual and overall asset class risk when no other assets are included. The GMV strategy has lower drawdowns over all MD measures than the RP strategy and the other strategies. This leads to believe that the included low-beta asset classes outperform the high-beta asset classes. GMV allocates most capital to these asset classes, therefore it has presumably performed the best. In this asset mix, the results show that the RP strategy does not always outperform with better Gini statistics, stressing that the choice of assets in the portfolio is important.

The Gini coefficients of the RCs is the last metric discussed in this section. By definition, the (levered) RP strategies have a RC Gini coefficient of 1 since the strategy equalizes RCs. The 60/40 portfolio performs the worst of all strategies with a RC Gini coefficient of 0.001, meaning that the source of risk is not diversified and the source is equity. Although the GMV minimizes risk, its RC Gini coefficient is 0.221 since it invests most capital in bonds, which is the source of the risk. The 1/N portfolio has a RC Gini coefficient of 0.377 which is relatively diversified compared to others. This is likely due to the equal allocation to each asset class, meaning that is has exposure to every asset class. Not every asset class has the same risk characteristics, the 1/N portfolio simply takes the average of every asset class' RC. The higher Gini coefficient for RC doesn't necessarily mean better results, since the GMV portfolio outperformed the other portfolios.

Return Origin

Where do the returns come from and why do some outperform others? To find an answer to that question, we will have to look how the portfolio is correlated with the asset classes' return and how asset classes performed in the sample period. Table 5.6 presents the correlations of the four strategies with their asset class components. In general, a high correlation tells us where return comes from and how sensitive a change in the return of an asset classes is to the return of a strategy. Preferably, a portfolio's return should not have extreme high correlations with asset classes to protect itself from adverse shocks in asset class returns.

For the 60/40 portfolio, we clearly see that the return is highly, nearly perfectly (98.94%), correlated with equity returns. This means that the returns are mostly explained by equity returns. Bond returns have a much lower correlation, so the 60/40 strategy relies more equity than bonds for returns. This strategy included bonds to diversify in order to protect against shocks. We see that most of the riskiness is incorporated by the equity, suggesting that this is poor diversification.

TABLE 5.6: Correlation Portfolio 2 with Asset Classes

	Equity(%)	**Bonds(%)**	**Comm.(%)**	**RE(%)**	**PE(%)**	**Infl.(%)**
60/40	98.94	18.25				
1/N	70.81	13.87	59.44	81.86	49.31	-0.47
GMV	24.50	95.69	19.79	24.20		4.67
RP	59.42	44.84	44.13	67.70	47.47	6.28

The 1/N strategy's return has less extreme correlations with equity, but they remain on the high side (70.81%). Bonds have a lower correlation (13.87% on average) than the 60/40 portfolio. The lower correlation is explained by the relative lower weights to the asset classes, 1/N allocates 16.67% while the 60/40 strategy allocates

40%. Furthermore, real estate and private equity both have high correlations (81.86 and 49.31%) with the 1/N strategy's return. These are both high-beta asset classes. Commodities follow with a correlation of 59.44% and finally inflation-linked bonds with a correlation lower than zero (-0.47%). By dividing capital equally across asset classes, we see that the correlations are higher for the high-beta classes, and lower for the low-beta classes. Dividing capital equally over classes might not be the ideal strategy when an investor wants to diversify from shocks regardless of the asset class.

The GMV strategy strongly overweights low-beta classes with respect to the high-beta classes. We see that equity, commodities and real estate all have much lower correlations with the returns of the GMV portfolio relative to previously discussed strategies. The correlations of private equity with GMV returns is not discussed since there is no capital allocation to this asset class. The weights of the asset classes can be found in table 3.1. The inflation-linked bonds have a small correlation with the strategy's returns. The capital allocation is very small, so no significance is given to this correlations. Bonds have a high capital allocation, which is why the bond asset class is highly correlated with the strategy's return. The strategy is minimizes the variance, however, it concentrates risk and dependency in one asset class, making it sensitive to idiosyncratic shocks.

Lastly, the RP strategy. This results in seemingly equal correlations for all strategies, which are very similar to the results found in asset mix 1. Of course, when equalizing RCs using volatilities, it results in more equal correlations since correlations are a product of volatilities, but these correlations show that there are no extreme dependencies on certain asset classes. It is interesting to see that the high-beta asset classes still have a higher correlation with returns than the low-beta asset classes. Most likely, this is due to the higher volatility of these assets and positive correlations between asset classes. The main takeaway is that the RP strategy has more balanced correlations with asset classes. Therefore, the portfolio is more diversified than its benchmarks in this portfolio.

We have identified how the asset classes were correlated with the strategies. This, however, does not tell us why the strategy has the return it has. We use figure 5.4, which presents the position of asset classes and strategies in a mean-variance framework; and table 5.7 which shows the statistics of the asset classes in portfolio 1. According to the CAPM theory, markets are efficient and all asset classes should have the same risk-return trade-off. The mean-variance diagram shows that this does not hold. Inflation-linked bonds, equity and private equity have performed better than bonds, commodities and real estate.

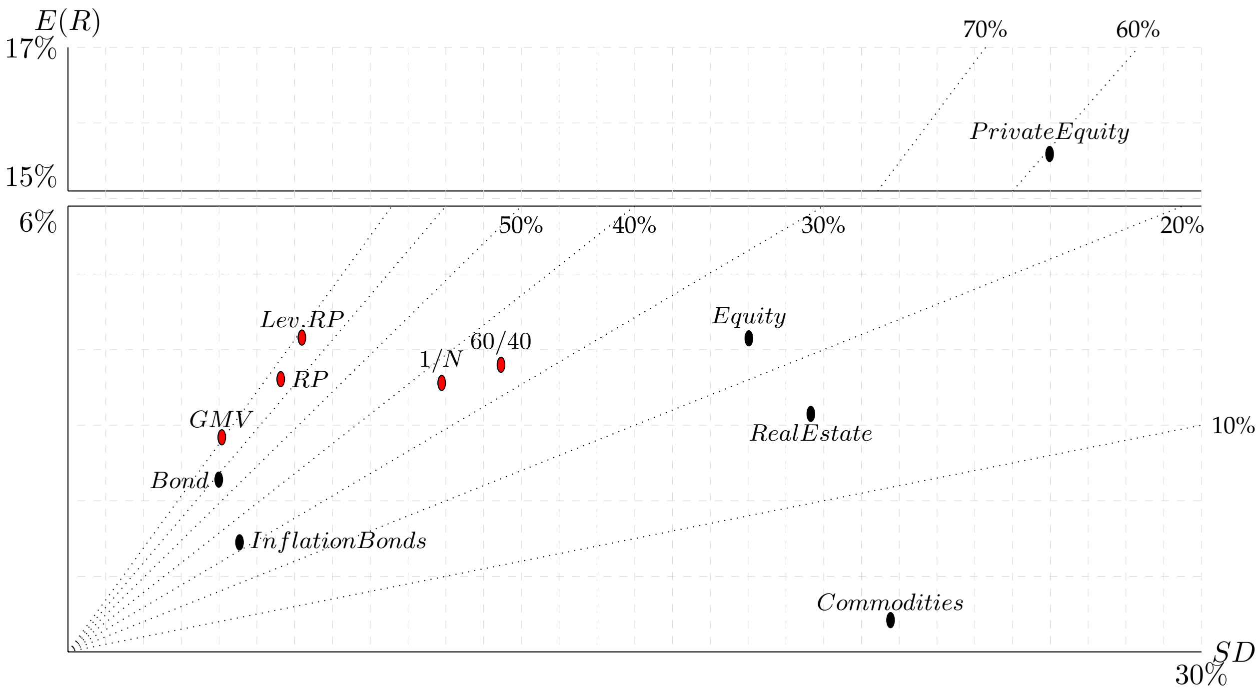
TABLE 5.7: Asset Class Statistics Portfolio 2

	Equity(%)	Bonds(%)	Comm.(%)	RE(%)	PE(%)	Infl.(%)
Excess Annual Return	4.15	2.28	0.42	3.15	15.59	1.45
Volatility	18.02	3.99	21.77	19.66	26.99	4.54
SR	23.03	57.27	1.91	16.02	57.77	31.95

Bonds and private equity outperform in terms of SR. Throughout the analysis of asset mix 2, we saw strategies which invest much capital in low-beta assets tend to outperform peer strategies. Figure 5.4 shows the position of asset classes and strategies in a mean-variance framework. That the GMV outperformed all the other strategies is the result of 92.45% allocation on average in the bond asset class. The reason it performed better than a 100% allocation in the bond asset class is because it is diversified with equity, commodities and real estate. Although the capital allocation is small, the correlations[1] between these classes is close to zero, meaning that it provides diversification benefits. RP realized a high SR of 64.05% due to high allocations to bonds and inflation linked bonds. It probably performed worse than the GMV portfolio since it has a 8.31% allocation to commodities and exposure to all of the other relatively worse performing asset classes. We see that the 1/N and 60/40 strategies roughly perform equal. The 60/40 thanks its return from the 40% bonds and small diversification benefit between equity and bonds. By having exposure to both high-yielding and low-yielding asset classes, the 1/N strategy has an average SR. All in all, we observe that the low-beta strategies outperformed high-beta strategies due to the high allocation to the high-yielding bond asset class. All strategies didn't take much advantage of the high-yielding private equity class since no strategy allocated much capital to it. Note that this analysis only holds for this asset mix, this is not a reflection of all asset classes in the world, since those have other (cor)relations with each other.

[1]Table B.1 shows the correlations between all assets.

FIGURE 5.4: Mean-Variance Diagram Portfolio 2



The excess return-volatility characteristics of the strategies and asset classes are plotted in this mean-variance graph. The dotted lines depict the different levels of SR.

Leverage

Until now, we have assumed that the use of leverage has no cost. Levering up a portfolio only makes sense if the used strategy has the highest award for volatility. However, in this mix of assets, the GMV strategy has the best reward for volatility. Therefore, it makes no sense to lever a RP portfolio since extra costs and concerns will be imposed. The general effects of levering up the portfolio will be higher MDs, more risk, but not necessarily higher returns since this is dependent on the costs.

5.1.3 European portfolios - Portfolio 3

Portfolio Statistics

The assets that are included in portfolio 3 can be found in table 4.7, the asset weights can be found in both B.2 and the asset weights over time can be found in figure B.7. Similar to the previous section, we start the analysis by observing the general portfolio characteristics presented in table 5.8. We see that the 1/N strategy resulted in the highest total return, followed by the 60/40 strategy, the RP strategy, and finally the GMV strategy. As the allocation to high-beta assets increases, the SRs of strategies decrease. The RP portfolio produced the highest SR over the whole sample. This is also reflected by the M^2, where strategies are treated as if it would have the same volatility as the RP strategy. We see that the RP strategy stands out with an at least 0.5% outperformance of the second-best strategy. In terms of reward-for-volatility, the RP strategy offers the best statistics relative to its peers.

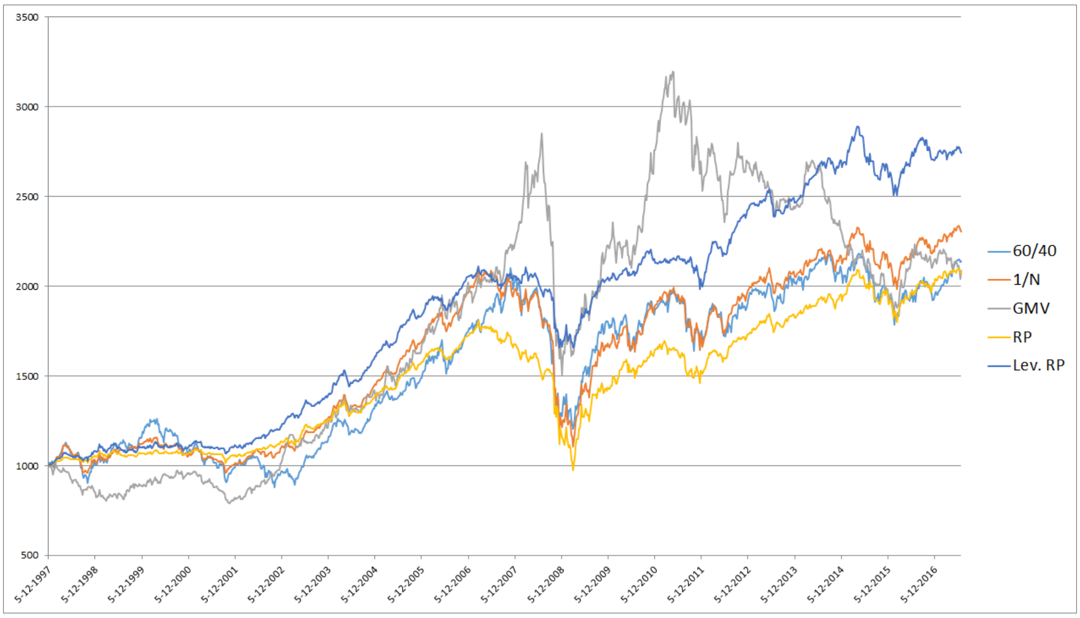
TABLE 5.8: Portfolio 3 Annualized Statistics

	60/40	**1/N**	**GMV**	**RP**	**Lev. RP**
Excess Return (%)	2.73	2.87	0.83	1.64	3.41
SD (%)	12.34	9.98	3.28	3.94	5.91
SR (%)	22.13	28.81	25.26	41.57	57.69
M^2 (%)	0.87	1.13	0.99	1.64	2.27
MD Full	-59.64	-58.99	-75.24	-50.86	-65.20
MD 5 year (%)	-57.60	-50.27	-60.57	-34.50	-46.89
MD 1 year (%)	-33.72	-36.25	-34.29	-13.87	-20.01
GINI weights	0.300	1.000	0.217	0.579	0.580
GINI RC	0.008	0.531	0.219	1.000	1.000

Table 5.8 shows us the return characteristics of a leveraged RP strategy with a leverage ratio of 1.5. This ratio is chosen for illustration purposes and will be varied in a later section. We see that with 'only' 50% extra capital invested, we can obtain a superior return with lower volatility than the 60/40 and 1/N strategy. Figure 5.5 shows how 1000 euro invested according to each strategy develops over time. It is interesting to see that the SR of a leveraged RP increases as leverage increases. This has a simple reason: the full benefits of return are absorbed while the volatility of the strategy's portfolio didn't grow at the same rate. We see that the M^2 measure is higher than the RP strategy since it has a higher SR, meaning it outperforms unlevered RP as well. The consequences of leverage have not been taken into account, meaning that the return characteristics or leveraged RP have not been taken into account yet. Overall in these portfolio of assets, the RP and levered RP have performed better than their peers.

Table 5.8 shows the MDs and Gini coefficients of all strategies. We see that the GMV strategy has lost the most value (75.24%) in the whole sample period and the 5-year rolling period. In the short run (1-year), the MD is lower than the 1/N, but comparable. For the GMV strategy, this is probably due to the concentration of asset weights, as shown by its Gini coefficient of 0.217. The high concentrations leads to higher idiosyncratic asset class risk because of the lower diversification. This means that adverse shocks within or across asset classes can lead to high losses. A well diversified portfolio in capital terms doesn't mean it doesn't face this risk. The 1/N portfolio has a Gini coefficient of 1 since capital is divided equally over all asset classes, but the MD over the whole period is still quite high: -58.99%. The 60/40 strategy has similar drawdown statistics as the 1/N strategy, most likely because of the 60% investment in high-beta assets in combination with a high concentration of assets (Gini coefficient of 0.300). The RP portfolio concentrates its assets seemingly well with a Gini coefficient of 0.579. This is partly due to the strategy that RP always invests something in the chosen asset classes. The MD statistics are the

FIGURE 5.5: Wealth plot Portfolio 3



lowest of all strategies, most likely because the strategy equalizes risk across asset classes, thereby minimizing the potential downfall of value. The levered portfolio does equally well in terms of the Gini coefficients, but has higher MD statistics since more capital is invested, so there is more sensitivity to individual and overall asset class risk when no other assets are included. The MD statistics are quite high for the entire sample, but decrease rapidly when the time period is shortened. In this portfolio, the RP portfolios perform promising in terms of asset concentration and drawdown of capital in at leas the short run, dependent on the level of leverage.

The Gini coefficients of the RCs is the last metric discussed in this paragraph. The (levered) RP strategies have a RC Gini coefficient of 1 since the strategy equalizes RCs. The 60/40 portfolio performs the worst of all strategies with a RC Gini coefficient of 0.008, meaning that the source of risk is not diversified and most of it comes from equity. Although the GMV minimizes risk, its RC Gini coefficient is 0.219 since it invests most capital in bonds, which is the source of the risk. The 1/N portfolio has a RC Gini coefficient of 0.531 which is pretty diversified when compared to the other strategies. This is likely due to the equal allocation to each asset class, meaning that is has exposure to every asset class. Not every asset class has the same risk characteristics, the 1/N portfolio simply takes the average of every asset class' RC. This is, however, not reflected in the characteristics as it has rather high MDs and performs worse than the RP strategy.

Return Origin

The previous findings seem to suggest that the RP strategy has performed better than the 60/40, 1/N and GMV strategy, but it does not explain the origin of RP's outperformance. Therefore, the question remains: where do the returns come from and why do some outperform others? To find an answer to that question, we will have to look how the portfolio is correlated with the asset classes' return and how asset classes performed in the sample period. Table 5.9 presents the correlations of the four strategies with their asset class components. In general, a high correlation tells us where return comes from and how sensitive a change in the return of an asset classes is to the return of a strategy. Preferably, a portfolio's return should not have extreme high correlations with asset classes to protect itself from adverse shocks in asset class returns.

For the 60/40 portfolio, we clearly see that the return is highly, nearly perfectly (99.36%), correlated with equity returns. This means that the returns are mostly explained by equity returns. Bond returns have a much lower correlation, so the 60/40 strategy relies more equity than bonds for returns. This strategy included bonds to diversify in order to protect against shocks. We see that most of the riskiness is caused by the equity, suggesting that this is poor diversification.

TABLE 5.9: Correlation Portfolio 3 with Asset Classes

	Equity(%)	**Bonds(%)**	**Comm.(%)**	**RE(%)**	**PE(%)**	**Infl.(%)**
60/40	99.36	16.16				
1/N	72.98	8.76	55.28	82.09	74.68	15.30
GMV	23.17	91.82	23.47	28.15		
RP	58.72	42.38	50.43	67.13	58.15	39.05

The 1/N strategy's return has less extreme correlations with equity, but they remain on the high side (72.98%). Bonds have a lower correlation (8.76% on average) than the 60/40 portfolio. The lower correlation is explained by the relative lower weights to the asset classes, 1/N allocates 16.67% while the 60/40 strategy allocates 40%. Furthermore, real estate and private equity both have high correlations (82.09 and 74.68%) with the 1/N strategy. These are both high-beta asset classes. This is followed by commodities with a correlation of 55.28%. The 1/N strategy has a correlation of 15.30% with inflation linked bonds due to the 16.67% allocation to this class. It is interesting to see that this number is higher than that of bonds, apparently inflation-linked bonds' returns had a higher or similar impact/effect on the return of the 1/N strategy. By dividing capital equally across asset classes, we see that the correlations are higher for the high-beta classes, and lower for the low-beta classes.

Dividing capital equally over classes might not be the ideal strategy when an investor wants to diversify himself from shocks regardless of the asset class.

The GMV strategy strongly overweights low-beta classes with respect to the high-beta classes. We see that equity, commodities and real estate all have much lower correlations with the returns of the GMV portfolio relative to previously discussed strategies. The correlations of inflation linked bonds and private equity with GMV returns are not discussed since the weights are zero or close to zero. The weights of the asset classes can be found in table 3.1. Bonds have a high capital allocation, which is why the bond asset class is highly correlated with the strategy's return. The strategy is minimizes the variance, but it concentrates risk and dependency in one asset class, making it sensitive to idiosyncratic shocks.

Lastly, the RP strategy. It is interesting to see that the high-beta asset classes still have a higher correlation with returns than the low-beta asset classes. Most likely, it is due to the higher volatility of these assets and positive correlations between asset classes. The main takeaway is that the RP strategy has more balanced correlations with asset classes, therefore the portfolio is more diversified than its benchmarks in this portfolio.

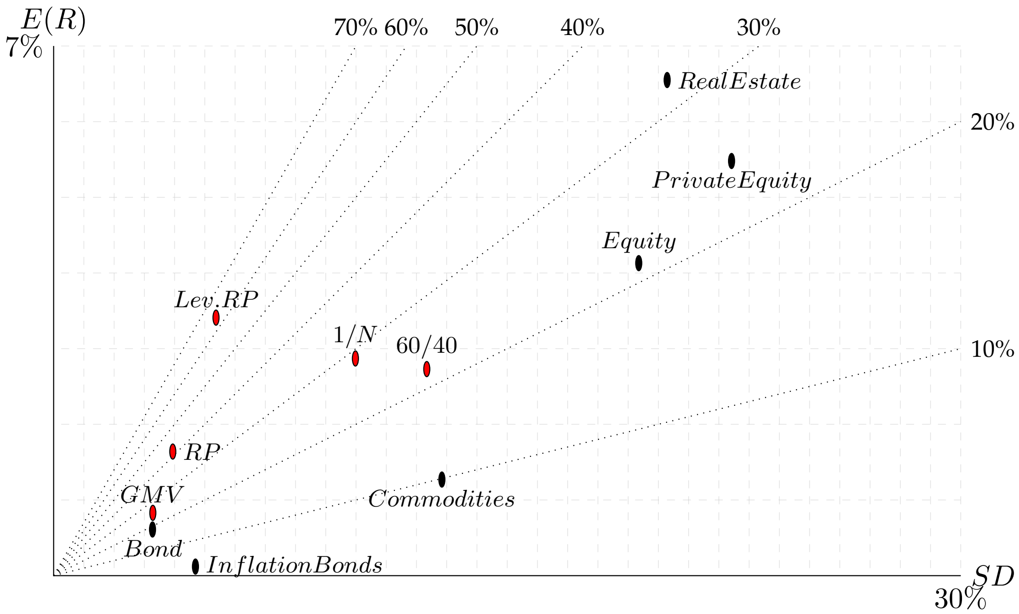
We have identified how the asset classes were correlated with the strategies. This, however, does not tell us why the strategy has the return it has. We use figure 5.6 which shows the position of asset classes in a mean-variance framework and table 5.10 which shows the statistics of the asset classes in portfolio 3. According to the CAPM theory, markets are efficient and all asset classes should have the same risk-return trade-off. The mean-variance diagram shows that this does not hold. Each asset class has a different volatility-return trade-off.

TABLE 5.10: Asset Class Statistics Portfolio 3

	Equity(%)	**Bonds(%)**	**Comm.(%)**	**RE(%)**	**PE(%)**	**Infl.(%)**
Excess Annual Return	4.13	0.61	1.24	6.55	5.48	0.12
Volatility	19.35	3.27	12.84	20.29	22.42	4.69
SR	21.33	18.77	9.92	32.29	24.45	2.47

In this mix of assets, the high-beta assets outperform the low-beta assets, although the bond asset class perform quite good. This leads to believe that strategies that invest more capital in these classes are prone outperform those who don't. The 60/40 strategy has a better SR than the bond and equity asset classes due to diversification. We see that the volatility of the strategy is reduced by the 40% allocation to bonds. The 1/N strategy outperforms the 60/40 strategy since it has a larger exposure to higher yielding asset classes. Again, the SR is much higher than the weighted average of the six asset classes combined[2]. The correlation between assets are not equal to one, therefore there exists a diversification benefit which propels the SR. The GMV strategy has performed somewhat similar to the 60/40 and 1/N strategy despite the large dollar allocation to the lower yielding bond and commodity asset class. The correlations between the commodities and bonds are negative (as can be seen in table B.1, which likely caused diversification benefits, leading to a higher SR. The RP portfolio outperformed in terms of SR when compared to its peers. Although it has a large capital allocation to bonds and commodities, the higher SR is likely caused by a positive exposure to every asset class, meaning that the benefits from high-yielding equity, private equity and real estate are reaped. Note that this analysis only holds for this portfolio, this is not a reflection of all asset classes in the world, since those have other (cor)relations with each other.

FIGURE 5.6: Mean-Variance Diagram Portfolio 3



The excess return-volatility characteristics of the strategies and asset classes are plotted in this mean-variance graph. The dotted lines depict the different levels of SR.

Leverage

Table 5.11 shows the characteristics of a levered RP portfolio with different levels of leverage. Similar to the results using asset mix 1, We observe that the source of leverage is important for the outcome. The use of a 5-year bond as a proxy for leverage cost leads to lower return as the level of level of leverage increases. In other words, the marginal cost of leverage higher than the marginal benefit of the use of leverage. When the real costs of leverage equal a 5-year European bond, the use of leverage is not beneficial. The opposite holds when the 3-month LIBOR rate is used as proxy for leverage costs. The net excess return increases as the level of leverage

[2] $(21.33 + 18.77 + 9.92 + 32.29 + 24.45 + 2.47)/6 = 18.21\%$ which is unequal to the SR 28.81% of the 1/N strategy

increases. We further see that the level of leverage positively affects the SR. The leverage causes the return to disproportionally rise relative to the rise in volatility. Furthermore, the MDs increase as leverage increase. This makes sense since more capital invested in the similar proportions means that the portfolio is more sensitive to (idiosyncratic) shocks. The MDs levels increase quite much. To illustrate, the 100% leveraged portfolio has an excess after leverage cost return of 2.52% in the optimistic case. This is lower than the 60/40 strategy. Their MD is also lower. So, even though the 100% portfolio has the highest SR, it cannot beat its peers in terms of real excess returns and downside risk.

TABLE 5.11: Portfolio 3 Leveraged Portfolio

	No lev.%	**10%**	**25%**	**50%**	**100%**
Excess annual Return (%)	1.64	1.99	2.52	3.41	5.18
SD (%)	3.94	4.33	4.92	5.91	7.88
SR(%)	41.57	45.97	51.24	57.69	65.75
MD Full (%)	-50.85	-54.16	-58.68	-65.20	-75.19
MD 5-year (%)	-34.50	-37.20	-41.03	-46.89	-56.88
MD 1-year (%)	-13.87	-15.14	-17.00	-20.01	-25.68
Cost leverage - 5-year (%)	0.00	0.39	0.94	1.87	3.79
Cost leverage - LIBOR (%)	0.00	0.28	0.66	1.31	2.66
Net Excess Return 5-year(%)	1.64	1.60	1.58	1.54	1.39
Net Excess Return LIBOR(%)	1.64	1.71	1.86	2.10	2.52

5.1.4 European portfolios - Portfolio 4

Portfolio Statistics

The assets that are included in portfolio 4 can be found in table 4.7, the asset weights can be found in both B.2 and the asset weights over time can be found in figure B.8. We start with observing the general portfolio characteristics presented in table 5.12. We see that the equal capital allocation strategy, 1/N, has the highest return. It has a lower volatility than the 60/40 strategy, resulting in a higher SR. The GMV strategy performs poorly by generating a SR of 12.63% and having a low (0.45%) total return. The RP strategy performs the best of all strategies. It has a total return of 1.97% excess of the risk-free rate while having a volatility of 4.01%, resulting in a SR of 49.17%. The result is also reflected by the M^2 measure introduced in section 2.1. The M^2 shows the return of a benchmark strategy as if it would have the same volatility as the RP strategy but with its own SR. Now it is easier to see what the higher SR of the RP strategy means and how it compares: the benchmark strategies all have lower returns and the RP strategy seems superior based on the previously discussed statistics.

TABLE 5.12: Portfolio 4 Annualized Statistics

	60/40	1/N	GMV	RP	Lev. RP
Excess Return (%)	2.73	3.18	0.45	1.97	3.91
SD (%)	11.72	9.76	3.48	4.01	6.02
SR (%)	23.33	32.55	12.63	49.17	64.98
M^2	0.94	1.31	0.52	1.97	2.61
MD Full (%)	-60.85	-68.59	-40.37	-54.70	-69.21
MD 5 year (%)	-54.51	-52.09	-21.46	-31.53	-43.21
MD 1 year (%)	-30.95	-33.15	-10.36	-15.84	-22.72
GINI weights	0.300	1.000	0.206	0.580	0.580
GINI RC	0.009	0.598	0.177	1.000	1.000

Table 5.1 shows us the return characteristics of a leveraged RP strategy with a leverage ratio of 1.5. We see that with 50% extra capital invested, we can obtain a superior return with lower volatility than the 60/40 and 1/N strategy. Figure 5.7 shows how 1000 euro invested according to each strategy develops over time. We see that the M^2 measure is higher than the RP strategy since it has a higher SR than other strategies, meaning it outperforms unlevered RP as well. The consequences of leverage have not been taken into account, meaning that the return characteristics or leveraged RP have not been punished yet. Overall in these portfolio of assets, the RP and levered RP have performed better than their peers.

FIGURE 5.7: Wealth plot Portfolio 4

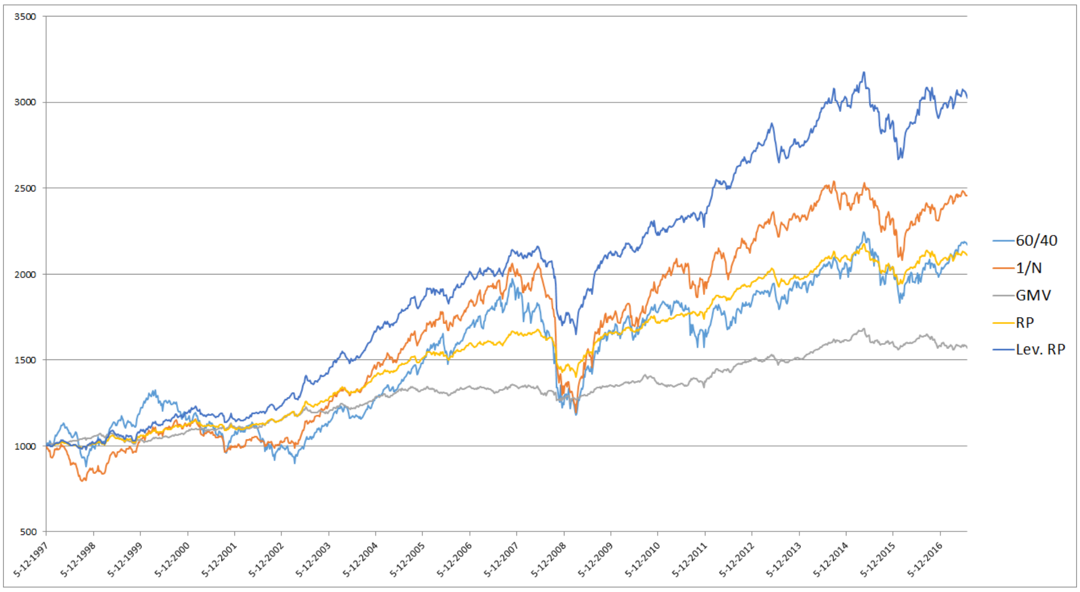


Table 5.12 shows the MDs and Gini coefficients of all strategies. The MD statistics is the cumulative loss of a specified period. We observe that strategies which invest more in high-beta assets tend to have higher MD statistics. The 1/N strategy has the highest whole sample and 1-year MD statistics. While the 60/40 strategy has a lower full sample MD statistic, its 5-year figure is the lowest and overall it is

quite similar to the 1/N strategy. The Gini coefficients of these strategies seem to be counter-intuitive: when capital is more equally invested, the potential downfall of capital is larger. Apparently, diversification doesn't pay off in this situation. As was the case with the result of the previous asset mixes, a well diversified portfolio in capital terms does not mean it doesn't face this risk. Therefore, it seems that equity and bonds together were more stable, or covered for each other's idiosyncratic shocks, than the six asset classes combined. This again stresses the point that asset selection for portfolios is very important. Also, as pointed out in the previous sections, the correlations between asset classes rose in the time of crisis[3]. Therefore a capital-diversified portfolio might still face severe downside risk. The low-beta strategies have a downside risk which is much lower than those of the high-beta strategies. Especially the GMV strategy has very low drawdowns, even though the Gini coefficient of the RP strategy is much higher (0.580 vs 0.206). Equally weighting the marginal contribution of risk of each asset class didn't result in the lowest drawdowns. The cause of this is the stable performance of bonds. Usually safer assets such as bonds are less volatile, which caused the bond-dominated GMV strategy to have stable returns relative to its peers. The bond asset class has performed very stable, this combined with the high capital-allocation of the GMV and RP strategies to bonds it results in these lower drawdowns. The 50% levered RP portfolio has a high MD over the whole period, but it drops rapidly as the period shortens. The MD statistics are worse since more capital is invested, so there is more sensitivity to individual and overall asset class risk when no other assets are included. In this portfolio, the RP portfolios perform promising in terms of asset concentration and drawdown of capital, in at least the short run, dependent on the level of leverage.

The Gini coefficients of the RCs is the last metric discussed in this paragraph. The (levered) RP strategies have a RC Gini coefficient of 1 since the strategy equalizes RCs. The 60/40 portfolio performs underperforms all strategies with a RC Gini coefficient of 0.008, meaning that the source of risk is not diversified and most of it comes from equity. Although the GMV minimizes risk, its RC Gini coefficient is 0.177 since it invests most capital in bonds, which is the source of the risk. The 1/N portfolio has a RC Gini coefficient of 0.598 which is rather diversified. This is likely due to the equal allocation to each asset class, meaning that is has exposure to every asset class. Not every asset class has the same risk characteristics, the 1/N portfolio simply takes the average of every asset class' RC. This is, however, has resulted in high drawdown statistics which are not desirable.

Return Origin

Where do the returns of strategies come from and why do some outperform others? To find an answer to that question, we will look how the portfolio is correlated with the asset classes' return and how asset classes performed in the sample

[3]See (Koestrich, 2015) and B.2

period. Table 5.13 presents the correlations of the four strategies with their asset class components. In general, a high correlation tells us where return comes from and how sensitive a change in the return of an asset classes is to the return of a strategy. Preferably, a portfolio's return should not have extreme high correlations with asset classes to protect itself from adverse shocks in asset class returns.

For the 60/40 portfolio, we clearly see that the return is highly, nearly perfectly (99.23%), correlated with equity returns. This means that the returns are mostly explained by equity returns. Bond returns have a much lower correlation of 16.73%, so the 60/40 strategy relies more equity than bonds for returns. This strategy included bonds to diversify in order to protect against shocks. We see that most of the riskiness is incorporated by the equity, suggesting that this is poor diversification.

TABLE 5.13: Correlation Portfolio 4 with Asset Classes

	Equity(%)	**Bonds(%)**	**Comm.(%)**	**RE(%)**	**PE(%)**	**Infl.(%)**
60/40	99.23	16.73				
1/N	69.35	1.72	58.53	81.26	49.54	-1.30
GMV	23.38	92.56	16.80	24.20		
RP	54.22	45.03	47.79	61.67	35.39	39.45

The 1/N strategy's return has less extreme correlations with equity, but they remain on the high side (around 69.35%). Bonds have a lower correlation (1.72%) than the 60/40 portfolio. The lower correlation is explained by the relative lower weights to the asset classes, 1/N allocates 16.67% while the 60/40 strategy allocates 40%. Furthermore, real estate returns are very highly correlated with the return of the 1/N strategy. Commodities and private equity both tend to move together with the 1/N strategy (58.53% and 49.54% respectively). These are both high-beta asset classes. Finally, inflation linked bonds have a negative correlation (-1.30%). By dividing capital equally across asset classes, we see that the correlations are higher for the high-beta classes, and lower for the low-beta classes. Dividing capital equally over classes might not be the ideal strategy when an investor wants to diversify himself from shocks regardless of the asset class.

The GMV strategy strongly overweights low-beta classes with respect to the high-beta classes. We see that equity, commodities and real estate all have much lower correlations with the returns of the GMV portfolio relative to previously discussed strategies. The correlations of inflation linked bonds and private equity with GMV returns are not discussed since the weights are zero or close to zero. The weights of the asset classes can be found in table 3.1. Bonds have a high capital allocation, which is why the bond asset class is highly correlated with the strategy's return. The strategy is minimizes the variance, yet it concentrates risk and dependency in one asset class, making it sensitive to idiosyncratic shocks.

Lastly, the RP strategy. Asset weights are determined by the volatility of an asset class and the main objective is to equalize RCs among them. This results in seemingly equal correlations for all strategies. Of course, when equalizing RCs using volatilities, this results in more equal correlations since correlations are a product of volatilities, but these correlations show that there are no extreme dependencies on certain asset classes. It is interesting to see that the high-beta asset classes still have a higher correlation with returns than the low-beta asset classes. Most likely, this is due to the higher volatility of these assets and positive correlations between asset classes. The main takeaway is that the RP strategy has more balanced correlations with asset classes, therefore the portfolio is more diversified than its benchmarks in this portfolio.

We've identified how the asset classes were correlated with the strategies. This, however, does not tell us why the strategy has the return it has. We use figure 5.8 which shows the position of asset classes in a mean-variance framework and table 5.14 which shows the statistics of the asset classes in portfolio 4. According to the CAPM theory, markets are efficient and all asset classes should have the same risk-return trade-off. The mean-variance diagram shows that this does not hold. Private equity has a very large SR followed by inflation linked bonds. Asset classes equity bonds and real estate have performed somewhat similar. Commodities the worst of all asset classes.

TABLE 5.14: Asset Class Statistics Portfolio 4

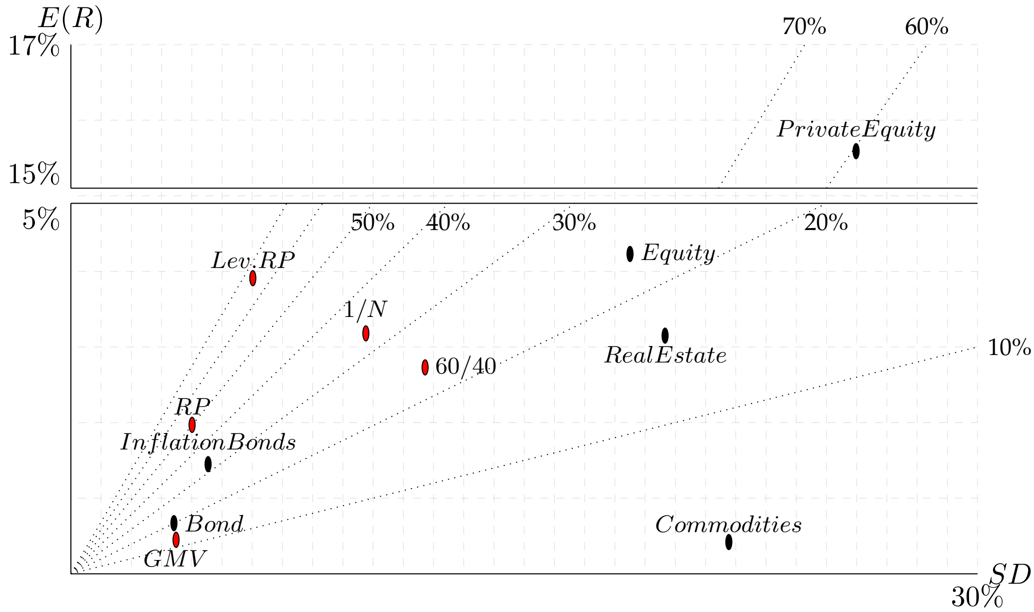
	Equity(%)	**Bonds(%)**	**Comm.(%)**	**RE(%)**	**PE(%)**	**Infl.(%)**
Excess Annual Return	4.23	0.67	0.42	3.15	15.59	1.45
Volatility	18.50	3.41	21.77	19.66	26.99	4.54
SR	22.88	19.69	1.91	16.02	57.77	31.95

The 1/N strategy outperformed the 60/40 strategy while lower-yielding classes such as commodities and real estate were included in this portfolio. The 1/N strategy reaped the returns of the higher-yielding private equity and real estate. Combined with diversification benefits[4], it could achieve a high SR. When looking at the Mean-variance diagram 5.8, we see that the 60/40 strategy lies between the equity class and bond class, thereby having a roughly similar SR. The actual SR is higher than either asset class because of imperfect correlations between the two assets. The GMV strategy performed relatively poorly due to a high allocation to bonds and an allocation to commodities. The RP strategy outperformed in terms of SR when compared to its peers. Although it has a large capital allocation to bonds and commodities,

[4]table B.1 shows that the correlation between assets is not equal to 1 over the whole sample.

the higher SR is likely caused by a large allocation to bonds and the high yielding inflation-linked bonds, while having an exposure to equity and private equity. Again, diversification plays a role since the correlations between assets is not equal to one. Note that this analysis only holds for this portfolio, this is not a reflection of all asset classes in the world, since those have other (cor)relations with each other.

FIGURE 5.8: Mean-Variance Diagram Portfolio 4



The excess return-volatility characteristics of the strategies and asset classes are plotted in this mean-variance graph. The dotted lines depict the different levels of SR.

Leverage

Table 5.15 shows the characteristics of a levered RP portfolio with different levels of leverage. We see that the source of leverage is important for the outcome. The use of a 5-year bond as a proxy for leverage cost increases the return as the level of level of leverage increases. In other words, the marginal cost of leverage is lower than the marginal benefit of the use of leverage. When the real costs of leverage equal a 5-year European bond, the use of leverage is beneficial. The same holds when the 3-month LIBOR rate is used as proxy for leverage costs. The net excess return increases as the level of leverage increases. We further see that the level of leverage positively affects the SR. The leverage causes the return to disproportionally rise relative to the rise in volatility. Furthermore, the MDs increase as leverage increase. This makes sense since more capital invested in the similar proportions means that the portfolio is more sensitive to (idiosyncratic) shocks. The MDs levels increase quite much. To illustrate, the 100% leveraged portfolio has an excess after leverage cost return of 3.21% in the optimistic case. This is the first level of leverage at which the leveraged RP strategy outperform all strategies. However, the MDs of peer strategies is much lower in the long run. It is interesting to see that the 50% levered RP portfolio has a lower 1-year MD than the 25% leveraged portfolio.

The increase in less-than-one-correlated assets caused them to cover-up losses in the short run. So, even though the lowest costing 100% leveraged strategy has the highest SR and highest return, the potential downside risk increases much. Since many pitfalls of leverage have not been discussed, this stresses that the use of leverage is not ideal.

TABLE 5.15: Portfolio 4 Leveraged Portfolio

	No lev.%	**10%**	**25%**	**50%**	**100%**
Excess annual Return (%)	1.97	2.36	2.94	3.91	5.85
SD (%)	4.01	4.42	5.02	6.02	8.03
SR(%)	49.17	53.48	58.66	64.98	72.89
MD Full (%)	-54.70	-58.09	-62.68	-69.21	-78.93
MD 5-year (%)	-31.53	-34.05	-37.65	-43.21	-52.83
MD 1-year (%)	-15.84	-17.26	-23.61	-19.36	-28.99
Cost leverage - 5-year (%)	0.00	0.38	0.92	1.85	3.78
Cost leverage - LIBOR (%)	0.00	0.27	0.64	1.29	2.64
Net Excess Return 5-year(%)	1.97	1.98	2.02	2.06	2.07
Net Excess Return LIBOR(%)	1.97	2.09	2.30	2.62	3.21

5.1.5 Aggregate results

The aggregate results of all strategies and asset mixes is discussed above, this section discusses a few observations on aggregate level. How does the RC affect the returns? If we regress the RC Gini coefficient with the SR using the data of all asset mixes, we see that higher value for the RC Gini coefficient results in a higher SR at a 1-percent significance. This can be found in Appendix B.9. The RC Gini coefficients are also regressed with the total returns and volatilities to examine how the positive relation between RC Gini coefficients and SR came to be. Only the regression between volatility and the RC Gini coefficients was significant; the SD and RC Gini coefficients have a negative relationship at a 5-percent significance level. It suggests that more equal RCs leads to a lower volatility rather than a higher return. This makes a case for the RP strategy, since it seems that a higher RC Gini coefficient lowers volatility and therefore increases the volatility-return trade-off.

The MD of each strategy with each combination of assets is generally high. As discussed before, the correlation of assets in the period 2006-2010 is statistically closer together than over the whole sample[5]. The study by (Koestrich, 2015) also suggests this. The high MD can be explained by the financial crisis of 2007-2009 which adversely affected financial markets and led to a large drop in value for all portfolios. The wealth plots of every asset mix show a large drawdown in this period. The drawdown data shows that most of the MDs indeed occurred in this period.

[5]These results are found in B.2

5.2 Part II - Robustness

Empirical analysis performed in financial literature are often subject to many assumptions and restrictions. For example, this thesis analyzes a set of four asset mixes to assess the performance of RP. Obviously, this number is not enough to have a concrete answer to questions about the theory. The goal of this thesis is not to give a concrete answer as to which portfolio is the best. Nor is it to give exact answers what the characteristics of RP are. It does, however, shed light on the subject and attempts to add to the current literature about RP. To do this in the best manner, the robustness of the backtests will be tested in this part. Backtesting portfolios has several pitfalls. The first pitfall of backtesting that is discussed is the choice of estimation period which could affect the outcome. Before assessing other robustness measures, we need to know whether the choice of estimation period affects the outcome. The only variable that needs to be estimated over a specified period is the variance-covariance matrix. In the initial setup, a period of three years was used. However, we want to determine the effect of changing estimation windows. Therefore table B.4 presents the results of all strategies when the estimation period of the VCV varies between 1, 3 and 5 years. Only asset mix 1 will be used to review the effects, it is assumed that the effect of changing periods will be linear. The only strategies that change due to the new estimation period are the GMV and RP strategy since these need the VCV as an input. We see that both the 1-year VCV and the 5-year VCV lead to the same result as the 3-year VCV. The alternative estimation periods do not lead to a change in relationships among portfolios. For this reason, the 3-year estimation period used is considered robust.

The second pitfall is that the obtained result is hugely dependent on the choice of assets. This thesis has incorporated a part of this pitfall by constructing four portfolios with each consisting of different assets. In the empirical setup, portfolios consisting of six asset classes have been chosen to be a proxy for a real-life portfolios. However, real-life portfolios are not limited to six asset classes. They can vary in number or the sorts of asset classes. The number and choice of assets/asset classes is arbitrary and should be taken into account when assessing performance. Would RP still perform the way it did in the previous analysis if we would use four asset classes instead of six? Table B.5 shows the performance statistics of asset mix 1 when private equity and inflation-linked bonds are omitted. We see some changes in the outcomes. The 1/N strategy's statistics differ somewhat but remain relatively equal. GMV statistics do not change since they allocate no capital to inflation-linked bonds and private equity. RP, however, produces significantly lower returns and higher SD. This leads to a portfolio that cannot outperform the others in volatility-return trade-off. It continues to have the best drawdown statistics, but these are irrelevant when the investor is not rewarded for the risk he takes. Table B.6 shows the performance statistics of asset mix 1 when commodities and real estate are omitted. Similar to the previous test, the RP strategy does not provide superior statistics. The GMV outperforms the

RP strategy and both strategies have similar SR's to the 1/N strategy, which has the highest total return. The last robustness check regarding asset classes is presented in table B.7. Only bonds and equities are included in this backtest. We see that RP produces a higher SR, has one of the most balanced weights and moderate drawdowns. All in all, we have seen that not only does the choice of assets matter for the outcome, but also the included asset classes strongly affect outcomes.

One of the major assumptions in the empirical analysis framework is the rebalancing frequency of the portfolios. The asset allocation is the result asset's characteristics in the past. As time passes, these characteristics change, therefore the optimal weights of these assets change as well. The frequency of rebalancing is therefore very important. Ideally, portfolio weights are altered instantaneously as time passes. However, this imposes several issues. First of all, there has to be a market for assets. We have assumed so far that we could buy/sell any assets every rebalancing period. However, this is not always the case. If an asset is not liquid, it means that the owner of the asset cannot get rid of it easily or that a potential buyer cannot obtain it easily. This means that the portfolio is suboptimal since the ideal weight of the (illiquid) asset cannot be achieved. Illiquidity poses a second problem, namely that the owner cannot get rid of it and bears full losses of the asset when it cannot be sold. Another problem with a market for assets is when getting rid of a portion of assets, this may adversely affect markets when the size is large enough, thereby harming future returns. The empirical framework could have been supplemented by imposing a cost for large market orders. Second, a concern is heightened when a proposed portfolio strategy is backtested by using historical data. Consider an investment strategy that can be adopted today with readily available securities. If those securities were unavailable in the past, then the strategy has no true antecedent. Backtesting must be conducted with proxies for the securities, and the choice of proxies can have a direct effect on measured returns. As described in the methodology, the rebalancing period will be varied to observe whether this has any effect on the outcomes. The new frequencies of rebalancing will be quarterly and yearly since these are reasonable frequencies. Any frequency lower than a month can be considered more speculating and trading rather than investing. Trading costs would erode profits if rebalancing happens too frequently. This thesis does not take trading costs into account, as the choice is arbitrary and it is beyond its scope. It remains an interesting idea for future research to impose trading costs for assets as a function of their liquidity. Table B.8 presents the results. We see that there are no major changes for the RP portfolios. The strategy produces similar volatility-return statistics and remains to outperform its peers. The yearly strategy yields a much higher SR due to a low SD, most likely caused by favorable holding positions. But there are two sides to this story, since it could have backfired when assets performed worse during the holding period. The GMV strategy's drawdowns increase in magnitude as the rebalancing period lengthens, indicating that long holding periods can lead to suboptimal results.

The last pitfall of portfolio backtesting is the choice of constraints. There were two constraints in this thesis: the sum of portfolio assets had to be equal to 1[6], and asset weights cannot be negative. In real life, individuals who invest could use upper-/lower-bounds for their asset classes. It restricts portfolio weights so that they cannot become too extreme and makes sure that there is a certain allocation to a specific asset. Asset weight restrictions allow for trimming a portfolio to one's preferences. Furthermore, allowing for negative weights could lead to better diversification benefits and higher returns. The general message is that a portfolio can be trimmed to the preferences of the investor. This thesis evaluates the general case. So, other constraints will not be tested, but the reader should keep in mind that other constraints lead to other outcomes.

[6]The levered RP portfolio is, by definition, an exception for this constraint.

Chapter 6

Discussion

Until now, four portfolios have been assessed which two different geographical constraints. The results have also been subject to robustness checks. This chapter combines the findings of chapter 5 and relates previous literature to the findings. This chapter first discusses the performance of RP, followed by a discussion of the asset choices and the use of leverage. Finally, it presents the contributions to the current literature, limitations of this thesis and presents some suggestions for future research.

6.1 Performance

To summarize the results of the four portfolios: unlevered RP outperformed in asset mixes 1, 3 and 4 in terms of SR. In the asset mix 2 setup, the bond asset class strongly outperformed other strategies, causing the GMV strategy to be superior. RP reaped its return mostly from low-beta asset classes, which is due to the propensity to invest in these assets. The positive investments in the other classes contributed to returns and diversification benefits. We saw that RP has much more balanced correlation with included asset classes' return and high-beta assets have higher correlations than low-beta assets. In all setups, RP could not obtain the highest total return. The strategy diversifies the RC of its assets, by definition, perfectly. The Gini coefficient varies between 0.547-0.629, similar to the findings of Maillard, Roncalli, and Teïletche (2010) where only the 1/N by definition had the least asset concentration. This is partly because RP always invests a positive amount of capital in each asset class. Volatility and risk are two concepts that are frequently used in this thesis. Although they may mean the same in some situations, they often are not identical. The risk of a RP strategy, represented as the MD in this thesis, was among the lowest of all strategies. Especially the 1-year MD was quite low when compared to other strategies. Since RP's weights are dominated by low-beta assets, it makes sense that the MD statistics are lower. Having a high Gini coefficient also helped in terms of diversification benefits. The latter existed since correlations weren't equal to 1 among asset classes. It has a low volatility compared to its peers, and the MDs are among the lowest of all tests, indicating that the strategy performs more smoothly over time. These findings are in line with the work of Maillard, Roncalli, and Teïletche

(2010). They also find lower tail risk for a RP strategy due to low asset concentration. In most of their backtests, RP produces better volatility-return statistics. Only the GMV strategy had a higher SR in their global sample, as was the case with one of the backtests of this thesis. The 60/40 and 1/N strategy were consistently outperformed by RP, similar to the findings of Chaves et al. (2011).

The benchmark strategies were chosen for a reason. To refresh: the 1/N strategy was chosen to test the RP strategy against a strategy that diversifies perfectly in terms of capital, the 60/40 strategy was chosen to compare a high-beta strategy with RP, and the GMV portfolio was chosen to compare RP with a portfolio that has the least volatility. A condition for all strategies was that they have to be constructed *ex ante*. The RP strategy had a higher SR than the 1/N strategy in every asset mix. This leads to the belief that a risk-diversified portfolio performs better than a capital diversified portfolio. A potential reason for this is given by Asness, Frazzini, and Pedersen (2012), namely that low-beta assets have a higher risk-return trade-off than high-beta assets. High-beta assets have lower risk/return characteristics since investors chase higher total returns and/or are leverage averse, meaning they will not invest in strategies they must lever to reap benefits. If most investors act in this way, low-beta assets will have a higher future earnings potential. This would explain why the low-beta strategies (RP and GMV) have outperformed in this empirical framework. Another reason could be the assets included as the papers of Chaves et al. (2011), Kunz (2011) and Inker (2011) suggest. The choice of assets in this thesis is arbitrary, other choices could have led to other outcomes. Similar to the 1/N strategy, the RP strategy also outperformed the high-beta 60/40 strategy in terms of volatility-return trade-off. It is not strange that the the GMV strategy generally underperforms the RP strategy in terms of volatility-return trade-off. The GMV strategy has the least volatility, it is on the extreme left side of Markowitz' bullet in a mean-variance framework. We know that the strategy that has the highest slope (tangency portfolio) is the portfolio with the highest risk-return trade-off. The GMV doesn't come close to that point since by definition it is on the extreme left side. That RP outperforms the GMV strategy is no surprise. Summing up the result of all backtests, RP outperformed its peer strategies, but its attractiveness is mostly valuable to an investor who looks for superior risk-return statistics.

In the analysis, the performance was assessed by looking at the build-up of capital, concentration risks and downside risks. There are, however, many more factors which can be incorporated to assess performance. Key to the choice is the focus of the authors. Having a strategy that generates a high total return performs very badly if the CPI's growth rate is higher. For example, a defined contribution pension fund that opts for a strategy that is not in line with the consumption pattern of its participants could potentially be wealth destroying (Merton, 1975). The choice of performance measure differs among studies, it is therefore important to see the results in relative terms instead of absolutes.

6.2 Asset Choice

The thesis used European portfolios due to the lack of focus on these in the literature. This choice comes with consequences. First of all, Europe is a combination of countries with their own fiscal policies. It is a big player in the world economy, comparable to the USA, but the decision making is not as centralized, the liberty of countries to make own decisions, diverse cultures and demographic characteristics complicates the process of finding the right causality in problems. That studies have chosen the USA as main 'research area' makes sense in many ways. The choice to investigate Europe is historically motivated, specifically by the lack of interest in this area in the financial literature. It might have affected the quality of the conclusions due to the higher economic complexity of the European Union. This should be taken into account. Besides the focus on Europe, global portfolios were also constructed, since this is more representative of the current age of broad global information access. For each region, two different asset mixes were used to construct the portfolios. The reason was that the literature suggests that the choice of assets in portfolio analysis is essential (Chaves et al., 2011).

The main difference between the global asset mixes is that unlevered RP has a higher SR in asset mix 1, but the GMV has the highest in asset mix 2. The included assets in each mix are considered to be comparable, so this means that the choice of assets matters, otherwise the results should have been similar. Other statistics, such as the drawdowns and the Gini coefficients. That GMV outperformed in asset mix 2 was due to high yield of bonds in that portfolio. In the European asset mixes, we see that unlevered RP has higher SRs. There does not seem to be much variance in the results, perhaps due to the high Gini coefficient making it less susceptible to individual asset changes. The GMV clearly suffers from this since asset mix 4 causes it to generate low volatility-return statistics. In both regions and all mixes, the 1/N strategy performs similarly, most likely due to equal weights making it robust. The 60/40 strategy has a large dependency on equity and bonds. This is highlighted by the low return in asset mix 4 where bonds and equity perform badly, and the high return in asset mix 2 where bonds outperform other asset classes. High-concentration strategies are prone to perform inconsistently when asset mixes change given their dependency on specific assets. The variability is seen in the Gini coefficients of the strategies: the lower the coefficient, the more susceptible it is to shocks. RP and 1/N have the highest Gini coefficients, which lead to the most stable characteristics across all asset mixes.

6.3 Leverage

Leverage does not propel performance. Asness, Frazzini, and Pedersen (2012) suggests that investors can benefit from using leverage. Their argument is that low-beta strategies tend to outperform because high-beta asset are bought too much, due to the fact that investors are leverage averse and thus chase high total returns. The results of the empirical backtests performed in this thesis show that leverage can boost performance and outperform high-beta strategies, but the cost of leverage needs to be low or non-existent. When a reasonable leverage cost is used, the benefit of using leverage washes away as the return only marginally increases or decreases. In this setup, RP cannot beat peer strategies that have high total returns. For some of the portfolios, the marginal benefit of leverage is higher than the marginal cost, but as pointed out by Kazemi (2012), a lot of leverage is necessary to make a competitive portfolio. Also, high leverage causes different issues. For example, if an individual acquires leverage to boost their portfolio, the timing is important since the economic environment is a factor that affects borrowing costs. If the investor acquired the leverage in a good economic regime, it could adversely affect the costs when the regime worsens. Another pitfall is the risk of a sudden drop in economic markets. When a shock hits the economy, assets are more correlated (Koestrich, 2015). If the loss of value causes a liquidity/solvency issue for the individual, the losses of leverage are much larger than the gains. The RP strategy did generate higher risk-return statistics than strategies that invest more in high-beta assets. However, the empirical evidence of this thesis does not support the theoretical justification (Frazzini and Pedersen (2014), Asness, Frazzini, and Pedersen (2012), Chaves et al. (2012), Qian (2011) and Kunz (2011)) of using leverage. As stated before, the choices for proxies of leverage costs are arbitrary, different choices for leverage could affect the results of this thesis.

6.4 Contributions and Limitations

This study contributes to the current body of literature on RP by providing backtests in a time period which has not previously been covered. As Thiagarajan and Schachter (2011) suggested in their study, more research had to be done regarding the sensitivity or RP performance regarding asset inclusion. This thesis has done so by varying assets and asset classes. Furthermore, it is the first study in RP literature that addresses European portfolios, where usually USA based or globally based portfolios are used. All in all, this thesis presents an overview of the RP method and literature and adds to the research on RP performance by investigating total return, reward-for-volatility, risk and weight concentrations, wealth drawdowns and asset choice sensitivity using four multi-regional backtests.

Four asset mixes is not enough to draw solid conclusions about the robustness of strategy's performances. Ideally, one would select a wide variety of assets and asset classes to make portfolios that consist of randomly selected assets. One could then simulate the performances of all the possible portfolios. The aggregate return of each portfolio method could then be compared to benchmarks strategies. In this way, the robustness could be verified in a correct manner. This is, however, outside, the scope of this portfolio and subject to computational complexity, therefore it remains a topic that deserve further investigation.

Furthermore, this thesis uses the volatility as a measure to equalize RCs and to calculate to total variances. As explained before, return data suffers from statistical moments, which makes the use of volatility undesirable. The volatility doesn't capture the total risk of a portfolio, merely the deviations from the mean. For this reason, the MD statistics were incorporated in this thesis. However, it remains an interesting subject to combine RP strategies with several, more complete, measures of risk.

The results should be interpreted with caution. The used method relies on the historical relationship of asset. Thereby we assume that these events are likely to happen in the future. This way of 'average' thinking is extremely dangerous when extreme events happen. Indeed, performance is highly dependent on results during these extreme events, particularly in the finance industry. One should be wise and thoroughly analyze possibilities of extreme events. The cost of average thinking is high for portfolio managers.

Chapter 7

Conclusion

The analysis and comparison of portfolios cannot result in a definitive answer or absolute superior suggestion. We can only observe the outcomes in this setting and draw conclusions based on the results. The suggestion for the 'best' portfolio is dependent on the preferences of the individual who decides which portfolio is chosen for implementation. Given the information about the different portfolios, they will make a choice depending on their risk tolerance, a preset specified set of goals and beliefs that they have. This thesis does not aim to serve as investment advice, nor will it provide that. Instead, its single purpose is investigating and supplementing the theory of RP.

This thesis investigated the attractiveness of RP in several performance dimensions. Four backtests in the period 1997 - 2017 were performed, varying the regional focus equally between Europe and the World. RP was compared with several benchmark strategies: 60/40, 1/N and GMV. First, this thesis finds that unlevered RP's total returns are lower than the high-beta strategies 1/N and 60/40. In terms of reward-for-volatility, RP achieved the highest SR and M^2 measure in 3 of the 4 backtests. This was complemented by relatively low MDs, stable and high Gini coefficients. The robustness tests of the empirical backtests showed that the parameter estimation and rebalancing frequencies didn't influence the outcomes. Furthermore, the choice of assets and asset classes changed the results of the backtests, therefore tactical asset allocation is of great importance. Changing the portfolio's scope from European to global does not impose major changes to the outcomes. Overall, an unlevered RP portfolio is attractive when low total returns are sufficient for the investor.

When leverage is applied, the results of RP worsen. Using several proxies for borrowing costs, the strategy is not able to produce total returns with respect to high-beta benchmark strategies. This makes it unattractive to invest in when high returns are desired, especially considering that disadvantages of leverage such as timing and solvency were discussed but not included in the analysis. Previous work suggests (Asness, Frazzini, and Pedersen (2012), Chaves et al. (2011) and Qian (2011) that the use of leverage can boost the return of a RP portfolio. This thesis finds that this is very challenging due to profit-eroding borrowing costs.

To conclude, the choice to invest capital in a RP style should come forth out from beliefs about the the market conditions. Investing RP-style thinking - that equalizing RCs makes sense - does not justify the choice enough. One should have beliefs about the risk-return trade-off of all assets and asset classes in the market. If an investor were to adopt a RP strategy, he should consider the following points: 1) given that RP overweights low-volatile assets, will low-beta assets have better volatility-return characteristics in the future?, 2) The choice of assets is of great importance, poorly performing individual assets can erode performance, 3) the use of leverage comes with a cost that is prone to dilute profits, and 4) RP is not a magical superior strategy, it is just a heuristic asset allocation method with interesting dynamics that are worth delving into.

Appendix A

Calculations

A.1 Table 2.1

A.1.1 60/40 Portfolio

$$RiskContribution_{AssetA} = \frac{W_A^2 \times \sigma_A^2 + 2 \times W_A \times W_B \times Cov_{A,B}}{W_A^2 \times \sigma_A^2 + W_B^2 \times \sigma_B^2 + 2 \times W_A \times W_B \times Cov_{A,B}}$$
$$= \frac{0.6^2 \times (20\%)^2 + 0.6 \times 0.4 \times 0.2 \times 20\% \times 8\%}{0.6^2 \times (20\%)^2 + 0.4^2 \times (8\%) + 2 \times 0.6 \times 0.4 \times 0.2 \times 20\% \times 8\%}$$
$$= 90.32\%$$

$$RiskContribution_{AssetB} = \frac{W_B^2 \times \sigma_B^2 + 2 \times W_A \times W_B \times Cov_{A,B}}{W_A^2 \times \sigma_A^2 + W_B^2 \times \sigma_B^2 + 2 \times W_A \times W_B \times Cov_{A,B}}$$
$$= \frac{0.4^2 \times (20\%)^2 + 0.6 \times 0.4 \times 0.2 \times 20\% \times 8\%}{0.4^2 \times (20\%)^2 + 0.6^2 \times (8\%) + 2 \times 0.6 \times 0.4 \times 0.2 \times 20\% \times 8\%}$$
$$= 9.38\%$$

$$StandardDeviation = W_A^2 \times \sigma_A^2 + W_B^2 \times \sigma_B^2 + 2 \times W_A \times W_B \times Cov_{A,B}$$
$$= 0.6^2 \times (20\%)^2 + 0.4^2 \times (8\%) + 2 \times 0.6 \times 0.4 \times 0.2 \times 20\% \times 8\%$$
$$= 12.94\%$$

A.1.2 Risk Parity Portfolio

$$RiskContribution_{AssetA} = \frac{W_A^2 \times \sigma_A^2 + 2 \times W_A \times W_B \times Cov_{A,B}}{W_A^2 \times \sigma_A^2 + W_B^2 \times \sigma_B^2 + 2 \times W_A \times W_B \times Cov_{A,B}}$$
$$= \frac{0.7273^2 \times (20\%)^2 + 0.7273 \times 0.2727 \times 0.2 \times 20\% \times 8\%}{0.7273^2 \times (20\%)^2 + 0.2727^2 \times (8\%) + 2 \times 0.7273 \times 0.2727 \times 0.2 \times 20\% \times 8\%}$$
$$= 50\%$$

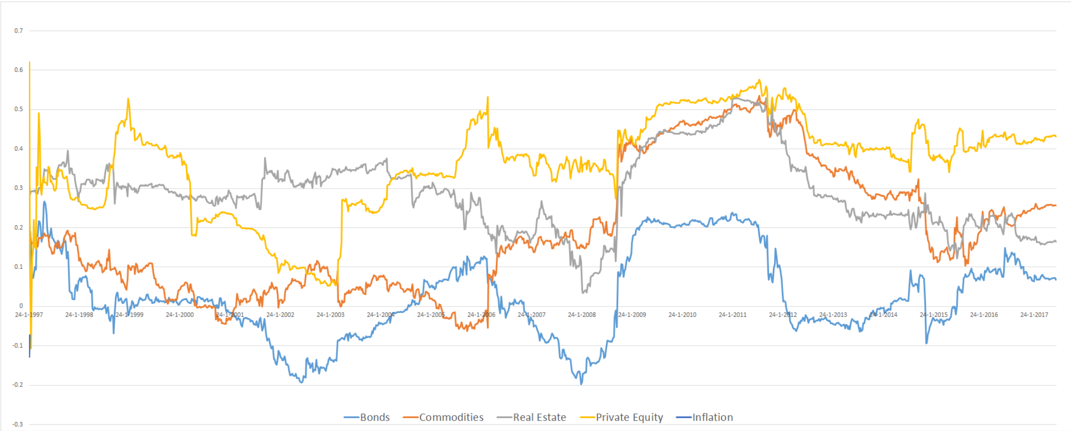
$$RiskContribution_{AssetB} = \frac{W_B^2 \times \sigma_B^2 + 2 \times W_A \times W_B \times Cov_{A,B}}{W_A^2 \times \sigma_A^2 + W_B^2 \times \sigma_B^2 + 2 \times W_A \times W_B \times Cov_{A,B}}$$
$$= \frac{0.2727^2 \times (20\%)^2 + 0.7273 \times 0.2727 \times 0.2 \times 20\% \times 8\%}{0.2727^2 \times (20\%)^2 + 0.7273^2 \times (8\%) + 2 \times 0.7273 \times 0.2727 \times 0.2 \times 20\% \times 8\%}$$
$$= 50\%$$

$$StandardDeviation = W_A^2 \times \sigma_A^2 + W_B^2 \times \sigma_B^2 + 2 \times W_A \times W_B \times Cov_{A,B}$$
$$= 0.7273^2 \times (20\%)^2 + 0.2727^2 \times (8\%) + 2 \times 0.7273 \times 0.2727 \times 0.2 \times 20\% \times 8\%$$
$$= 12.94\%$$

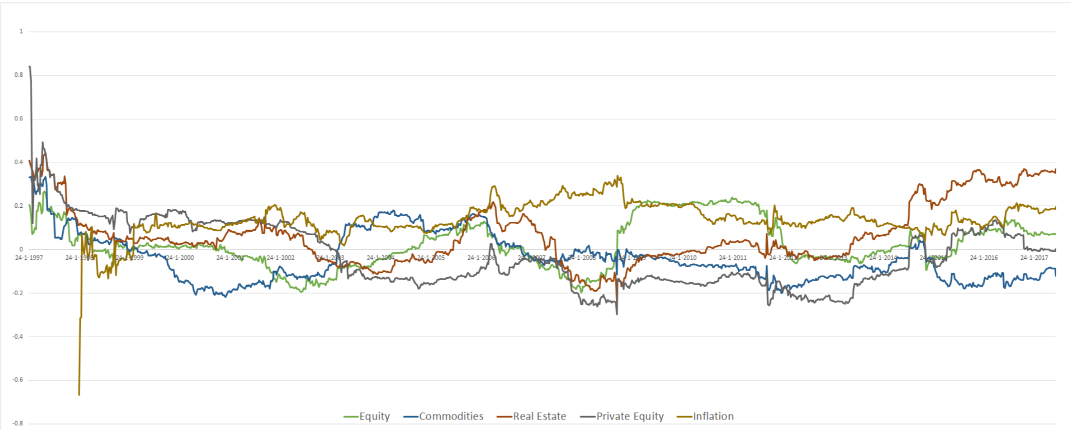
Appendix B

Data

FIGURE B.1: Correlations with equity over time



(A) Correlations with Equity over time



(B) Correlations with Bonds over time

TABLE B.1: Correlation Table Asset Classes

Assets		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	1. MSCI World	1																		
	2. MSCI Emerging Markets	0.718	1																	
Equi.	3. FTSE All World	0.997	0.801	1																
	4. MSCI Europe	0.830	0.695	0.911	1															
	5. STOXX Europe	0.820	0.652	0.856	0.844	1														
	6. JPM Global All	-0.042	-0.015	-0.038	-0.044	-0.044	1													
	7. BofA Ml Global Broad	0.044	0.049	0.047	0.057	-0.018	0.0472	1												
	8. BofA Ml Global Gov	-0.012	0.009	-0.010	-0.007	-0.006	-0.009	-0.034	1											
Bnds	9. Barclays EU Agg	0.084	0.081	0.086	0.087	0.085	0.704	0.489	0.039	1										
	10. IBOXX EU Overall	0.053	0.053	0.055	0.059	0.054	0.841	0.403	0.059	0.834	1									
	11. BofA ML EU Gov	0.020	0.008	0.021	0.019	0.063	0.809	0.310	0.049	0.779	0.927	1								
Com.	12. TR Equal Weight	0.193	0.261	0.286	0.190	0.155	-0.116	0.181	-0.068	-0.029	-0.054	-0.123	1							
	13. S&P GSCI Commodity	0.129	0.176	0.217	0.139	0.118	-0.163	0.093	-0.028	-0.056	-0.075	-0.100	0.734	1						
RE	14. MSCI World Real Estate	0.526	0.425	0.523	0.461	0.401	-0.061	0.140	0.019	0.078	0.063	0.017	0.344	0.254	1					
	15. MSCI EU Real Estate	0.438	0.370	0.438	0.410	0.405	-0.054	0.041	0.032	0.112	0.090	0.071	0.294	0.233	0.603	1				
PE	16. S&P Listed Private Equity	0.510	0.400	0.504	0.414	0.485	-0.223	-0.081	0.004	0.046	-0.017	-0.023	0.313	0.325	0.721	0.666	1			
	17. TR Private Equity Buyout	0.306	0.182	0.300	0.241	0.260	-0.307	-0.016	0.021	-0.018	-0.062	-0.054	0.097	0.050	0.410	0.392	0.578	1		
Infl.	18. BofA ML Global Gov	-0.117	-0.047	-0.109	-0.110	-0.203	0.325	0.110	-0.002	0.112	0.247	0.224	0.007	-0.009	-0.028	-0.036	-0.144	-0.095	1	
	19. BofA ML EU Gov	-0.023	-0.016	-0.022	-0.034	-0.013	-0.033	0.055	0.020	-0.002	-0.002	-0.012	0.036	0.039	0.081	0.091	0.142	0.044	-0.046	1

TABLE B.2: Asset weights Portfolios

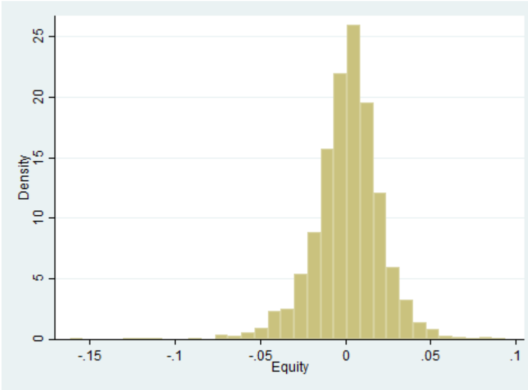
		Equity(%)	Bonds(%)	Comm.(%)	RE(%)	PE(%)	Inflation(%)
60/40	Portfolio 1	60.00	40.00	0	0	0	0
	Portfolio 2	60.00	40.00	0	0	0	0
	Portfolio 3	60.00	40.00	0	0	0	0
	Portfolio 4	60.00	40.00	0	0	0	0
1/N	Portfolio 1	16.67	16.67	16.67	16.67	16.67	16.67
	Portfolio 2	16.67	16.67	16.67	16.67	16.67	16.67
	Portfolio 3	16.67	16.67	16.67	16.67	16.67	16.67
	Portfolio 4	16.67	16.67	16.67	16.67	16.67	16.67
GMV	Portfolio 1	2.12	88.85	8.17	0.87	0	0
	Portfolio 2	3.95	92.45	2.35	1.01	0	0.55
	Portfolio 3	1.23	89.05	7.96	1.77	0	0
	Portfolio 4	1.77	93.09	3.72	1.43	0	0
RP	Portfolio 1	6.97	51.38	11.28	5.59	5.97	18.82
	Portfolio 2	8.92	50.90	8.31	7.41	9.53	14.93
	Portfolio 3	5.75	46.20	10.47	5.56	5.30	26.72
	Portfolio 4	6.47	44.17	6.58	5.41	5.32	32.05

FIGURE B.2: T-test Result of difference two correlation tables

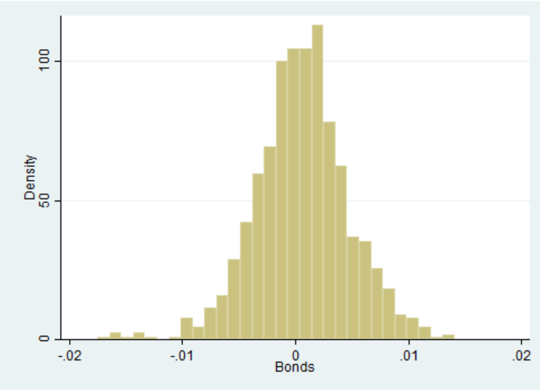
```
One-sample t test

Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+----------------------------------------------------------------------
    var1 |     190    .0378684    .0059687    .0822728    .0260946    .0496422
--------------------------------------------------------------------------------
    mean = mean(var1)                                             t =   6.3445
Ho: mean = 0                                     degrees of freedom =      189

    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 1.0000         Pr(|T| > |t|) = 0.0000          Pr(T > t) = 0.0000
```

The left diagonal values of the correlation matrix in the period 2006 - 2010 were subtracted by the left diagonal values of the correlation matrix of the full sample. The result was tested if it was higher than zero.

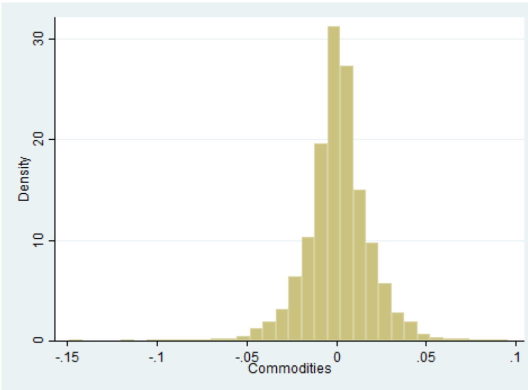
FIGURE B.3: Distribution Returns Asset Classes



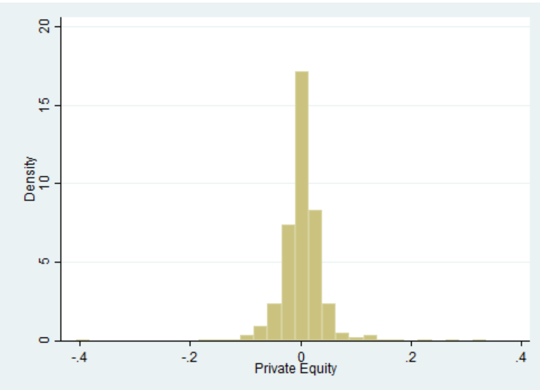
(A) Distribution Equity



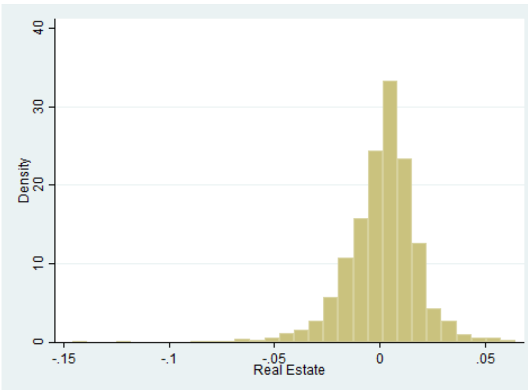
(B) Distribution Bonds



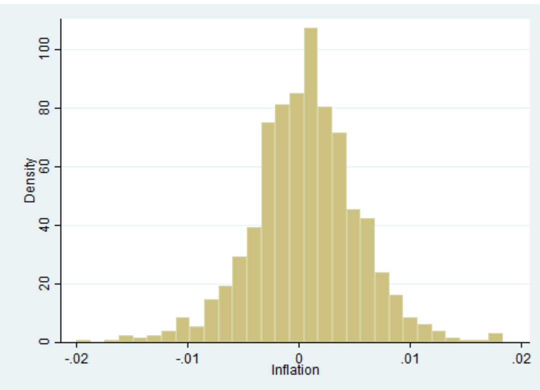
(C) Distribution Commodities



(D) Distribution Private Equity



(E) Distribution Real Estate



(F) Distribution Inflation-Linked

FIGURE B.4: Distribution Returns Asset Classes

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
GENERAL_eq~y	2.5e+03	0.0000	0.0000	.	0.0000
EQ_msci_wo~d	2.5e+03	0.0000	0.0000	.	0.0000
EQ_msci_em	1.5e+03	0.0000	0.0000	.	0.0000
EQ_ftse_al~d	1.2e+03	0.0000	0.0000	.	0.0000
EQ_msci_eu~e	2.5e+03	0.0000	0.0000	.	0.0000
EQ_stoxx_eu	1.6e+03	0.0000	0.0000	.	0.0000

(A) Skewness and Kurtosis test Equity indices

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
GENERAL_bo~s	1,083	0.0006	0.0000	35.30	0.0000
BOND_jpmg_~l	722	0.0041	0.0000	23.76	0.0000
BOND_bofa_~L	1,070	0.8346	0.0000	22.32	0.0000
BOND_bofa_~v	1,067	0.5199	0.0000	24.42	0.0000
BOND_barcl~g	987	0.0008	0.0000	48.57	0.0000
BOND_iboxx~l	965	0.0000	0.0000	57.71	0.0000
BOND_bofa_~l	1,083	0.0000	0.0000	.	0.0000

(B) Skewness and Kurtosis test Bonds indices

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
GENERAL_co~s	2,977	0.0000	0.0000	.	0.0000
COMM_thomp~w	2,977	0.0000	0.0000	.	0.0000
COMM_sp_co~y	2,478	0.0000	0.0000	.	0.0000

(C) Skewness and Kurtosis test Commodities indices

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
GENERAL_pr~y	1,069	0.0139	0.0000	.	0.0000
PE_sp_priv~y	710	0.0000	0.0000	.	0.0000
PE_tr_priv~y	1,069	0.0006	0.0000	.	0.0000

(D) Skewness and Kurtosis test Private Equity indices

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
GENERAL_re~e	1,175	0.0000	0.0000	.	0.0000
RE_msci_wo~e	1,174	0.0000	0.0000	.	0.0000
RE_msci_eu~e	1,174	0.0000	0.0000	.	0.0000

(E) Skewness and Kurtosis test Real Estate indices

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
GENERAL_in~n	1,017	0.1463	0.0000	28.26	0.0000
INF_bofa_e~n	975	0.0071	0.0000	69.58	0.0000
INFL_bofa_~n	1,017	0.0955	0.0000	73.47	0.0000

(F) Skewness and Kurtosis test Inflation-Linked indices

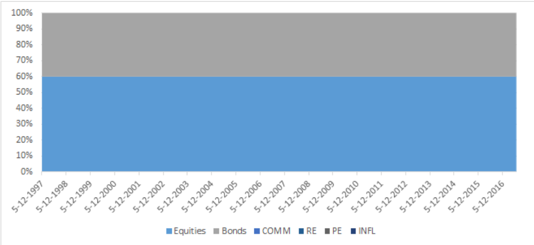
B.1 Risk-free Rate and Borrowing Rate

Table B.3 presents the statistics of the risk free rate and the borrowing rates. The first is used to calculate the excess returns. The 90-day European risk-free rate was chosen due to the European focus, and the assumption that the rate faces no liquidity or solvency issues due to the short-term. The latter are used as a proxy for the cost of using leverage. Two rates were chosen to compare the results of a short-term paper costs and the cost for borrowing money for a long term.

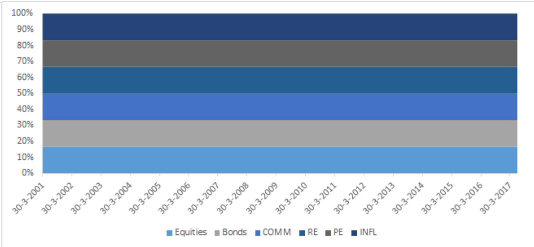
TABLE B.3: Summary statistics Risk-free Rate and Borrowing Rate

	Ann. Return(%)	**SD(%)**	**Start date**
90-day EU risk-free rate	1.90	2.04	05-12-1997
90-day LIBOR rate	2.42	2.20	05-12-1997
EU 5-year Government Bond	3.46	6.43	05-12-1997

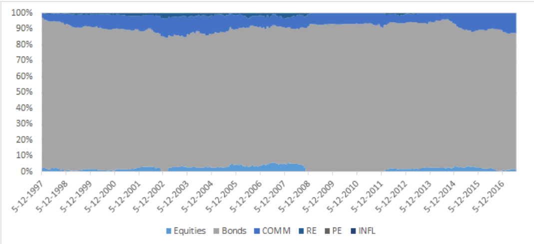
FIGURE B.5: Asset Weights over Time Portfolio 1



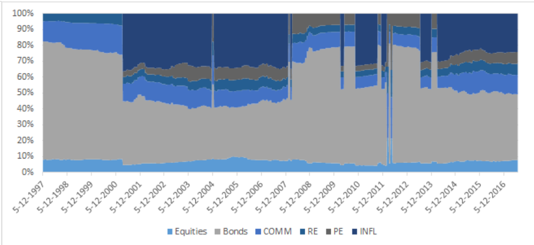
(A) Asset Weights 60/40 Strategy



(B) Asset Weights 1/N Strategy

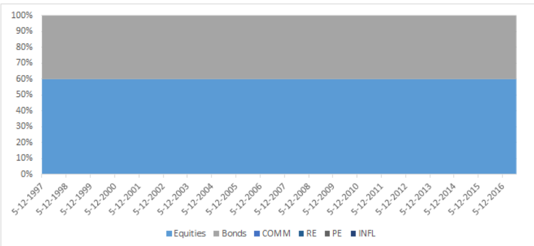


(C) Asset Weights GMV Strategy

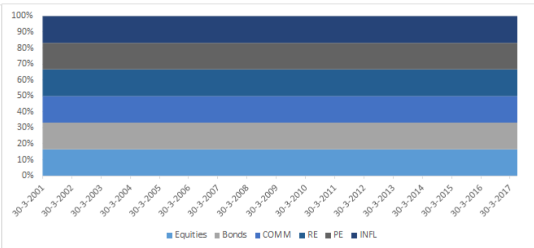


(D) Asset Weights RP Strategy

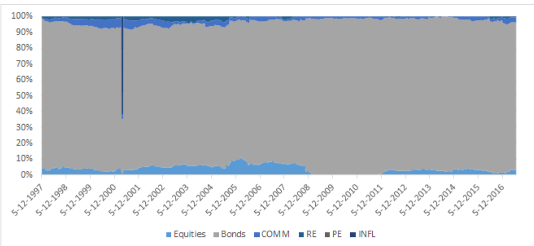
FIGURE B.6: Asset Weights over Time Portfolio 2



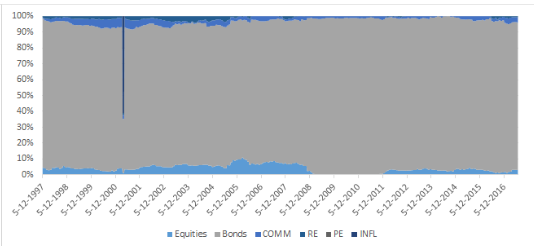
(A) Asset Weights 60/40 Strategy



(B) Asset Weights 1/N Strategy

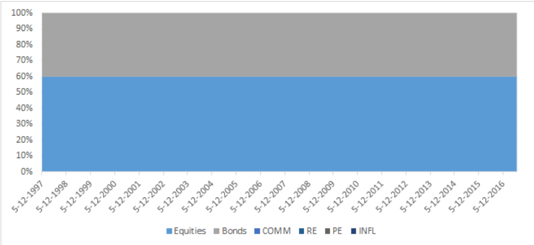


(C) Asset Weights GMV Strategy

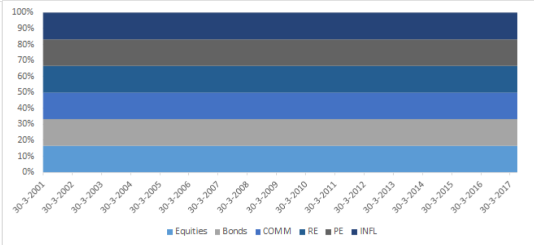


(D) Asset Weights RP Strategy

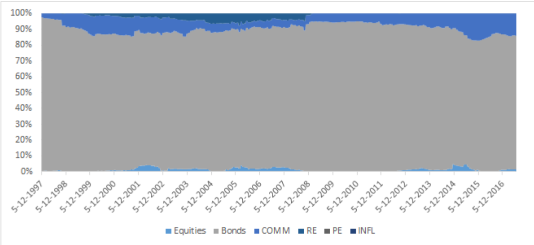
FIGURE B.7: Asset Weights over Time Portfolio 3



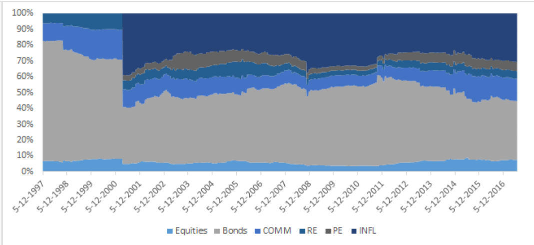
(A) Asset Weights 60/40 Strategy



(B) Asset Weights 1/N Strategy

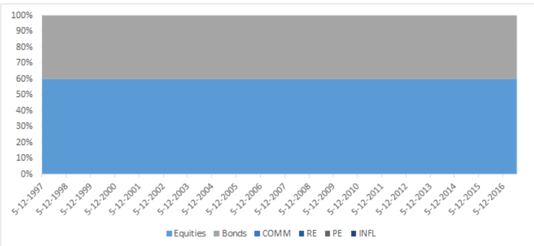


(C) Asset Weights GMV Strategy

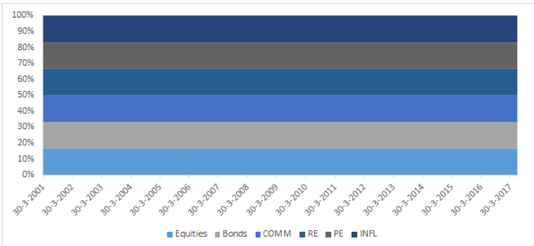


(D) Asset Weights RP Strategy

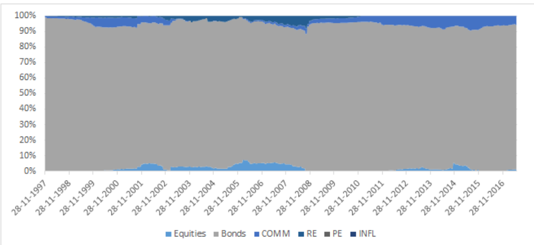
FIGURE B.8: Asset Weights over Time Portfolio 4



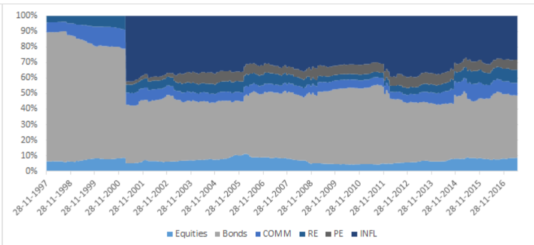
(A) Asset Weights 60/40 Strategy



(B) Asset Weights 1/N Strategy



(C) Asset Weights GMV Strategy



(D) Asset Weights RP Strategy

FIGURE B.9: RC Gini coefficients regressions with SR, volatility and total return

```
. reg sr gini_rc

      Source |       SS           df       MS      Number of obs   =        30
-------------+----------------------------------   F(1, 28)        =      8.15
       Model |  .407948919         1  .407948919   Prob > F        =    0.0080
    Residual |  1.40139676        28  .050049884   R-squared       =    0.2255
-------------+----------------------------------   Adj R-squared   =    0.1978
       Total |  1.80934568        29   .06239123   Root MSE        =    .22372

------------------------------------------------------------------------------
          sr |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     gini_rc |   .2854004   .0999662     2.85   0.008     .0806289    .4901719
       _cons |   .1623008   .0676255     2.40   0.023     .0237762    .3008254
------------------------------------------------------------------------------

. reg sd gini_rc

      Source |       SS           df       MS      Number of obs   =        30
-------------+----------------------------------   F(1, 28)        =      6.04
       Model |  .005953461         1  .005953461   Prob > F        =    0.0204
    Residual |  .027578271        28  .000984938   R-squared       =    0.1775
-------------+----------------------------------   Adj R-squared   =    0.1482
       Total |  .033531731        29  .001156267   Root MSE        =    .03138

------------------------------------------------------------------------------
          sd |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     gini_rc |  -.0344776   .0140235    -2.46   0.020    -.0632034   -.0057517
       _cons |   .0888943   .0094867     9.37   0.000     .0694617    .1083268
------------------------------------------------------------------------------

. reg return gini_rc

      Source |       SS           df       MS      Number of obs   =        30
-------------+----------------------------------   F(1, 28)        =      1.13
       Model |  .000227869         1  .000227869   Prob > F        =    0.2970
    Residual |  .005650648        28  .000201809   R-squared       =    0.0388
-------------+----------------------------------   Adj R-squared   =    0.0044
       Total |  .005878517        29  .000202707   Root MSE        =    .01421

------------------------------------------------------------------------------
      return |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     gini_rc |   .0067452   .0063478     1.06   0.297    -.0062576     .019748
       _cons |   .0186438   .0042942     4.34   0.000     .0098476      .02744
------------------------------------------------------------------------------
```

FIGURE B.10: Spread 3-month LIBOR and 3-month risk-free EU rate

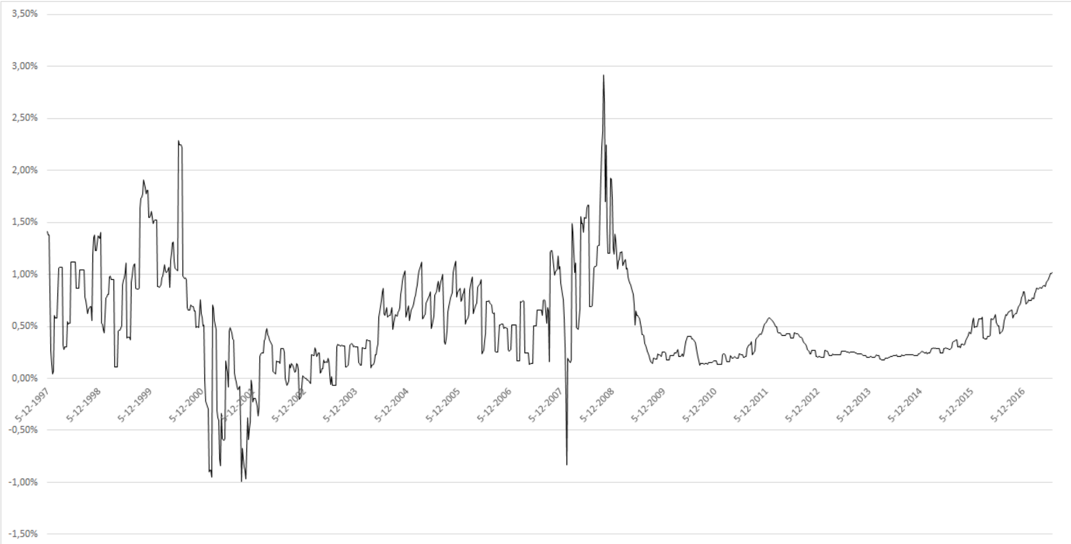


TABLE B.4: Robustness - Estimation Periods

	1-Year VCV				3-Year VCV				5-Year VCV			
	60/40	**1/N**	**GMV**	**RP**	**60/40**	**1/N**	**GMV**	**RP**	**60/40**	**1/N**	**GMV**	**RP**
Return (%)	2.86	2.34	0.88	1.22	2.86	2.34	0.84	1.47	2.86	2.34	0.89	1.44
SD (%)	11.20	9.73	3.35	4.23	11.20	9.73	3.33	4.50	11.20	9.73	3.33	4.24
SR (%)	25.53	24.06	26.39	28.76	25.53	24.06	25.26	32.66	25.53	24.06	26.76	34.04
M^2	1.08	1.02	1.12	1.22	1.15	1.09	1.14	1.47	1.08	1.02	1.14	1.44
MD Full (%)	-61.21	-60.90	-71.28	-48.46	-61.21	-60.90	-72.05	-50.57	-61.21	-60.90	-72.18	-50.47
MD 5 year (%)	-55.28	-49.90	-58.74	-35.15	-55.28	-49.90	-59.95	-34.88	-55.28	-49.90	-60.10	-34.04
MD 1 year (%)	-32.27	-37.23	-33.17	-15.91	-32.27	-37.23	-34.09	-19.42	-32.27	-37.23	-34.01	-19.00
GINI weights	0.3000	1.000	0.227	0.566	0.3000	1.000	0.209	0.547	0.3000	1.000	0.213	0.547
GINI RC	0.0008	0.500	0.235	1.000	0.0008	0.500	0.221	1.000	0.0008	0.500	0.258	1.000

[1)] All returns are excess of the 90-days Risk-free Rate.

TABLE B.5: Robustness - Four Asset Class Portfolio 1 Annualized Returns - Excluding Private Equity and Infl.-Linked Bonds

	Portfolio 1 - Original				**Portfolio 1 - Four Asset Class (1)**			
	60/40	**1/N**	**GMV**	**RP**	**60/40**	**1/N**	**GMV**	**RP**
Return (%)	2.86	2.34	0.84	1.47	2.86	2.37	0.84	1.09
SD (%)	11.20	9.73	3.33	4.50	11.20	10.15	3.33	5.18
SR (%)	25.53	24.06	25.26	32.66	25.53	23.37	25.26	21.00
M^2	1.15	1.08	1.14	1.47	1.32	1.21	1.31	1.09
MD Full (%)	-61.21	-60.90	-72.05	-50.57	-61.21	-62.17	-72.05	-47.85
MD 5 year (%)	-55.28	-49.90	-59.95	-34.88	-55.28	-54.40	-59.95	-38.74
MD 1 year (%)	-32.27	-37.23	-34.09	-19.42	-32.27	-35.13	-34.09	-14.83
GINI weights	0.3000	1.000	0.209	0.547	0.3000	1.000	0.209	0.574
GINI RC	0.0008	0.500	0.221	1.000	0.0008	0.482	0.221	1.000

[1)] All returns are excess of the 90-days Risk-free Rate.

TABLE B.6: Robustness - Four Asset Class Portfolio 1 Annualized Returns - Excluding Real Estate and Commodities

	Portfolio 1 - Original				**Portfolio 1 - Four Asset Class (2)**			
	60/40	**1/N**	**GMV**	**RP**	**60/40**	**1/N**	**GMV**	**RP**
Return (%)	2.86	2.34	0.84	1.47	2.86	3.38	1.24	1.24
SD (%)	11.20	9.73	3.33	4.50	11.20	8.87	3.12	3.54
SR (%)	25.53	24.06	25.26	32.66	25.53	38.11	39.75	39.35
M^2	1.15	1.08	1.14	1.47	0.90	1.35	1.41	1.39
MD Full (%)	-61.21	-60.90	-72.05	-50.57	-61.21	-63.83	-47.88	-48.65
MD 5 year (%)	-55.28	-49.90	-59.95	-34.88	-55.28	-47.93	-25.72	-30.16
MD 1 year (%)	-32.27	-37.23	-34.09	-19.42	-32.27	-35.15	-11.22	-12.88
GINI weights	0.3000	1.000	0.209	0.547	0.3000	1.000	0.493	0.706
GINI RC	0.0008	0.500	0.221	1.000	0.0008	0.476	0.285	1.000

[1)] All returns are excess of the 90-days Risk-free Rate.

TABLE B.7: Robustness - Two Asset Class Portfolio 1 Annualized Returns

	Portfolio 1 - Original				Portfolio 1 - Two Asset Class			
	60/40	**1/N**	**GMV**	**RP**	**60/40**	**1/N**	**GMV**	**RP**
Return (%)	2.86	2.34	0.84	1.47	2.86	2.50	0.79	1.47
SD (%)	11.20	9.73	3.33	4.50	11.20	9.42	3.51	4.24
SR (%)	25.53	24.06	25.26	32.66	25.53	26.50	22.59	34.12
M^2	1.15	1.08	1.14	1.47	1.08	1.12	0.96	1.47
MD Full (%)	-61.21	-60.90	-72.05	-50.57	-61.21	-58.39	-43.49	-49.13
MD 5 year (%)	-55.28	-49.90	-59.95	-34.88	-55.28	-50.03	-22.92	-32.10
MD 1 year (%)	-32.27	-37.23	-34.09	-19.42	-32.27	-28.26	-11.00	-13.75
GINI weights	0.3000	1.000	0.209	0.547	0.3000	1.000	0.194	0.547
GINI RC	0.0008	0.500	0.221	1.000	0.0008	0.383	0.215	1.000

[1)] All returns are excess of the 90-days Risk-free Rate.

TABLE B.8: Robustness - Rebalancing Periods

	Monthly				Quarterly				Yearly			
	60/40	**1/N**	**GMV**	**RP**	**60/40**	**1/N**	**GMV**	**RP**	**60/40**	**1/N**	**GMV**	**RP**
Return (%)	2.86	2.34	0.84	1.47	2.86	2.34	0.83	1.44	2.86	2.34	0.73	1.50
SD (%)	11.20	9.73	3.33	4.23	11.20	9.73	3.35	4.51	11.20	9.73	3.35	3.80
SR (%)	25.53	24.06	25.43	32.66	25.53	24.06	24.70	31.88	25.53	24.06	21.91	39.35
M^2	1.15	1.08	1.14	1.47	1.15	1.08	1.11	1.44	0.97	0.92	0.83	1.50
MD Full (%)	-61.21	-60.90	-72.05	-50.57	-61.21	-60.90	-73.11	-50.24	-61.21	-60.90	-82.76	-51.16
MD 5 year (%)	-55.28	-49.90	-59.95	-34.88	-55.28	-49.90	-60.01	-34.70	-55.28	-49.90	-70.89	-33.81
MD 1 year (%)	-32.27	-37.23	-34.09	-19.42	-32.27	-37.23	-34.10	-19.47	-32.27	-37.23	-43.56	-15.23
GINI weights	0.3000	1.000	0.209	0.547	0.3000	1.000	0.224	0.547	0.3000	1.000	0.218	0.572
GINI RC	0.0008	0.500	0.221	1.000	0.0008	0.500	0.213	1.000	0.0008	0.500	0.324	1.000

[1)] All returns are excess of the 90-days Risk-free Rate.

Appendix C

Code

The scripts used to obtain the data are presented below. For the calculations, I used the program Excel from Microsoft Office, I used an Excel Marco that iterated over the data to obtain the weights of the RP and GMV portfolios. The script is the general script used, for some of the data had to be adjusted, yet the general function remained the same.

C.1 Risk Parity Script

```
Sub RP()

Dim covrange As String
Dim solverange As String
Dim solvevalue As String
Dim solveconstr As String
Dim SD As String
Dim SD2 As String
Dim SD3 As String
Dim weights As String
Dim weights2 As String
Dim weights3 As String
Dim weights4 As String
Dim i As Integer

Startrow = 3
Secondstartrow = 156
Lastrow = 2
sv = 155
weightrow = 15
columnrow = 3
sv2 = 327
weightrow = 15
weightrow2 = 195
columnrow = 3
columnrow2 = 5
x = 0

Let covrange = "R" & Startrow & "C" & Lastrow & ":" & "R" & Secondstartrow & "C" & Lastrow
Let solverange = "AL" & sv & ":" & "AO" & sv
```

```
Let solveconstr = "AS" & sv
Let sleep1 = "AL" & sv & ":" & "AO" & sv
Let sleep2 = "AL" & sv & ":" & "AO" & sv + 3
Let SD = "R[" & weightrow & "]C[" & 6 & "]"
Let SD2 = "R[" & weightrow & "]C[" & 4 & "]"
Let weights = "R[" & weightrow & "]C:R[" & weightrow & "]C[" & columnrow & "]"
Let weights2 = "R[" & weightrow & "]C[-1]:R[" & weightrow & "]C[" & columnrow - 1 & "]"

For i = 1 To 43
    Range("AL148").Select
    ActiveCell.FormulaR1C1 = "=COVAR(OFFSET(" & covrange & ",0,R147C-1),OFFSET(" & covrange
    Range("AL148").Select
    Selection.AutoFill Destination:=Range("AL148:AO148"), Type:=xlFillDefault
    Range("AL148:AO148").Select
    Selection.AutoFill Destination:=Range("AL148:AO151"), Type:=xlFillDefault
    Range("AL148:AO151").Select

    Range("AL140").Select
    Selection.FormulaArray = "=MMULT(R[8]C:R[11]C[3],TRANSPOSE(" & weights & ")/" & SD & ")
    Range("AM140").Select
    Selection.FormulaArray = "=RC[-1]:R[3]C[-1]*TRANSPOSE(" & weights2 & ")"
    Range("AN140").Select
    Selection.FormulaArray = "=RC[-1]:R[3]C[-1]/" & SD2
    Range("AO140").Select

    SolverReset
    SolverOk SetCell:="$AN$140", MaxMinVal:=3, ValueOf:=0.25, ByChange:=solverange, Engine:
    SolverAdd CellRef:=solveconstr, Relation:=2, FormulaText:="1"
    SolverAdd CellRef:="$AN$141", Relation:=2, FormulaText:="$AN$140"
    SolverAdd CellRef:="$AN$142", Relation:=2, FormulaText:="$AN$140"
    SolverAdd CellRef:="$AN$143", Relation:=2, FormulaText:="$AN$140"
    SolverSolve userFinish:=True

    Range(sleep1).Select
    Selection.AutoFill Destination:=Range(sleep2), Type:=xlFillDefault
    Range(sleep2).Select

    Startrow = Startrow + 4
    Secondstartrow = Secondstartrow + 4
    sv = sv + 4
    weightrow = weightrow + 4
    x = x + 1

    Let covrange = "R" & Startrow & "C" & Lastrow & ":" & "R" & Secondstartrow & "C" & Last
    Let solverange = "AL" & sv & ":" & "AO" & sv
    Let solveconstr = "AS" & sv
    Let sleep1 = "AL" & sv & ":" & "AO" & sv
    Let sleep2 = "AL" & sv & ":" & "AO" & sv + 3
    Let SD = "R[" & weightrow & "]C[" & 6 & "]"
    Let SD2 = "R[" & weightrow & "]C[" & 4 & "]"
    Let weights = "R[" & weightrow & "]C:R[" & weightrow & "]C[" & columnrow & "]"
    Let weights2 = "R[" & weightrow & "]C[-1]:R[" & weightrow & "]C[" & columnrow - 1 & "]"
```

```
    Range("AT148").Value = x
Next i

Let covrange = "R" & Startrow & "C" & Lastrow & ":" & "R" & Secondstartrow & "C" & Lastrow
Let solveconstr = "AS" & sv
Let SD = "R[" & weightrow & "]C[" & 6 & "]"
Let SD2 = "R[" & weightrow & "]C[" & 4 & "]"
Let SD3 = "R[" & weightrow2 & "]C[" & 6 & "]"
Let SD4 = "R[" & weightrow2 & "]C[" & 4 & "]"
Let weights = "R[" & weightrow & "]C:R[" & weightrow & "]C[" & columnrow & "]"
Let weights2 = "R[" & weightrow & "]C[-1]:R[" & weightrow & "]C[" & columnrow - 1 & "]"
Let weights3 = "R[" & weightrow2 & "]C:R[" & weightrow2 & "]C[" & columnrow2 & "]"
Let weights4 = "R[" & weightrow2 & "]C[-1]:R[" & weightrow2 & "]C[" & columnrow2 - 1 & "]"

Let sleep1 = "AL" & sv & ":" & "AQ" & sv
Let sleep2 = "AL" & sv & ":" & "AQ" & sv + 3
Let solverange = "AL" & sv & ":" & "AQ" & sv

For i = 1 To 213

    Range("AL148").Select
    ActiveCell.FormulaR1C1 = "=COVAR(OFFSET(" & covrange & ",0,R147C-1),OFFSET(" & covrange & "
    Range("AL148").Select
    Selection.AutoFill Destination:=Range("AL148:AQ148"), Type:=xlFillDefault
    Range("AL148:AQ148").Select
    Selection.AutoFill Destination:=Range("AL148:AQ153"), Type:=xlFillDefault
    Range("AL148:AQ153").Select

    Range("AL132").Select
    Selection.FormulaArray = "=MMULT(R148C38:R153C43,TRANSPOSE(" & weights3 & ")/" & SD3 & ")"
    Range("AM132").Select
    Selection.FormulaArray = "=R132C38:R137C38*TRANSPOSE(" & weights4 & ")"
    Range("AN132").Select
    Selection.FormulaArray = "=RC[-1]:R[5]C[-1]/" & SD4
    Range("AO132").Select

    SolverReset
    SolverOk SetCell:="$AN$132", MaxMinVal:=3, ValueOf:=0.166666666666667, ByChange:=solverange
    SolverAdd CellRef:=solveconstr, Relation:=2, FormulaText:="1"
    SolverAdd CellRef:="$AN$133", Relation:=2, FormulaText:="$AN$132"
    SolverAdd CellRef:="$AN$134", Relation:=2, FormulaText:="$AN$132"
    SolverAdd CellRef:="$AN$135", Relation:=2, FormulaText:="$AN$132"
    SolverAdd CellRef:="$AN$136", Relation:=2, FormulaText:="$AN$132"
    SolverAdd CellRef:="$AN$137", Relation:=2, FormulaText:="$AN$132"
    SolverSolve userFinish:=True

    Range(sleep1).Select
    Selection.AutoFill Destination:=Range(sleep2), Type:=xlFillDefault
    Range(sleep2).Select

    Startrow = Startrow + 4
    Secondstartrow = Secondstartrow + 4
```

```
    sv = sv + 4
    sv2 = sv2 + 4
    weightrow = weightrow + 4
    weightrow2 = weightrow2 + 4
    x = x + 1

    Let solverange = "AL" & sv & ":" & "AQ" & sv
    Let covrange = "R" & Startrow & "C" & Lastrow & ":" & "R" & Secondstartrow & "C" & Last
    Let solveconstr = "AS" & sv
    Let SD = "R[" & weightrow & "]C[" & 6 & "]"
    Let SD2 = "R[" & weightrow & "]C[" & 4 & "]"
    Let SD3 = "R[" & weightrow2 & "]C[" & 6 & "]"
    Let SD4 = "R[" & weightrow2 & "]C[" & 4 & "]"
    Let sleep1 = "AL" & sv & ":" & "AQ" & sv
    Let sleep2 = "AL" & sv & ":" & "AQ" & sv + 3
    Let weights = "R[" & weightrow & "]C:R[" & weightrow & "]C[" & columnrow & "]"
    Let weights2 = "R[" & weightrow & "]C[-1]:R[" & weightrow & "]C[" & columnrow - 1 & "]"
    Let weights3 = "R[" & weightrow2 & "]C:R[" & weightrow2 & "]C[" & columnrow2 & "]"
    Let weights4 = "R[" & weightrow2 & "]C[-1]:R[" & weightrow2 & "]C[" & columnrow2 - 1 &
    Range("AT148").Value = x
Next i

End Sub
```

C.2 Global Minimum Variance Script

```
Sub GMV()

Dim covrange As String
Dim solverange As String
Dim solvevalue As String
Dim solveconstr As String
Dim solveconstr2 As String
Dim i As Integer

Startrow = 3
Secondstartrow = 156
Lastrow = 2
sv = 155
x = 0

Let covrange = "R" & Startrow & "C" & Lastrow & ":" & "R" & Secondstartrow & "C" & Lastrow
Let solvevalue = "$AG$" & sv
Let solverange = "AA" & sv & ":" & "AD" & sv
Let solveconstr = "AH" & sv
Let solveconstr2 = "AB" & sv
Let sleep1 = "AA" & sv & ":" & "AG" & sv
Let sleep2 = "AA" & sv & ":" & "AG" & sv + 3

For i = 1 To 43
    Range("AA148").Select
    ActiveCell.FormulaR1C1 = "=COVAR(OFFSET(" & covrange & ",0,R147C-1),OFFSET(" & covrange
```

```
    Range("AA148").Select
    Selection.AutoFill Destination:=Range("AA148:AD148"), Type:=xlFillDefault
    Range("AA148:AD148").Select
    Selection.AutoFill Destination:=Range("AA148:AD151"), Type:=xlFillDefault
    Range("AA148:AD151").Select

    SolverReset
    SolverOk SetCell:=solvevalue, MaxMinVal:=2, ValueOf:=0, ByChange:=solverange, Engine:=1, Er
    SolverAdd CellRef:=solveconstr, Relation:=2, FormulaText:="1"
    SolverSolve userFinish:=True

    Range(sleep1).Select
    Selection.AutoFill Destination:=Range(sleep2), Type:=xlFillDefault
    Range(sleep2).Select

    Startrow = Startrow + 4
    Secondstartrow = Secondstartrow + 4
    sv = sv + 4
    x = x + 1
    Let covrange = "R" & Startrow & "C" & Lastrow & ":" & "R" & Secondstartrow & "C" & Lastrow
    Let solvevalue = "$AG$" & sv
    Let solverange = "AA" & sv & ":" & "AD" & sv
    Let solveconstr = "AH" & sv
    Let solveconstr2 = "AB" & sv
    Let sleep1 = "AA" & sv & ":" & "AG" & sv
    Let sleep2 = "AA" & sv & ":" & "AG" & sv + 3
    Range("AH148").Value = x
Next i

Let solverange = "AA" & sv & ":" & "AF" & sv
Let sleep1 = "AA" & sv & ":" & "AF" & sv
Let sleep2 = "AA" & sv & ":" & "AF" & sv + 3

For i = 1 To 213
    Range("AA148").Select
    ActiveCell.FormulaR1C1 = "=COVAR(OFFSET(" & covrange & ",0,R147C-1),OFFSET(" & covrange & "
    Range("AA148").Select
    Selection.AutoFill Destination:=Range("AA148:AF148"), Type:=xlFillDefault
    Range("AA148:AF148").Select
    Selection.AutoFill Destination:=Range("AA148:AF153"), Type:=xlFillDefault
    Range("AA148:AF153").Select

    SolverReset
    SolverOk SetCell:=solvevalue, MaxMinVal:=2, ValueOf:=0, ByChange:=solverange, Engine:=1, Er
    SolverAdd CellRef:=solveconstr, Relation:=2, FormulaText:="1"
    SolverSolve userFinish:=True

    Range(sleep1).Select
    Selection.AutoFill Destination:=Range(sleep2), Type:=xlFillDefault
    Range(sleep2).Select

    Startrow = Startrow + 4
```

```
    Secondstartrow = Secondstartrow + 4
    sv = sv + 4
    x = x + 1
    Let covrange = "R" & Startrow & "C" & Lastrow & ":" & "R" & Secondstartrow & "C" & Last
    Let solvevalue = "$AG$" & sv
    Let solverange = "AA" & sv & ":" & "AD" & sv
    Let solveconstr = "AH" & sv
    Let solveconstr2 = "AB" & sv
    Let sleep1 = "AA" & sv & ":" & "AG" & sv
    Let sleep2 = "AA" & sv & ":" & "AG" & sv + 3
    Range("AH148").Value = x
Next i

End Sub
```

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