The Implication of Currency Hedging Strategies for Pension Funds

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Abstract

In this paper, we analyze currency risk by taking advantage of Markowitz’s mean-variance model based on two OECD suggested pension fund investment guidelines. We include two groups of investment instruments for Euro currency based pension fund investors: (1) hedged and unhedged indices, (2) unhedged indices with forward contracts. Based on our data and its period, we find hedging currency risk by using forward contacts reduces the risk in our portfolio. In addition, we come up with some recommendations for the pension fund analysts regarding the difficulties in implementing Markowitz Mean-Variance Models.
1. Introduction

Pension fund analysts have many international classes of investment to choose from. The vast selection of asset classes allows investors to easily diversify their portfolio risk with using certain financial models; however, currency risk arises during this diversification process. The uncertainty in exchange rate often risks projected returns of the portfolios and affects the cash flow of a pension fund. To avoid such risk, pension fund analysts face the decision of whether to hedge currency risk and at what percentage.

There are various strategies to tackle the currency optimization problem. To some, currency hedging is about the problem of tactical asset allocation problem while for others the problem is to do with strategic asset allocation. Regardless of how analysts see the currency hedging strategies, one must implement such strategies in complement to the benchmark and investment guidelines on an institutional level.

From the perspective of public pension funds, guidelines often require transparency to safeguard the integrity and governance structure of the fund. It is often the task of internal analysts or other contracted financial institutions to derive a set of investment allocation strategies in relation to the fund’s position and its complexity. According to the OECD Guidelines, it recommends its member countries manage pension funds following a risk-based benchmark:

“As good pension fund governance should be ‘risk-based’, the division of responsibilities should reflect the nature and extent of the risks posed by the fund”. (OECD Guidelines for Pension Fund Governance 2009, p5).

The ‘risk-based’ investment principle proposed by the OECD Guideline clarifies that investment strategy be implemented with the objective of minimizing risk given the objective of the fund. In addition, the OECD Guideline on Pension Fund Asset Management further sets out the jurisdiction regarding the variables as follows:

“In the case of defined benefit plans, the goal of the investment function is to generate the highest possible returns consistent with the liabilities and liquidity needs of the pension plan, and in light of the risk tolerances of affected parties” (OECD Guidelines on Pension Fund Asset Management 2006, p2).

With the recommended investment guideline set up by the OECD, we analyze the currency hedging with the effect on risk reduction/return maximization based on the risk tolerances and the asset/liability performance of the pension fund. Nevertheless, many global fund managers offer to manage currency with an active currency overlay program. The program claims extra returns through currency volatility and can enchant the performance of returns at greater risk. However, in this paper, to protect the pension fund’s resilience to risk, we conduct the research with a passive currency management style.

1.1. Background: A Brief Review on Pension Fund Investment Policies and Governance

A well-defined guideline is important for pension analysts to follow. It outlines the investment principles of the fund and sets out the benchmark for pension managers/analysts to follow. Often, managing a pension fund is similar in approach to managing a global
investment fund: achieving a steady stream of cash flow to sustain the obligations to beneficiaries. Global asset managers are more inclined to achieve maximum return by taking risks, while pension fund managers tend to focus on balancing risk and return in terms of future income rather than current capita. To balance risk and return in pension funds, it is important to install adequate supervisions and to manage the trade-off between income and risk to fulfill its obligations. In addition, a pension fund requires an effective investment policy guideline to govern the analyst’s good practice and monitoring. For example, a guideline can enforce the inclusion of Socially Responsible Investing (SRI) projects for ethical investments. Norwegian pension fund is an outstanding endorser by constructing its policy guidelines in avoidance of investing in unethical acts and omissions\(^1\). In addition, a guideline simplifies the monitoring and the evaluation process of the performance on asset and liability by setting up an explicit benchmark for investment strategies to align with. Therefore, such a guideline should be constantly reviewed and mandated by the Management Board on an evolving basis for the pension funds and financial market.

In order to derive a sound investment policy guideline for pension funds, analysts need to follow the objective of the investment portfolio and carry out an examination based on an asset and liabilities management (ALM) study. The analysts should examine and quantify the risk of the financial market with realistic parameters and then project the cash flow of the liabilities. Once the parameters have been defined, the analysts can derive a reasonable set of assumptions and results based on the studies and estimations. Using these assumptions, the analysts can further match the risk/return and obligations between the asset and liabilities of pension funds in regard to the portfolio goals, objectives, and constraints. The final results should be presented to the Board for review and comments. If it is approved, the assumptions shall be reviewed by the Budget Committee for further approval. If not in compliance with the Board recommendation, analysts should additionally revise and amend the investment assumptions accordingly for further review.

1.2 Thesis Motivation

Institutional investors among 34 OECD member countries had totaled USD 78.2 trillion in 2012 with USD 30.0 trillion coming from investment funds according to OECD Pension Markets in Focus 2013. This large pool of funds, therefore, needs to be responsibly managed. The investment strategies are crucial for the growth of defined benefit pension funds to meet obligations for the plan’s participants as they serve an important part of the steady retirement income stream. The large body of the member countries at OECD is accustomed to matching their pension investment strategies based on the domestic components such as demography, geopolitics, and economy. However, under the integration of the global financial market, many pension fund analysts widely diversify their investments seeking stable and low-risk returns.

Diversification can result in a large purchase of financial products in foreign currencies. To address the currency exposure, analysts need to estimate the currency risk to predict the outcome of currency exposure to unhedged portfolios. Besides analyzing the

\(^1\) According to the Government Pension Fund of Norway, the fund endorses responsible investment in the areas of equal treatment of shareholders, shareholder influence and Board accountability well-functioning, legitimate and efficient markets, children's rights, climate change risk management, and water management.
currency impact, analysts should address issues such as the possibility of currency hedging adding additional value to the fund in risk reduction or gains in return. Given the large literature contribution to the pension fund asset investment guide; this paper is drawing on the OECD recommended investment guidelines to develop ideas and recommendations for the implication of theoretical models in pension plans among OECD countries. In addition, the experience of an internship at a pension fund company, with an interdisciplinary knowledge between theoretical research and practical studying, has helped the author to identify the obstacles of implementing models. The paper, therefore, aims to provide adaptive solutions to the difficulties in practical use of the financial models.

1.3 Research Question

There are many solutions for deriving a set of optimized hedge ratios. Nevertheless, the benefit is mainly investor specific. Instead of optimizing a set of hedge ratios, some investors choose to apply a simple ratio such as 50% to avoid “regret risk”\(^2\). Others may hedge 100% currency risk to simply ignore the exchange rate risk of the portfolio. These ratios depend heavily on an investor’s risk aversion or utility function. Investors with a mean variance utility function can implement currency hedging strategy by treating currency as a separate asset or simply mixing an unhedged index to its parental index during the portfolio optimization process. While investors with other utility functions can benefit from currency hedging or simply ignore currency risk to take advantage of the local currency growth. For institutional investors such as pension funds, it can be difficult to define such utility function given its large composition of members which are composed of different sets of utilities. However, investment guidelines solve this issue. Given the recommended investment policy, a risk based risk aversion level or a utility function can be derived to fit into the utility mean-variance model and can further help the analysts to develop the portfolio weight optimization problem.

Once the utility is defined, analysts can use it to develop the Markowitz mean variance portfolio to meet the risk minimization objective of a pension fund. The objectives such as risk minimization and return maximization can be incorporated into the utility equation during the optimization process. It will depend on the role of currencies in relation to the entire portfolio. Currency hedging can reduce the portfolio’s risk. However, it can also lead to an undesired reduction in the portfolio’s expected return if the currencies’ return makes up a significant portion of the portfolio. Therefore, a risk/reward ratio such as the Sharpe ratio should be considered. In addition, optimizing asset allocation for a pension fund is a challenging issue with great future uncertainties. Historical analysis or simulation programs can lead to various hedging results due to the data, base currency, and period difference. Therefore, the first part of the paper will address the probability of an under-funded pension fund to meet its target wealth objective in a given year by studying the asset side. To estimate the probability, we will model currency risk with the mean-variance portfolio optimization approach under the OECD Guidelines on Pension Fund Asset Management 2006. The second part of the paper will take into account the pension fund liability by treating the pension benefit payment as a periodic bond with an internal rate of return to the portfolio bond return. By including the liability return in the currency hedged portfolio, we can derive a unique set

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\(^2\) Regret risk is caused by the emotion of pain and anger due to bad decisions in the past that affects the performance of ex-post investment (Michenaud et.al 2008)
of the optimal allocation assumptions to re-examine the probability to meet the target’s full funding level, which analysts can present to the Board for review.

Using this framework, we examine the impact of regulations on pension investment decisions based on a hypothetical pension fund with pre-selected asset classes. Two objectives shall be achieved. First, we intend to compare the risk-adjusted return of the pension fund with and without fully hedging currency risk. Secondly, we want to investigate the effect of using the optimal hedging ratios we derived to analyze the probability of a pension fund’s wealth growth. The currency risk can add extra volatility based on the time horizon, economic cycles, and risks in the financial market. To achieve a clear transparency of the assumptions to other governing bodies of the fund, a standard model such as Markowitz mean-variance portfolio model is an adequate start.

1.4 Outline of Analytical Framework of Asset Liability Management

The structure of the thesis will be the following: literature reviews, asset allocation analysis, asset/liability optimization, and conclusion. In the literature review section, we review the historical debate regarding currency hedging and discuss relevant results. In the asset allocation analysis section, we first examine the currency risk and hedging strategies. Then, we shall review some of the relevant models such as Dividend Discounting Model and Markowitz Mean-Variance Model. At last, we will incorporate these models to drive an optimal set of asset allocations along with the hedge ratios. During the optimization process, we mix hedge and hedged indices. In addition, we will treat currency as a separate set of the assets. We then include the liability component in our existing models to further analyze the impact of liability on the funding rate of pension funds. In the last part of the thesis, we conclude the findings and recommend some topics for further research.

2. Literature

There is a large body of literature devoted to the topic of currency hedging regardless of private or public settings. The results are quite controversial and different. The difference in results is due to the fact that the researchers are relying on various objectives: maximizing of returns, minimizing of risks, using active/passive currency overlay strategies for extra returns, and diversifying portfolios. Each objective can lead to a different set of results. Most importantly, time horizons, exchange rates, currency pairs, asset classes and underlying assumptions in the model are the main determinants of hedge ratios. However, the key purpose of the research lies in the hypothesis that currency hedging can reduce volatility by diversifying into global assets to offset the covariance among assets through the interaction of currency with other assets. In practice, projected hedge ratios have limited predicting power given currency’s volatile property. Some investors instead conventionally hedge fully or fifty percent of the portfolio to protect their investments. This often leads to different outcomes depending on analysts’ objectives. In this section, the papers will examine two sets of arguments in history that lead to the present debate regarding currency hedging in relation to risk and returns. In addition, some imperative results in early research will be discussed pertaining to the objective of risk reduction.
2.1 “Free Lunch” Arguments and Universal Currency Hedge Ratio

Because of the differences in base currency, time period, hedging methods, underlying assumptions and objectives, there are several sets of results regarding currency hedging. In accordance to the research topic of this paper, we focus on the work by claiming currency hedging reduces risk under the Asset Pricing Theory. Other proposals such as global equilibrium hedging ratio for all investors will also be included (Solnik 1974; Adler and Dumas 1983; Perold and Schulman 1988; Black 1988-1990).

In terms of considering risk minimization, Solnik (1974) published the results of analysis for various stocks and bonds from different developed countries. In his research, he found fully hedging exchange risk slightly reduced risks assuming foreign currencies are uncorrelated with other assets’ returns. Similar results were also discovered by Eun and Resnick (1988) when using forward contracts or options with unitary hedge ratios. The result of the currency hedged portfolio, relative to its unhedged portfolio, showed reduction in the portfolio’s volatility without lowering the returns of the portfolio. Later, this argument has been called a “free lunch” by Perold and Schulman (1988), since it is costless for investors to hedge currency.3 Other researchers point out that the “free lunch” argument only holds under the assumption that the exchange rate is uncorrelated with local asset returns (Ajaoutel Tuchschmid 1996). This assumption is unrealistic and the argument does not hold in the long run. Researchers such as Glen and Jorin (1993), Rudolf and Zimmermann (1998), Ajaoutel and Tuchschmid (1996) and etc. also showed that currency itself can be treated as a separate set of assets and positioned in the mean-variance portfolio. De Roon and et al. finds that currency lowers the volatility of international equity/bond portfolio at the same time it has negative effects on returns, Sharpe ratio and skewness.

Under the Purchasing Power Parity theory (PPP), Froot (1993) and point out that the “free lunch” argument can work in the short-horizon, but may not hold in the long horizon. Instead, hedging does not reduce the variance in the long horizon but can add extra volatility with the occurrence of an interest surprise shock. If PPP theory holds for the exchange rate, Froot implies that the real exchange rates and asset prices display mean reversion, the only factor can affect the exchange rates is the unexpected inflation and real interest differentials of a country relative to another. His conclusion is drawn from U.K based investors investing in U.S by applying 200 years of U.S. financial returns data. The currency itself is naturally hedged, and the hedge ratio is only a speculation of future inflation shock and real interest differential. So, it is better for unhedged currencies if the investors have a long horizon.

Empirical studies conducted by Campbell, Viceira, and White (2003) shows that the long-term investors are interested in holding international bills or currency exposure in reserve currencies such as dollars and Euros. The stable interest rates of the two currencies can help minimize the real interest rate risk, which is in support of Froot’s conclusion that the hedging in the long horizon reflects the investors’ concern with real interest differentials and inflations. In addition, Campbell Medeiros and Viceira (2007), also find that full hedging equity does not necessarily reduce portfolio risk in the long run given that the currency displays correlation with the equity market. For bonds, full hedging tends to be optimal in the sample. There are similar findings by other methods with different sets of data and time periods.

3 There is a small fee for currency hedging. However, it can very low depending on the scale of investments.
Instead of customizing a set of hedging ratios for a portfolio; Black (1989, 1990) introduced a universal hedge ratio for all global investors based on the Siegel Paradox and some simplified assumptions. The Siegel Paradox argues against the currency having a zero expected return. The quarterly exchange rate between dollars and marks in Black’s work indicates that the majority of the currency pairs returns are positive. Extending the Siegel Paradox to the international CAPM model, Black estimated the equilibrium in the fraction of currency that should be hedged is approximately 77% based on the degree of risk aversion of investors. The ratio should be less than 100%. Critics claim that the universal hedging ratio is shortsighted due to some of its over simplified assumptions. Solnik (1993) states, “hedge ratios are not universal and depend on investors’ preferences and relative wealth. So-called universal hedging rules are devoid of solid theoretical underpinning and practical applicability.” Bailey et al. (2000) also points out that a constant universal hedging ratio for all is unrealistic provided the model does not take into account inflation, the consumption basket, and different risk.

2.2 Other Relevant Literatures

A large portion of the literature has addressed hedging from the perspective of global investors. Abken et al. (1997) using forward contracts for a portfolio that consisted of equity and bonds during 1986 to 1996 found that the hedged equity portfolio’s efficiency frontier was more efficient than the unhedged portfolio efficiency frontier line in 1980-1985. However the result reverses in the following ten years. Hedging in the bond market is more effective than the equity market given the improvement of dollar returns. Glen et al. (1993) also used forward contracts showing that U.S. investors can benefit from volatility reduction in the bond market in contrast to less pronounced effects in the equity market. Other researchers and this paper also found similar results which could mean that the bond market is relatively stable compared to the equity market. Bond returns co-varies with the exchange rates more than the equity market. However, hedging bond portfolios can also reduce the return given historical data. Therefore, whether or not to hedge will depend on the investment strategies and investor’s utility curve which this paper will address in section 4.

3. Analytical Framework of Asset Allocation

In this section, we first construct a general model of the wealth growth of a hypothetical pension fund. By analyzing the model, we will go into the optimization process to improve the risk/return of the individual return variables with the inclusion of currency hedging. After the optimization process, we examine if the improved optimized variables have an effect on the original wealth model.

To understand the wealth management of a pension fund, analysts must make realistic assumptions. To derive such assumptions, one must consider the current funding level and risks based on the guidelines. Then reasonable models and constraints should be applied to prevent extreme asset weight (corner solutions). During the investment procedure, asset selection falling outside the benchmark should be considered as risky strategy, and the objective of volatility minimization in the portfolio should be incorporated during the optimization process. To begin our analysis, we first apply statistic models to analyze the growth of an underfunded hypothetical pension fund.
We assume that under the OECD guidelines a pension fund’s annual target return is 5.0\% with a required risk of 10.0\% and the initial wealth of the current pension fund is 1 billion Euros. Analysts are tasked to estimate the fund growth to a target full funding level, for instance, 3.0 billion Euros in the following 30 years. To estimate the growth of the fund, analysts can estimate the probability of the fund wealth growth by applying the statistic model such as cumulative distribution density function (cdf). Doing so can help the analysts to see the likelihood of the fund meeting the target wealth, assuming annual growth is under normal distribution, where the expected return is independently and identically distributed (i.i.d). Suppose a pension fund has the initial wealth of $W_1$ including the current year of realized gain/loss of the investments and all initial wealth is invested in the fund portfolio in the following 30 years without further withdrawals. Here we can use the following mathematical expressions to model the wealth growth of a pension fund:

At the beginning, the wealth of current pension fund is $W_1 = 1$ billion Euros. Then,

- At the end of the period 1, the fund grows to $W_1(1 + x'r_{1+1})$;
- At the end of the period 2, the fund grows to $W_1W_{1+1}(1 + x'r_{1+2})$;
- ....
- At the end of the period 30, the fund grows to $W_1W_{1+30}(1 + x'r_{1+30})$;

Therefore, for any period, the fund can be expressed as follows:

$$W_{1+T} = W_1 \times \prod_{t=1}^{T}(1 + x'r_{1+t})$$

Omit the terms of the higher order, the above expression approximates to

$$W_{1+T} \approx W_1 \times \sum_{t=1}^{T}(1 + x'r_{1+t}) \quad (3.1)$$

Under the i.i.d. assumption, we can project the total wealth of the fund using equation 3.1. Let $\sum_{t=1}^{T} x'r_{1+t}$ be the estimated total portfolio return for the next 30 years, where $T$ is the total estimated years, $x$ is the optimized asset allocation, and $r_{1+t}$ is the corresponding asset return to $x$. Given the i.i.d property of the annual return along with a fixed set of portfolio weight, we can easily factor out the initial wealth, $W_1$ and have the total expected return within $T$ years as the following:

$$E(W_{1+T} \approx E(W_1 + W_1x'r_{1+1} + W_1x'r_{1+2} + \cdots + W_1x'r_{1+T})$$

$$E(W_{1+T} \approx W_1 + W_1E(x'r_{1+1}) + \cdots + W_1E(x'r_{1+T})$$

$$E(W_{1+T} \approx W_1(1 + Tx'r) \quad (3.2)$$

Under the assumption of annual return is independently and identically distributed (i.i.d), we express $x'r_{1+t} = E(x'r_{1+t}) = 5.0\%$. Thus, the expected total wealth in $T=30$ years can be written as:

$$E(W_{1+30} \approx E(W_1) + E(W_1x'r_{1+1}) + \cdots + E(W_1x'r_{1+30})$$

$$E(W_{1+30} \approx 1.0(1 + 30 \times 5.0)$$

$$E(W_{1+30} \approx 2.5 billion$$
In addition, we can express the required 10% risk as volatility as $\sigma_{1+T} = 10$. Under the i.i.d assumption, we can arrive the following by the sum of normally distributed random variable theorem:

\[
\sigma^2_{W_{1+T}} = \sigma^2_{1+1} + \sigma^2_{1+2} + \cdots + \sigma^2_{1+30} = 30 \times \sigma^2_{1+30} = 30.0
\]

\[
\sigma_{W_{1+30}} = \sqrt{30} \times \sigma^2_{1+30} = 54.8
\]

where $\sigma_{W_{1+30}}$ is the expected risk of the fund in 30 years. Summing up all, we get $N(\mu_{W_{1+30}} = 2.5, \sigma W_{1+30} = 30.0$ or $N(\mu W_{1+30} = 2.5, \sigma W_{1+30} = 54.8$, where $\mu W_{1+30}$ is the mean wealth in 30 years. Under these set of assumptions, the probability of the pension fund to meet the target wealth accumulation will be expressed as:

\[
P_{r}(W_{1+T} < W_{\text{target}})
\]

(3.3)

where $W_{\text{target}}$ is the 3.0 billion target wealth, and $P_{r}$ is the probability. To find the probability of the fund’s wealth accumulation after 30 years, we employ the cumulative distribution functions to analyze the expected growth of the fund given $E(W_{1+T})$ is a continuous function. Using the computing program, Matlab, we derive the probability of the expected wealth accumulation as shown in Figure 3.1., where we have $P_{r}(E(W_{1+30}) < 3.0 \text{ billion}) = 82\%$, which can be interpreted as there is 82% chance that the fund will accumulate less than 3.0 billion target wealth in next 30 years.

**Figure 3.1:** Cumulative Distribution Function of the Assumed Fund Growth.

Notes: the calculation and graph have been implemented by author and Matlab program. We apply equation 3.2 under $N(2.5, 54.8)$ i.i.d, where there is 82% chance that fund will accumulate less than 3.0 billion target wealth in next 30 years.

There is 82% probability that the pension fund will not accumulate the target wealth. In order to improve the probability of the fund to meet its target wealth by 30 years, pension fund
analysts can optimize its asset allocation strategy by achieving risk reduction or return improvement. Diversification is one of the tools used by many asset managers to enchant the portfolio performance. Following the OECD recommended investment guidelines, analysts can consider including currency hedging assets to either reduce the risk or improve the return of the fund. Due to investing for a pension fund, we will mainly follow a passive strategy to reduce the risk of using currency hedging.

3.1. Asset Allocation Models and Currency Hedging

In the light of the Modern Portfolio Theory—mean variance portfolio approach proposed by Harry Markowitz (1952), and Capital Asset Pricing Model (CAPM) later developed by William Sharpe, John Lintner, and Jan Mossin (1964), a defined benefit pension fund can utilize mean variance portfolio approach and its extension to construct a pension fund portfolio. The benefit of using mean-variance is that it extends quite naturally to the global context by allowing investors to construct a diversified global market portfolio. Since defined benefit plans consist of a guaranteed future payment to a large number of retirees and/or their partners, analysts treat each individual retiree’s risk tolerance by replicating a pension fund portfolio return to the global market portfolio. This is a good starting point because the global market portfolio represents each retiree investments in global assets and receives an equal portion of returns from the market. This market neutral position generates transparency and stability for each beneficiary and it is easy for the governing body to evaluate the portfolio.

Investing in the global market using Markowitz is a reasonable start; however, a new issue arises from global investing—currency risk. In an ideal world, where Purchasing Power Parity Theory (PPP)\(^4\) holds, pension fund managers should disregard the foreign currencies in the liabilities. However, this is not the case in the real world—price levels fluctuate among countries and institutional investors have different objectives and interests around exchange rates. Therefore, the analysts of a pension fund should tailor the Markowitz model to alternatives or complex models depending on the risks, funding, contributions, and policy guidelines. In order to deal with a multiple currency issue, the analysts must draw a clear conclusion on the base currency which significantly affects the outcome of the assumptions.

In the case of pension funds, the base currency should be the underlying currency paid out to the retirees in the pension plan. This is obvious for reasons of reducing liquidity risks, currency exposure, and clarifying actuarial calculation. Once the base currency is defined, analysts should consider the exposure of foreign currencies of the portfolio based on the risks, periods, and asset classes within pension funds.

Researchers have used various approaches to optimize the hedging ratio. Most studies are carried out from the historical perspective by examining historical returns of portfolios and other currency instruments. For instance, De Roon et al (2001) shows that a U.S dollar

\(^{4}\) According to OECD, Statistics Directorate, Purchasing Price Parties are the rates of currency conversion that equalize the purchasing power of different currencies by eliminating the differences in price levels between countries. In their simplest form, PPPs are simply price relatives that show the ratio of the prices in national currencies of the same good or service in different countries. PPPs are also calculated for product groups and for each of the various levels of aggregation up to and including GDP.
international equity portfolio during 1977-1998 can be enchanted by dynamic hedging, whereas a static hedging portfolio remains the same for mean variance investors. Cambellet al (2010) believes that several currencies (U.S. dollar, Swiss Franc, and Euro) should be favoured by risk-mean variance investors to lower the risk of the portfolio given these currencies move against the world equity market. Vecchio (1999) concludes that in the context of a portfolio with U.S. equities, leaving foreign equity exposure unhedged provides a greater degree of diversification rather than leaving foreign equity hedged. In addition, Vecchio incorporates the data in 1973 to 1997 equity market into a portfolio optimizer. The optimizer ignores the currency hedged portfolio instead includes the unhedged portfolio. These analyses are very useful for helping investors understand the importance of currency hedging in relation to the asset’s return. Before implementing appropriate optimization models, analysts should first examine the currency risk. By doing so, it promotes transparency for members and the Board. Also, it can provide an estimation of the magnitude of risk reductions in the portfolio. Therefore, we will investigate the currency exposure in a fixed portfolio based on historical performance.

3.2. Data

Before analyzing currency risk, we will review and summarize our data. The data that we use in this paper are index funds existing on Data Stream by Thomson Reuters Company. The selection of index funds are aimed at ensuring that the investment portfolio achieves the following: (i) diversification regionally and globally, (ii) low transaction costs, (iii) stable returns with low risks, and (iv) easy to be replicated by other index funds. We choose to use the equity index return data from Morgan Stanley Capital International (MSCI), and the bond index return data from Citigroup Fixed Income Index LLC. These data series are chosen on a monthly basis, and for equity index we additionally divide it into European Monetary Union equity, emerging market equity, and developed market equity. All indices are value-weighted and selected on a minimum requirement for size, liquidity, free-floated, and adjusted in accordance to dividends payments on a daily basis. In addition, we set the Euro as our portfolio and pension payment base currency. Along with the two main assets, we also select some alternative assets which include real estate market and risk free asset. The index for real estate is taken from MSCI Europe Real Estate Index given that the interest of this paper is on currency hedging within equity and bond asset classes. The risk free asset is in Euro cash and we apply the Euro Interbank offered Rate.

The time span is a ten year horizon for our first part of analysis. This is relatively short for multiple reasons. First, the base currency Euro was circulated in 1999 with the creation of the European Monetary Union. This affects some of the availability of the data. One may argue some indices have Euro currency data before 1999 by simulation. This raises further questions regarding the conversion methodologies which vary from index providers and some conversion scales are not fully presented in reports. These limitations affect the accuracy of the results. Secondly, some of the essential asset classes are fairly new such as the inclusion

5 Formerly was branded under the name, Salomon Smith Barney Family of Fixed Income Indices.

6 Although it was first introduced at Maastricht Treaty in 1992.
of currency hedged indices in our sample portfolios especially in emerging markets. In the MSCI hedged index, the index hedges the foreign currencies to domestic denominations by selling currency forward on a daily or monthly basis. This reduces the volatility of currency risk, and it replicates the index return to a domestic index. It is practical investing in currency hedged indices rather than buying and selling currency forward individually given the management of large scale pension funds and the cost reduction achievement in a hedged index fund. However, the currency hedged indices are quite “opaque” and illiquid in the sense that some of the constituents are not listed in the public reports or in the data. Also currently, the MSCI index is only hedged to U.S dollars and Euros. Thirdly, we employ some of the main indicating figures such as 2018-2060 GDP growth and expected inflation by the OECD annual outlook. These data will help us to predict the future returns of underlying assets in our model. We also use the 15 years data for the forward contracts from 6 major countries across the globe released by WM/Reuters Company based on the introduction date of our base currency Euro. With the selection of these global data, it helps us to achieve diversification for the members and allows us to replicate portfolio in a practical approach.

3.3. Understanding Currency Risk

In the context of the mean-variance approach, we examine the effect of currency risk on a pension fund asset return during two short term periods: 2008-2010 coined by the period of the financial crisis and 2002-2008 a relatively stable period of economic growth. Here we apply the average monthly return of the MSCI indices along with the Citigroup world bond index to a 60/40% stock to a fixed-income portfolio. The 60/40 rule has been a reference model for long-term investors started in the 1950s after the introduction of the Markowitz mean variance portfolio theoretical framework. The mix is based on the universe investable markets which consist of stock and bond in the light of the limited investment vehicles during the time. Nowadays 60/40 has become quite shortsighted. Fraser et al. (2000) points out, the mix does not recognize wealth, risk aversion and investment horizon. Many critics of managers have claimed the mix lacks other assets to diversify risk, and it is impossible to replicate the strong returns of bonds that delivered over the last 30 years. However, the simplicity of the mix and the ease of incorporating currency hedged assets can help us to identify how the exchange-rate volatility relates to the entire portfolio. Applying the data, we arrive at the following set of returns, volatility and Sharpe ratios.

Table I. 1) Mean and Volatilities for All Asset Classes in 2002-2010

<table>
<thead>
<tr>
<th></th>
<th>EME (unhedged)</th>
<th>World Equity (unhedged)</th>
<th>World Equity (hedged)</th>
<th>Emerging Market</th>
<th>Global Gov't Bond (unhedged)</th>
<th>Global Gov't Bond (unhedged)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>9.8%</td>
<td>4.58%</td>
<td>2.59%</td>
<td>8.20%</td>
<td>5.98%</td>
<td>6.78%</td>
</tr>
<tr>
<td>Volatility ($\sigma$)</td>
<td>18.2%</td>
<td>16.5%</td>
<td>14.4%</td>
<td>21.2%</td>
<td>7.1%</td>
<td>3.0%</td>
</tr>
</tbody>
</table>
Table I. 2) Hedge ratios regarding 60%-40% portfolio in 2002-2008 and 2008-2010

<table>
<thead>
<tr>
<th>Hedge Ratios</th>
<th>2002-2008</th>
<th>2008-2010</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0%</td>
<td>50%</td>
</tr>
<tr>
<td>Real Annual Return</td>
<td>8.2%</td>
<td>7.7%</td>
</tr>
<tr>
<td>Volatility</td>
<td>9.3%</td>
<td>8.4%</td>
</tr>
<tr>
<td>Sharpe</td>
<td>88.4%</td>
<td>92.1%</td>
</tr>
</tbody>
</table>

Note: The above results is derived from Data Stream and calculated by the author. Table 1 shows the means and volatilities of the underlying asset classes. Table 2 shows the optimal currency hedge ratios for 2002-2008 are 100% in developed market equity, 0% in emerging market. While this ratio changes to 0% for all equity assets during 2008-2010 financial crisis.

The fixed-income portfolio shows a significant risk reduction by almost half of the volatility from 7.1% to 3.31% with a margin of 0.08% loss in risk premium for every 10% increment in hedge ratio. The large effect of risk reduction in fixed-income portfolio could be the fact that government bond returns are largely affected by the movement of interest and exchange rates. Investors choose to buy/sell foreign bonds carefully when interest rates change. This implies that exchange rates tend to move together with interest rates. Since interest and exchange rate fluctuated a lot, currency hedging in bond portfolio largely eliminates the fluctuation by securing the foreign bond returns in domestic currency. Therefore, it adds diversification. In an equity market, hedge currency is less effective with a margin of 0.35% loss in risk premium for every 10% increment in hedge ratio since the equity is sensitive to multiple risk factors: global factors, geopolitical factors and firm’s individual risk factors. Aside from examining each market individually, we also observe the effect on hedging currency risk as a whole. The results are presented by comparing the efficiency frontier lines of mixing of the hedged indices and unhedged indices with the same data above.

Aside from studying the impact with a fix hedging ratio, we model the equity/bond risk in relation to the currency risk to further induce the effect of currency risk on the entire portfolio. The exchange rate movement is affected by a variety of factors such as economic policies, geopolitical changes, and financial developments. Besides the complexity of currency movements, the equity and bond market react differently to currency risk. Vecchio (1998) finds that in the context of a portfolio with U.S. equities, leaving foreign equity exposure unhedged provides a greater degree of diversification rather than leaving foreign equity hedged. However, Solnik’s finding is very different. For the bond market, it is recommended to be fully hedged because the bond market is highly correlated with change in exchange rate, yet for the equity market an optimal ratio should be derived (Solnik, 1999). Given the complexity of exchange rate movements, we examine the risk reduction effect of equity and bond markets separately during 2004-2014 by varying hedge ratios. The result on the impact of currency hedging is presented in Figure 3.2.
Figure 3.2: Volatility Reduction in Equity/Bond Market with Currency Hedging

Note: the graph is derived from author’s calculation without short sale. The equity market data is taken from MSCI developed market Index, MSCI Emerging Market index, and their corresponding hedged indices. For fixed-income we used Citigroup World Government Bond Index and its hedged index make up the bond market. Currency hedging reduces about 2% risk in equity market and 4% risk in bond market.

The fixed-income portfolio shows a significant risk reduction by almost half of the volatility from 7.1% to 3.31% with a margin of 0.08% loss in risk premium for every 10% increment in hedge ratio. The large effect of risk reduction in fixed-income portfolio could be the fact that government bond returns are largely affected by the movement of interest and exchange rates. Investors choose to buy/sell foreign bonds carefully when interest rates change. This implies that exchange rates tend to move together with interest rates. Since interest and exchange rate fluctuated a lot, currency hedging in bond portfolio largely eliminates the fluctuation by securing the foreign bond returns in domestic currency. Therefore, it adds diversification. In an equity market, hedge currency is less effective with a margin of 0.35% loss in risk premium for every 10% increment in hedge ratio since the equity is sensitive to multiple risk factors: global factors, geopolitical factors and firm’s individual risk factors. Aside from examining each market individually, we also observe the effect on hedging currency risk as a whole. The results are presented by comparing the efficiency frontier lines of mixing of the hedged indices and unhedged indices with the same data above.
Figure 3.3: Efficiency Frontiers with Currency Hedging

Note: the graph is derived from author’s calculation without short sales. The equity market data is taken from MSCI developed market Index, MSCI Emerging Market index, and their corresponding hedged indices. For fixed-income, we used Citigroup World Government Bond Index and its hedged index make up the bond market.

Combining both equity and bond data, we find the efficiency frontiers also show hedge currency risk can decrease volatility while reserving return to a certain extent as the two efficient frontiers eventually converge in Figure 3.3. Since hedging for the bond market is more efficient, this implies that the optimizer would reward bonds over stocks in order to minimize risks and produce same level of return. Pension funds have different polices on asset allocations. If the fund specifies to invest only in equity, the “free lunch” argument may not hold based on our data and the period. With these observations and the preliminary results, we should choose our models carefully and further investigate the impact of currency risk on pension funds given its long investment horizon. It seems beneficial to fully hedge bonds in order to achieve the risk minimization objective without incurring loss in risk premiums.

4. Hedging Currency Risk: Selection of Finance Models

Based on the study in chapter 3, we can see currency risk has an impact on portfolio performance. Therefore investors should consider hedging such risk. In this section, we will optimize the portfolio by considering the currency exposure. We will start with an unhedged portfolio and include currency hedged assets to derive hedge ratios. We will employ several financial linear models that promote a transparent analysis following the OECD guidelines’ requirements. In additional some supplementary works based on academic studies and theories will be introduced for further investigation and research in the implication of pension funds.

In the context of model selection, first of all, we will apply Dividend Discount Model (DDM) by Gordon and Shapiro (1956) to estimate the asset return for the next 46 years setting Gross Domestic Product as the driver of the projection. The estimated currency hedging strategies
based on history does not offer a solution to investors in term of the future perspective especially in the context of pension funds with long investment horizons. Therefore, it would be realistic to apply the DDM to model an appropriate investment strategy. Secondly, we choose the Markowitz’s mean-variance to model currency hedging strategy. The Markowitz’s model will be ideal to satisfy the guideline’s objective by studying risk based return of the investment portfolio along with an optimal currency hedging strategy. Also, it can help us to find an optimal risk aversion level along the efficient frontier given the fact many pension funds do not explicitly define a risk aversion level or a utility function. All these models will cover the topic of deriving an optimal hedging ratio.

4.1. Dividend Discount Model

In 1956 Gordon and Shapiro published a paper that made a reference to the model. The Dividend Discount Model (DDM) assumes that the cash flow an investor receives from a firm when buying stocks is dividends. This model is useful to evaluate the equity returns by discounting the future dividend returns to a required rate of return:

\[ V_o = \sum_i \frac{D}{(1+r)^i} \]  

(4.1)

where the \( v \) is the intrinsic value of a share of the index, \( D \) is the expected dividend per share, \( i \) is the number of the period, and \( r \) is the required rate of return (Bodie, 2009). Despite the simplicity of the model, it embedded many assumptions that are capable of expansion in including extra variables such as GDP growth. The expansion of the DDM model- Constant-Growth Model allows the analysts to further build in the assumptions of GDP growth within countries. The constant growth model which is similar to the DDM with the presence of growth term:

\[ V_o = \sum_i \frac{D(1+g)^{i-1}}{(1+r)^i} \]  

(4.2)

The extra term \( g \) is a stable growth rate of the capital gain yield. The model can rewrite it into the form (see appendix I):

\[ V_o = \frac{D}{(r-g)} \]  

(4.3)

\[ E(r) = \frac{D}{p_0} + g \]  

(4.4)

where the new variable \( p_0 \) is current stock price, and \( r > g \). The required rate of return of an index therefore is just the return on a share of index that we want to estimate with the respect of the growth rate. In this context, we assume the growth rate is the same as the GDP growth of the regional investments since we are projecting the growth of companies over a long period of horizon. Therefore, the model can decompose to the following:

\[ E(r) = \frac{D}{p_0} + realGDP + inflation + \varepsilon \]  

(4.5)

the real GDP and inflation growth of the region and an error term, \( \varepsilon \). The strength of applying the Gordon-Shapiro constant growth model is that analysts can take advantage of the organizational forecasted long-run GDP growth and inflation rate such as the publication from the International Monetary Fund (IMF) or OECD as long-term views. This eliminates
the problem of the individual forecast. Also analysts can avoid relying on a third private entity’s views on long-term returns of equity index. The dividends and price are data within the last 10 years from the MSCI index.

Here we apply the OECD outlook forecasted real GDP growth rate and inflation rate from 2018-2060 as our long-term growth rate with the historical data on dividends and price:

**Table II. Projection of Equity Return by Region**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>EMU</td>
<td>1.6%</td>
<td>1.5%</td>
<td>2.0%</td>
<td>-</td>
<td>5.1%</td>
</tr>
<tr>
<td>Developed (unhedged)</td>
<td>3.0%</td>
<td>2.6%</td>
<td>2.8%</td>
<td>-</td>
<td>8.4%</td>
</tr>
<tr>
<td>Developed (hedged)</td>
<td>3.0%</td>
<td>2.6%</td>
<td>2.8%</td>
<td>-2%</td>
<td>6.4%</td>
</tr>
<tr>
<td>Emerging Market (unhedged)</td>
<td>4.1%</td>
<td>3.1%</td>
<td>4.0%</td>
<td>-</td>
<td>11.2%</td>
</tr>
<tr>
<td>Emerging Market (hedged)</td>
<td>4.1%</td>
<td>3.1%</td>
<td>4.0%</td>
<td>-7.9%</td>
<td>3.3%</td>
</tr>
</tbody>
</table>

**Bond**

| Sovereign Bond (unhedged) | -              | 1.8%      | 2.1%                 | -                           | 3.9%                       | 6.0%                        |
| Sovereign Bond (hedged)   | -              | 1.8%      | 2.1%                 | 0.7%                        | 4.6%                       | 6.7%                        |

**Other**

| Real Estate | 5.8%              | 1.5%      | 2.0%                 | -                           | 9.3%                       | 7.1%                        |

Note: the GDP growth is derived from OECD outlook 2013 and historical returns from Data Stream. The equity index projection consists of Europe, developed market, and emerging market which is corresponding to the regional MSCI indices we have selected from our data. The bond is corresponding to the world bond index from CITI group. The majority of constituent countries in the index are considered developed countries. Here, we apply the OECD countries GDP growth and inflation forecasted data.

The projected data includes the return of the hedged index; this will help us to study the impact of currency hedging by applying index funds to construct a portfolio. All hedging costs are referring to the ten year difference of hedged and unhedged historical return of the indices. The forecasted return indicates that the emerging market companies will continue to outgrow companies in developed countries given the strong economic driver. The developed market equity index return appears to be very optimistic; this is because the global inflation rate is quite high which appears to be coming from the fast growing emerging market. Global bonds display reasonable returns given it is relatively stable in comparison to equity and the real estate market, while the volatile nature of the real estate market causes it to post a high projected return.
With the help of the DDM model, we are able to project the return based on economic growth, historical dividends, and hedging differences. This unique set of returns does not rely on a particular analyst’s or manager’s perspective. The assumptions of this set of returns, therefore, mainly rest on the OECD organization’s projection, which is subjected to the OECD’s estimation error. However, provided with ongoing issues of OECD outlook reports, the analysts can update and monitor the assumptions according to the change in the outlook reports and historical data. The objective view of the projected return can further build into other models such as the Mean-variance portfolio model and Black-Litterman Model published in 1992. In addition the currency hedged assets become a new set of assets which can be included during the optimization process.

4.2 Markowitz Mean-Variance Model: Theoretical and Practical Approach

Markowitz’s model uses a mean-variance approach of returns to allow the investors to derive an optimal portfolio. The research work was ground breaking in 1952, because there were limited studies on the relation between the portfolio as a whole and asset selection. The mean-variance model cannot only be used for stock selection; it can also be extended to multiple assets such as bonds, mutual funds, and other derivative instruments by constructing an efficient frontier with the selected assets. The input variables in the model are asset weights, returns, and variance for individual assets over a given period of time, along with covariance among all assets within the model’s allocation universe. The model can be optimized in a way that maximizes investors’ expected returns while minimizing the risk. The framework of the model is ideal for the requirement of the OECD’s guideline where it explicitly quantifies the risk within a portfolio. Moreover, the analysts can include currencies in the model to further derive an optimal currency hedging ratio. To derive the attainable portfolio, analysts need to apply the formulas as proposed by Markowitz:

Max: \( E(\eta_p) = \sum_i x_i E(r_{i+1}) \) \hspace{1cm} (4.6)

Min: \( \sigma_p^2 = x_i^\prime \Sigma x_i \) \hspace{1cm} (4.7)

where \( E(r_i) \) is the expected return of each asset within the portfolio, \( W_i \) is the weight on each asset which can be written as \( x_i = (x_{1i}, x_{2i}, \ldots, x_{ni}) \), and \( x_i \cdot x_i^\prime = 1 \). \( \Sigma \) is variance-covariance matrix which can be written as:

\[
\begin{pmatrix}
\sigma_1^2 & \sigma_{12} & \ldots & \sigma_{1n} \\
\sigma_{21} & \sigma_2^2 & \ldots & \sigma_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{ni} & \sigma_{2n} & \ldots & \sigma_n^2
\end{pmatrix}
\]

\( E(r_p) \) is the total expected return of the portfolio, and \( \sigma_p^2(\eta_p) \) is the volatility or the risk of the entire portfolio. Often an alternative function which includes the investor’s utilities is applied in deriving the optimal portfolio:

Max: \( U = E(\eta_p) - .5 \times A \times \sigma_p^2 \) \hspace{1cm} (4.8)

\[
\text{Sharpe Ratio} = \frac{r_p - r_f}{\sigma_p}
\] \hspace{1cm} (4.9)

\footnote{There are other possible optimization solutions by introducing variables targeting investors’ preference, risk aversion, and utilities. However, maximization of the utility function is the same as a minimizing the risk. A proof can be found in the Appendix II.}
where $A$ is the risk aversion level of an individual investor, and $U$ is the total utilities level of the mean-variance portfolio offer to the investors. Theoretically, the analysts can further define a utility function or a risk aversion level upon the wealth accumulation function. In our case, we use some mathematical models to arrive at wealth utility function as:

$$
E(U(W_{1+t})) \approx E(r_p) + 0.5 \frac{U''(W_1)W_1}{U'(W_1)} \sigma_p^2
$$

(4.10)

Based on the utility theory laid out by von Neumann and Morgenstern (1947), Arrow-Pratt (1964-1965) derived the measurement of relative risk aversion (RRA) as the following:

$$
A := -\frac{U''(W)}{U'(W)}
$$

(4.11)

Applying Arrow-Pratt’s RRA measurement into the wealth utility function (4.10) we derived earlier, we get:

$$
U(E(W_{1+t})) \approx E(r_p) - 0.5RRA \sigma_p^2
$$

(4.12)

RRA can be a useful tool to help analysts find a risk aversion level of a pension fund given its utility function. However, one of the problems encountered by pension fund analysts is that the fund itself cannot explicitly offer an adequate utility function. To define one, analyst can consider the groundwork laid out by von Neumann and Morgenstern, Arrow, Pratt, De Finetti and etc. to form a convexity-based or linear-based utility curve. However, extra parameters need to be estimated and to be approved by the board members. An alternative method can also be applied. In Black (1992)’s work of constructing the Black-Litterman model, Black implicitly used an alternative method to derive the investor’s utility level, where he has the risk aversion level defined as:

$$
A = \frac{\mu_p}{\sigma_p^2}
$$

(4.13)

The risk aversion level has become an optimization process under the Lagrange multipliers. A proof can be found along with the mean-variance optimization study in Appendix II. Applying the targeted return and volatility from our hypothetical pension fund, we can derive the risk aversion level as $A=\frac{\mu_p}{\sigma_p^2} = \frac{5}{10^2} = 5$. Once we have the risk aversion level, we can complete the mean variance optimization process by maximization utility function. As presented in equation 4.6, 4.7, and 4.8, there are two options to optimize a pension fund’s portfolio. Analysts can either employ model 4.6 and 4.7 by minimizing the variance and maximizing returns or simply maximize the utility function as 4.8 provided with the risk aversion.

---

*Mathematical work is presented in Appendix II*
Note: the graph is derived from author’s calculation without short sales. The equity market data is taken from MSCI developed market Index, MSCI Emerging Market index, and their corresponding hedged indices. For fixed-income, we used Citigroup World Government Bond Index and its hedged index make up the bond market. The minimum variance portfolio has a standard deviation of 2.29%.

Practically, analysts can simplify some of the assumptions of the defining utility function to the board members. By doing so, analysts can suggest investing along the efficient frontier curve given that each point on the curve is embedded in the utility level which is just the market utility level. In this case, we can start from the minimum variance portfolio as in Figure 4.1. Suppose that the board desires to invest in a portfolio with the least amount of risk, without care about the return. The trade-off between the return and risk can be accustomed to the Board’s request as we move along the curve. As long as the portfolio is on the frontier, each portfolio is efficient and risk is minimized.

### 4.2.1 Markowitz Mean-Variance Model: Applying Historical Returns

An exercise of portfolio construction is done by applying the Markowitz model with currency hedging—mixing the hedged and unhedged mutual funds with the Euro as the base currency. We include an extra set of data in our global portfolio by dividing the investments in regions: European Monetary Union, developed market and emerging market. Using the historical data, we conduct the study by deriving two sets of sample portfolios each with different risk aversion level: a constrained portfolio and unconstrained portfolio. Sample portfolio 1 is a constrained portfolio following an approximated 60/40 split in equity and fixed income allocation. Sample portfolio 2 is an unconstrained portfolio; we forbid short positions given the risk of going short. The reason we use a 60/40 split constraint is due to the wide acceptance of the split from pension fund analysts, supported by a large number of academic and non-academic publications on such a ratio. In the optimization process, the objective of the portfolio is to seek for maximum utility function. The result of the exercise is presented in the Table III below.
Table III. Historical Return Based Asset Allocation Weights

<table>
<thead>
<tr>
<th>Equity</th>
<th>Portfolio 1 (60-40 equity-bond)</th>
<th>Portfolio 2 (unconstrained)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMU</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Developed market (unhedged)</td>
<td>0%</td>
<td>17%</td>
</tr>
<tr>
<td>Developed market (hedged)</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Emerging Market (unhedged)</td>
<td>60%</td>
<td>25%</td>
</tr>
<tr>
<td>Emerging Market (hedged)</td>
<td>0%</td>
<td>18%</td>
</tr>
<tr>
<td>Global Sovereign Bond</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Bond (unhedged)</td>
<td>40%</td>
<td>40%</td>
</tr>
<tr>
<td>Global Sovereign Bond (hedged)</td>
<td>40%</td>
<td>40%</td>
</tr>
<tr>
<td>Total return</td>
<td>8.56%</td>
<td>6.65%</td>
</tr>
<tr>
<td>Volatility</td>
<td>13.0%</td>
<td>7.17%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>66.1%</td>
<td>92.6%</td>
</tr>
</tbody>
</table>

Note: The above results is derived from Data Stream and calculated by the author. First column identifies the asset type; the second column states the specific asset. Two sets of portfolios have been presented: one with 60/40 split and other without constraint. Each portfolio also comes with a different set of risk aversion level.

The result of the Markowitz-model appears to be extreme due to some of the flaws in the model. One of the flaws is that it produces extreme weights or “corner” solutions such as the unconstrained portfolio. In the exercise, the portfolio produced a high percentage of weight in investments of emerging and bond markets. It is highly unrealistic for pension funds to invest for the future due to the lack of diversification and high concentration of weight in a single class asset, for example, equity and bond investment. Regardless of the risk aversion level, the 60/40 split portfolio displays a strong regionally biased portfolio where most of the investments go into emerging market equity and bond market, while the unconstrained portfolio is even more extreme in investing in the emerging equity market. The reason for arriving at such extreme portfolios is because the expected return is challenging to estimate. Given the outstanding performance of the emerging and bond markets in the last 10 years, the model is very generous in rewarding high returned assets—emerging market and bond index. The extreme weight in asset allocation lacks of diversification which can hinder the approval of strategic asset allocation from the Board members. It can further delay the investment opportunities by reconstruction of the portfolio. Therefore, some adjustments must be made in strategic asset allocation studies.

One of the solutions to the “corner” portfolio problems such as the risk neutral portfolio in sample two is that the analysts can add to the portfolio. On one hand, adding constraints may help analysts to derive a reasonable set of asset allocation with a good set of diversifications. But, on the other hand, the problem is that if too many constraints are included, the portfolio is heavily tailored in a way based on analysts’ view to secure an approval. The heavily
constrained portfolio lost its authenticity for taking advantage of the efficient frontier. The point of portfolio construction becomes trivial rather than analysts implicitly expressing views on investment. Therefore, a few constraints should be used to help the portfolio remain in the general framework under the investment guideline and to achieve diversification. If the current pension fund is underfunded, the portfolio may be forced to allocate large portion of assets in the equity market in order to boost asset growth. If the fund has surplus, the investment strategy may focus on the bond market to help the fund retain its funding ratio.

Another problem is the historical return: the existence of large estimation errors due to limited historical returns can further put the model in question. The historical data offered some insights to the question of hedging. However, the past performance cannot extend towards the future. Therefore, the historical return based portfolio only offers an unconditional hedging solution (static). In order to include the outlook for the growth fund with world GDP growth, analysts should be able to construct reliable future portfolio returns. To do this, a projected equity and bond returns could be substituted into the mean-variance model which we used for future global GDP growth. Markowitz implies that his model is also applicable to the “beliefs of one or more security analysts concerning future performance” (Markowitz, 1991, p3). The belief/view of an analyst or a third party on securities is unwanted in some pension organizations considering risks such as moral hazard and adverse selections. To eliminate or minimize the analysts’ view is partly to protect the integrity and reduce the liability of the governing body. The issues with expressing an individual or a private party’s investment view are very “political” in public pension funds and in the event that an internal staff leaves his/her position, the ex-ante views cannot be resumed by other internal staff. Also expert views are difficult to evaluate. In terms of forecasting, there are a vast number of statistical models and stimulation programs allowing analysts to follow. The Majority of simulation programs are based on theories from the time series model or the Monte Carlo—Geometric Brownian Motion process and eventually develop into stimulation programs. However, some critics point out the time series model is also based on the past observations of variance-covariance to project the future stock returns. The Geometric Brownian Motion (GBM) process is not a completely realistic tool given the volatility change in real stock prices, where it is assumed to be constant in GBM. These approaches may further cause confusion and delay in investment process. To avoid an inclusion of views and simplify assumptions, we apply the expected returns we derived from Gordon Shapiro’s Dividend Discount Model on the account of OECD’s projected global GDP growth to analyze the index fund in this case.

4.2.2 Markowitz Mean-Variance Model: Applying DDM-Expected Returns

Using the expected return from section 4.2, we derive a new set of optimal weights by maximizing the mean-variance utility function with the pension fund’s risk aversion level A=5. Three different sets of constraints are used. The first set of constraint represents current requirement of investment guidelines. Constraint 2 and 3 are relatively relaxed in order for us to observe the improvements in return and volatility.

1) Constrain 1 (Investment guidelines): EMU equity-20% with +/-4%, global equity-30% with +/-4%, emerging market equity-10% with +/-4%, global fixed-income-30% with +/-4%, and Euro real estate market-10% with +/-4%. Short sale is restricted. These constraints apply for portfolio 1 and 4.
2) Constraint 2: Equity and real estate-60% with +/-4%, and global fixed-income-40% with +/-4%. Short sale is restricted. These constrains applies for portfolio 2 and 5.

3) Constraint 3: We restrict short sale of the portfolio. This constraint applies to portfolio 3 and 6.

To compare the results of hedged and unhedged portfolios, we let portfolio 1, 2, and 3 to be unhedged, while portfolio 4, 5, and 6 to be hedged. Table IV displays our findings:

**Table IV. Asset Allocation for Expected Returns**

<table>
<thead>
<tr>
<th></th>
<th>Unhedged</th>
<th></th>
<th></th>
<th>Hedged</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Port. 1</td>
<td>Port. 2</td>
<td>Port. 3</td>
<td>Port. 4</td>
<td>Port. 5</td>
<td>Port. 6</td>
</tr>
<tr>
<td>Equity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EMU</td>
<td>16%</td>
<td>0%</td>
<td>0%</td>
<td>16%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>developed market (unhedged)</td>
<td>26%</td>
<td>12.9%</td>
<td>2.7%</td>
<td>26%</td>
<td>9.4%</td>
<td>0%</td>
</tr>
<tr>
<td>developed market (hedged)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Emerging Market (unhedged)</td>
<td>10%</td>
<td>21.87%</td>
<td>24.97%</td>
<td>4.1%</td>
<td>22.5%</td>
<td>24.7%</td>
</tr>
<tr>
<td>Emerging Market (hedged)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>9.9%</td>
<td>10.7%</td>
<td>0%</td>
</tr>
<tr>
<td>Bond</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Global Sovereign Bond (unhedged)</td>
<td>34%</td>
<td>44%</td>
<td>52.48%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Global Sovereign Bond (hedged)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>34%</td>
<td>44%</td>
<td>64%</td>
</tr>
<tr>
<td>Others</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Estate</td>
<td>14%</td>
<td>21.2%</td>
<td>19.9%</td>
<td>10%</td>
<td>16%</td>
<td>12.7%</td>
</tr>
<tr>
<td>Total return</td>
<td>6.75%</td>
<td>7.22%</td>
<td>6.92%</td>
<td>6.3%</td>
<td>6.94%</td>
<td>6.8%</td>
</tr>
<tr>
<td>Volatility</td>
<td>10.14%</td>
<td>9.22%</td>
<td>8.41%</td>
<td>8.4%</td>
<td>7.46%</td>
<td>6.5%</td>
</tr>
<tr>
<td>Sharpe</td>
<td>66.6%</td>
<td>78.35%</td>
<td>82.23%</td>
<td>74.8%</td>
<td>93%</td>
<td>103.8%</td>
</tr>
</tbody>
</table>

Note: Results are taken from Data Stream and author’s calculation. First column identifies the asset type; the second column states the specific asset. The risk aversion used to maximize the utility function is A=5, also different constrains as described from above have set for each portfolio.

- Portfolio 4 optimal hedge ratio (referring to constraint 1): 0% in developed market equity, 29.2% in emerging market equity and full hedge in bond.

- Portfolio 5 optimal hedge ratio (referring to constraint 2): 0% in developed market equity, 32% in emerging market equity, and full hedge in bond.

- Portfolio 6 optimal hedge ratio (referring to constraint 3): 0% for all assets.

All portfolios tend to favor fixed-income assets. This is because of the low risk and relatively high returns associated with this class of asset. However, moderate allocation in high risk assets such as emerging market equity and real estate index are included in the portfolios. This tendency is quite obvious in Portfolio 2 and 3. Especially portfolio 3 has a large portion
of the asset invested in unhedged bond and emerging market equity which makes the portfolio vulnerable to currency and geographical risk.

In hedged portfolios, all portfolios are fully hedged to fixed-income asset. This outcome is aligned to our early observation in hedging strategy: the bond market is more vulnerable to currency risk. Nevertheless, hedge ratios for the equity market are varying in relation to the portfolio constraints. Currency hedged portfolios 4 & 5 are similar in hedge ratios. However due to the difference in constraint portfolio 5 invest more in international asset, while portfolio 4 have a 16% investment in domestic equity. The use of constraints reduces the Sharpe ratio of the fund’s portfolio. However, given the volatile economic growth and geopolitical issues in the regions, both portfolio 4 or 5 can be more resilient to risk in the long run.

4.2.3 Markowitz Mean-Variance Model: Applying Forward Contacts

The Markowitz’s mean-variance portfolio model can also extend to derive hedge ratios by treating currency as a distinct asset class. Solnik (1997) along with other researchers proposed that investors can approach currency management as a separate decision or they can combine currency management into the overall asset allocation process. However, currency movement can affect the performance of the indices to a certain degree depending on the time horizon, base currency, and specific assets, and correlation. From these observations, we choose to optimize forward contracts together with the unhedged indices in the context of Markowitz mean-variance portfolio theory. By doing so, analysts can set the currency weight as the hedge ratios as the number of forward contracts that an investor buy/sell.

In our case, working with a portfolio consists of multi-currency indices, we find it is difficult to estimate individual currency weight given that MSCI and Citigroup indices are adjusted to the market capitalization or price index level. Along with the daily adjustment of the indices, often a significant portion of the local currencies are grouped into one category in the index reports. It is useful to considering the “original sin” argument introduced by Eichengreen and Hausmann (1999) when we choose to use forward contracts. They hypothesize that the emerging market countries lack the capacity to trade in their own currency, whereas only advanced economies can dominate such transaction. Indeed, it is difficult to replicate emerging market portfolios provided with a statement in the MSCI Index Calculation Methodology:

“While the local currency series of regional indices cannot be replicated in the real world, it represents the theoretical performance of an index”. (MSCI Index Calculation Methodology, May 2012, p6)

Even though other scholars argue that improved macroeconomic policies and globalization have made local currency more attractive to investors, it is still very hard to trade local currency denominated financial products. Therefore, we choose only to observe the major currencies in our indices.

Based on the available information from fact sheets, we include six major foreign currencies in our analysis: U.S. Dollar (USD), UK Pound (GBP), Canadian Dollar (CAN), Australian Dollar (AUD), Japanese Yen (JPY), Chinese Yuan (RMB), and South Korean Won (KRW). All forward exchange rate data are taken from WM/Reuters Company on Data Stream on a monthly basis from 1999 to 2014. The beginning year of the data refers to the creation of the
Euro currencies in 1999. Using the same concept of minimizing the volatility of the indices and currency return, we construct the following model:

\[ E(W_{1+30}) = W_i (1 + TX_{1+30}^f + TX_{1+30}^f) \]  

(4.14)

where \( x_{1}^{uh}, x_{1}^{f} \) is the unhedged asset allocation and the forward weight, and \( r_{1+30}^{uh}, r_{1+30}^{f} \) is the corresponding ex-ante estimated return and historical return for the asset and the currency. In the framework of Markowitz we choose to construct the same mean variance portfolio by minimizing the volatility:

\[
\text{Min: } \sigma_p^2(r_p) = (x_{1}^{uh} x_{1}^{f}) \left[ \Sigma_{uh} \Sigma_{uh,f} \right] (x_{1}^{uh} x_{1}^{f})',
\]  

(4.15)

where \( \sigma_p^2(r_p) \) is the variance of the portfolio including the currency forward, \( \Sigma_{uh}, \Sigma_f \) is the variance covariance matrix of the unhedged indices and forward contracts, and \( \sigma_{uh,f} \) is the covariance matrix of the forward and unhedged indices. Our objective is to minimize the risk of the portfolio. To do so, we allow pension fund analysts to take both short and long positions to hedge the currency exposure of the unhedged portfolio by selling/buying an appropriate number of future currency contracts denominated in our sample currencies. Each optimized currency weight implies what the optimal hedging ratios investors should hold in their portfolio. In addition, we separate the currencies between developed market and emerging market to further induce the regional difference between hedging ratios. We follow the same investment strategies proposed in the previous section to come up with three constrained portfolios. In this framework, a positive value of currency weight indicates that the investor could reduce the volatility of the portfolio by holding a short position of the forward, while a negative value means the opposite, a long position of the contract. Table V summarizes our findings.

From the above results, we find significant improvements in risk reduction while the return remains almost at the same level to Table IV-hedged portfolio. The results justify the “free lunch” argument in currency hedging based upon our data, period, and base currency. The Bond market, considered as a safety asset, still appears to be very attractive to invest in our sample. As a result of this characteristic, all three portfolios appear to reward a substantial portion of the portfolio in such an asset. In contrast to the bond, equity plays an interesting role in asset allocation and the selection of asset class. All three sets of the portfolio are moving away from the domestic equity market (European Monetary Union) to global assets such as developed markets or emerging markets with respect to the constraints. This tendency validates that currency hedging strategy works in complement to global asset diversification. The main difference in equity asset class is the allocations between emerging market and world market equity assets. When mixing hedged and unhedged indices, our strategy to achieve diversification is to take advantage of the unhedged emerging market equity with high returns and balance the risk by investing in safe assets such as bonds. While in the context of currency hedging by forwards, we diversify our portfolio by further reducing risk in investing in developed markets. The two hedging methods, therefore, result in different hedging strategies, which is reflected in asset class selection, volatility, and Sharpe ratios. In terms of volatility to hedging ratios, we do not observe any differences between emerging market currencies and developed market currencies.

After finding the optimal weight of the portfolio, we analyze the two sets of the results with the cumulative distribution function to see if we can improve the probability of the fund to meet its 3.0 billion targeted wealth in the 30 years. In Figure 4.2 we present the cumulative
distribution function for the strategies of mixing hedged and unhedged asset and using forward contracts:

Table V. Asset Allocation for Currency and Underlying Assets

<table>
<thead>
<tr>
<th>Equity</th>
<th>Hedge by Forward Contracts</th>
<th>Hedge by Mixing Benchmark Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Port. 1</td>
<td>Port. 2</td>
</tr>
<tr>
<td>EMU</td>
<td>16%</td>
<td>0%</td>
</tr>
<tr>
<td>Developed Market</td>
<td>29%</td>
<td>38%</td>
</tr>
<tr>
<td>Developed Market (Hdg)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Emerging Market</td>
<td>14%</td>
<td>12%</td>
</tr>
<tr>
<td>Emerging Market (Hdg)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bond</th>
<th>Hedge by Forward Contracts</th>
<th>Hedge by Mixing Benchmark Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Port. 1</td>
<td>Port. 2</td>
</tr>
<tr>
<td>Global Sovereign Bond</td>
<td>34%</td>
<td>44%</td>
</tr>
<tr>
<td>Global Sov. Bond (Hdg)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Others</th>
<th>Hedge by Forward Contracts</th>
<th>Hedge by Mixing Benchmark Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Port. 1</td>
<td>Port. 2</td>
</tr>
<tr>
<td>Real Estate</td>
<td>7%</td>
<td>6%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Develop Market Currency</th>
<th>Hedge by Forward Contracts</th>
<th>Hedge by Mixing Benchmark Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td>US/Euro</td>
<td>-2%</td>
<td>5%</td>
</tr>
<tr>
<td>Japan/Euro</td>
<td>-10%</td>
<td>-1%</td>
</tr>
<tr>
<td>UK/Euro</td>
<td>-8%</td>
<td>-5%</td>
</tr>
<tr>
<td>CAN/Euro</td>
<td>37%</td>
<td>29%</td>
</tr>
<tr>
<td>AUS/Euro</td>
<td>23%</td>
<td>21%</td>
</tr>
<tr>
<td>Swiss/Euro</td>
<td>-21%</td>
<td>-14%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EM Currency</th>
<th>Hedge by Forward Contracts</th>
<th>Hedge by Mixing Benchmark Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chinese Yuan/Euro</td>
<td>-16%</td>
<td>-10%</td>
</tr>
<tr>
<td>Korean Won/Euro</td>
<td>43%</td>
<td>35%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Hedge by Forward Contracts</th>
<th>Hedge by Mixing Benchmark Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Port. 1</td>
<td>Port. 2</td>
</tr>
<tr>
<td>Total return</td>
<td>7.3%</td>
<td>7.7%</td>
</tr>
<tr>
<td>Volatility</td>
<td>6.7%</td>
<td>5.2%</td>
</tr>
<tr>
<td>Sharpe</td>
<td>110%</td>
<td>149%</td>
</tr>
</tbody>
</table>

Note: Results are taken from Data Stream and author’s calculation. The table summarizes the return and risk characteristics for the benchmarks and three portfolios with different constraints. Constraint for each portfolio is set to the percentage as the description presented in section 4.3.2 Markowitz’s Mean-Variance Model. Positive ratios imply a short position in the corresponding currency, while negative ratios in forward contracts imply a long position in corresponding currency.
Figure 4.2.1: Cumulative Distribution Graph of the Assumed Fund Growth by Applying Forward Contracts

Notes: the calculation and graph have been implemented by author and Matlab program. We apply the expected return and volatility in Table V under i.i.d. return assumption. There are 29.4%, 26.1%, and 13.1% chances that the fund will accumulate less than 3.0 billion target wealth in next 30 billion target wealth in next 30 years for each corresponding constrained portfolios 1, 2, and 3 by using forward to hedge underlying assets.

Figure 4.2.2: Cumulative Distribution Graph of the Assumed Fund Growth by Mixing Hedged/Unhedged Indices:

Notes: the calculation and graph have been implemented by author and Matlab program. We apply the expected return and volatility in Table V under i.i.d. return assumption. There are 59.1%, 47.3%, and 42.9% chances that the fund will accumulate less than 3.0 billion target wealth in next 30 years for each corresponding constrained portfolios 4, 5, and 6 by mixing unhedged and hedged assets.

By comparing the two graphs, it appears that hedge currency risk with forward contracts have improved fund growth substantially rather than mixing the hedged and unhedged assets. The reduction of uncertainties goes down by 50% of the probability, which is very significant. In particular, for the unconstrained case that there are 13.1% likelihood the fund will meet its target. The superiority of applying forward contacts is that the negative correlation between exchange rate return and asset return reduces the volatility of the entire portfolio, especially if
we treat each currency as an individual asset class. This leads to further diversification in contrast to only including a limited number of currency hedged assets. However, in reality hedging multi-currency indices with forward contacts can be very difficult to implement. Some MSCI regional indices only serve as benchmarks; there are very few funds actually exist. Moreover, currency weights change frequently given the capitalization based calculation methods and undeclared currency categories in the reports.

4.3 Asset and Liability Studies

The nature of the pension fund existence is to ensure its members receive a payment for their retirement at present and in the future. In the framework of strategic asset allocation analysis, analysts often only focus on the asset values and abstracts only. The presence of pension liabilities has often been left out which make the analysis impractical to implement and the asset allocation results are suboptimal. This can contribute to bewilderment for other committee members and can jeopardize the pension fund’s projected asset/liability growth in the future. In this section of the thesis, we investigate the strategic asset allocation process in the presence of liabilities. To include liability, it brings in an interesting complexity in the asset allocation framework. Depending on the characteristic of the fund along with other factors such as financial market outlook, funding ratios, and actuarial studies, the best investment strategies regarding the fund growth may not be to simply invest in the asset with the highest return. For example, many funds use the duration matching technique by matching the duration of the fund future payment to its current portfolio duration. By investing in this manner, analysts can align the growth of the fund to the payments to retirees. So the fund will not disappear in the long run, and there will be enough surpluses to ensure the future employees receive a pension at retirement. It is practical to consider how asset is correlated with liabilities.

In our context of the analysis, we consider each pension instalment as a coupon payment of a bond. The return of the bond and its correlation with other assets can be calculated by discounting the future obligations with an internal rate return. Even though it appears straightforward for actuaries to project this discount rate, there are many uncertainties since there are a number of unknown factors that can affect the projections. These factors are mortality rates, future salary growth, employee’s demographics, and other pension funds idiosyncratic factors such as payment denomination, allowance programs, and other undesired events. We focus on the strategic asset allocation framework by investigating three components in regard to the 30 year performance of a pension fund: portfolio growth/pension payment, liability accumulation, and currency hedging in the presence of liability.

To start with our analysis, we select the liability growth model which is identical to our wealth accumulation model in section 3:

\[
L_{1+t} = L_1 \times \sum_{t=1}^{T} (1 + \lambda r_{1+t})
\]

(4.16)

where \(L_{1+t}\) is the total liability growth from each period \(t\), \(T\) is the total number of the estimated periods, \(\lambda\) is the weight of the asset in liability. In this context of only bond in our liability portfolio, we let \(\lambda = (0,1,0,\ldots)\) where there is only bond exists in the liabilities since we treat pension fund’s period payment as a coupon bond.

To examine the correlation between asset and liability, we denote FR as the funding ratio which is a ratio between the value of the fund’s wealth (asset) and the fund’s liabilities, and it can be expressed as:

\[
FR = \frac{\text{asset}}{\text{liability}}
\]
We substitute equation 3.1 and 4.16 in FR such that:

\[
FR = \frac{W}{L}
\]  

To simplify this model, we omit the higher order and approximate to:

\[
FR_{1+t} \approx \frac{W_1 \times (1 + T \lambda')}{L_1} \times \frac{(1 + T x' \tau_l)}{(1 + T \lambda' \tau_l)}
\]

Under our early i.i.d assumption, we then take the expectation of the funding ratio which it becomes:

\[
E(FR_{1+t}) \approx E(FR_1) \times E((1 + T(x' - \lambda')\tau_l))
\]

Under the normal distribution assumption, the \( E(FR_{1+t}) \) becomes the following:

\[
N \approx \left( \mu_{FR_{1+t}} = E(FR_{1+t}), \sigma^2_{FR_{1+t}} = T \times \sigma^2_{FR_1} \right)
\]

For the simplicity of the calculation, we assume the fund is underfunded with a current liability level of 2.0 billion Euros. The internal rate of return is 3.5\%\(^9\). In 30 years, the fund liability is targeted to become 3 billion. This implies a targeted funding ratio of 100\% in respect to the target fund’s wealth accumulation in 30 years. With the presence of liabilities and internal rate of return, we first adjust the optimal asset allocation of our portfolio and then we use our new asset weight to estimate the probability of the pension fund to meet its full funding objective:

\[
Pr \left( E(FR_{1+t}) < FR_{target} \right)
\]

Using the Markowitz mean variance approach, we again minimize, \( \sigma^2_{FR_{1+t}} = (x - \lambda)\Sigma(x' - \lambda') \) of the risk of the portfolio.

\(^9\)Since the framework of this thesis is regarding the strategic asset allocation, we choose to focus on the same subject throughout this paper. There are many approaches to calculate an appropriate discount rate depending on countries, financial market, and the purpose of reporting. Given the complexity of this issue, we took a universal fixed discount rate of 3.5\% for Euro area based OECD’s working papers by Pond et.al (2011) for underfunded pension which is derived from the sum of 1.5\% real interest rate and 2\% expected inflation rate. However, further research along with existing work should carry out to improve such discount rate.
The return on portfolio has greatly diminished; this is because we include the liabilities in our model. Comparing the two set of results, we find that currency hedging using forward slightly improves the return while substantially reducing the risk level of the portfolio in our sample. Mixing hedged/unhedged funds also has the risk reduction effect; yet, it reduces the return of the portfolio. All unconstrained portfolios yield the best return with the lowest risk for investors, and bond market appears to be a safe haven based on its low historical volatility and correlation. This can lead to less diversification. Therefore, some constraints should limit all investment going to the bond market.

Table VI. Asset Allocation including Liability:

<table>
<thead>
<tr>
<th></th>
<th>Hedge by Forward Contracts</th>
<th></th>
<th>Hedge by Mixing Benchmark Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Port. 1</td>
<td>Port. 2</td>
<td>Port. 3</td>
</tr>
<tr>
<td><strong>Equity</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EMU</td>
<td>16%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Developed Market</td>
<td>30%</td>
<td>37%</td>
<td>0%</td>
</tr>
<tr>
<td>Developed Market (Hdg)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Emerging Market</td>
<td>14%</td>
<td>13%</td>
<td>40.1%</td>
</tr>
<tr>
<td>Emerging Market (Hdg)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Bond</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Global Sovereign Bond</td>
<td>-49%</td>
<td>-39%</td>
<td>-23%</td>
</tr>
<tr>
<td>Global Sov. Bond (Hdg)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Others</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Estate</td>
<td>6%</td>
<td>6%</td>
<td>0%</td>
</tr>
<tr>
<td><strong>Develop Market Currency</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US/Euro</td>
<td>-33%</td>
<td>-22%</td>
<td>-26%</td>
</tr>
<tr>
<td>Japan/Euro</td>
<td>-47%</td>
<td>-37%</td>
<td>-20%</td>
</tr>
<tr>
<td>UK/Euro</td>
<td>5.02%</td>
<td>2%</td>
<td>7%</td>
</tr>
<tr>
<td>CAN/Euro</td>
<td>41.4%</td>
<td>33%</td>
<td>28%</td>
</tr>
<tr>
<td>AUS/Euro</td>
<td>31%</td>
<td>25%</td>
<td>31%</td>
</tr>
<tr>
<td>Swiss/Euro</td>
<td>0%</td>
<td>-3%</td>
<td>-24%</td>
</tr>
<tr>
<td><strong>EM Currency</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chinese Yuan/Euro</td>
<td>7%</td>
<td>6%</td>
<td>4%</td>
</tr>
<tr>
<td>Korean Won/Euro</td>
<td>45%</td>
<td>36%</td>
<td>25%</td>
</tr>
<tr>
<td><strong>Total return</strong></td>
<td>2.4%</td>
<td>2.9%</td>
<td>3.1%</td>
</tr>
<tr>
<td><strong>Volatility</strong></td>
<td>7.75%</td>
<td>5.94%</td>
<td>5.28%</td>
</tr>
<tr>
<td><strong>Sharpe</strong></td>
<td>30.6%</td>
<td>48.3%</td>
<td>58.5%</td>
</tr>
</tbody>
</table>

Note: The results are taken from Data Stream and author’s calculation. The table summarizes the return and risk characteristics for the three portfolios with different constraints by including liabilities payment as a coupon bond. Each portfolio’s constraint is set to the percentage as described presented in section 4.3.2 Markowitz’s Mean-Variance Model. Positive ratios in currencies imply a short position, while negative ratios in forward contracts imply a long position.

To determine the best strategies to implement, analysts should consider both asset and liability growth of the fund. Some fund analysts evaluate fund asset/liability growth in terms of a targeted wealth level during a certain period, while others rather to focus on a targeted funding ratio. Here, we analyze our fund’s asset/liability long term growth in respect to a targeted funding ratio for the following reasons. First, funding ratio reflects how risk on side of the pension fund can affect the other. For instance, crisis periods like the tech bubble in 2000 and the financial crisis of 2008 have significantly reduced the funding ratios. In
response to the crisis, pension fund analysts can adjust fund’s discount rate to maintain a stable funding ratio. Secondly, it gives a complete roadmap of a pension fund, which makes it easier for analysts to present the fund performance to the management board.

Relying on the models we derived in this section, we construct two cumulative distribution graphs of the projected funding ratios, based on the results from Table VI. In figure 4.4.1, the probability of the fund’s failure to meet full funding level in the next 30 years is at 50% to 63% for all portfolios. The currency hedging component indeed reduces the downside distribution very differently. Yet, the reduction in probability is very minor. This is due to the reason that hedging currency risk by forward contracts only reduces the volatility; but do not improve the return even when we relax constraints. Therefore, if fund analysts choose to use forward contracts given our sample, they should either consider extend the length of the funding period or lowering the funding ratio within 30 years. Figure 4.3.2 shows the cumulative distribution graph of currency hedging by mixing hedged/unhedged indices. By relaxing constraints, we see improvements on portfolio returns and reduction in volatility. The chance of the pension fund failing to meet its targeted funding ratio is reduced by almost 30%. The potential problem of executing unconstrained portfolio is a lack of diversification. Therefore, moderate constraint should be exercised. The 60/40 split portfolio can be a good starting point to implement constraint.

**Figure 4.4.1: Cumulative Distribution Graph of the Assumed Funding Ratio by Applying Forward Contracts**

Notes: the calculation and graph have been implemented by author and Matlab program. We apply the expected return and volatility in Table VI under i.i.d., return assumption. There are 63.3%, 58.5%, and 55.1% chances that the fund ratio be less than 100% in next 30 years for each corresponding portfolios 1, 2, and 3 by using forward to hedge underlying assets.
Figure 4.4.2: Cumulative Distribution Graph of the Assumed Funding Ratio by Mixing Hedged/Unhedged Indices

Notes: the calculation and graph have been implemented by author and Matlab program. We apply the expected return and volatility in Table V under i.i.d. return assumption. There are 70.0%, 55.0%, and 50.7% chances that the fund ratio be less than 100% in next 30 years for each corresponding portfolios 5, 6, and 7 by mixing hedged/unhedged indices.

4.4 Interpretation of the Results

Throughout the paper, we presented two studies on hedge currency risk—asset allocation study and asset liability management study. For each type of study, we also used two methods of hedging currency risk—mixing hedged and unhedged assets and using forward contracts. Table VII provides a summary for each method:

Table VII. Probability of Pension Fund Become Less than Fully Funded

<table>
<thead>
<tr>
<th></th>
<th>Asset Allocation Study</th>
<th>ALM study</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Currency Hedging</td>
<td>82%</td>
<td>80%</td>
</tr>
<tr>
<td>Forward Contracts</td>
<td>29.4%</td>
<td>63.3%</td>
</tr>
<tr>
<td>Hedged/Unhedged Assets</td>
<td>59.1%</td>
<td>70%</td>
</tr>
</tbody>
</table>

Note: Each probability is taken in respect to the current investment guidelines as shown in section 4.3.2. For Asset Allocation Study column, we measure the probability of the fund fails to meet its targeted wealth level of 3 billion Euros. While, ALM study shows the results of the probability of the fund become less than fully funded.

It appears that the uncertainty of the fund to meet its objective was reduced substantially by using forward contacts in an asset allocation study. However, only studying the asset side can simply lead to a suboptimal or unrealistic asset allocation if analysts pursue a liability driven target funding ratio. In our case, hedging currency risk with forwards illustrates such problems by displaying an optimistic result of 88% reduction in uncertainty in an asset
allocation study, although the ALM study argues otherwise with the presence of ongoing liabilities of a pension fund.

From the cumulative distribution graphs, analysts can further induce other risk assessments. For instance, one way to measure the portfolio risk is by applying Value at Risk (VaR). Such measurement provides the analysts an estimation of the potential loss of a risky investment at a given confidence interval during a certain time horizon. For instance, referring the 5.0% expected return and 10% of risk for the 30 year wealth accumulation in section 3.1. The interpretation of 5% VaR is that on a 95% confidence level, the loss of the portfolio’s value over a 30 year will be 1.3 billion Euro or more. The estimation can be found in plotted graph through Matlab as presented in Figure 4.4.3. As we convert the cumulative distribution function (cdf) back into the probability density function(pdf) under the i.i.d assumption. In addition, we can further extend the VaR analysis to the funding ratio with a similar interpretation that the on the 95% confidence level the reduction of funding ratio is 0.8% or more due to the loss in portfolio for the next 30 years where we summarize these in the Table VIII.

**Figure 4.4.3. VaR of the Wealth Accumulation Portfolio.**

Notes: the calculation and graph have been implemented by author through Matlab program. We apply the expected return and volatility of the target wealth objective under i.i.d. assumption. The VaR presents that on a 95 percent confidence level the loss is more than 1.3 billion Euros for the next 30 years.
Table VIII. VaR for Asset Allocation Study and ALM Study

<table>
<thead>
<tr>
<th>VaR</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset Allocation Study (in billion Euros)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Target</td>
<td>-1.6</td>
<td>-1.3</td>
<td>-1.0</td>
</tr>
<tr>
<td>Forwards</td>
<td>-1.1</td>
<td>-0.9</td>
<td>-0.8</td>
</tr>
<tr>
<td>Mixed</td>
<td>-1.3</td>
<td>-1.1</td>
<td>-0.9</td>
</tr>
<tr>
<td>ALM Study (funding ratio)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Target</td>
<td>-1.1%</td>
<td>-0.8%</td>
<td>-0.6%</td>
</tr>
<tr>
<td>Forwards</td>
<td>-0.4%</td>
<td>-0.1%</td>
<td>-0.02%</td>
</tr>
<tr>
<td>Mixed</td>
<td>-0.8%</td>
<td>-0.5%</td>
<td>-0.3%</td>
</tr>
</tbody>
</table>

Notes: the calculation has been done by the author. We used the results based on the investment guidelines as stated in section 4.3.2. For Asset Allocation Study, the VaR presents that on the 95 percent confidence level the loss of the portfolio value for the next 30 years. While, for ALM studies it presents that on the 95 percent confidence level the reduction in funding ratio due to the loss in portfolio for the next 30 years.

By evaluating the VaR, we conclude that using forward contracts to hedge portfolio risk mitigates the risk for the next 30 years in both asset and ALM studies. However, VaR often comes to its limitations, such as accuracy for estimating the portfolio risk. In our case, if the normally distributed return assumption is violated, the VaR can lose its power in its accuracy. However, given the pension funds facing a long investment horizon, VaR and underlying assumptions should be updated during its periodic review to ensure its accuracy, especially for pension funds use historical mean-variance or Monte Carlo simulation methods.

5. Conclusion

In this paper, we studied the impact of currency hedging in regard to the performance of pension funds by following OECD recommended pension guidelines. We began with a wealth accumulation model which allowed us to estimate the pension funds asset growth. The thesis eventually expanded into asset and liability analysis with respect to currency risk.

5.1. Findings and Limitations

In the empirical study, we analyzed currency risk by taking advantage of Markowitz’s mean-variance model. We included two groups of investment instruments for a Euro-currency based pension fund investor: (1) hedged and unhedged indices, (2) unhedged indices with forward contract of six global currencies. These currency hedged instruments are fairly novel. A great portion of them especially started to surface after the financial crisis. Based on our data and its period, we found the following evidence can help the analysts to make strategic investment assumptions regarding the OECD pension fund guideline:

a) When mixing hedged and unhedged funds, it improves portfolio’s risk reduction yet it also forgoes a portion of the return. Therefore, unconstrained portfolios tend to produce extreme solutions with low returns if the objective is to minimize risks.
b) Currency hedging with forward contracts offer investor a “free lunch” by reducing volatility while keep the return at the relatively same level.

c) When analyzing funding ratios of a pension fund, mixing hedged/unhedged indices can be more effective in improving such funding ratio. The downside of this strategy is that asset allocation may achieve less diversification. In contrast to using indices, hedging currency risk with forward contacts offers less certainty in achieving targeted funding ratio, but it deceases risk and achieves more diversification.

It is also noteworthy to point out that some of the obstacles and how we overcame them in our studies are practical. First of all we suggested estimating expected return by Dividend Discounting Growth Model given it can project the stock return based GDP growth and expected future inflation. We further chose to incorporate the expected 42 year GDP and inflation data from OECD outlook into DDG model. This offers analysts the reliability to project return with trustworthy sources especially when the organization is a member of OECD countries.

Secondly, many pension funds are short of a general utility function or a risk aversion level which makes it difficult for analysts to implement utility based investment models. We suggest analysts should invest along the efficiency frontier curve which represents the market equilibrium portfolio given the nature of defined benefit scheme is to invest in the interest of its large body of members.

At last, within the scope of this thesis, we mainly focused on static currency hedging; the complexity of the models can be further developed. For instance, as presented by Campbell and Viceira (2004), we can further use interest rates to estimate exchange rates and apply the results to other dynamic hedging models. However, the extensive assumptions to build such models become questionable to the public and the management board. In addition, all models have a limited predicting power, which raises the question of whether complex models are necessarily more reliable than simple models. Subsequently, the selection of an adequate model depends on the guideline provided by the individual organization.

### 5.2 Further Research

In this thesis, we mainly focused on applying the Markowitz model to examine the currency hedging effect on a risk basis portfolio construction. However, other existing models can also be applied to the analysis of the strategic allocation. For instance, it is interesting to use Black-Litterman model to find a set of asset allocations based on historical returns and projected returns since the only using historical return is widely under criticism.

In addition, referring to a lack of utility functions or a risk aversion level for defined benefit pension funds in section 3, scholars can expand economic/financial models to further analyze a utility function for pension fund members. Once it is defined, analysts can include it into other models.
References


Appendix:

Appendix I: DDM Constant Growth Model

Proof of the transformation of the Constant-Growth DDM (3.5):

\[ V_0 = \sum_{i} \frac{D(1+g)^{i-1}}{(1+r)^i} \]

\[ V_0 = \frac{D}{1+r} + \frac{D(1+g)}{(1+r)^2} + \frac{D(1+g)^2}{(1+r)^3} + \ldots \]

Applying geometric series, where we assume \( k<1 \)

\[ \frac{1 + r}{1 + g} V_0 = \frac{D}{1 + g} + V_0 \]

\[ \frac{1 + r}{1 + g} V_0 = \frac{D}{1 + g} + V_0 \]

\[ \frac{1 + r}{1 + g} - 1)V_0 = \frac{D}{1 + g} \]

\[ \frac{r - g}{1 + g} V_0 = \frac{D}{1 + g} \]

\[ V_0 = \frac{D}{r - g} \]

Appendix II: Mean-Variance Portfolio Utility Function and Risk Aversion Level:

2.1 Derivation of Utility Function for Mean Variance Portfolio by Taylor Expansion Series and Arrow-Pitt Measurement

In respect to expected total wealth accumulation function \( E(W_{1+r}) \approx W_1(1 + T \mathbf{x}' \mathbf{r}) \); we show the risk aversion level in \( U = E\left(r_p\right) - .5 \times A \times \sigma_p^2 \) as a form of utility function with 2\(^{nd}\) order of Taylor expansion series:

\[ E\left(U(W_{1+r})\right) = E[U(W_1)(1 + r_p)]^{10} \]

\[ \approx E[U(W_1) + U'(W_1)r_pW_1 + .5U''(W_1)r_p^2W_1^2] \]

\[ = U(W_1) + U'(W_1)E(r_p)W_1 + .5U''(W_1)W_1^2[(E(r_p)^2 + \sigma_p^2)] \]

\(^{10}\) Here we treat \( \mathbf{x}' \mathbf{r}_{1+j} \) as \( r_p \)
To simplify the expected utility, we subtract $U(W_1)$ and divide by $U'(W_1)W_1$ to arrive at the mean variance utility function:

$$E(r_p) + 0.5 \frac{U''(W_1)W_1}{U'(W_1)} [(E(r_p)^2 + \sigma_p^2)$$

$$\approx E(r_p) + 0.5 \frac{U''(W_1)W_1}{U'(W_1)} \sigma_p^2$$

where $-\frac{U''(W_1)}{U'(W_1)} = A$ is defined by Arrow-Pratt as relative risk aversion measurement.

### 2.2 Identical Approach between Utility Maximization and Risk Minimization by Lagrange Function

The mean-variance equation suggests the following:

Max: $E(r_p) = \sum x_i E(r_i) = \sum x_i \mu$

Min $\sigma_p^2(r_p) = x_i^T \Sigma x_i$

By applying Lagrange function, we derive the following:

$L = x_i \mu - \lambda (x_i^T \Sigma x_i)$

Differentiating the Lagrange function, we get:

$$\frac{dL}{dx} = \mu - 2\lambda x_i \Sigma = 0$$

Instead of deriving for $\lambda$, we let the $\lambda=0.5*A$ from utility function $E(r_p) + 0.5A \sigma_p^2$ as the multiplier. So, we have:

$$\frac{dL}{dx} = 0 = \mu - 2\lambda x_i \Sigma$$

$$\frac{dL}{dx} = 0 = \mu - 2(0.5A)x_i \Sigma$$

$$\frac{dL}{dx} = 0 = \mu - Ax_i \Sigma$$

$$\mu = Ax_i \Sigma$$

$$x_i = (A \Sigma)^{-1} \mu$$

Then, we plug the $w$ into the variance equation:

$$\sigma_p^2(r_p) = x_i^T \Sigma x_i$$
\[ \sigma_p^2(r_p) = (A\Sigma)^{-1} \mu^T \Sigma i (A\Sigma)^{-1} \mu = A^{-2} \mu^T \Sigma i^{-1} \mu \]  \hspace{1cm} (A.2.1)

To maximize utility equation, we have:

\[ \text{Max: } U = E(r_p) - 0.5 \times A \times \sigma_p^2 = x_i \mu_p - 0.5 \times A \times \sigma_p^2 \]

First of all, we take the derivative of the utility function to maximize the given function as the following:

\[ \frac{du}{dx} = x_i \mu_i - 0.5 \times A \times \sigma_p^2 \]

\[ \frac{du}{dx} = x_i \mu_i - 0.5 \times A \times x_i^T \Sigma i x_i \]

\[ \frac{du}{dx} = \mu_i - A \sum_i x_i \]

\[ \frac{du}{dx} = 0 = \mu_i - A \sum_i x_i \]

\[ \mu_i = A \sum_i x_i \]

\[ x_i = (A\Sigma)^{-1} \mu_i \]

Without losing of generality, we get:

\[ \sigma_p^2 = x_i^T \sum_i x_i = (A\Sigma)^{-1} \mu^T \Sigma i (A\Sigma)^{-1} \mu = A^{-2} \mu^T \Sigma i^{-1} \mu \]

which is the same as A.2.1, the variance equation for maximization of return and minimization of risk. Therefore, the 0.5A is just a Lagrange multiplier. Both maximization equations produce the same results. However, utility maximization function allows investor to impute the risk aversion level in the mean-variance framework.

To go further, one may derive the equation for risk aversion level \( A^{11} \)

\[ \sigma_p^2 = A^{-2} \mu^T \sum_i^{-1} \mu \text{, given } \mu = Ax_i \sum_i \]

\[ \sigma_p^2 = A^{-2} \mu^T \sum_i^{-1} Ax_i \sum_i \]

\[ \sigma_p^2 = A^{-1} \mu^T x_i \]

\[ A = \frac{\mu^T x_i}{\sigma_p^2} \]

\[ A = \frac{\mu_p}{\sigma_p^2} \]

\[ 11 \text{ This is the same risk aversion formula used in Black-Litterman model} \]
This result is also used in the Black’s paper in estimating the risk aversion level.