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The Minimum Variance Portfolio

An Exploitable Anomaly?

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Last decade's reported findings on the outperformance of minimum variance strategies seem to be in violation with traditional principles of risk and return. A long-only "unconstrained" constructed Minimum Variance (MV) portfolio outperforms its market capitalization weighted (MCW) counterpart on the basis of the Carhart (1997) four-factor model from January 1976 to December 2012. The dominance in efficiency and the significant alpha return provokes the perception of an exploitable MV anomaly. The composition and construction methodology of MV portfolios are at the heart of the perceived "Minimum Variance Puzzle". The MV alpha return could reflect a rational premium that MV investors require as compensation for the exposure to unobserved risk factors. The former, significant alpha return is statistically insignificant when controlling for BAB, the absence of implied protection, and industry-specific risk. The emerging underlying risk properties question the low-risk appearance of minimum variance strategies.

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1. Introduction

Mean variance efficient portfolios have been the backbone of traditional finance concepts since the first enunciation of modern portfolio theory in the 1960s (Samuelson, 1965) and (Fama and Malkiel, 1970). The efficient frontier procedure: estimating expected returns and corresponding (co)variances for individual assets, and then minimizing ex-ante portfolio risk for a given return level by altering individual security weights, as described by Markowitz (1952), appears to approach the end of its lifetime. The implementation of the Markowitz (1952) theory gives rise to some practicality issues such as the complexity of econometric techniques necessary for larger security sets and the problems resulting from expected returns estimations (Clarke, de Silva, and Thorley, 2006). In addition, the assumption that market-matching portfolios are efficient is debatable. “Matching the market is an inefficient investment strategy, even in an informationally efficient market.” Haugen and Baker (1991) were among the first researchers who questioned the efficiency of market capitalization weighted portfolios. The general prediction that these portfolios are mean-variance efficient holds true only in the presence of some strong assumptions. Haugen and Baker (1991) summarize the following assumptions; (i) all investors agree about the risk and return distribution for all investable securities; (ii) no restrictions exist on short-selling; (iii) returns are not exposed to taxes; and (iv) the investment universe is restricted to the securities in the capitalization weighted index. They argue that it is unrealistic and highly unlikely that these assumptions hold. Even in the presence of some assumptions, investing in a capitalization weighted index might be a suboptimal investment strategy. The conclusion from this theory calls for identifying investment strategies that are characterized by proper risk management and efficient risk-return properties.

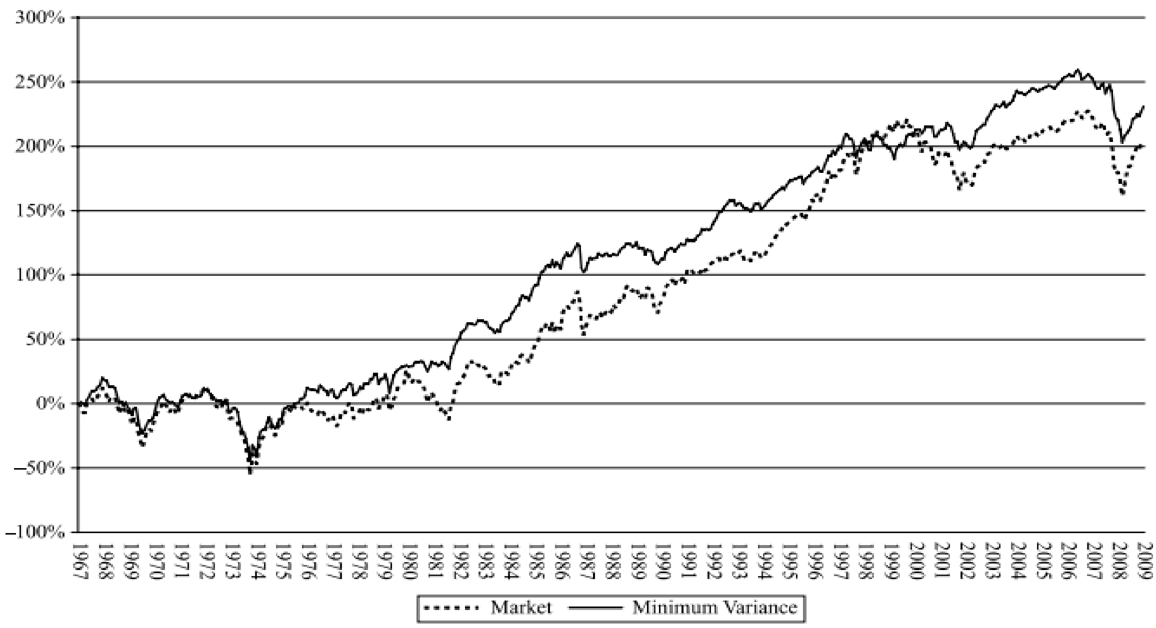
1.1 Research Problem

One of the investment strategies gaining popularity is the Minimum Variance (MV) portfolio. This portfolio is depicted at the outmost left point on the efficient part of the mean variance frontier (MSCI Index Research, 2010). It is designed to minimize risk without a given target return. In simple terms, the MV portfolio can be defined as the fully invested portfolio containing the lowest level of portfolio variance, or risk. The portfolio’s construction methodology is purely based on the universe of assets and the complete covariance matrix (Luo, Cahan, Jussa, Chen, and Alvarez, 2011). In contrast to the market capitalization weighted portfolio, the MV portfolio weights do not depend on any stock return forecasts.

Strategies relying on expected return estimates are often oversensitive to small changes in forecasted returns and could lead to highly unbalanced portfolio weights (Clarke et al., 2006). In contrast, the MV portfolio is merely based on the estimation of the covariance matrix. As an asset's variance and covariance are likely to be persistent and rather predictable (Clarke et al., 2006), conducting this minimization problem has more favorable properties from a practical point of view. Furthermore, MV portfolio's independency of expected return estimates is appealing for investors who do not want to be engaged in actively chasing alpha returns.

Following the findings of Haugen and Baker (1991) that instigated a departure from market-matching portfolio strategies, many other researchers attempted to detect alternative efficient investment strategies. However, as mentioned before, the minimum variance strategy is likely to be one of the most popular alternatives. Clarke, de Silva, and Thorley (2011) conducted research similar to the study done by Haugen and Baker. They analyzed the performance of equity portfolios having the lowest possible variance through an examination of large-scale minimum variance portfolios from 1968 to 2009. The realized cumulative excess return of the MV portfolio has been higher than the market as presented in figure 1. The dotted line is the cumulative excess return of a Market Capitalization Weighted (MCW) portfolio containing the 1000 largest U.S. stocks. The solid line represents the cumulative excess return of the MV portfolio with a long-only constraint.

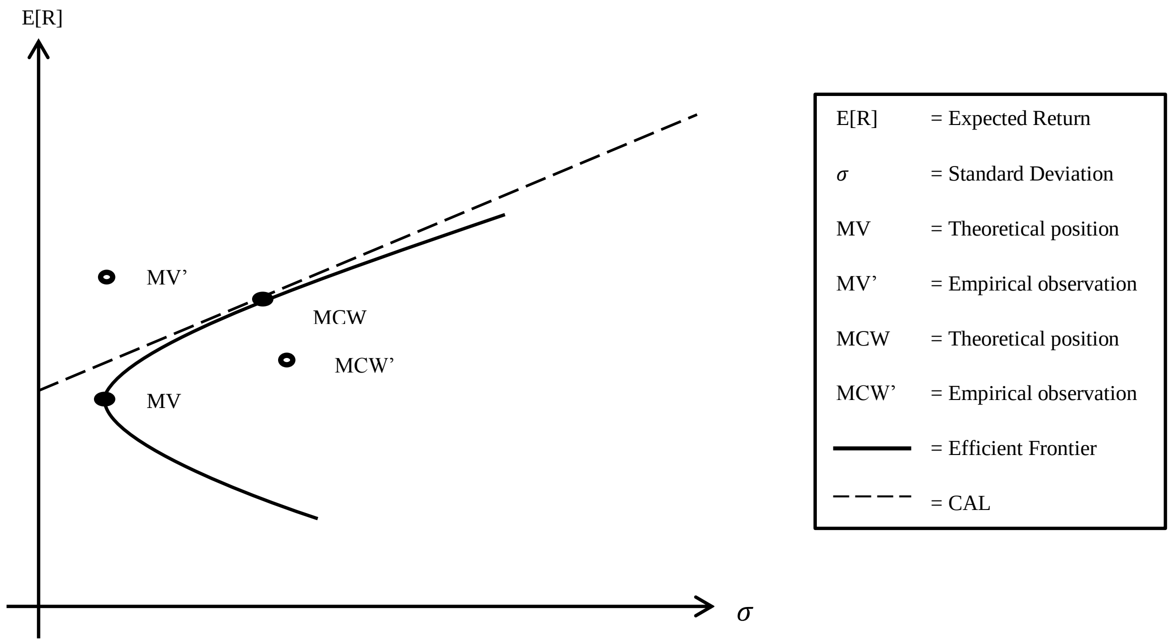
Figure 1: Cumulative Excess Returns of Market and Minimum Variance Portfolios, 1968-2009.



Source: Clarke, de Silva, and Thorley (2011).

Additionally, their paper indicates that the realized risk of the MV portfolio is still below the risk of the market portfolio. Many other researchers encountered similar results on the perceived outperformance of the MV portfolio (Jagannathan and Ma, 2003; Behr, Güttler, and Miebs, 2008; Nielsen and Aylursubramanian, 2008; Baker, Bradley, and Wurgler, 2011; Blitz and van Vliet, 2011; Blitz, Pang, and van Vliet 2013; and Kuo and Li, 2013). In sum, these findings provide evidence for an overall extraordinary performance of MV portfolios with respect to their capitalization-weighted benchmarks. Figure 2 depicts the difference in the empirical observation and the theoretical position of both portfolios. This gives rise to the general perception of an existing anomaly in traditional finance theories. The MV portfolio seems to offer a higher return than the MCW portfolio, while having a lower level of risk. In order to explore whether this anomaly can be attributed to the existence of unobserved risk sources, thorough insight must be gained in the characteristics of MV portfolios.

Figure 2: Difference in the Risk-Return Properties of the Theoretical and Empirical Positions of the MV and MCW Portfolio.



1.2 Research Aim

The first aim of this paper is to test whether (on the basis of large-scale portfolio optimization) the risk-return properties of the unconstrained, long-only MV portfolio are more favorable than those of its market capitalization weighted (MCW) counterpart over the period January 1976 to December 2012. In this respect, this research examines whether the MV portfolio exhibits significant and risk-adjusted out-of-sample outperformance with respect to the MCW portfolio based on the Carhart (1997) four-factor model. The MV portfolio and the MCW benchmark portfolio are constructed from the same set of securities; the 1000 largest U.S. market capitalization stocks.

Secondly, this paper proposes a set of unobserved risk factors that may have the ability to declare the potential risk-adjusted outperformance under the four-factor (Carhart, 1997) model. The empirical observation that low-beta stocks outperformed high-beta stocks gave birth to the Betting-against-Beta (BAB) factor as documented by Frazzini and Pedersen (2010). The BAB factor captures the excess return differential between low and high-beta stocks. Following from the expectation that the MV portfolio is concentrated towards low-beta stocks, it is of high interest to test whether the BAB factor and related (or underlying) risk sources affect the performance of the MV portfolio. The impact of the related risk sources, namely leverage constraints, absence of implied protection, and industry-specific risk will be narrowly defined and explored. To this end, the paper attempts to declare whether the long-only "unconstrained" MV portfolio is prone to these risk factors. Consequently, these unobserved risk sources try to explain the MV outperformance and the corresponding perception of a MV anomaly.

1.3 Relevance

In this paper, the performance, composition, and risk-return properties of the MV portfolio are identified and compared with amongst others the MCW benchmark portfolio. In this respect, this field of research discusses the role of the MV portfolio in contrast to the market portfolio as benchmark for conducting properly risk-managed equity strategies.

Interest in the minimum variance strategy has increased substantially, particularly in the past decade. The financial crisis and the prolonged economic downturn that went along with stricter regulations in the pension and banking sector caused higher aversion to asset volatility (MSCI, 2012). This development sets the trend for low-volatility equity strategies such as the minimum variance strategy. The popularity of MV portfolios is expressed in the emergence of MSCI Minimum Volatility Indices for several domestic, international, and global markets (Luo et al., 2011). Recently, even Germany and the United Kingdom have launched minimum volatility indices, the DAXplus minimum variance, and the FTSE 100 Minimum Variance Index respectively. A more important reason that contributed to the sudden popularity for MV strategies is the perceived risk-adjusted outperformance relative to the market portfolio. In an efficient market, where betas are linearly related to equity returns, this would not be possible. Designing a portfolio to reduce portfolio risk must in theory generate lower returns. It is of high relevance to test whether the perception of an MV anomaly is not just a compensation for a set of unobserved risk factors. The historical violation of risk and return principles and the perceived outperformance of MV strategies will be scrutinized. It is important to be able to answer whether the MV strategy is a valuable future investment opportunity and one that could serve as a relevant benchmark instead of a market capitalization weighted benchmark. In order to answer these questions, a large-scale, long-only "unconstrained" MV portfolio is constructed in a similar way as the study conducted by Clarke et al. (2006, 2010). In contrast to their papers, this research goes beyond the effect of the market factors, SMB, HML, and MOM on the MV performance. The impact of leverage constraints, the absence of implied protection, and industry-specific risk are introduced as potential determinants of the extraordinary performance of the MV portfolio. Additionally, this paper extends the study on the long-only MV portfolio to December 2012. It is key to gain a better understanding of the portfolio's exposure to underlying risk factors in the light of the increased appreciation for risk management.

1.4 Research Question

Is the risk-adjusted outperformance of the MV portfolio an exploitable anomaly or a rational compensation for a set of unobserved risk factors?

1.5 Structure

Section 2 starts describing the historical developments that led to the emergence of the four-factor (Carhart, 1997) model. This model is implemented in order to test for significant outperformance. Subsection 2.3 describes empirical findings of similar studies in the field of MV strategies and theoretical derivations of the MV portfolio. Subsection 2.4 discusses the departure from traditional risk sources. Section 3 draws a complete overview of the empirical framework and the corresponding five hypotheses. Section 4 provides a specification about the data used for every variable. The research methodology and the construction of the long-only MV portfolio and other factors are elaborately explained in section 5. Subsequently, the empirical findings are discussed in section 6. Results are complemented by a sensitivity analysis that is conducted in section 7. Section 8 gives a detailed conclusion. At the end, limitations and recommendations are provided in section 9 and 10 respectively.

2. Literature Review

This section presents a comprehensive overview of existing literature on the performance of Minimum Variance (MV) portfolios and the emergence of new risk sources. This review starts discussing the developments in traditional finance that eventually led to the birth of the Carhart (1997) four-factor model. This asset pricing theory forms the base case model of this paper and plays a key role in portfolio performance evaluation. Secondly, market capitalization weighted portfolios and their role as proxy for the market are discussed. Additionally, this review reflects the findings of researchers and academics concerning the performance and the risk-return properties of the MV portfolio. At the end of this section, three additional sources of risk are explored and proposed as risk factors that potentially capture the MV alpha return.

2.1 Carhart Four-Factor Model

The general prediction of the Capital Asset Pricing Model (CAPM) developed by Sharpe (1964), Lintner (1965), and Black (1972) is that the market portfolio is mean-variance efficient. As a consequence, Jensen's alpha (1967), the intercept term in a time-series regression, is zero on average. This also implies that individual stock returns are a positive linear function of their beta, which fulfills the function to explain cross-sectional variation in returns. Giving the implicit assumptions, investors should allocate their investment portfolio to the risk-free asset and the market portfolio. As risk aversion differs among individuals, each investor could move along the capital market line such that their risk-return preferences are satisfied.

However, later developments have caused the need for some adjustments to traditional finance theories. The market beta did not suffice to explain cross-sectional variation in asset returns. Fama and French (1992) showed the existence of some risk factors in addition to market risk. They confirmed that firms with a smaller capitalization and a high market-to-book ratio on average had a positive abnormal risk-adjusted return. For this reason, factors are constructed on the basis of long-short portfolios that capture the return differences. This finding has led to the emergence of the small-minus-big (SMB) and high-minus-low (HML) factors. These factors can be interpreted as priced risk factors; any exposure to one of these risk factors will be compensated with a risk premium. However, another risk factor remains unexplained by the three-factor model. The momentum effect of Jegadeesh and Titman (1993) show that stocks that performed relatively well over the last three to twelve months tend to continue doing well for the subsequent months, and stocks that demonstrated relative underperformance continue performing poorly. In response, Carhart (1997) included the momentum (MOM) factor that measures the excess return differential by implementing a trading strategy that goes long in recent winners and short in recent losers. The Carhart (1997) four-factor model is represented by regression equation 1.

$$R_t^p - R_t^f = \alpha + \beta\left[R_t^m - R_t^f\right] + sSMB_t + hHML_t + mMOM_t \quad [1]$$

2.2 A Proxy for the Market Portfolio

For many decades, the financial industry relied on the perception that the market portfolio is mean-variance efficient and that market capitalization-weighted indices form a good proxy for the efficient market portfolio (Arnott, Kalesnik, Moghtader, and Schol, 2010). Advocates of this theoretical perception have fundamental reasons to stick to this view. Hsu (2006) lists some notable benefits of cap-weighting portfolios. Portfolio selection based on market capitalization could be advantageous because of automatic rebalancing and the low level active management that is required. This makes it an attractive trading strategy from a cost perspective. As the greatest portfolio weights are assigned to stocks with the largest market capitalization, this investment strategy results in an efficient diversification within the stock market. Lastly, it is stated that a market portfolio is automatically mean-variance optimal in the sense that the Sharpe Ratio is maximized. Though, the latter only holds in the presence of a strict set of CAPM assumptions. Although the benefits are powerful, the unlikeliness of the required conditions inevitably caused the search for investment strategies departing from the market capitalization weighted strategy.

2.3 Minimum Variance Portfolio

The MV portfolio is positioned at the most left point on the mean variance frontier as can be seen in figure 2. This point indicates the efficient portfolio that has the lowest possible level of return variation, or risk. It is designed by minimizing portfolio variance without a given target return. The portfolio's construction is purely based on the universe of assets and the complete covariance matrix (Luo et al., 2011). In contrast to the MWC portfolio, the MV portfolio weights do not depend on any stock return forecasts. Strategies relying on expected return estimates are often oversensitive to small changes in forecasted returns and could lead to highly unbalanced portfolio weights (Clarke et al., 2006). In contrast, the MV portfolio is merely based on the estimation of the covariance matrix. As an asset's variance and covariance are likely to be persistent and rather predictable (Clarke et al., 2006), conducting this minimization problem has more favorable properties from a practical point of view. Furthermore, MV portfolio's independency of expected security estimates is an appealing characteristic for investors who do not want be engaged in the practice of actively chasing alpha returns.

2.3.1 Construction Minimum Variance Portfolio

The main challenge of efficient portfolio formation is estimating expected returns accurately. Weights of efficient portfolios are highly dependent on expected return measures. A small change in return estimation could cause a severe shift in the stock weight allocation process. MV portfolios have some construction advantages. In contrast to other efficient portfolios, MV portfolio weights only rely on an ex-ante variance-covariance matrix. This does stress the importance of applying an appropriate method to estimate stock variances and pairwise covariances correctly. In this paper, (co)variance estimation is done through the Bayesian shrinkage method of Ledoit and Wolf (2004). The construction of the MV portfolio could be presented in a constrained minimization problem. In equation 2, this portfolio optimization problem is defined in matrix notation.

$$\min_{w} \quad w'\Omega w \qquad\qquad s.t.\ w'\iota = 1 \qquad [2]$$

The objective function is minimized with respect to the constraint that the sum of individual stock weights is equal to 1. In the minimization function, w refers to the individual stock weights, and Ω to the estimated covariance matrix. The constraint function is a multiplication of a row vector of stock weights by a vector ι consisting of ones. In order to find the solution for the MV portfolio weights, the Langrage multiplier is employed (Chincarini and Kim, 2006). The Lagrange function can be observed in equation 3.

$$L = w'\Omega w - \lambda(w'\iota - 1) \qquad [3]$$

The Langrage function as represented above, is a combination of the minimization function and the constraint, where λ is the Lagrange multiplier. Taking the partial derivatives of the

Lagrange function with respect to w and λ provides equation 4 and 5. Accordingly, these first order conditions are set equal to zero in order to solve for the parameters w and λ.

$$\frac{\partial L}{\partial w} = 2\Omega w - \lambda\iota = 0 \qquad [4]$$

$$\frac{\partial L}{\partial \lambda} = w'\iota - 1 = 0 \qquad [5]$$

In addition to this, the system of linear equations can be solved by means of substitution. The first partial derivative results into equation 6. The latter results in equation 7, and simply states that the sum of individual weights is equal to one.

$$w = \lambda(2\Omega)^{-1}\iota \qquad [6]$$

$$w'\iota = 1 \qquad [7]$$

Further, equation 7 is plugged into equation 6 by multiplying both sides of the equation by iota as could be observed in equation 8. From this, the Lagrange multiplier can easily be derived (equation 9).

$$w'\iota = \lambda\iota'(2\Omega)^{-1}\iota \qquad [8]$$

$$\lambda = \frac{1}{\iota'(2\Omega)^{-1}\iota} \qquad [9]$$

In order to compute the final MV weight vector, the function of the Lagrange parameter is used in equation 8 to substitute out for lambda. Finally, both the nominator and denominator are divided by two in order to derive the final weight vector as represented in equation 10.

$$w_{MVP} = \frac{1}{\iota'(2\Omega)^{-1}\iota}(2\Omega)^{-1}\iota = \frac{\Omega^{-1}\iota}{\iota'\Omega^{-1}\iota} \qquad [10]$$

This provides the function for the portfolio variance:

$$w^{*\prime}\Omega w^{*} = \frac{1}{\iota'\Omega^{-1}\iota} \qquad [11]$$

2.3.2 Historical Performance

In the past decade, many academics have reported remarkable results concerning the performance of MV strategies. Figure 1 displays the outperformance of the MV portfolio compared to its market benchmark portfolio on the basis of cumulative excess returns found by Clarke et al. (2010). For almost the complete data period (1967-2009), the solid line that represents the cumulative returns of the MV portfolio lies above the dotted line, the market's cumulative excess return. In contrast to traditional finance, the higher average return of the MV portfolio does not come at the cost of more risk. This relatively recent empirical finding has connection with research dating back to 1992. Fama and French (1992) documented that average returns of high-market-beta stocks are not commensurate with the risk related to these stocks. Even before this finding, Black, Jensen, and Scholes (1972) suggested that the security market line for U.S. stocks is too flat in comparison to the CAPM prediction.

In the past few years, academics continued reporting extraordinary returns to low-volatility portfolios. Clarke et al. (2006, 2010) proved significant outperformance of their constructed long-only unconstrained minimum variance portfolio compared to the market benchmark portfolio. Behr, Güttler, and Miebs (2008) conducted the constrained minimum variance approach of Jagannathan and Ma (2003). This constraint assures a certain level of diversification within the investment portfolio. They provide robust evidence that constrained minimum variance portfolios perform relatively better than their value-weighted benchmarks on a risk-adjusted basis. Arnot et al. (2010) concluded that MV portfolios dominated the capitalization weighted portfolio in returns but also when controlling for the four factors of Carhart (1997). The approach used by Blitz and van Vliet (2011) also recognized a superior risk-return tradeoff established by a low-volatility index compared to passively investing in the capitalization-weighted market index. Baker, Bradley, and Wurgler (2011) add to the existing literature by providing additional evidence for the long term outperformance of low-volatility and low beta-stock portfolios. In addition, Geiger and Plagge (2007) conducted a study on the performance of MV portfolios on a broader cross-section of countries by including the stock markets of Germany, France, Switzerland, Japan, and the U.S. In terms of Sharpe Ratio, the improvement caused by implementing a minimum variance approach was largest for stock markets outside the U.S. Nielsen and Aylursubramanian (2008) extend the study on MV portfolios by focusing on the global universe. Their findings suggest a strong risk-adjusted outperformance of the MSCI global MV Index compared to the MSCI World

Index over the period June 1995 to December 2007. The MV Index of the MSCI demonstrated an overall excess return of 6.5% in contrast to a 6% of the MSCI World. Additionally, the MSCI global MV Index reported almost 30% lower volatility. This resulted in a superior Sharpe Ratio in favor of the MV component relative to its market proxy of 0.67 versus 0.45. Linzmeier (2010) conducts thorough research on three risk balanced investment approaches, namely minimum variance, equal risk contribution, and maximum diversification. All strategies are exposed to regular risk factors but favor the benefits of lower volatility. The minimum variance approach exhibits both superior annualized return and superior volatility for the sample period from August 1991 to December 2010. Kuo and Li (2013) also indicate that in times of financial market volatility and economic downturn, performing a MV strategy offers considerably more added value than their proxy for a market portfolio. They reason that in recessionary times, investors require additional compensation for taking up risk. A more recent study conducted by Blitz et al. (2013), examines the volatility effect in emerging markets. The authors demonstrate that a flat or even negative relation exist between risk and return in emerging equity markets. Additionally, they state that the volatility effect tends to be more severe over time.

2.4 Rationale for other Risk Sources

The empirical observation that MV portfolios outperform their market benchmark portfolio in terms of risk and return might stem from additional risk sources inherent to a MV strategy. In other words, the MV portfolio might be exposed to other types of risks than the market portfolio which might arise from differences in portfolio construction and the corresponding portfolio composition. The MV weight allocation process is purely dependent on the estimated covariance matrix. This fact results into the expectation that MV portfolios are biased towards assets having low covariation with the market, namely low beta-stocks. This guides towards the search for risk sources that prevail in low-beta strategies. This research departs from current risk factors and test whether the observed violation of risk and return principles still holds in the presence of alternative (low-beta) risk factors. The focus is on three additional risk sources: leverage constraints, the absence of implied protection, and industry-specific risk. Frazzini and Pedersen (2010) propose the Betting-against-Beta (BAB) factor. They demonstrate that portfolios consisting of a long levered component of low-beta assets and a short de-levered component of high-beta assets produce significant positive risk-adjusted returns. An alternative explanation for the MV premium might come from the finding of Cowan and Wilderman (2011), who illustrate the absence of implied protection in low-beta stocks. They state that the absence of implied protection is the main driver of the concave return distribution in low-beta stocks. Thirdly, MV's concentration towards specific industries will be assessed.

2.4.1 Betting-against-Beta Factor

The BAB factor came into live with the empirical observation that low-beta stocks performed better compared to the predictions of the CAPM, while high-beta stocks performed worse (Black et al., 1972) and (Miller and Scholes, 1972). Many other authors, even Fama and French (1992) came to conclude that the security market line, demonstrating the relationship between risk and return, is flatter than what the CAPM indicates. The fact that low-beta stocks had performed so well in the past motivated Frazzini and Pedersen (2010) to explore the reward of conducting a Betting-against-Beta strategy. The BAB factor has a long position in low beta-assets and a short position in high-beta assets. The BAB factor is market neutral in the sense that the long component is leveraged up to a beta of one and the short component is de-leveraged to a beta of one. In this way, the BAB's exposure to the market portfolio is equal to zero. This zero-cost portfolio expresses the excess return differential similar to the Carhart (1997) factors. The authors find significant positive risk-adjusted BAB returns that are consistent both over time and over the cross-section of countries and asset classes.

According to Frazinni and Pedersen (2010), leverage constraints are at the core of the positive risk-adjusted BAB return. The primitive thought of the CAPM is that all investors invest in the portfolio providing the highest Sharpe ratio, and then leverage or de-leverage their portfolio until they have reached their preferred risk-return profile. However, the CAPM ignores that some institutional investors are constrained in the degree of leverage they can take, while other investors are even averse to use it. This results in a general preference for the use of high-beta stocks (implicit leverage) over the use of leveraged low-beta stocks (explicit leverage) to get at the preferred level of market exposure. Excessive demand for risky high-beta stocks drives up their prices and lowers their required risk-adjusted return. The opposite line of reasoning goes for low-beta stocks that require higher risk-adjusted returns.

Frazzini and Pedersen (2010) argue that real-world investors are threatened by funding constraints like margin requirements and leverage constraints. They state that BAB returns are dependent on the tightness of credit constraints and on the cross-sectional beta dispersion between the long and short sides of the factor. Their findings suggest that the TED spread (employed as proxy for liquidity risk) negatively predicts BAB returns. Intuitively, agents may have to de-leverage their betting-against-beta position or even buy market exposure by investing in higher beta-stocks in times of increased liquidity risk. The factor loses as constraints become more binding. Investors are financially more restricted and require even higher returns. In times of illiquidity, characterized by a high TED spread, there exists a lower cross-sectional dispersion of betas. The ex-ante spread between the beta of the long and the short component of the factor positively affects BAB returns. Less-constrained investors can take advantage by the BAB effect by leveraging low-beta positions; they are simply being compensated by more-constrained investors that invest in high-beta stocks.

Frazinni and Pedesen consider leverage constraints as the main source of risk that validates the risk-adjusted outperformance of the BAB factor. It is unlikely that leverage constraints are the only source of risk that is captured by the BAB factor. However, it is unclear whether the alternative risk elements fully constitute this factor or have an individual effect on the performance of MV portfolios. Therefore, the theoretical characteristics of these factors are discussed individually in the following subsections.

2.4.2 Risk from the Absence of Implied Protection

Cowan and Wilderman (2011) do not perceive the counterintuitive outperformance of low-beta stocks compared to high-beta stocks as an anomaly. The phenomenon they refer to as the "beta puzzle" can be attributed to a general misconception of risk. In addition, they do not fully agree with the theory of Frazzini and Pedersen (2010). It is not the general preference for high-beta stocks (implicit leverage) over leveraged low-beta stocks (explicit leverage), but the desirability of protection that the implicit leverage of high-beta stocks offer. The implied protection present in high-beta stocks is reflected in the convex payoff pattern. According to this view, the "beta puzzle" is not an exploitable arbitrage opportunity, but instead a rational outcome of the difference in the payoff patterns between low and high beta-type assets during different market conditions caused by the different types of leverage (figure 3 and 4).

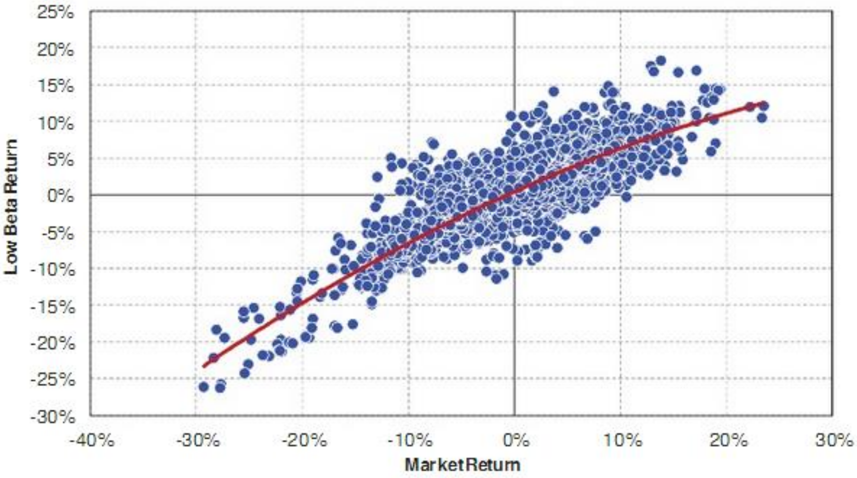
Accordingly, high-beta stocks offer implicit exposure to the market. In order to reach a similar level of market exposure with low-beta stocks, one must use explicit leverage (borrowings). The difference in the type of leverage creates different payoff patterns during different market conditions. As the example will demonstrate, payoffs in a market upstate will be almost similar, while they differ during market down states.

Example:

Suppose that we have two investors, A and B, who both have $100 to invest. The risk-free rate is equal to 3%. Investor A invests $100 in stock X with a $\beta = 1$. *Investor B invests in stock Y with a* $\beta = 0.5$. *In order to synthetically create the same level of market exposure, investor B borrows an additional $100 to invest a total of $200 in stock Y. Investor A makes use of the implicit leverage offered by the market, while investor B uses explicit leverage by borrowing money. In the case of a market upstate, both investors will observe similar positive returns. However, if the market falls by for instance 60%, investor A has still left $40 from his initial investment while investor B has nothing left or (even worse in the case of a margin call) has a negative cash position. The value of his investment in stock Y has dropped to $80, while he has to pay off debt of $103 (after one year).*

The example indicates that the high performance of (leveraged) low-beta stocks is due to a higher return as compensation for a high down market equity exposure. The potential loss for low-beta stocks in the case of a stock market crash is much larger than for high-beta stocks as these low-beta stocks have to hold a higher amount of equity to arrive at the same level of market exposure as high-beta stocks. Vice versa, high-beta stocks offer an implicitly leveraged market position. Cowan and Wilderman (2011) reason that high-beta investors benefit from the upside potential during bull markets, while their downside risk is limited in bear markets. As a consequence, high-beta investors will accept a lower return. The implicit leverage of high-beta stocks is also referred to as implied protection and results in a convex relation with the market. MV portfolios are expected to be concentrated towards low-beta stocks that are characterized by an absence of implied protection (caused by the explicit form of leverage). Low-beta stocks have a relatively high beta in bear markets and a relatively low beta in bull markets (Cowan and Wilderman, 2011). Intuitively, in case of an extreme negative event, betas converge to one regardless of the historical beta level. As a consequence, low-beta stocks benefit less from upside potential, while suffering from more downside risk. This unfavorable property of low-beta stocks could be a justification for the premium demanded for holding these stocks. In other words, the absence of implied protection in low-beta stocks and the corresponding concave payoff pattern (figure 3) may explain the risk-adjusted outperformance of MV portfolios.

Figure 3: Concave Payoff Patter of Low-Beta Stocks.

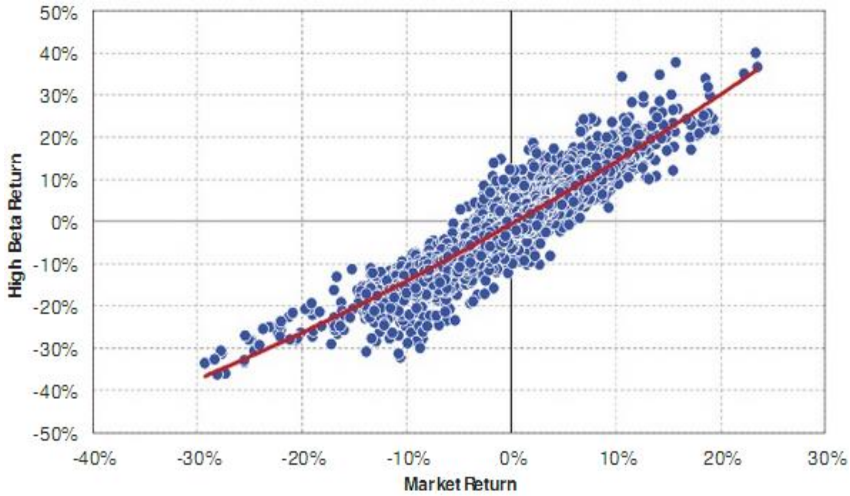


Source: Cowan and Wilderman (2011).

Cowan and Wilderman (2011) claim that the beta level of low and high beta-type stocks is conditional on the performance of the market. This is in conflict with the CAPM assumption stating that stock returns exhibit a linear relation with their corresponding betas irrespective of the market state. The absence of implied protection in low-beta stocks results in an unfavorable concave return relation with the market. In times of high market volatility, the return on low-beta stocks and low-volatility portfolios is likely to be worse than what the CAPM predicts, and investors require a premium in the form of positive alpha.

The authors note that the asymmetry in the payoff patterns is equivalent to the payoff profiles observed in the option market. The convex relation of high-beta stocks with the market (figure 4) is similar to the payoff profile constructed from holding a long position in the market and owning a call option on the market. Investors fully profit in the upstate when the extra payoff of the call is received, while their loss is limited to the option premium in the downstate. This is exactly what can be observed in figure 4. The opposite line of reasoning can be applied for low-beta stocks that can be replicated through holding a combination of a long market position and a short call. Important to notice is that the explicit concavity from the long market-short call position is incorporated in the price through the option premium. Hypothetically, the implicit concavity of low-beta stocks must be priced as well (in the form of higher return). Similarly, a MV strategy must be compensated by a positive alpha return.

Figure 4: Convex Payoff Pattern of High-Beta Stocks.



Source: Cowan and Wilderman (2011).

2.4.3 Industry-specific Risk

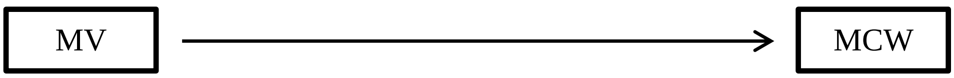
The weight composition of the MV portfolio is adjusted in such a way that the ex-ante portfolio variance is minimized. Because of this, it would not be unrealistic to assume that the MV portfolio is biased to only few securities that are featured by a low return variance (low-beta stocks). The same line of reasoning can be applied to industry selection; variation of returns is not only different across assets, but also varies across industries. In this respect, it is of interest to test whether the MV portfolio is biased towards a small number of industries. This type of risk can be referred to as industry-specific risk. Melas, Brian, and Urwin (2011) indicate that their minimum volatility index overweight utility, industrial, and consumer good industry sectors. This type of idiosyncratic risk can be translated into the exposure to a negative shock or collapse of a particular industry. Although these industries may be characterized as low-risky, being exposed to only a small number of industries is a risk-seeking and above all inefficient strategy from a diversification perspective. Investors will not take on this risk without being compensated by a positive alpha return. The risk-adjusted outperformance of the MV portfolio might be partly due to the bias towards a small number of industries. This directs towards exploring the effect of industry-specific risk on the MV portfolio's performance.

3. Theoretical Framework

This section does not further provide detailed variable descriptions since these have already been discussed in the previous section. Additional information on the source and construction of the variables are explained in the data and methodology section of this paper. The paper addresses the research question in two steps. Each step is explained by a graphical representation of the model that is employed. On the basis of these steps, the paper formulates five corresponding hypotheses. All hypotheses are accommodated with a theoretical motivation.

3.1 Hypotheses

Step 1: Testing whether the Minimum Variance (MV) portfolio is more (Sharpe) efficient than the Market Capitalization Weighted (MCW) portfolio.



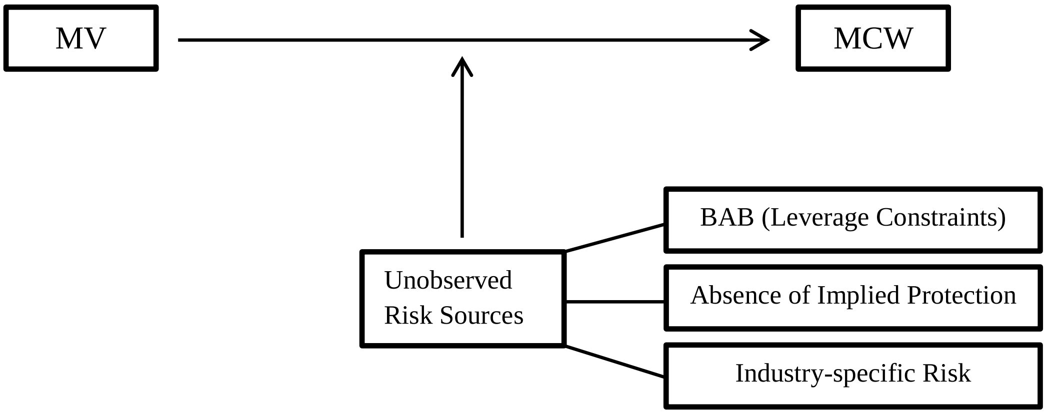
Hypothesis 1:

H_0: The MV portfolio does not exhibit a significant risk-adjusted outperformance with respect to its MCW benchmark portfolio.

H_1: The MV portfolio exhibits a significant risk-adjusted outperformance with respect to its MCW benchmark portfolio.

Motivation 1: *Many MV studies indicated a significant outperformance of the MV portfolio compared to a MCW benchmark in terms of risk return. This study tests whether the MV outperformance holds for a (semi-annually rebalanced), unconstrained long-only MV portfolio, constructed from the 1000 largest U.S. CRSP stocks during time period 1976 - 2012. The performance is adjusted for the Carhart (1997) four factors: SMB, HML, and MOM.*

<u>Step 2:</u> Testing to what extent the MV "anomaly" can be attributed to a set of unobserved risk sources.



Hypothesis 2:

H_0: The BAB factor does not capture the significant alpha performance of MV portfolios.

H_1: The BAB factor captures the significant alpha performance of MV portfolios.

Motivation 2: *A higher risk-adjusted MV return without the sacrifice of a lower risk level seems to be an exploitable anomaly in finance. However, the construction of the MV portfolio and its corresponding composition tends to impress that this portfolio is prone to other, alternative risk sources. The expectation that the MV portfolio is concentrated towards low-beta stocks calls for a close look into its bias to a BAB strategy. The BAB factor captures the return differential between leveraging a long, low-beta position and de-levering a short, high-beta position. This risk factor might explain the outperformance of conducting a MV strategy as it potentially captures other (low-beta related) risk sources.*

Hypothesis 3:

H_0: The return of the MV portfolio is linearly related to the return of the benchmark portfolio.

H_1: The return of the MV portfolio is quadrangular related to the return of the benchmark portfolio.

Motivation 3: *According to Frazzini and Pedersen (2010), leverage constraints are the main source of the "beta puzzle". However, Cowan and Wilderman (2011) argue that it is the difference between implicit and explicit leverage that solves the puzzle. In a down market, the potential loss for explicitly leveraged low-beta stocks is larger than for implicitly leveraged high-beta stocks. Additionally, the cross-sectional beta dispersion is higher in bull markets than in bear markets, making low-beta stocks unattractive. The absence of implied protection in low-beta strategies (resulting in an unfavorable concave payoff pattern), could cause investors to require a premium for holding this type of stocks. Since it is unclear whether the BAB factor fully captures this risk source, its impact on the MV portfolio is studied individually.*

Hypothesis 4:

H_0: The outperformance of the long-only MV portfolio cannot be explained by an exposure to industry-specific risk.

H_1: The outperformance of the long-only MV portfolio can be explained by an exposure to industry-specific risk.

Motivation 4: *Minimizing ex-ante portfolio variance could result in an over-allocation of a small number of industries that are characterized by a low variation of return. The potential bias of the MV portfolio towards a small number of industries causes exposure to industry-specific risk factors that are not captured by the Carhart (1997) four-factor model. Investors simply require a premium in order to compensate for idiosyncratic risk inherent to these industries. Adjustment for industry-specific risk could reduce the MV outperformance.*

Hypothesis 5:

H_0: The MV portfolio offers significant protection during the occurrence of extreme market conditions.

H_1: The MV portfolio does not offer significant protection during the occurrence of extreme market conditions.

Motivation 5: *From a theoretical point of view, low-beta strategies seem to lose during extreme down markets. Frazzini and Pedersen (2010) reason that a liquidity shock causes credit constraints to be more tightened. Subsequently, investors are constrained in the use of leverage which could force them to deleverage their BAB position. As a result, the required return on the BAB factor increases, which put a downward pressure on the realized BAB* *return. Similarly, the absence of implied protection becomes visible during extreme down markets. Cowan and Wilderman (2011) state that the relatively high performance of low-beta strategies is simply a compensation for the loss of insurance caused by extreme "downside market exposure" during market down-movements. In this view, the risk-adjusted MV outperformance could be seen as a rational compensation in exchange for the increased market exposure and the corresponding loss of protection during extreme events. A confirmation of this hypothesis would suggest that an MV anomaly does not exist; the positive alpha return solely reflects compensation required by MV investors for the exposure to potential large losses during extreme market conditions.*

4. Data

The most important input source necessary to derive the MV portfolio weights is the covariance matrix. A covariance matrix for the 1000 largest market capitalization U.S. stocks will be estimated similar to Clarke et al. (2006; 2010). This research applies covariance estimation at the beginning of every six months, starting from December 1975 through June 2012. Each covariance matrix is based on the prior 60 months of historical excess return data that is retrieved from the Center of Research in Security Prices (CRSP). The MV out-of-sample excess returns are computed through the multiplication of the MV weight matrix (following from the covariance matrix) by the return matrix, which consists of the corresponding excess returns in the 6 subsequent months. Hence, this procedure requires firms to have 66 months of non-missing return data. June 1976 is the first month reporting more than 1000 firms having at least 66 months of non-missing return data. Using the holding period return variable as input source for return measurement relieves from stock split events. These events are included in stock prices, and hence would severely suppress stock returns in the case of simply dividing stock prices. Occasionally, holding period return data displays extreme negative observations. However, these often follow from announcements of unexpected losses, plans in future merger or sale of businesses or business units, uncertainty in obtaining regulatory approvals, uncertainty in future profitability, or situations where financial results fell short of earlier projections. Consequently, immediate action by shareholders could cause months of extremely negative returns (for which no correction should be made). Following the retrieval of historical return data, logarithmic returns are computed and reported in U.S. dollars. The one-month U.S. Treasury bill rate from Ibbotson Associates is subtracted from each individual monthly stock return. Consequently, all stock returns are in excess of the contemporaneous risk-free rate. The sample covariance matrix is referred to as the product of the N x T matrix of historical excess returns (with N = 1000, and T = 60) and its transpose. This results in a N x N (one million element) matrix containing 1000 individual stock volatility estimates on the diagonal elements, and 499500 (1000 x 999/2) covariance estimates on the off-diagonal elements. According to the procedure of French, Schwert, and Stambaugh (1987), statistical variance is computed without subtracting the time series' average realized stock excess return. Therefore, sample covariance estimation is not completely in accordance with the general description of historical pair-wise stock covariance. Individual stock weights of the MV portfolio are determined in such way that they would have minimized the in-sample portfolio variance over the preceding 60-month period.

4.1 Data Problems

A problem forthcoming out of conducting this procedure of portfolio optimization is the non-invertability of the sample covariance matrix. The phenomenon of non-invertability arises because the number of stocks within the parent index (N = 1000) is larger than the number of historical data periods (T = 60) that are used for each sample covariance estimation (Clarke et al., 2006). In addition to non-invertability, the size of the covariance matrix could cause error maximization (Clarke et al., 2006). Having a large number of variance and covariance estimations, could potentially lead to estimation outliers resulting in an unbalanced portfolio. In order to both assure matrix invertability and reduce the effect of extreme values in the diagonal and off-diagonal elements, the Bayesian shrinkage method of Ledoit and Wolf (2004) is applied. Following this procedure, the final covariance matrix is a weighted average between the sample covariance matrix and the so-called Bayesian prior matrix, where the shrinkage parameter λ determines the weight on the prior. For simplicity, the shrinkage constant is assumed to be 0.5 over time. That is, equal weight is placed on historical stock return data and on the Bayesian prior. Wang (2005) states this is an appropriate assumption since historical data is used to estimate the prior. Practitioners often apply the average of both estimates when using various models. In fact, the exact value of the shrinkage factor is time-dependent where on average 54.4% of the individual covariance elements are determined by their cross-sectional means (Ledoit and Wolf, 2004).

4.2 Factor Data

In order to assess the performance of the long-only “unconstrained” MV portfolio, having an appropriate benchmark portfolio is of high importance. Main interest is in the return of the MV portfolio in excess of the risk-adjusted return based on the Carhart (1997) four-factor model, and its exposure to additional risk factors. The market benchmark portfolio is the market capitalization weighted (MCW) portfolio consisting of all 1000 stocks that jointly form the parent index. Important to notice is that this is the same set of securities used for the construction of the MV portfolio. The other variables are factors that measure the exposure to risk premiums based on the construction of long-short portfolios.

Size Factor (SMB): The excess return resulting from going long in firms with a small market capitalization and short in firms with a large market capitalization. The small-minus-big factor captures the effect of the relative outperformance of small firms over large firms.

Value Factor (HML): The excess return resulting from going long in firms with a high book-to-market value and short in firms with a low book-to-market value. The high-minus-low factor captures the effect of the relative outperformance of value firms over growth firms.

Momentum Factor (MOM): The excess return resulting from going long in firms with high prior returns and short in firms with low prior returns. The momentum factor captures the effect of the relative outperformance of past winners over past losers.

In order to defy comparison with similar studies, monthly factor data on SMB, HML, and MOM is directly retrieved from Kenneth French’s data library. Using this data, the historical sensitivity of the MV portfolio with respect to these risk factors could be measured appropriately.

In addition to testing for a risk-adjusted outperformance of the MV, three other risk factors are proposed. First of all, the BAB factor is constructed by creating long-short portfolios on the basis of beta-decile portfolios. The CRSP database has a unique set of beta-decile portfolios for which return data is available. These beta-deciles can be defined as beta-ranked portfolios, where portfolio 1 consists of the lowest beta securities, and portfolio 10 is composed of the highest beta securities. Portfolio 1 and portfolio 10 are employed as critical input for the construction of the BAB factor.

The impact of the risk sources coming from industry-specific risk and the absence of implied protection will be explored as well. Industry-specific risk follows from the bias towards particular industries. Monthly industry returns are directly retrieved from Kenneth French’s data library. This return data is derived from portfolios that are constructed on the basis of 49 different industries. Subsequently, industry returns are subtracted by the contemporaneous risk-free rate. The absence of implied protection implies a concave pattern in the payoff distribution of low-beta stocks. In order to observe whether the returns of the MV portfolio follow an asymmetric pattern, the excess returns of the MCW portfolio are squared and included (as dependent variable) in the regression model.

5. Methodology

In order to be able to answer the research question, the performance and risk-return properties of a long-only “unconstrained” MV portfolio are studied over the period January 1976 to December 2012. This paper conducts two basic steps in order to analyze whether the (potential) risk-adjusted outperformance of the long-only MV portfolio can be attributed to a rational compensation for a set of unobserved risk sources:

1.) Examining the risk-adjusted performance of the long-only MV portfolio compared to the MCW benchmark portfolio on the basis of the Carhart (1997) four-factor model.

2.) Assessing the effect of leverage constraints (BAB), the absence of implied protection, and industry-specific risk on the performance of the MV portfolio.

Regarding the first step, focus is on the construction of large-scale covariance matrices as critical input for the portfolio’s minimization problem. Employing the Bayesian shrinkage method to structure the covariance matrix ameliorates problems of matrix non-invertability and error maximization (Clarke et al., 2006). The second step involves the construction of the three risk factors derived from the impact of leverage constraints, the absence of implied protection, and industry-specific risk. This additional set of risk sources are proposed as determinants of the risk-adjusted alpha (outperformance) under the Carhart (1997) model. In this respect, it is evaluated if their properties have an effect on the performance of the MV investment strategy.

5.1 MV Portfolio Construction

In order to build the MV portfolio for time period January 1976 to December 2012, the same five steps are conducted as in the paper of Clarke et al. (2006). (i) First of all, non-missing historical return data is saved for the 1000 largest U.S. capitalization-weighted stocks of the CRSP database. As the covariance matrix (based on 60 months of return data) is multiplied by a return matrix consisting of the 6 subsequent months of excess return data, the condition of 66 months of non-missing return data must be satisfied. (ii) Secondly, individual stock returns are translated into logarithmic excess returns which are employed as main input for the sample covariance matrix. (iii) Thirdly, the sample covariance matrix is structured by applying the Bayesian shrinkage method. (iv) Fourthly, the final structured covariance matrix is plugged into the optimizer to determine the optimal MV stock weights (for which the in-sample portfolio variance is minimized). Finally, the row vector of optimal MV security weights is multiplied by the subsequent 6 months of realized excess returns to obtain the MV portfolio excess returns. This procedure is repeated every 6 months. The following subsections discuss the methodology of the five steps in more detail.

5.1.1 Covariance Matrix Estimation

Estimation of the covariance matrix is solely based on historical excess return data. In this study, X is assumed to be the N x T matrix of historical excess returns, with N is 1000 stocks and T is 60 months. Multiplication of the excess return matrix X with its transpose X' results in the sample covariance matrix. As mentioned in the previous section, the matrix elements are not completely in line with the standard description of pairwise stock covariance. Conducting the covariance estimation procedure used by French et al. (1987) results in the following sample covariance matrix (equation 12):

$$\Omega_{sample} = \begin{bmatrix} R_{1,1} & \cdots & R_{1,T} \\ \vdots & \ddots & \vdots \\ R_{N,1} & \cdots & R_{N,T} \end{bmatrix} * \begin{bmatrix} R_{1,1} & \cdots & R_{1,T} \\ \vdots & \ddots & \vdots \\ R_{N,1} & \cdots & R_{N,T} \end{bmatrix}^{T} = XX^{T} \qquad [12]$$

The Bayesian shrinkage technique is employed in order to produce an invertable covariance matrix that is suitable for solving the minimization problem. This structuring procedure creates a final estimation covariance matrix on the basis of a weighted average of the prior covariance matrix and the sample covariance matrix. For the matter of practicality, the Bayesian prior matrix's diagonal and off-diagonal elements are composed of their cross-sectional averages (equation 13), similar to Clarke et al. (2006, 2010). In this way, the effect of estimation outliers that could have led to highly unbalanced security weights is reduced.

$$\Omega_{prior} = \begin{bmatrix} \sigma_1^2 & \cdots & \sigma_{1,N} \\ \vdots & \ddots & \vdots \\ \sigma_{1,N} & \cdots & \sigma_N^2 \end{bmatrix} = Y \qquad \text{with: } \sigma_1^2 = \ldots = \sigma_N^2,\ \sigma_{1,2} = \ldots = \sigma_{1,N} \qquad [13]$$

This prior matrix is combined with the sample covariance matrix in order to construct a scaled estimation matrix where the shrinkage factor λ and (1- λ) indicate the weights that are placed on the prior and the sample covariance matrix respectively. The shrinkage parameter must be a number between zero and one. In this study, equal weight is placed on the prior and the sample covariance matrix, so λ =0.5.

$$\Omega = \lambda\Omega_{prior} + (1-\lambda)\Omega_{sample} \qquad [14]$$

5.1.2 Minimum Variance Weights and Returns

The final structured covariance estimation is plugged into the portfolio's minimization problem as presented in equation 15. The transpose of the weight vector that consists of the (N = 1000) individual stock weights within the parent index is multiplied by the structured covariance matrix and the corresponding weight vector. The portfolio optimization function is subject to two constraints. First of all, the sum of all individual security weights must equal 100%, so that all is invested in the risky portfolio (equation 16). Secondly, the weight in each individual stock must be either equal or larger than zero which is in accordance with the restriction on short-selling (equation 17).

$$\underset{w}{min}\ [w_1 \ldots w_N]_t * \Omega_t * \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}_t = \underset{w}{min}\ w_t{}'\Omega_t w_t \qquad [15]$$

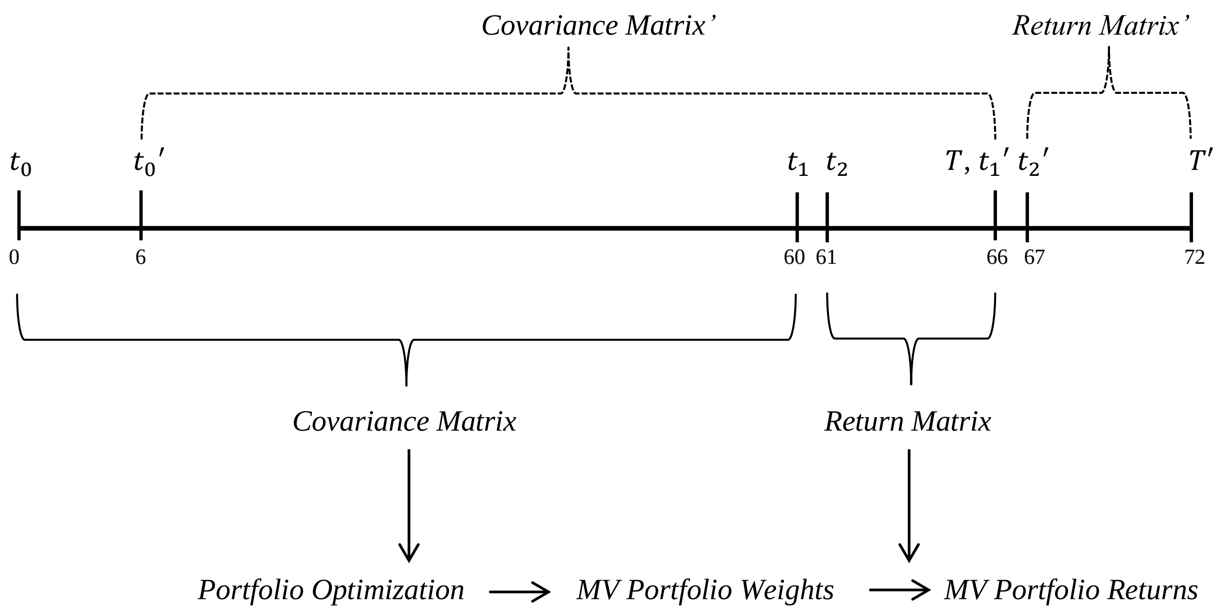
i) $$\sum_{i=1}^{N} w_{i,t} = w_t{}'\iota = 1 \qquad [16]$$

ii) $$w_{i,t} \geq 0 \qquad [17]$$

Conducting this minimization problem provides the individual MV stock weights for which the in-sample portfolio variance is minimized. Important to notice is that the full data sample covers the period starting from January 1971 and ending in December 2012. The first optimization process takes at the beginning of January 1976 which uses the historical excess returns of the preceding 60 months (five years). In total, 74 individual estimation windows are created to execute 74 portfolio optimizations. Each estimation window consists of 66 months of return data. The first 60 months of returns are the critical input source for the construction of the structured covariance matrix. The returns retrieved from the last 6 months, form the elements of the (1000 x 6) excess return matrix. This distinction is clearly shown in figure 5.

As mentioned in the previous section, the in-sample portfolio variance is based on 60 months of excess return data derived from the 1000 stocks that make up asset universe. As can be observed in the figure 5, the solid line demonstrates the first covariance matrix estimation which includes the estimation period t_0 = January 1971 to t_1 = December 1975. As a semi-annual rebalancing procedure is conducted, the second estimation is rolled over from ${t_0}'$ = July 1971 up to ${t_1}'$ = June 1976 which is depicted as the upper and dashed line. Additionally, the in-sample MV weight vector (following from the optimization) is multiplied by the out-of-sample 6-month excess return matrix R (equation 18). This excess return matrix is the (1000 x 6) matrix consisting of the 6 subsequent months of 1000 individual stock excess returns. Accordingly, the first excess return matrix estimation corresponds to the period t_2 = January 1976 to T = June 1976. This procedure is repeated every 6 months, 74 times for the complete data period. Following from this methodology, an estimation of the out-of-sample MV excess returns is obtained. In order to trace the MV performance, a market capitalization weighted benchmark portfolio will be constructed. The portfolio's construction will be explained in the next subsection.

Figure 5: Methodology Used for Obtaining the Out-of-Sample MV Excess Return Matrix.



$$R_t^{MVP} - R_t^f = \begin{bmatrix} w_{1,t}^{mv} \\ \vdots \\ w_{N,t}^{mv} \end{bmatrix}^T * \begin{bmatrix} R_{1,t+1} & \cdots & R_{1,T} \\ \vdots & \ddots & \vdots \\ R_{N,t+1} & \cdots & R_{N,T} \end{bmatrix} \quad [18]$$

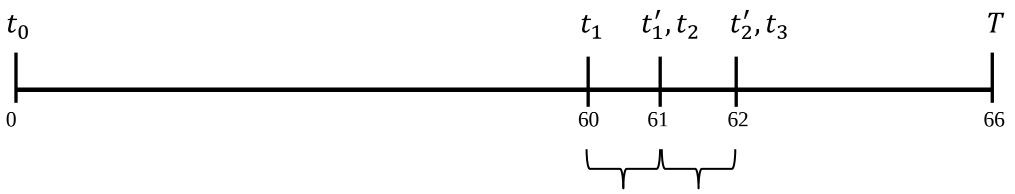
5.1.3 Market Capitalization Weights and Returns

In order to be able to draw valid conclusions about the relative performance of the MV portfolio, MV excess return data is compared with the excess returns of the MCW benchmark portfolio. This benchmark portfolio is built from the same set of securities used for the construction of the MV portfolio (1000 largest U.S. market capitalization weighted stocks) derived from the CRSP database. The individual market capitalization can be computed through the multiplication of the stock price by the corresponding number of outstanding shares (equation 19). The weight that is assigned to each individual stock in month *t* is equal to the individual market capitalization in percentage of the total market capitalization in the previous month $t - 1$ of the parent index (equation 20).

$$mc_{i,t-1} = p_{i,t-1} * shrout_{i,t-1} \quad [19]$$

$$w_{i,t} = \frac{mc_{i,t-1}}{\sum_{i=1}^{N} mc_{i,t-1}} \quad [20]$$

Figure 6: Methodology Used for Obtaining the Out-of-Sample MCW Excess Return Matrix.



For each individual estimation window, the first MCW weight vector is obtained in t_1. Similar to obtaining the MV excess return, the transpose of this market capitalization weight vector is multiplied by the vector of excess returns in the subsequent month t_2 to derive the monthly out-of-sample MCW excess return (equation 21). The second weight vector of the

MCW portfolio is rolled over to t_1' and multiplied with the excess returns in month t_2'. Conducting this approach gives a monthly-rebalanced MCW (out-of-sample) excess return. The specifications of the estimations of all matrices are summarized in the table 1.

$$R_t^{MCW} - R_t^f = \begin{bmatrix} w_{1,t}^{mcw} \\ \vdots \\ w_{N,t}^{mcw} \end{bmatrix}^T * \begin{bmatrix} R_{1,t+1} \\ \vdots \\ R_{N,t+1} \end{bmatrix} \quad [21]$$

Table 1: Individual Specifications of the Construction of the MV Portfolio and the MCW Portfolio.

Index	**Estimation Type**	**Characteristic**	**Specification**
MV Portfolio	Covariance matrix	Full estimation period Number of periods Rebalancing	January 1971 – June 2012 74 Semiannually
	Weight vector	Full estimation period Number of periods Rebalancing Constraints	January 1971 – June 2012 74 Semiannually - $w_i \geq 0$ - $\sum_{i=1}^{N} w_i = 1$
	Return matrix	Full estimation period Number of periods Rebalancing	January 1976 – December 2012 444 Monthly
MCW Portfolio	Weight vector	Full estimation period Number of periods Rebalancing Constraints	December 1975 – November 2012 444 Monthly - $w_{i,t} = \frac{1}{N} \sum_{i=1}^{N} mc_{i,t-1}$
	Return matrix	Full estimation period Number of periods Rebalancing	January 1976 – December 2012 444 Monthly

5.2 Performance Metrics

This subsection discusses the performance metrics applied to measure the risk-adjusted outperformance of the MV portfolio. The Sharpe ratio gives information on the risk and return properties of the MV portfolio and its MCW counterpart. In subsections 5.2.1, 5.2.2, and 5.2.3, the construction methodology of the BAB factor, the absence of implied protection, and industry-specific risk is explained respectively. In order to test the effect of this set of risk sources on the MV performance, the Carhart (1997) four-factor regression model will be extended by these factors.

The Carhart (1997) four-factor model is employed in order to test for risk-adjusted outperformance of the MV portfolio and to observe its exposure to traditionally included risk factors; market beta, SMB, HML, and MOM (equation 22). The Sharpe Ratio (Sharpe, 1964) is a good indicator of a portfolio's efficiency in terms of the risk-return tradeoff. The ratio specifies a portfolio's excess return per unit of standard deviation of return, or risk (equation 23).

$$R_t^{MVP} - R_t^f = \alpha + \beta\left[R_t^m - R_t^f\right] + sSMB_t + hHML_t + mMOM_t \qquad [22]$$

$$\frac{E\left[R_t^p\right] - R_t^f}{\sigma_t^p} \qquad [23]$$

5.2.1 BAB Construction

According to Frazinni and Pedersen (2010), the BAB factor captures the effect of leverage. A common preference exists for high-beta stocks over leveraged low-beta stocks to arrive at a preferred level of market exposure. They reason is that either (institutional) investors prefer to avoid leverage or they are constrained to use it. As a consequence, investors require a premium on low-beta stocks which explains their relative historical outperformance. Important to mention is that the BAB factor captures the outperformance of low-beta stocks relative to high-beta stocks. Cowan and Wilderman (2011) give a slightly different reason for the extraordinary performance of low-beta stocks. The absence of implied protection in low-beta stocks must give reason for a rational compensation required by low-beta investors.

The main point is that the BAB factor may not only capture the effect of leverage. The absence of implied protection, and even industry-specific risk might be related as well. Including the BAB factor into the regression model, may give several meaningful insights. Most importantly is to clarify whether the potential risk-adjusted outperformance of the MV strategy disappears through adding the BAB factor (hypothesis 2). Secondly, the factor provides clear notion on the composition of the MV portfolio and its bias towards low-beta stocks.

The BAB factor finds its origin in the construction of beta-decile portfolios. The CRSP offers beta-decile portfolio returns for each NYSE/AMEX and NASDAQ security. The construction methodology of the NYSE/AMEX beta-deciles follows the procedure of Scholes and Williams (1977) described in their paper "Estimating Betas from Nonsynchronous Data". Stocks are ranked on their previous year's estimated annual beta and are subsequently sorted into ten equally weighted portfolios. Beta-deciles of the NASDAQ securities are not based on this technique as a majority of these securities were not required to report transactions until 1992. For this reason, only the beta-decile portfolio returns created from NYSE/AMEX securities are directly retrieved from the CRSP database.

$$R_t^{BAB} = \frac{1}{\beta_{t-1}^L}\left[R_t^{low} - R_t^f\right] - \frac{1}{\beta_{t-1}^H}\left[R_t^{high} - R_t^f\right] \quad [24]$$

The method to calculate BAB returns is similar to the one employed by Frazzini and Pedersen (2010). The beta-decile portfolios are ranked from the lowest betas (portfolio 1) to the highest betas (portfolio 10). In addition, the betas of portfolio 1 and 10 are estimated through conducting a rolling regression on the corresponding equally-weighted portfolio. In order to assure market neutrality, the long position in low-beta stocks (β^L) is leveraged up to 1, and the short position in high-beta stocks (β^H) is de-levered down to 1. Subsequently, the long and short beta positions in month $t-1$ are multiplied by the corresponding portfolio beta-decile returns in the subsequent month. In order to derive the monthly out-of-sample BAB excess return, the excess returns from the position in high-beta stocks (portfolio 10) are subtracted from the position in low-beta stocks (portfolio 1) as can be observed in equation 24.

$$R_t^{MVP} - R_t^f = \alpha + \beta\left[R_t^m - R_t^f\right] + sSMB_t + hHML_t + mMOM_t + bBAB_t \quad [25]$$

Regression equation 25 captures the effect of the BAB factor. A significant BAB factor indicates that the MV returns are positively related to the BAB returns. A positive and significant risk premium could declare potential alpha performance when executing this regression.

5.2.2 Industry-specific Risk Construction

The presence of industry-specific risk and the absence of implied protection may be partly captured by the BAB factor. However, these individual risk sources could also have a direct effect on the MV strategy. Industry-specific risk arises from the overrepresentation of particular industries within the MV portfolio. Industry return data is obtained from Kenneth French's data library. The excess returns of the most dominant industries are included in the regression equation as can be observed in equation 26. A significant value for lambda indicates a bias or overrepresentation of the individual industry *i* within the MV portfolio. In order to test for specific industry risk that is not captured by the BAB factor, the same regression is conducted including the BAB factor.

$$R_t^{MVP} - R_t^f = \alpha + \beta\left[R_t^m - R_t^f\right] + sSMB_t + hHML_t + mMOM_t + \lambda_i\left[R_t^i - R_t^f\right] \qquad [26]$$

5.2.3 Absence of Implied Protection Construction

Cowan and Wilderman (2011) state that high-beta positions provide implicit leverage and protection in contrast to low-beta positions. These characteristics create more upside potential in bull markets while limiting the loss in down markets compared to the market portfolio. Accordingly, the main reason why high-beta stocks are traded at a premium stems from the convex or call-like payoff pattern. The favorable properties of high-beta stocks come at the cost of lower return that the investor must pay. The assumption of implied protection is hypothesized to be absent in the MV portfolio. This follows from the fact that low-beta stocks are expected to account for the largest part of the MV composition. In this view, the MV portfolio exhibits an opposite, concave return payoff distribution. In accordance with traditional finance theories, long-term returns must be commensurate with the corresponding risks an investor bears. This guides towards the hypothesis that MV investors require additional compensation (in the form of alpha) in exchange for being exposed to the unfavorable attributes of low-beta stocks present in the MV portfolio. Underperformance of the MV portfolio compared to its MCW counterpart in more volatile market circumstances would imply a concave relation between the MV and the MCW portfolio returns (hypothesis 3).

$$R_t^{MVP} - R_t^f = \alpha + \beta\left[R_t^m - R_t^f\right] + \beta^2\left[R_t^m - R_t^f\right]^2 + sSMB_t + hHML_t + mMOM_t \qquad [27]$$

In order to test for the presence of a concave pattern in the MV portfolio returns, the above regression (equation 27) is conducted. The squared excess returns of the MCW are added as independent variable. A significantly negative coefficient for this factor would imply both that investors do require compensation for the risk of underperformance in volatile periods and hence that implied protection is absent in a MV strategy.

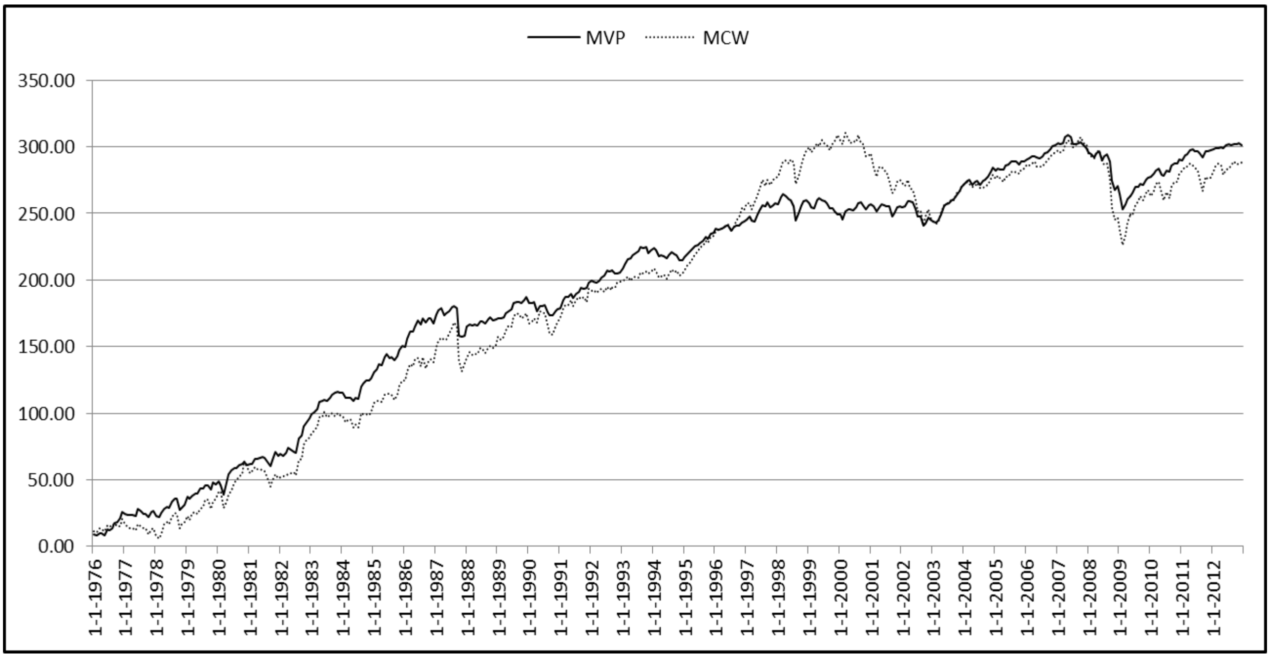
6. Results

The main question in this paper is whether the possible risk-adjusted outperformance of the long-only unconstrained MV portfolio still holds in the presence of a relevant bundle of alternative risk sources. The starting point is to demonstrate the risk-adjusted outperformance of the MV portfolio compared to the MCW benchmark portfolio on the basis of the four-factor model (Carhart, 1997). In addition, the results of the risk-return properties of both portfolios are provided to get insight into relative efficiency performances. Hereafter, in subsections 6.2, 6.3, and 6.4, the effects of BAB, the absence of implied protection, and industry-specific risk are assessed. These sections demonstrate whether the risk-adjusted outperformance also survives in the presence of these additional risk sources.

6.1 Outperformance of the MV Portfolio

The relative out-of-sample outperformance of the long-only unconstrained MV portfolio is graphically presented in figure 7. The solid line, representing the cumulative excess returns of the MV portfolio, lies above the cumulative excess returns of the market portfolio (dotted line) for most of the data period. The period January 1976 up to September 2007 exhibits a significant growth pattern.

Figure 7: Cumulative Excess Returns of the MV Portfolio and the MCW Portfolio between January 1976 and December 1976.



The cumulative excess return levels of both the MV and the Market Capitalization Weigthed (MCW) portfolio experience enormous improvement. At the end of 1987, many large international equity markets around the world fell by a significant margin with the stock market crash in October 1987, often referred to as "Black Monday". As a consequence, the MV portfolio lost around 2000 basis points, while the MCW portfolio lost more than 2500 basis points. In the early nineties, the MCW portfolio starts to overtake the cumulative return level of the MV portfolio which is not able to move along with the growth of the market, and lags behind. The stock market continues observing growth until January 2000, when the MCW portfolio faces some months of extremely negative returns. In contrast, the level of the cumulative returns of the MV portfolio remains stable. The turbulence in the stock market during the years 2001 and 2002 is followed up by a period of solid growth enjoyed by both portfolios. The collapse of the U.S. housing market and the corresponding financial crisis pressed down stock returns dramatically. A sequence of negative returns followed, with an intense loss in September 2008 with the bankruptcy of the Lehman Brothers, where the MV and MCW portfolio reported monthly return fallings of 14.49% and 21.43% respectively. From February 2009 onwards, the stock market shows a period of recovery. At the end of 2012, the MCW portfolio has reached a cumulative excess return of 288.4, where the MV portfolio already surpassed the 300 level.

The outperformance of the MV portfolio with respect to its market benchmark portfolio also holds in terms of efficiency. Table 2 reports statistics on the realized portfolio excess return and risk levels for the MV, MCW, and equally-weighted (EQW) portfolios based on a time series of 444 months. The monthly return of the MCW portfolio in excess of its contemporaneous risk-free rate is 0.65%, with an annualized excess return of 7.79%.

Table 2: Monthly and Annualized Risk and Return Measurers for the Minimum Variance portfolio, Market Capitalization Weighted Portfolio, and the Equally Weighted Portfolio.

	Monthly			Annualized		
	MVP	MCW	EQW	MVP	MCW	EQW
Excess Return	0.679%	0.649%	0.645%	8.148%	7.788%	7.740%
Std. Dev.	3.106%	4.514%	5.145%	10.759%	15.637%	17.822%
Sharpe-Ratio	-	-	-	0.757	0.498	0.434

This is somewhat higher than the annualized excess return of the EQW portfolio (7.74%). In terms of realized risk, as measured by the standard deviation of portfolio excess returns, the market experienced a lower level of risk than the EWQ portfolio. The monthly standard deviation of the market portfolio and the equally weighted portfolio are 4.51%, and 5.15% respectively. The monthly risk levels are multiplied by the square root of twelve to obtain the corresponding annualized risk levels. The MV portfolio demonstrates a slightly higher realized (annual) excess return compared to the market portfolio, 8.15% versus 7.79%. Despite the higher return, the realized risk level of the MV portfolio (10.76%) remains far below the risk level of the market. Appendix 1 shows that (on the basis of a 30-month moving average) the standard deviation of the MV portfolio is structurally lower for the complete data period. The Sharpe Ratio of the MV portfolio is equal to 0.757, which is far more Sharpe efficient than the Sharpe Ratio of the market (0.498). This results in an efficiency improvement of 52%. The large improvement can be attributed to a higher realized excess return, and a much lower realized level of risk in favor of the MV portfolio (appendix 3).

The results confirm that the MV portfolio is more efficient in terms of Sharpe Ratio than its market benchmark portfolio. It is of great importance to determine whether the outperformance of the MV portfolio is still present with the adjustment for traditional risk factors. The outcomes of three regression analyses (CAPM, three-factor model, and the four-factor model) are presented in table 3. The first regression analysis (specification 1) is the CAPM regression where the monthly excess returns of the MV portfolio are regressed on the monthly excess returns of the MCW portfolio. The annualized CAPM-alpha return is equal to 3.62% and is significant at a 2% level with a t-value of 3.77. Specifications 2 and 3 adjust for traditional risk factors, namely SMB, HML, and MOM. The alpha, reporting the risk-adjusted outperformance of the MV portfolio, is still highly significant in both specifications. The three-factor regression produces an annualized alpha of 2.28%, and the four-factor regression model exhibits an annualized alpha equal to 2.47%. The corresponding t-values are 2.55 and 2.73, and hence significant at the 2% level.

On the basis of the results demonstrated in table 3, the MV portfolio reports statistical significant outperformance with respect to the market portfolio. The positive and significant (risk-adjusted) alpha in specification 1, 2, and 3 confirms the rejection of hypothesis 1. The statistics in table 2 show that the MV outperformance is not limited to alpha, but also holds in terms of Sharpe efficiency.

Table 3: "MV Portfolio Outperformance - Regression Analyses". The dependent variable is the monthly excess return of the MV portfolio. The sample includes 444 months of return data from the 1000 largest capitalization U.S. stocks from January 1976 until December 2012. The benchmark portfolio is the MCW portfolio constructed from the same dataset. The annualized alpha is the variable of interest. The t-values are in parentheses.

	(1) CAPM	(2) 3-Factor Model	(3) 4-Factor Model
α	3.622%**	2.279%**	2.468%**
	(3.77)	(2.55)	(2.73)
β_M	0.581	0.601	0.599
	(33.11)	(35.86)	(35.56)
S_{SMB}		0.135	0.131
		(5.61)	(5.42)
H_{HML}		0.221	0.218
		(8.49)	(8.31)
M_{MOM}			-0.021
			(-1.36)
$Adj. R^2$	0.712	0.757	0.758
N	444	444	444

** Significance at the 2% level.

6.2 Betting-against-Beta

The Betting-against-beta (BAB) factor consists of a long component of low-beta stocks and a short component of high-beta stocks. The factor is market neutral in the sense that both components are (de)-levered up (down) to a beta of one. Consequently, the factor has zero exposure to the market and expresses the excess return differential of a long-short strategy. The historical outperformance of low-beta stocks (decile 1) compared to high-beta stocks (decile 10) is demonstrated in appendix 9, 11, 12, and 13. However, more important is to verify whether the significant alpha performance found in the previous section is also present when controlling for the BAB factor in the regression models. Table 4 summarizes the results on the basis of six regression models. Specification 1, 3, and 5 are similar to specification 1, 2, and 3 in table 3. Specification 2, 4, and 6 represent regressions based on the CAPM, three-factor model, and four-factor model with the addition of the BAB factor. In this way, it is clearly depicted how the alpha behaves when adjusting for the BAB factor. Similar to table 3, all specifications that are not extended with a BAB factor (1, 3, and 5), exhibit a positive and significant alpha. Specification 2 demonstrates that the extension of the BAB factor reduces the annualized CAPM-alpha from 3.79% to 1.68%. The alpha is not significant anymore at the 2% level. Important to notice is that the MV portfolio has a positive and significant exposure to the BAB factor. The t-value of 5.62 and the improvement in the adjusted R^2 indicate that this factor is a significant explanatory variable.

Table 4: "BAB Factor Exposure - Regression Analyses". The dependent variable is the monthly excess return of the MV portfolio. The sample includes 436 months of return data between January 1976 and December 2012. The benchmark portfolio is the MCW portfolio. The annualized alpha and the BAB factor are the variables of interest. The t-values are in parentheses.

	(1)	(2)	(3)	(4)	(5)	(6)
α	3.792%	1.683%	2.312%	1.034%	2.526%	1.218%
	(3.91)**	(1.67)	(2.55)**	(1.09)	(2.75)**	(1.26)
β_M	0.578	0.582	0.603	0.603	0.600	0.601
	(32.68)	(33.98)	(35.45)	(36.05)	(35.09)	(35.69)
S_{SMB}			0.136	0.121	0.132	0.118
			(5.62)	(5.01)	(5.40)	(4.88)
H_{HML}			0.227	0.205	0.222	0.202
			(8.48)	(7.62)	(8.26)	(7.50)
M_{MOM}					-0.023	-0.015
					(-1.46)	(-0.98)
B_{BAB}		0.058		0.039		0.038
		(5.62)**		(3.95)**		(3.79)**
$Adj. R^2$	0.710	0.730	0.757	0.765	0.758	0.765
N	436	436	436	436	436	436

** Significance at the 2% level.

This means that some part of the variation in the excess returns of the MV portfolio can be explained by the variation of the BAB excess return. Specifications 4 and 6 display comparable results. For both specification 4 and 6, the BAB factor is positively significant based on a 2% confidence level. The positive factor loading on the BAB factor confirms the prediction that the MV portfolio is biased towards low-beta stocks. This is supported by appendix 4 that clearly depicts the portfolio's concentration around the 0.5 beta level. The adjustment for the BAB factor significantly reduces the alpha return that was present in the base case (table 3). The alpha performance in specification 4 amounts to an annual return of 1.03%, while this is around 1.22% in specification 6. In addition, both alpha values are statistically insignificant as the table reports t-values of 1.09 and 1.26 respectively. The significant alpha performance in the previous section is completely removed with the addition of the BAB factor. The MV portfolio is exposed to this risk factor that potentially captures different risk sources. In sum, these results are in favor of the rejection of hypothesis 2.

Table 5: Monthly and Annualized Risk and Return Reasurers for the MV Portfolio, MCW portfolio, and the Betting-against-Beta Factor.

	Monthly			Annualized		
	MVP	MCW	BAB	MVP	MCW	BAB
Excess Return	0.679%	0.0.649%	2.985%	8.148%	7.788%	35.816%
Std. Dev.	3.106%	4.514%	7.479%	10.759%	15.759%	25.908%
Sharpe-Ratio	-	-	-	0.757	0.498	1.382

The BAB risk and return properties are shown in table 5. The factor dominates both the MV and MCW portfolio in terms of return. The BAB factor reports an annual excess return of 35.82%, compared to 8.15% and 7.79% of the MV and the MCW portfolio respectively. In contrast, its realized risk level (as measured by the standard deviation) is much higher. The realized risk of the MV portfolio of 10.76% is more than 40% lower than the risk level of the BAB factor (25.91%). Though, the BAB factor still prevails in terms of Sharpe Ratio. This is completely due to the overwhelming realized excess return on the factor.

The BAB factor probably captures a bundle of risk sources, amongst others leverage constraints (Frazzini and Pedersen, 2010). The absence of implied protection and industry-specific risk are also proposed as potential sources of risk that could affect the performance of the MV portfolio. As it is unclear whether the BAB factor fully captures these risk sources, it is of high relevance to study whether these additional sources of risk are able to explain the MV outperformance individually. The results of the relation between the MV portfolio and these two risk sources are discussed in the following two subsections.

6.3 Absence of Implied Protection

Cowan and Wilderman (2011) state that it is not leverage itself but the type of leverage that causes implied protection. They reason that the protection in extreme market circumstances is a favorable feature of high-beta stocks that provide implicit leverage. As a consequence, high-beta stocks fully profit from the upside potential in the good state of the market, while being protected in the downstate of the market. In contrast to high-beta stocks, implied protection is absent in low-beta stocks. In the previous subsection, the results confirm that the MV portfolio has a positive loading towards low-beta stocks. Hypothetically, MV investors require an additional premium in order to be compensated for the absence of implied protection and the corresponding concave relation with the market. In order to test for the absence of implied protection in the MV portfolio, three regression analyses are employed (table 6). Important to note, a test for the absence of implied protection is simultaneously a test for the presence of a concave payoff structure inherent to conducting a MV strategy. The focus is on the latter as the squared excess returns of the benchmark portfolio are added as explanatory variable into the basic three pricing models. Note that the expectation of a concave payoff structure and a corresponding (significant) negative value for β_{M^2} is in conflict with the CAPM, which states that a positive linear relation with the market exists, and hence a zero value for β_{M^2} must hold.

The results of the regression analyses are summarized in table 6. The specifications demonstrate the regression results based on the three basic pricing models, namely the CAPM, the three-factor model (including SMB and HML), and the Carhart (1997) four-factor model. It can be observed that the β_{M^2} coefficient is significantly negative for all specifications at the 5% level. The negative values for the squared excess returns of the benchmark portfolio imply that the payoff structure of the MV portfolio exhibits a concave pattern. The significant values for alpha indicate that investors require an additional premium as compensation for the absence of implied protection inherent to the MV portfolio. The alpha return compensates MV investors for potentially large losses during extreme market volatility.

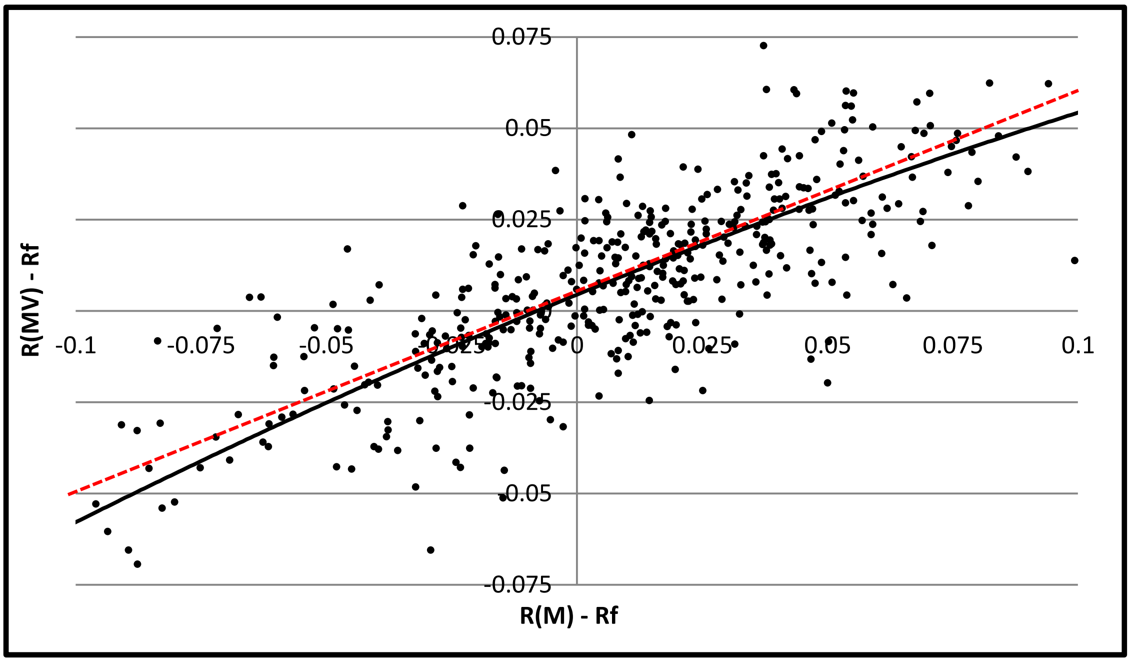
Table 6: "MV Portfolio and Concavity - Regression Analyses". The dependent variable is the monthly excess return of the MV portfolio. The sample includes 444 months of return data between January 1976 and December 2012. The benchmark portfolio is the MCW portfolio constructed from the same dataset. The annualized alpha and the squared excess returns of the market portfolio are the variables of interest. A negative value for the squared excess return implies a concave relationship with the market. The t-values are in parentheses.

	(1) CAPM	(2) 3-Factor Model	(3) 4-Factor Model
α	5.332%**	3.245%**	3.597%**
	(5.01)	(3.20)	(3.48)
β_M	0.561	0.590	0.586
	(30.68)	(33.49)	(32.96)
S_{SMB}		0.126	0.120
		(5.15)	(4.87)
H_{HML}		0.213	0.208
		(8.12)	(7.86)
M_{MOM}			-0.026
			(-1.68)
β_{M^2}	-0.623**	-0.334*	-0.375*
	(-3.53)	(-2.00)	(-2.22)
$Adj.R^2$	0.719	0.759	0.761
N	444	444	444

* Significance at the 5% level.
** Significance at the 2% level.

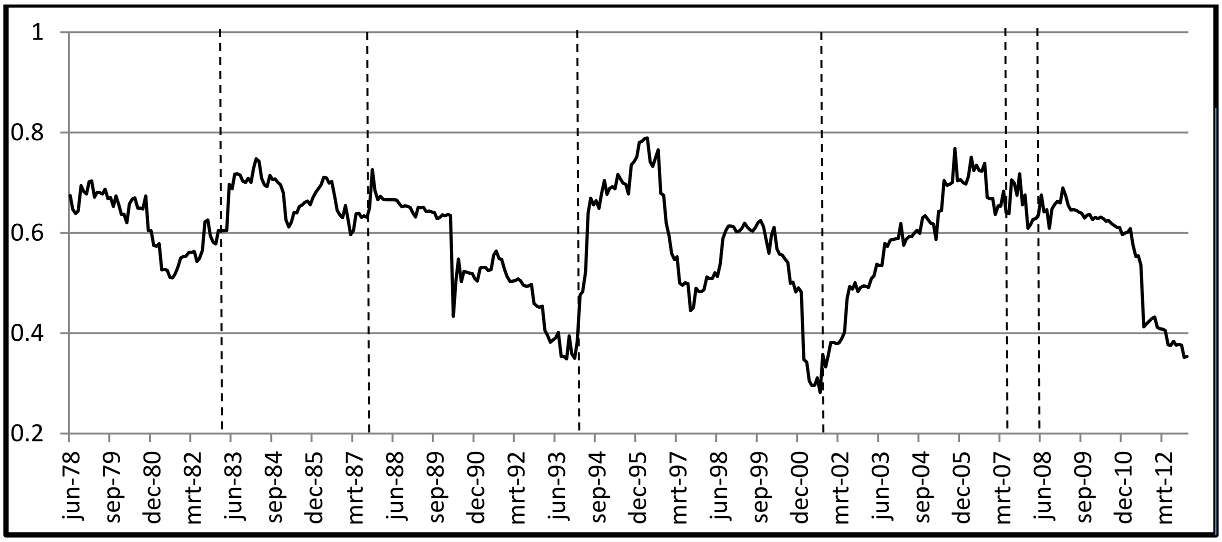
The concave relation between the excess returns of the MV portfolio and the excess returns of the benchmark portfolio is illustrated in figure 8. The x-axis represents the excess return of the MCW portfolio, and the y-axis represents the excess return of the MV portfolio. The solid line demonstrates the concavity that is present in the returns of the MV portfolio. During the occurrence of an extreme negative event like the collapse of a financial system or a natural disaster, the loss on high-beta stocks is equal to the loss of low beta stocks, namely the value of the equity investment. With a relatively lower loss per unit of beta, high-beta stocks offer some implied protection. On the other hand, low-beta investors need a higher level of investment (explicit leverage) in order to synthetically create the same market exposure as high-beta investors. This could make situations of extreme market down movements more severe, as the investor can lose even more (caused by a margin call for example).

Figure 8: A scatter-plot showing the concave relation between the excess returns of the MV portfolio and the excess returns of the market portfolio. The sample consists of 444 months of excess return data from both portfolios between January 1976 and December 2012. The black solid line represents the concave trend of the MV portfolio returns.



Intuitively, with the collapse of a complete financial system, all betas converge to one. Figure 9 presents the relation between historical extreme events and the MV level of beta based on a 30-month moving estimation window. The dotted lines represent extreme negative events. An example of such an event is the Coalinga earthquake took place on May 2, 1983. The earthquake caused severe damage especially in Coalinga, where several commercial districts were almost fully destroyed (Manos and Clough, 2006). As a response, the beta level of the MV portfolio exhibits a jump of high magnitude from a beta of 0.6 to a beta of almost 0.7. A similar jump can be observed after Black Monday (19 October, 1987). The crash of the U.S. stock market results in an increase of the MV beta level from 0.65 to 0.73. In addition, June 1994 was a heating month concerning the North-Korean nuclear crisis, as the U.S. almost got into war in order to prevent North-Korea from continuing their nuclear program. The threat of an atom war led to severe growth in the beta level of the MV portfolio from 0.52 to 0.64. Other events that caused market turmoil were the entrance of the financial crisis and the bankruptcy of the Lehman Brothers in September 2008 which also caused a rise in the level of beta. Additionally, the periods March 2000 to March 2003 and October 2007 to February 2009 that were specified by Melas, Brian, and Urwin (2011) as the dotcom and the subprime crisis respectively, also demonstrate structurally higher levels in the beta of the MV portfolio.

Figure 9: Rolling Regression of the CAPM-beta of the MV Portfolio. The beta level is constructed on the basis of a 30-month moving window between June 1978 and December 2012. The dotted lines represent extreme events, namely the Coallinga Earthquake (May 1983), Black Monday (October 1987), North-Korean Nuclear Crisis (June 1994), Terrorist Attacks 9/11 (September 2001), Begin of the Financial Crisis (October 2007), and the Bankruptcy of the Lehman Brothers (September 2008).



From combining the results of table 6 and figure 8 and 9, it could be concluded that the MV portfolio returns are not linearly related to the returns of the market portfolio. The MV portfolio is biased towards low-beta stocks that are characterized by the absence of implied protection. As a consequence, MV investors could require an additional premium in the form of a positive alpha return (table 6). This results into the rejection of hypothesis 3.

6.4 Industry-specific Risk

Section 6.2 confirmed the bias of the MV portfolio towards low-beta stocks (appendix 4). The MV construction methodology aims for minimizing portfolio variance or risk. As low-beta stocks have relatively low co-variation with the market, and thus a lower level of market risk, the concentration towards low-beta stocks does not come as a surprise. More important is to question whether the concentration of MV portfolios does also occur at industry level. It might be even more interesting to consider whether the effect of a potential bias towards certain industries and related risk sources are strong enough to explain the risk-adjusted alpha performance. Table 7 presents the performance of the MV portfolio with the adjustment for a subset of industries.

The first two specifications of table 7 are similar to specifications 5 and 6 in table 4 and are only employed in order to draw valid comparisons with specifications 3 and 4 that adjust for industry factors. Specification 3 indicates that the MV portfolio is significantly exposed to some industry-specific risk. All industry factors have coefficients that are different from zero based on a 2% significance level (except from Electronic Equipment which is significant at the 5% level). The significant annualized alpha return of 2.53% in specification 1 transforms into an insignificant annual alpha performance of 1.32% with the adjustment for industry factors (specification 3). Additionally, extending the four-factor model with the industry factors leads to an adjusted R^2 improvement from 0.758 to 0.846. It may also be noted that the MV portfolio has some negative factor loadings, namely for Shipbuilding and Railroad Equipment, Coal, and Computer Software. A positive industry bias is highest for Utilities with a t-value of 11.72. This sector exhibits the highest Sharpe Ratio and the lowest standard deviation (appendix 14 and 15). Additionally, the Utility sector demonstrates a positive risk-adjusted outperformance (appendix 16). The effect of the correction for industry factors on the alpha reduction is not as large as for the addition of the BAB factor. It could be observed that the outperformance of the MV portfolio in specification 3 is still significant at the 10% level while this is not the case in specification 2. Specification 4 makes clear that industry-specific is not (fully) captured by the BAB factor. The significant values for the industry factors remain unchanged with the correction for the BAB factor. Vice versa, the MV portfolio still demonstrates a significantly positive loading to the BAB factor with the adjustment for industry factors. In addition, specification 5 shows the exposure of the BAB factor towards the industry factors. The BAB factor exhibits a positive loading towards gold, while the other industry factors remain insignificant. This is in line with the findings associated with specification 4.

The significant loading of the MV portfolio towards industry factors, the insignificant alpha performance, and the factor's independency of the BAB factor, all act in favor of the rejection of hypothesis 4. The rejection can be reinforced by the high adjusted R^2 improvement. On the other side, it must be noted that the risk-adjusted outperformance of the MV portfolio is not fully removed with the correction for industry factors, as indicated by the positive t-value of 1.75. Consequently, the annualized alpha return is still significant at the 10% level. Only when controlling for the BAB factor, the risk-adjusted outperformance of the MV portfolio with respect to its benchmark portfolio is completely removed.

Table 7: "Industry-specific Risk - Regression Analyses". The dependent variable is the monthly excess return of the MV portfolio. The sample includes 436 months of return data between January 1976 and December 2012. The benchmark is the MCW portfolio. Specification 1 denotes the base case regression model (Carhart four-factor model). Specification 2 includes the BAB factor. Specifications 3 and 4 are extended with the adjustment for industry factors. Specification 5 demonstrates the relation between BAB factor returns and industry portfolio returns. Industry portfolio returns are retrieved from Kenneth French's Data Library. The t-values are in parentheses. Abbreviations note: (HC = Healthcare, MD = Medical Equipment, SH = Shipbuilding and Railroad Equipment, GLD = Gold, CL = Coal, UTLS = Utilities, PER = Personal Services, SFTW = Computer Software, EE = Electronic Equipment).

	(1)	(2)	(3)	(4)	(5)
α	2.526%** (2.75)	1.218% (1.26)	1.316% (1.75)	0.386% (0.49)	34.89%** (7.82)
β_M	0.600 (35.09)	0.601 (35.69)	0.406 (10.64)	0.414 (10.96)	-0.362 (-1.66)
S_{SMB}	0.132 (5.40)	0.118 (4.88)	0.108 (4.72)	0.098 (4.29)	
H_{HML}	0.222 (8.26)	0.202 (7.50)	0.121 (4.81)	0.103 (4.09)	
M_{MOM}	-0.023 (-1.46)	-0.015 (-0.98)	-0.053 (-3.99)	-0.047 (-3.54)	
λ_{HC}			0.035** (2.78)	0.038** (3.02)	-0.060 (-0.77)
λ_{MD}			0.056** (3.02)	0.049** (2.63)	0.173 (1.53)
λ_{SH}			-0.034** (-3.03)	-0.032** (-2.82)	-0.015 (-0.22)
λ_{GLD}			0.032** (5.41)	0.030** (5.11)	0.075** (2.12)
λ_{CL}			-0.028** (-3.96)	-0.027** (-3.90)	-0.014 (-0.33)
λ_{UTLS}			0.244** (11.72)	0.241** (11.73)	0.160 (1.29)
λ_{PER}			0.045** (2.86)	0.043** (2.81)	0.097 (1.06)
λ_{SFTW}			-0.046** (-3.86)	-0.046** (-3.88)	-0.090 (-1.31)
λ_{EE}			0.032* (2.16)	0.030* (2.00)	0.110 (1.22)
BAB		0.038** (3.79)		0.028** (3.47)	
$Adj. R^2$	0.758	0.765	0.846	0.848	0.003
N	436	436	436	436	436

* Significance at the 5% level.
** Significance at the 2% level.

7. Sensitivity Analysis

A key element of this research paper is to answer whether the risk-adjusted outperformance of the MV portfolio can be devoted to underlying risk sources. In other words, is the significant alpha return a rational premium that is required by MV investors as compensation for the exposure to these additional risk sources? In order to answer this question, special interest is in the explanatory power of additional risk factors and their unique contributions to existing models. Important to note is that it is assumed that the risk of these unobserved factors mainly prevails in periods of down markets. Frazinni and Pedersen (2010) state that an increase in liquidity constraints (as caused by financial crises) leads to losses on the BAB factor as its required return rises. In their view, a liquidity shock results in a drop of all security prices and consequently into beta convergence. Intuitively, leverage-constrained investors are constrained in applying leverage and may have to de-leverage their low-beta position. Additionally, the risk of the absence of implied protection also becomes visible during extreme market conditions. Cowan and Wilderman (2011) argue that the risk of investment strategies having a concave payoff pattern (as the MV portfolio, figure 8) dominates during big down markets. The relatively high performance of these strategies is simply a compensation for the loss of insurance caused by "downside market exposure" during extreme market down-movements. Therefore, this section explores the performance of the MV portfolio during bad market circumstances.

The U.S. National Bureau of Economic Research (2010) reported a list of U.S. business cycles expansions and extractions. Table 8 provides an overview of all crises periods that occurred between January 1976 and December 212. The third and fourth column report the average excess returns of the MV and MCW portfolios during these crises. As could be observed, only the early 2000s and the great recession show negative returns for both portfolios. The latter indicates that the loss of the MV portfolio (1.75%) is quite close to the loss of the MCW portfolio (2.28%). In order to observe the effect of economic crises on the performance of the MV portfolio, several sub-sample regression analyses have been conducted. Table 9 demonstrates the difference in the annualized MV alpha performance between crises and non-crises periods. The crisis sub-sample consists of the 94 months belonging to one of the crises that are identified in table 8. The non-crisis sub-sample applies to the remaining 350 months defined as non-crisis months.

Table 8: Average excess returns of the MV and MCW portfolio during the periods of economic crises between January 1976 and December 2012.

Crisis	Period	Months	$\bar{R}^{MV}$	$\bar{R}^{MCW}$
1980 recession	January 1980 – July 1980	7	1.73%	1.82%
Early 1980s recession	July 1981 – November 1982	17	1.56%	1.35%
Early 1990s recession	July 1990 – March 1991	9	0.79%	0.57%
Early 2000s recession	March 2000 – March 2003	37	-0.01%	-1.60%
Great recession	July 2007 – June 2009	24	-1.75%	-2.28%
Total		94		

Source: U.S. National Bureau of Economic Research (2010).

Table 9: "Impact of Crises on MV Performance – Regression Analyses". The dependent variable is the monthly excess return of the MV portfolio. A distinction has been made between crises periods and non-crises periods. The full sample includes 444 months of return data between January 1976 and December 2012. Specification 1, 3, and 5 report the annualized alpha performance of the MV portfolio during crises periods under the CAPM, three-factor, and four-factor model respectively. Similarly, specification 2, 4, and 6 indicate the MV performance during normal market circumstances. The benchmark portfolio is the market capitalization weighted portfolio.

	(1)	(2)	(3)	(4)	(5)	(6)
α	6.108% (2.50)**	3.137% (2.99)**	4.529% (2.03)*	1.755% (1.77)	4.531% (2.02)*	2.172% (2.16)*
β_M	0.607 (17.71)	0.572 (26.88)	0.616 (19.53)	0.597 (28.59)	0.617 (18.11)	0.601 (28.81)
S_{SMB}			0.119 (2.40)	0.144 (5.17)	0.120 (2.37)	0.136 (4.88)
H_{HML}			0.227 (4.70)	0.213 (6.59)	0.227 (4.68)	0.199 (6.07)
M_{MOM}					0.000 (0.02)	-0.053 (-2.14)
$Adj. R^2$	0.771	0.674	0.813	0.720	0.819	0.723
N	94	350	94	350	94	350

* Significance at the 5% level.

** Significance at the 2% level.

Specification 1 and 2 indicate the difference in the MV alpha return on the basis of the CAPM model. It is quite striking that the MV portfolio exhibits a significantly positive outperformance with respect to its MCW benchmark portfolio in both specifications. In contrast to what is expected, this regression model demonstrates that the annualized alpha performance in crises periods is even higher than in normal periods (6.11% versus 3.14%) while both are significant at the 2% level. This remarkable finding also holds after adjusting for SMB, HML, and MOM risk factors. Specifications 3 and 5 also prove that the MV portfolio offers a significant risk-adjusted outperformance during periods of economic crises that is even higher compared to the outperformance during normal periods (specification 4 and 6). This result questions the hypothesis that the MV risk-adjusted outperformance can be fully attributed to a compensation required for the underperformance of the MV portfolio during down markets.

As table 8 and 9 demonstrate that the MV portfolio still outperforms the MCW portfolio over longer periods of economic crises, it is of high interest to test the behavior of the MV portfolio during specific months of extreme market conditions. Table 10 presents an overview of the excess return of the MV portfolio during extreme months, namely the Coallinga Earthquake (May 1983), Black Monday (October 1987), North-Korean Nuclear Crisis (June 1994), Terrorist Attacks 9/11 (September 2001), the start of the Financial Crisis (July 2007), and the Bankruptcy of the Lehman Brothers (September 2008). In addition to these historical happenings, six other months are included. These months reported volatilities that were at least three times as large as the average volatility (estimated on daily data on the S&P500). Column 1 and 2 represent the monthly excess returns of the MV and the MCW portfolio in the corresponding (extreme) months.

It could be observed that in most of the extreme months, the negative return of the MV portfolio is very close to the return of the MCW portfolio, and also more negative than what its long-term beta would suggest. The MV portfolio reports a negative average excess return of 7.88%. This corresponds to 75% of the average loss on the market portfolio (-10.51%). The average beta level of 0.63 (in extreme months) is approximately 500 basis points higher than the long-term beta level of 0.58. Additionally, the change in the beta level with respect to its previous month is equal to an average of 8.15% which indicates cross-sectional beta convergence. Especially during the months of Black Monday, the terrorist attacks of 9/11, and February 2009, the MV portfolio is not able to offer sufficient protection. The loss of the MV portfolio contributes to almost 84% of the return loss on the market portfolio. The return loss significantly deviates from the expected return derived from a long-term beta level of 0.58 (as shown in specification 3). The beta level in these months increases on average by 12.08% with respect to the previous month.

In contrast to the findings reported in table 8 and 9, table 10 indicates that the MV portfolio does not offer much protection in months of extreme market conditions. The MV portfolio appears to closely track the performance of market portfolio. The beta level and its contemporaneous change seem to suggest that the MV beta reaches significantly higher levels during extreme down movements, resulting in more down market exposure. The higher level of beta exposes the MV portfolio to more covariation with the returns of the market.

Table 10: The specifications present an overview of the performance of the MV and MCW portfolios during months of extreme market situations. Specification 1 and 2 report monthly excess returns. Specification 3 indicates the expected excess return of the MV portfolio based on a long-term beta of 0.5802. The difference between the realized excess return and the expected return of the MV portfolio can be observed in specification 4. Specification 5 demonstrates the beta level of the MV portfolio based on a 30-month rolling regression. Specification 6 shows the change of the level of beta compared to the previous month.

Date	(1) MV	(2) MCW	(3) Exp. Return	(4) Difference (1 - 3)	(5) Beta Level	(6) Beta Change
October 1978	-8.16%	-10.60%	-6.15%	-2.01%	0.6942	7.32%
March 1980	-6.61%	-11.90%	-6.90%	0.29%	0.6200	-2.77%
May 1983	1.12%	-0.18%	-0.01%	1.13%	0.6969	13.24%
October 1987	**-20.15%**	**-25.61%**	**-14.88%**	**-5.27%**	**0.7259**	**10.62%**
June 1994	-1.55%	-2.74%	-1.59%	0.04%	0.6395	18.40%
September 2001	**-7.97%**	**-9.72%**	**-5.65%**	**-2.32%**	**0.3577**	**21.28%**
September 2002	-7.28%	-11.47%	-6.66%	-0.62%	0.5009	2.61%
July 2007	-4.82%	-3.22%	-1.87%	-2.95%	0.7056	9.52%
June 2008	-6.93%	-8.77%	-5.02%	-1.91%	0.6757	5.86%
September 2008	-6.55%	-8.95%	-5.12%	-1.43%	0.6470	6.01%
October 2008	-14.49%	-21.43%	-12.45%	-2.04%	0.6570	1.40%
February 2009	**-11.14%**	**-11.51%**	**-6.47%**	**-4.67%**	**0.6895**	**4.35%**
Average	-7.88%	-10.51%	-6.64%	-1.81%	0.6342	8.15%

Due to the lack of observations on extreme market conditions, the results above have to be taken with great care. This prevents from drawing premature and inaccurate conclusions. For this reason, hypothesis 5 will not be rejected. Nevertheless, the findings seem to suggest that return does not come for free or without risk. MV investors appear to pay the bill in the occurrence of an extreme event such as the collapse of an entire stock market. The positive risk-adjusted alpha performance on the basis of the Carhart (1997) four-factor model may not be a compensation for longer periods of economic downturn but rather a compensation for worst case scenarios. From a diversification perspective, the MV portfolio might be perceived as a risky project. This is mainly due to the bias towards low-beta stocks and the concentration to specific industries. This effect is reinforced by the fact that relatively few stocks form the MV portfolio. Appendix 5 shows that the actual number of stocks within the MV portfolio, ranges from only 70 to 200 individual stocks out of the total asset universe (1000 stocks). In addition, the number of equally weighted stocks necessary to generate an equal level of diversification benefit is much lower for the MV portfolio compared to the market portfolio (appendix 6). On average the MCW portfolio is approximately two times as diversified as the MV portfolio. Further, the MV portfolio is not able to track the performance of the MCW portfolio during high up states of the market. As can be seen in appendix 17, the MV alpha return is statistically insignificant during the months of highest market return. In addition, appendix 18 shows that the market portfolio dominates in terms of Sharpe Ratio which is due to both a higher realized excess return, and a lower level of standard deviation.

7.1 Statistical Relevance

This subsection focuses on the statistical relevance of the factors in the model. This paper introduces several risk factors that extend the standard asset pricing models. For this, special interest is in the explanatory power of the additional risk factors and their unique contributions to existing models. Standardized coefficients measure the effect on the expected value of the dependent variable in terms of a one standard deviation change in the independent variable ceteris paribus (Greenland, Schlesselman, and Criqui, 1986). Standardized coefficients makes the coefficients of the model (or the effect of risk factors) more comparable. The first column of table 11 presents the standardized coefficients of the complete model. Naturally, the standardized effect is largest for market risk. A one standard deviation change in the excess return of the MCW portfolio causes a 0.87 standard deviation increase in the expected excess return of the MV portfolio. The utility sector has the greatest standardized effect among all industry factors. In addition, it could be observed that the HML and SMB risk factors have a larger impact than the BAB factor. However, the standardized coefficient of the BAB factor is much larger than the standardized coefficient of the MOM factor. The negative standardized effect of the squared excess returns expresses the concave relation between the MV portfolio and the market portfolio.

Another way to compare the unique contributions of all risk factors to the regression model is to measure the partial R^2 and the semi-partial R^2. The semi-partial R^2 (equation 27) indicates the difference between the R^2 in the complete model (R_C^2) and the R^2 in the restricted model (R_{RS}^2). The restricted model can be defined as the model without the specific risk factor for which the effect is measured. The partial R^2 is defined as the part of maximum improvement in R^2 achieved through including the variable of interest (Shedden, 2010).

$$R_C^2 - R_{RS}^2 \quad [27]$$

$$\frac{R_C^2 - R_{RS}^2}{1 - R_{RS}^2} \quad [28]$$

The partial R^2 and the semi-partial R^2 are represented in the second and third column of table 11. The results are similar to the ones indicated by the magnitude of standardized effects. The unique contribution of market risk to the regression model is highest as indicated by both measures. The BAB factor also significantly contributes to the model. Including the BAB factor into the regression model, results in a maximum R^2 improvement of 3.50%. This effect is almost as high as when adding the SMB factor. Additionally, the MV portfolio seems to be concentrated towards value stocks. The standardized coefficient and the R^2 measures for the HML factor point out the significant effect of this risk factor to the model. The impacts of the MOM factor and the absence of implied protection are relatively small. The aggregate of all industry specific risk factors does add significant value to the regression model.

Table 11: "Summary Statistics". The statistics are based on the full sample (N=444). Standardized coefficients are computed by multiplying the coefficients by the ratio of the standard deviation of the risk factor (independent variable) over the standard deviation of the MV excess return (dependent variable). The semi-partial R^2 measures the difference in R^2 between including and excluding the variable of interest. The partial R^2 is defined as the part of maximum improvement in R^2 achieved through including the variable of interest.

Factor	Standardized Coefficient	Semi-Partial R^2	Partial R^2
β_M	0.870	0.583	71.81%
S_{SMB}	0.133	0.009	4.11%
H_{HML}	0.208	0.026	10.24%
M_{MOM}	-0.032	0.001	0.44%
BAB	0.075	0.008	3.50%
β_M^2	-0.056	0.004	1.51%
C_{Risk}		0.085	36.99%
λ_{HC}	0.082		
λ_{MD}	0.096		
λ_{SH}	-0.083		
λ_{GLD}	0.110		
λ_{CL}	0.095		
λ_{UTLS}	0.307		
λ_{PER}	0.092		
λ_{SFTW}	-0.132		
λ_{EE}	0.092		
λ_{HC}	0.082		

8. Conclusion

The practical uniqueness of the Minimum Variance (MV) portfolio stems from the construction's independency of expected return estimates. The ex-ante minimization of portfolio variance is achieved through adjusting individual security weights. The process of security weight allocation is solely dependent on the estimated covariance matrix. This research applies a large-scale MV portfolio construction methodology similar to Clarke et al. (2006). The parent index is composed of excess return data from 1000 largest U.S. capitalization weighted stocks from the CRSP database. The MV variant is the long-only "unconstrained" portfolio constructed on the basis of a semi-annual rebalanced stock weight procedure. The optimal individual stock weights are determined as if they would have minimized portfolio variance in the preceding 60 months.

The long-only "unconstrained" large-scale MV portfolio demonstrates a risk-adjusted out-of-sample outperformance with respect to its Market Capitalization Weighted (MCW) benchmark portfolio between January 1976 and December 2012. The annualized alpha returns of CAPM, three-factor, and four-factor regression models are statistically significant. In addition, the MV portfolio indicates an outperformance in terms of efficiency measures. The Sharpe Ratio of the MV portfolio exhibits an efficiency improvement of 52% compared to the MCW portfolio (0.757 versus 0.498) which can be attributed to a slightly higher annually realized return (8.15% versus 7.79%) and a much lower realized level of risk (10.76% versus 16.64%). These findings are analogous to the results of previous studies.

The MV risk-adjusted outperformance both in risk and return provokes to conclude that an exploitable MV anomaly exists. Nevertheless, the composition and the construction methodology of the MV portfolio presumes that the portfolio is prone to other, unobserved risk factors that have the potential to reject this perceived anomaly and to solve the corresponding minimum variance puzzle. The historical risk-adjusted outperformance of low-beta stocks versus high-beta stocks called for the implementation of new investment strategies. Frazzini and Pedersen (2010) propose the BAB factor that captures the excess return differential between low-beta and high-beta stocks. The extraordinary performance of the factor may be ascribed to several underlying risk sources, namely leverage constraints, absence of implied protection, and industry-specific risk. With respect to the expected bias towards low-beta stocks, this paper tests whether the risk-adjusted outperformance of the MV portfolio can be devoted to these unobserved risk factors.

The long-only “unconstrained” MV portfolio indicates a significant exposure to the BAB factor, and hence to low-beta stocks. A statistically insignificant alpha return concludes that the former risk-adjusted outperformance of the MV portfolio completely disappears with the addition of the BAB factor.

The implied protection property and the related call-like (convex) payoff structure inherent to high-beta stocks (Cowan and Wilderman, 2011) is not present in the MV portfolio. A negative (and significant) relation exists between the excess returns of the MV portfolio and the squared excess returns of the MCW portfolio. The absence of implied protection indicates a concave relationship with the market resulting in a higher level of downside market exposure than suggested by the CAPM. Due to beta convergence, the explicitly levered low-beta stocks pay a higher cost per unit of beta in times of market turmoil than implicitly levered high-beta stocks. As a consequence, the emergence of a significant alpha seems to reflect an additional premium that MV investors require as a compensation for the concave payoff pattern in their investment.

The MV portfolio is exposed to industry-specific risk. The portfolio experiences an extreme positive bias towards utilities and gold industries, while a significant negative bias exists towards shipbuilding, coal, and software industries. Adjusting for industry factors leads to a large adjusted R^2 improvement and an insignificant alpha performance. This in combination with the independency of the BAB factor confirms that industry factors influence the performance of the MV portfolio individually. Though, it has to be mentioned that the risk-adjusted outperformance is completely removed with the correction for the BAB factor.

Although the significant alpha return turns out to be statistically insignificant when adjusting for additional risk sources, it seems premature to conclude that the initially perceived MV anomaly (positive risk-adjusted MV alpha return) is primarily the result of a compensation for unobserved risk factors. Months of extreme market conditions indeed demonstrate extreme downside market exposure and a corresponding loss of insurance, but a positive significant MV alpha return still persists during crises periods. Nevertheless, the concave payoff structure and the bias towards particular industries and betas shed light on the related riskiness of the MV portfolio. The perception that MV portfolios could function as a safe and low-risky benchmark is highly questioned from a diversification perspective.

9. Limitations

This research paper focusses on the performance of a large-scale long-only "unconstrained" MV portfolio. The main procedure to create this MV variant is the optimization of 1000 individual stock weights in such a way that the ex-ante portfolio variance is minimized. That is, the portfolio variance in the preceding 60 months. The optimization's most important input source is the structured covariance matrix. This N x N covariance matrix can be defined as a weighted average of the sample covariance matrix and the prior matrix, having the same number of elements. The prior matrix consists of two values, with the average of cross-sectional variance on the diagonal elements, and the average of the cross-sectional covariance on the off-diagonal elements. The practicality of this procedure ignores the differences of return variation across industries. Technically, it would be more realistic to structure the prior matrix with the industry averages of (co)variances.

Estimating the security covariance matrix is conducted on a monthly basis. The past five years of monthly returns are used at each half year's optimization of security weights. However, the use of higher frequency data in covariance estimation would be more optimal, as demonstrated by Gosier, Madhavan, Serbin, and Jian (2005). Using one year of daily data would allow for significantly more observations (T = 250) instead of (T = 60) which could lead to more precise stock weight allocations. Moreover, a monthly-rebalancing optimization procedure would provide a more accurate view of the MV portfolio's out-of-sample excess return compared to applying a semi-annual rebalancing procedure. Due to time constraints and the priority for studying the impact of unobserved risk sources, these procedures have not been implemented.

Two constraints are imposed on the composition of this long-only MV variant. In accordance with what its name suggests, short-selling is not allowed and hence each individual stock must have a zero or positive weight allocation. This is a realistic constraint as it would be quite costly to undertake short-selling practices in the real world. Potentially high transaction costs could press down realized returns. Except from the condition that the MV weights have to sum up to 100%, no other constraints are imposed and the MV portfolio can be called rather "unconstrained". The unconstrained character of the MV portfolio provides foundation for the emergence of extreme stock weight allocations. The results have shed light on the concentration towards low-beta stocks and specific industries that could expose the MV portfolio to high levels of risk. From a diversification point of view, it would be extremely relevant to test whether the MV outperformance still holds in the presence of some applicable industry-related or market-deviation constraints. Theoretically, implementing these constraints could result in a more diversified portfolio that offers somewhat more insurance in times of extreme market volatility than the current format of the MV portfolio. It would be interesting to observe whether this would be at the expense of a lower return.

The survival of the Carhart (1997) four-factor model is intriguing and seems to be in violation with traditional principles of risk and return. Shedding new light on unobserved risk sources that could potentially explain the extraordinary performance of the MV portfolio is of high importance. Leverage constraints, the absence of implied protection, and industry-specific risk are shown to be related to MV strategies. Theoretically, an MV investor requires an additional compensation for the use of leverage, the concave payoff pattern, and the concentration towards specific industries. Concerning the absence of implied protection, it is difficult to detect whether alpha is indeed a required compensation for the underperformance of low-beta stocks during the occurrence of an extreme market event. The limited number of observations on extreme market conditions does not sufficiently allow concluding that the MV outperformance (alpha) is just a rational outcome for the bill that MV investors pay in times of extreme market down movements.

10. Recommendations

In the light of the increased appreciation for risk management, it is of high value to discuss the function of minimum variance strategies. In the case that the MV portfolios are suggested to provide abnormal returns and offer insurance in market down states, an MV investor might end up with some unforeseen and unfavorable outcomes. In other words, the MV strategy might not be the appropriate strategy to serve as safe and low-volatility benchmark at all time. In this respect it would be of high interest to observe the performance of MV portfolios in the presence of some constraints that guarantee some level of diversification and protection.

Further, the sample of this research is limited to the 1000 U.S. largest market capitalization stocks. The practicality and replicability of the covariance estimation procedure call for research on the performance of minimum variance strategies using the same method. The performance of the MV portfolio and the effect of related risk sources may be different in other data samples, regions and asset classes. For this reason, it would be interesting to extend this research into other directions.

With respect to the absence of implied protection, a more detailed view is necessary on the performance of minimum variance strategies during extreme market events. The small number of observations on these events prevents from drawing accurate conclusions on this matter. For this, it might be an option to conduct an event study on the performance of MV portfolios during extreme market down movements. It would be of high value to demonstrate that MV portfolios indeed lack in the ability to offer reasonable insurance during down markets, and that the perception of a "risk-free return" and the related anomaly does not exist.

The MV portfolio seems to offer a reasonable return over the complete data period. The significant alpha outperformance and the dominance in efficiency are remarkable. Even in crises periods, the MV portfolio exhibits a positive alpha performance. On the other hand, in big up markets, the MV portfolio is not able to track the performance of the market, and lags behind (appendix 17 and 18). Extreme months of excessive market down movement demonstrate that the MV portfolio does not offer optimal insurance. The behavior of the MV portfolio differs during different states of the market which creates uncertainty. In contrast to what its name suggests, a minimum variance strategy can be very risky.

For this reason, products derived from this strategy are not recommended for pension funds or insurance companies, which need to closely track the performance of their liabilities at all time. A month of extremely negative returns could have a large impact on their investment portfolio. A severe decline in the level of assets puts high pressure on funding ratios and the ability to cover current and future liabilities. Individual investors must be aware that the MV portfolio is not always a low-risky project. The historical outperformance is no guarantee for similar future results. The popularity of low-volatility strategies can cause transaction costs to be high, and the MV construction methodology offers little room for diversification. Though, MV investments that include some strong constraints to assure a certain level of diversification might be beneficial.

11. References

Ang, Andrew, Joseph Chen, and Yuhang Xing (2006), "*Downside Risk*," Review of Financial Studies, 19(4), 1191-1239.

Arnott, R, Kalesnik, V., Moghtader, P. and Scholl, C., (2010), "*Beyond cap weight: the empirical evidence for a diversified beta"*, Journal of Indexes, 13(1), pp.16-29.

Baker, B. Wurgler., (2011), "*Benchmarks as Limits to Arbitrage: Understanding the Low-volatility Anomaly", Financial Analysis Journal*, *67*(1).

Behr, P., Güttler, A., and Miebs, F., (2008) "*Is Minimum-Variance Investing Really Worth the While*?" An Analysis with Robust Performance Inference," Working Paper.

Benartzi, Schlomo and Thaler, Richard H., (February 1995) "*Myopic Loss Aversion and the Equity Premium Puzzle*." Quarterly Journal of Economics, 110(1), pp. 73-92.

Black, F., (1972), "*Capital market equilibrium with restricted borrowing*", Journal of Business, 45(2), pp.444-455.

Black, F., M.C. Jensen, and M. Scholes (1972), "*The Capital Asset Pricing Model: Some Empirical Tests*." In Michael C. Jensen (ed.), Studies in the Theory of Capital Markets, New York, pp. 79-121.

Blitz, D., and Vliet, P. V., (2011), "*Benchmarking Low-Volatility Strategies*", Journal of Index Investing, 2(1), 44-49.

Blitz, D., Pang, J., and Vliet, P. V., (2013), "*The Volatility Effect in Emerging Markets*", Emerging Markets Review.

Brennan, M.J., (1971), "*Capital market equilibrium with divergent borrowing and lending rates*." Journal of Financial and Quantitative Analysis 6, 1197-1205.

Carhart, M., (1997), "*On persistence in mutual fund performance",* The Journal of Finance, 52 (1), pp.57-82.

Chincarini, L. B. and D. Kim, (2006), "*Quantitative Equity Portfolio Management*" New York, NY, USA: McGraw-Hill.

Clarke, R. G., de Silva, H., and Thorley, S., (2006), "*Minimum-variance portfolios in the US equity market*", The journal of portfolio management, 33(1), 10-24.

Clarke R., de Silva H., Thorley S., (2011, "*Minimum-Variance Portfolio Composition*", The Journal of Portfolio Management, Winter.

Cowan, D. and Wilderman, S., (2011), "*What the Beta Puzzle Tells Us about Investing*" [online] Retrieved at: <http://www.scribd.com/doc/73200528/ReThinking-Risk What-the-Beta-puzzle-Tells-Us-About-Investing-GMO> [Accessed 18 April 2013].

Fama, E., and French, K., (1992), "*The cross-section of expected stock returns*", Journal of Finance, 46, pp.427-465.

Fama, E., and French, L.,, (2004), "*The capital asset pricing model: Theory and evidence"*, Journal of Economic Perspectives 18:3, 25-46.

Frazzini, A., and Pedersen, L. H. (2010), "*Betting against beta"*, (No. w16601). National Bureau of Economic Research.

French, K. R., Schwert, G. W., and Stambaugh, R. F., (1987), "*Expected stock returns and volatility*", Journal of financial Economics, 19(1), 3-29.

Geiger, H., and Plagge, J., (2007). "*Minimum variance indices"*, Frankfurt: Deutsche Börse AG.

Gosier, K., Madhavan, A. N., Serbin, V., and Yang, J., (2005), "*Toward Better Risk Forecasts*", The Journal of Portfolio Management, 31(3), 82-91.

Greenland, S., Schlesselman, J. J., and Criqui, M. H., (1986), "*The fallacy of employing standardized regression coefficients and correlations as measures of effect*", American journal of epidemiology, 123(2), 203-208.

Haugen, R. A., and Baker, N. L., (1991), "*The efficient market inefficiency of capitalization-weighted stock portfolios*", The Journal of Portfolio Management, 17(3), 35-40.

Hsu J.C., (2006), "*Cap-Weighted Portfolios are Sub-Optimal Portfolios*", Journal of Investment Management, 4(3), pp. 1-10.

Jagannathan, R., and T. Ma., (2003), "*Risk Reduction in Large Portfolios: Why Imposing the Wrong Constraints Helps",* Journal of Finance 58:1651–84.

Jegadeesh, N., and S. Titman, (1993), "*Returns to buying winners and selling losers: Implications for stock market efficiency*," Journal of Finance 48, 65-91.

Jensen, M., (1968), "*The performance of mutual funds 1945-1964*", Journal of Finance, 83(2), pp.389-416.

Kuo, L. L., and Li, F., (2012), "*An Investor's Low Volatility Strategy*", The Journal of Index Investing, 3(4), 8-22.

Ledoit, O., and Wolf, M., (2003), "*Honey, I shrunk the sample covariance matrix*" *UPF* Economics and Business Working Paper, (691).

Lintner, J., (1965), "*The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets*", Review of Economics and Statistics, 47(1), pp.13-37.

Luo, Y., R. Cahan, J. Jussa, Z. Chen and M. Álvarez, (2011), *"Minimum Variance: Exposing the 'magic"*, Global Markets Quantitative Research, Deutsche Bank.

Malkiel, B. G., and Fama, E. F. (1970). "*Efficient Capital Markets: A Review Of Theory And Empirical Work*". The journal of Finance, 25(2), 383-417.

Manos, G. C., and Clough, R. W., (1985), "*Tank damage during the May 1983 Coalinga Earthquake*", Journal of Earthquake Engineering and Structural Dynamics, Vol. 1, No. 4, 1985, pp. 449–466.

Markowitz, H., (1952), "*Portfolio selection*", Journal of Finance, 7(1), pp.77-91.

Melas, D., Briand, R., and Urwin, R., (2011), *"Harvesting Risk Premia with Strategy Indices."* MSCI Research Insights.

Mehrling, P. (2005), "Fischer Black and the Revolutionary Idea of Finance," Wiley: New Jersey.

Miller, M. H., and Scholes, M., (1972), "*Rates of return in relation to risk: A reexamination of some recent findings*", Studies in the theory of capital markets, 23.

MSCI, (2010), *"MSCI Global Minimum Volatility Indices Methodology"* [online] Retrieved at:<http://www.msci.com/eqb/methodology/meth_docs/MSCI_Minimum_Volatility Methodology_Nov10.pdf> [Accessed 9 April 2013].

MSCI, (2012), "*MSCI Global Minimum Volatility Indices Methodology*", [online] Retrieved at: <http://www.msci.com/resources/factsheets/index_fact_sheet/msci-world-minimum volatility-index.pdf> [Accessed 10 April 2013].

Business Cycle Dating Committee, (2010), "*NBER Business Cycle Expansions and Contractions*", National Bureau of Economic Research (NBER), [online] Retreived at: <http://www.nber.org/cycles.html> [Accessed 1 August 2013].

Nielsen, F., and Aylursubramanian, R., (2008), "*Far From the Madding Crowd—Volatility Efficient Indices*", MSCI Barra Research Insights.

Samuelson, P. A. (1965). Proof that properly anticipated prices fluctuate randomly. *Industrial management review*, *6*(2).

Scholes, M., and Williams, J., (1977), “*Estimating betas from nonsynchronous data*”, Journal of financial economics, 5(3), 309-327.

Sharpe, W., (1964), “*Capital asset prices: a theory of market equilibrium under conditions of risk*”, American Finance Association, 19(3), pp.425-442.

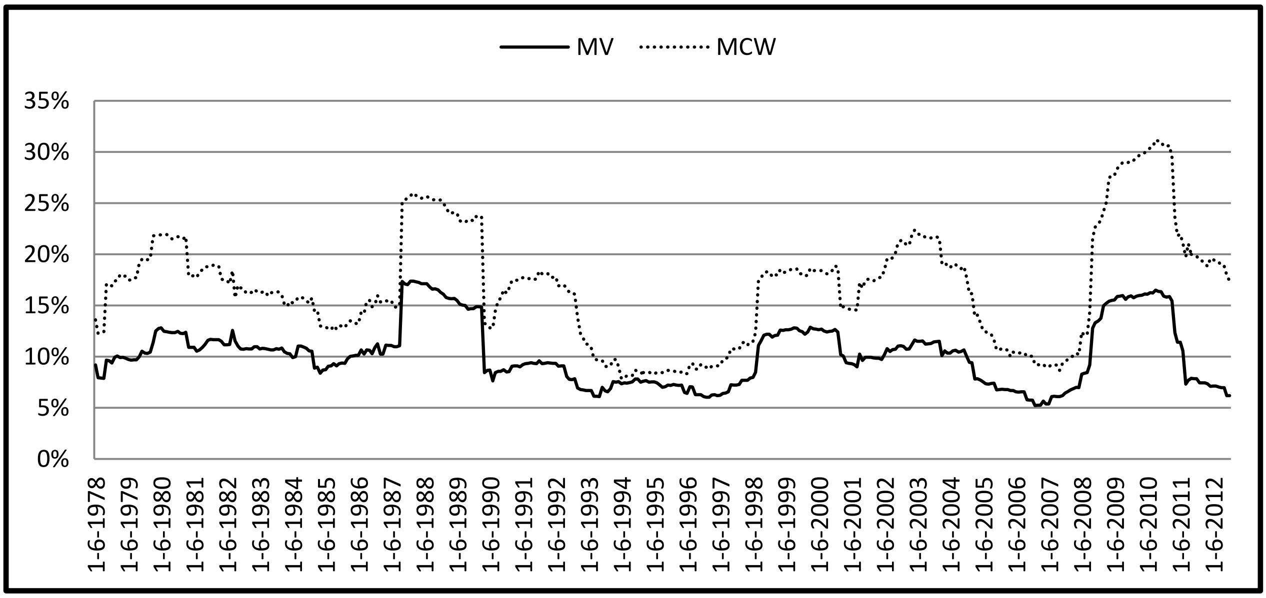
Shedden, K., (2010), “*Decomposing Variance*”, [online] Retrieved at: http://www.stat .lsa.umich.edu/~kshedden/Courses/Stat600/Notes/decomposing-variance.pdf Accessed 3 August 2013].

Wang, Z. (2005), “*A shrinkage approach to model uncertainty and asset allocation*”, Review of Financial Studies, 18(2), 673-705.

12. Appendix

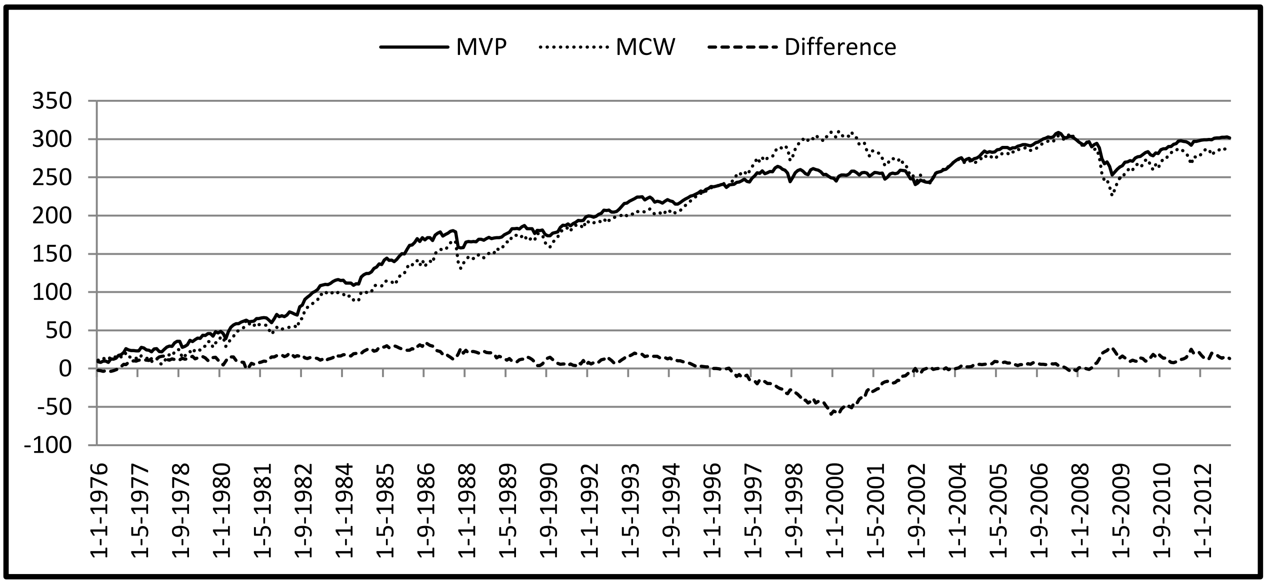
Appendix 1: Realized Level of Portfolio Risk.

Realized portfolio risk is estimated through a 30-month moving average of the standard deviation of returns. The estimation uses the complete data sample of monthly excess returns from January 1976 to December 2012. The solid line represents the realized risk level of the MV portfolio, and the dashed line shows the realized risk level of the MCW benchmark portfolio.

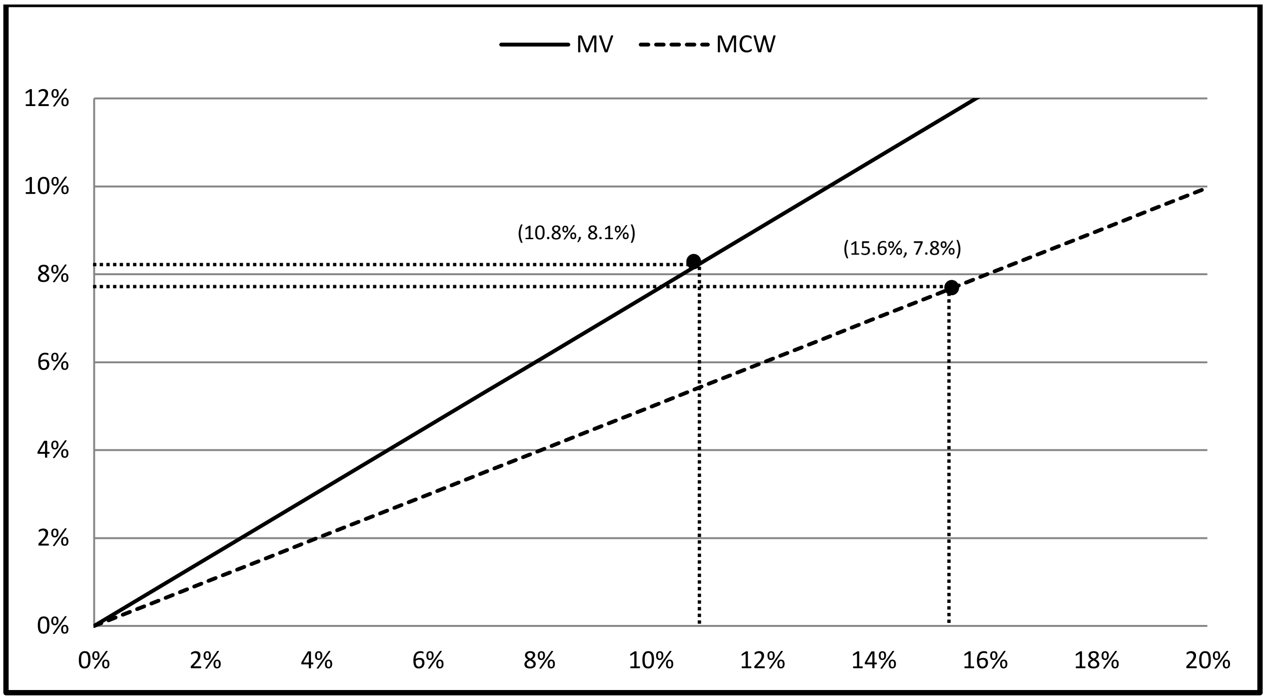


Appendix 2: Cumulative Excess Returns.

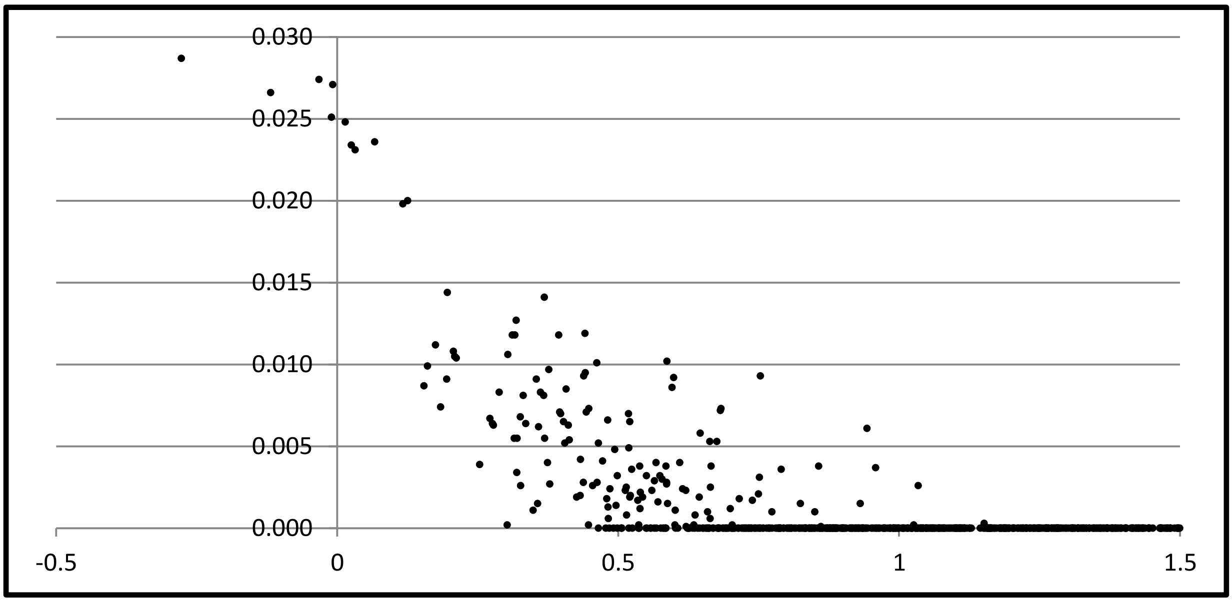
Cumulative excess returns of the MV Portfolio and the MCW Portfolio between January 1976 and December 2012. The lower line indicates the difference in the level of cumulative excess returns between both portfolios.



Appendix 3: Empirical Positions of the MV and MCW portfolio.
The x-axis and the y-axis represent the level of annualized standard deviation and excess returns respectively. The solid line shows risk-return relation of the MV portfolio, while the dashed line represents the same relation for the MCW portfolio. The slope of each line equals the corresponding Sharpe ratio of each portfolio. Results are based on the completed data period (1976 -2012).

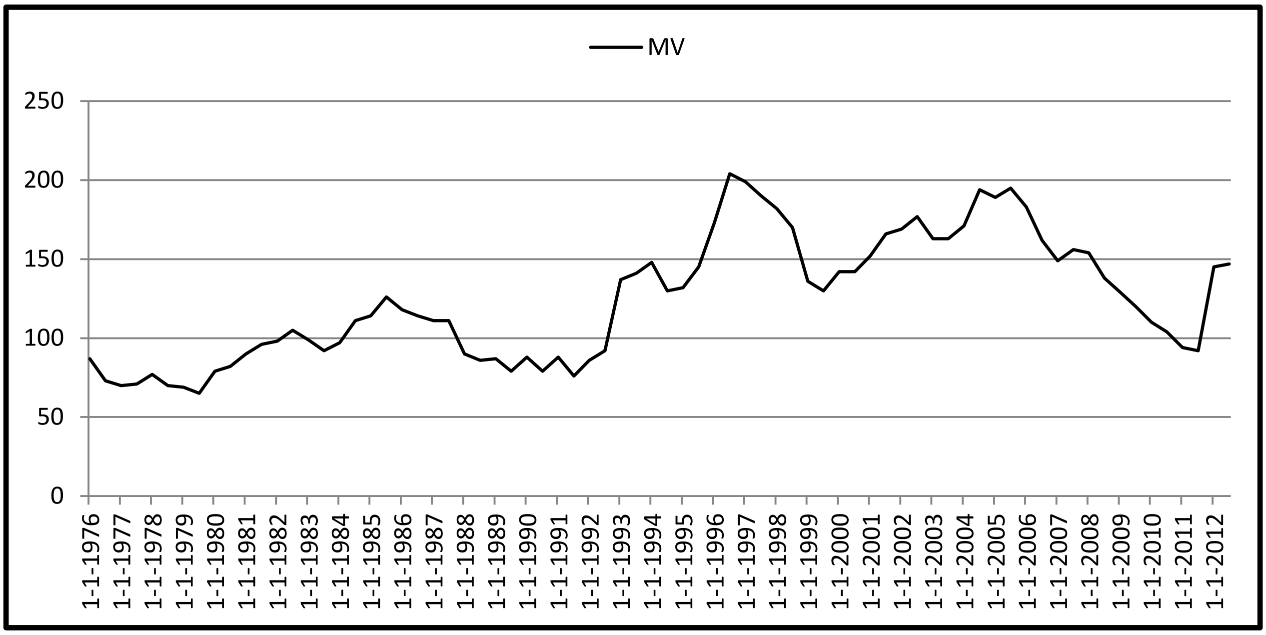


Appendix 4: Relation MV Stock Weights and Corresponding Betas.
The figure presents the relation between the 1000 MV individual stocks weights and their corresponding level of beta. This scatterplot is the result of the final MV weight optimization (June 2012). The y-axis represents the weight of an individual stock within the MV portfolio. The x-axis indicates the corresponding beta level.



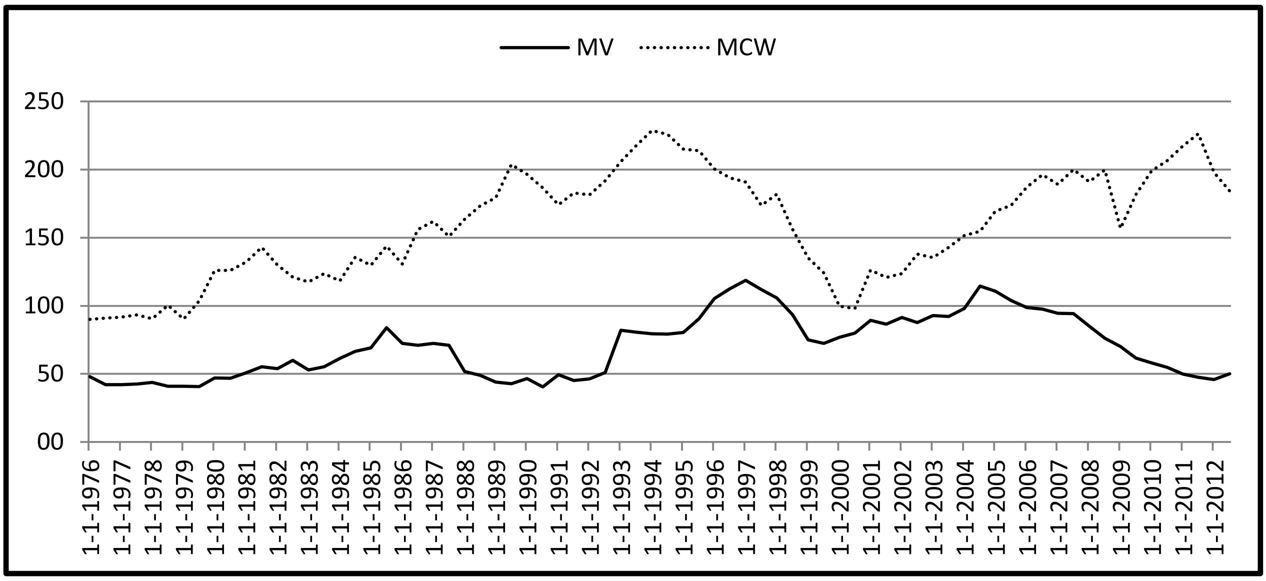
Appendix 5: Actual Number of Individual Stocks within the MV Portfolio.

The figure plots the number of individual stocks that jointly form hte MV portfolio over the complete data sample. The asset universe consists of the 1000 largest U.S. market capitalization weighted stocks. The MV optimization procedure (weight allocation) is conducted on a semi-annual basis. The actual number of stocks present in the MV portfolio ranges from 70 to approximately 200. Results are based on the completed data period (1976 - 2012).



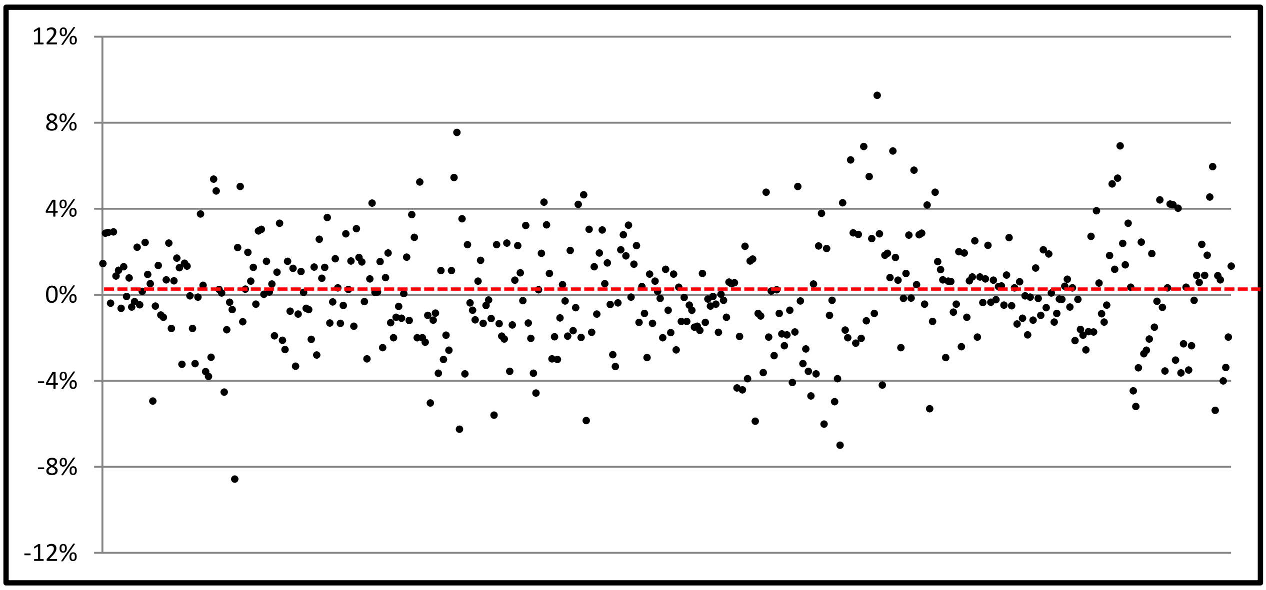
Appendix 6: Effective Number of Individual Stocks

The effective number of stocks gives an indication of the number of equally weighed stocks that would generate an equal level of diversification benefit (Clarke et al., 2006). The number equals the inverse of the sum of squared individual stock weights. The dashed MCW line demonstrates that the MCW portfolio is significantly more diversified than the MV portfolio (solid line). Results are based on the completed data period (1976-2012).



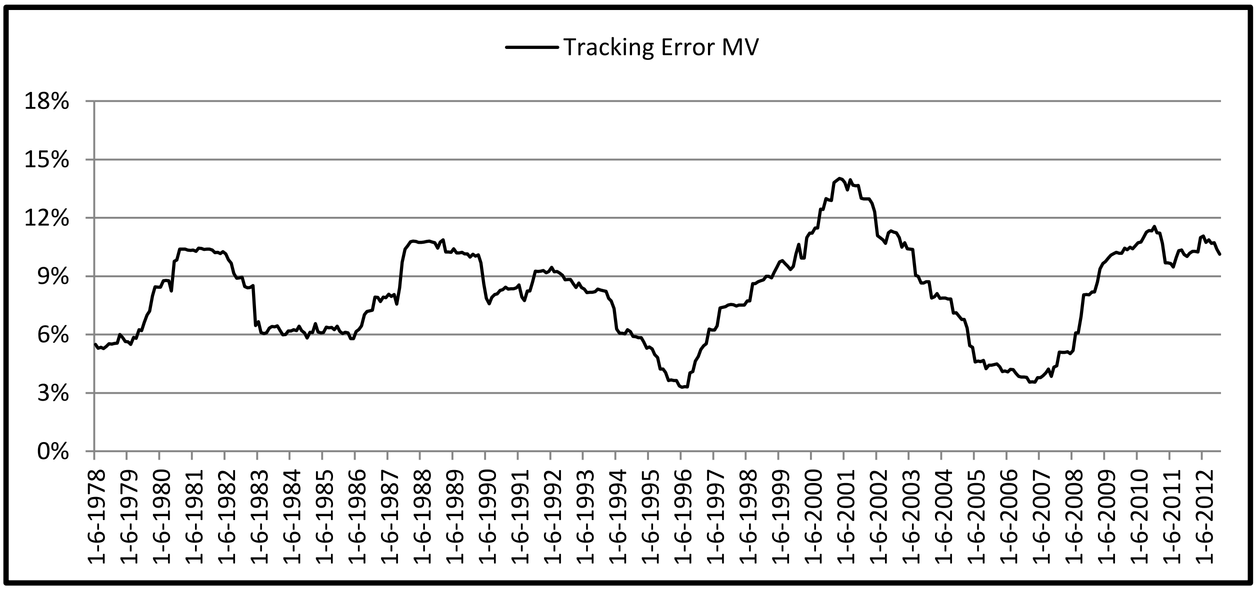
Appendix 7: Return Differences between the MV Portfolio and the MCW Portfolio.

The scatterplot represents the difference in the excess returns of the MV portfolio in comparison with the MCW benchmark portfolio from January 1976 to December 2012. The total range of return differences gives an indication of the tracking error of the MV portfolio. The tracking error of the MV portfolio can be defined as the annualized standard deviation of all return differences between the MV portfolio and its benchmark, the MCW portfolio. The dashed red line indicates the mean level of return difference.



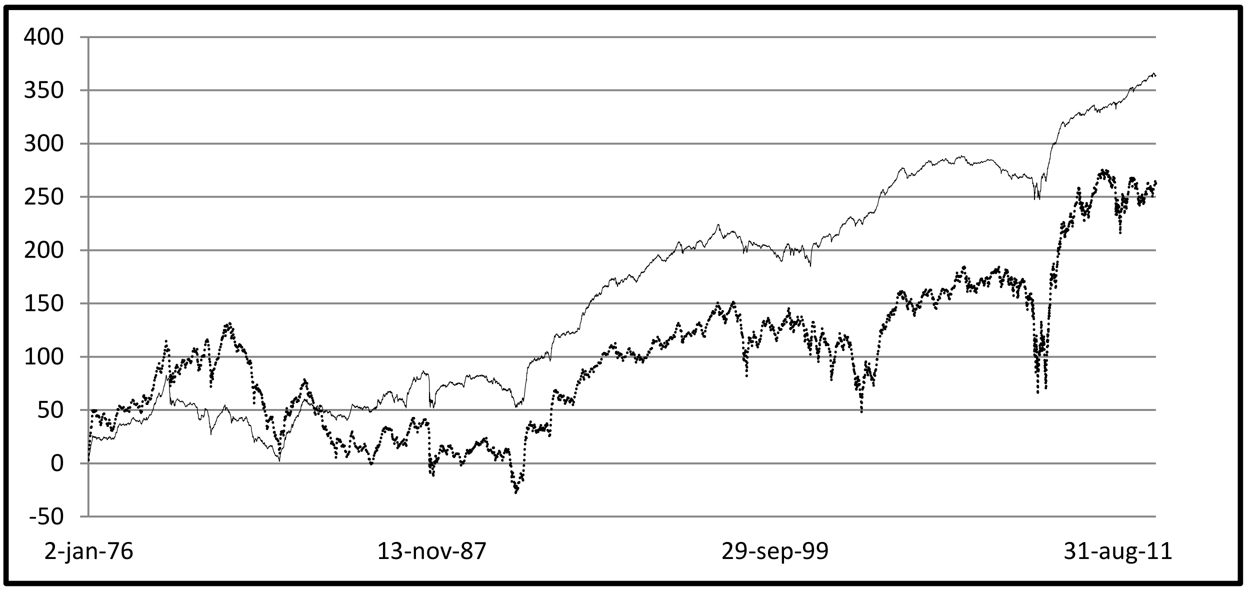
Appendix 8: Annualized Tracking Error of the MV Portfolio.

The line represents a 30-month moving average level of the annualized MV tracking error. The tracking error of the MV portfolio can be defined as the annualized standard deviation of all return differences between the MV portfolio and its benchmark, the MCW portfolio. Results are based on the completed data period (1976-2012).



Appendix 9: Level of Cumulative Excess Returns of the Lowest and Highest Beta-decile Portfolio.

The portfolio that consists of the lowest betas (portfolio 1) is represented by the solid line. The dashed line shows the cumulative excess return of the portfolio that consists of the highest betas (portfolio 10). Portfolio10 reaches the 250 level, while the level of cumulative excess returns of portfolio 1 are higher than 350 at the end of 2012. Results are based on the complete data period (1976 – 2012).



Appendix 10: Summary Statistics. The table presents the annualized outcomes of the excess return, standard deviation, Sharpe ratio, mean excess return, tracking error, and information ratio for the MV portfolio, MCW portfolio, and the EQW portfolio. The mean excess return is the average annualized difference between the portfolio of interest and the MCW portfolio. The tracking error is the annualized standard deviation of the return differences between the portfolio of interest and the MCW portfolio. The information ratio can be defined as the ratio of mean excess return to tracking error. Results are based on the complete data period (1976 – 2012).

	Annualized		
	MCW	MV	EQW
Excess Return	7.788%	8.148%	7.740%
Std. Dev.	15.637%	10.759%	17.822%
Sharpe-Ratio	0.498	0.757	0.434
Mean Excess Return	-	0.356%	-0.055%
Tracking Error	-	8.719%	4.082%
Information Ratio	-	0.041	-0.013

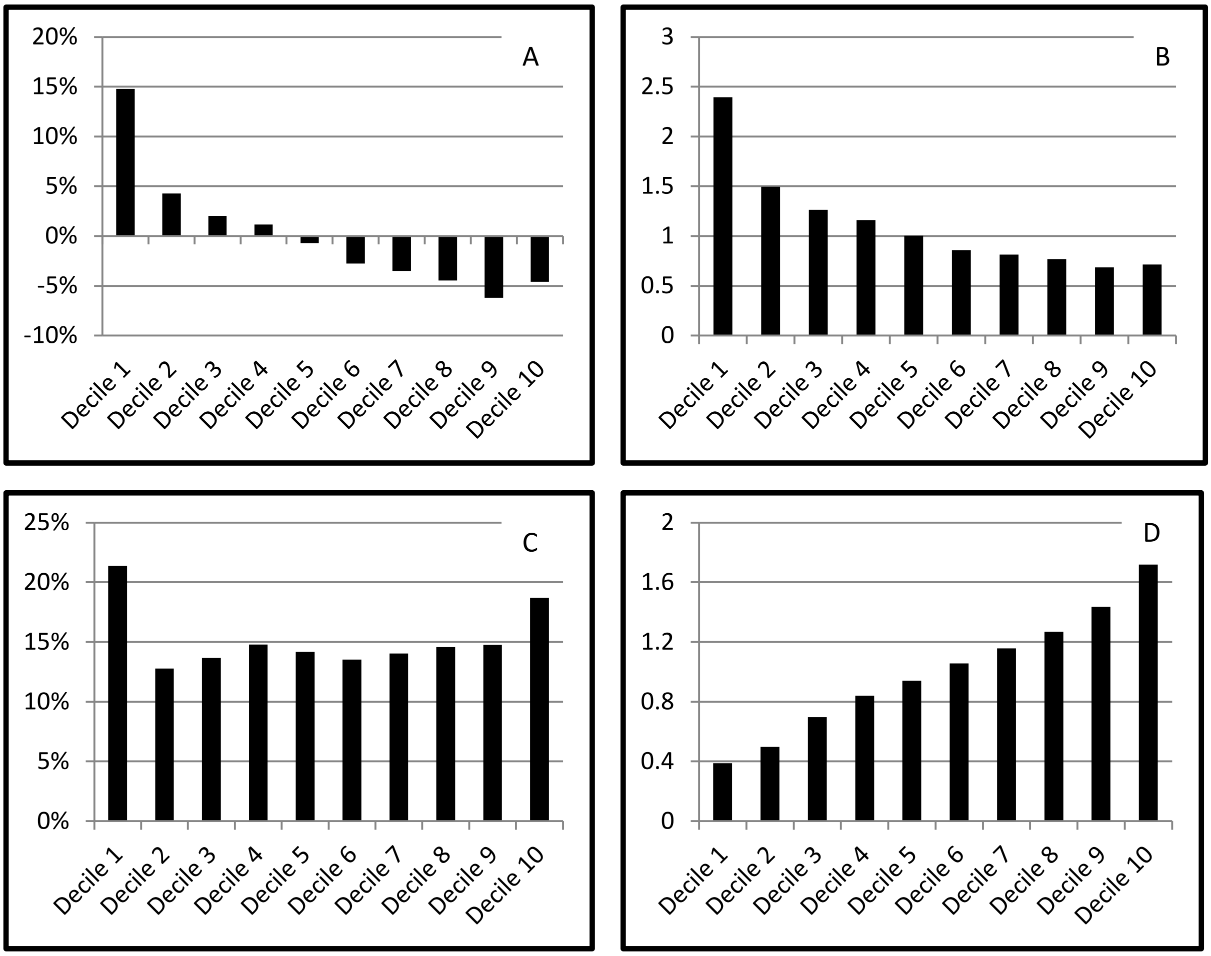
Appendix 11: Summary Statistics. The table presents annualized excess returns, standard deviations, and Sharpe ratios of all beta-decile portfolios. Results are based on daily return data from January 1976 to December 2012.

Decile Portfolio	Ex. Return	St. Dev.	Sharpe Ratio
1 (Low Beta)	21.38%	8.93%	2.394
2	12.78%	8.54%	1.496
3	13.65%	10.82%	1.262
4	14.78%	12.75%	1.159
5	14.18%	14.12%	1.004
6	13.52%	15.78%	0.857
7	14.04%	17.26%	0.813
8	14.58%	18.96%	0.769
9	14.75%	21.49%	0.686
10 (High Beta)	18.69%	26.23%	0.713
EQW	15.23%	14.64%	1.040
BAB	35.82%	25.91%	1.382

Appendix 12: Regression Analyses. The table presents the exposure to the four factors of all beta-decile portfolios based on a four-factor (Carhart, 1997) regression model. The alpha return is annualized on the basis of 252 trading days. The results are based on daily return data for the complete data period (1976 – 2012.)

Decile Portfolio	α	β_M	S_{SMB}	H_{HML}	M_{MOM}	$Adj.R^2$
1 (Low Beta)	14.78%	0.395	0.069	-0.048	0.083	0.424
	(13.21)	(79.58)	(8.56)	(-5.42)	(13.29)	
2	4.28%	0.511	-0.011	0.039	0.074	0.733
	(5.87)	(158.12)	(-2.08)	(6.75)	(18.26)	
3	2.01%	0.709	0.006	0.079	0.061	0.890
	(3.40)	(270.23)	(1.52)	(17.11)	(18.74)	
4	1.17%	0.850	0.005	0.082	0.038	0.934
	(2.15)	(354.18)	(1.37)	(19.29)	(12.67)	
5	-0.72%	0.949	-0.005	0.058	0.026	0.953
	(-1.42)	(422.27)	(-1.35)	(14.60)	(9.37)	
6	-2.76%	1.060	-0.018	0.036	0.002	0.962
	(-5.43)	(470.51)	(-4.82)	(8.97)	(0.64)	
7	-3.50%	1.158	-0.032	0.109	-0.012	0.965
	(-6.59)	(492.13)	(-8.26)	(2.62)	(-4.24)	
8	-4.46%	1.267	-0.031	0.002	-0.027	0.962
	(-7.29)	(466.95)	(-6.94)	(0.40)	(-7.89)	
9	-6.19%	1.423	-0.019	-0.042	-0.068	0.958
	(-8.53)	(442.43)	(-3.72)	(-7.28)	(-16.90)	
10 (High Beta)	-4.60%	1.677	0.034	-0.218	-0.177	0.928
	(-3.96)	(325.77)	(4.10)	(-23.89)	(-27.53)	
BAB	34.48%	-0.024	0.348	0.521	-0.200	0.062
	(7.93)	(-0.30)	(3.02)	(4.08)	(-2.68)	

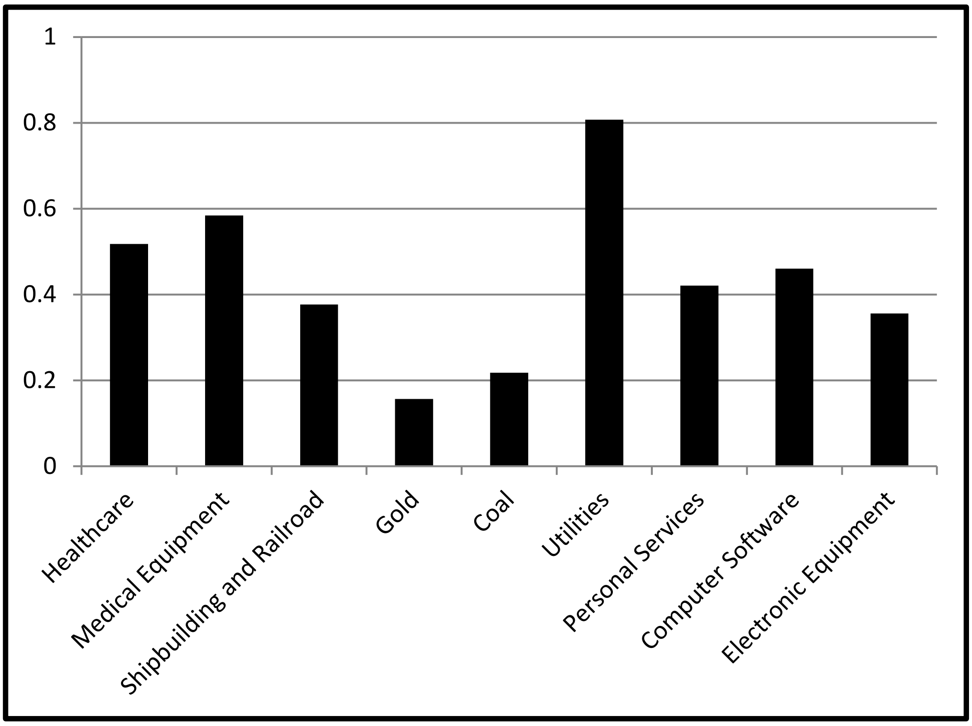
Appendix 13: Beta-Decile Figures. All figures based on daily return data from January 1976 to December 2012 derived from the CRSP database. Figure A displays the annualized alpha that is based on 252 trading days. Figure B indicates the Sharpe Ratio of each beta-decile portfolio. Figure C and D show the annualized average excess return and the beta level respectively. Lowest betas are in decile 1, the highest beta belong to decile 10.



Appendix 14: Summary Statistics. The table presents annualized excess returns, standard deviations, and Sharpe Ratios for all 9 industry portfolios. Results are based on monthly return data from January 1976 to December 2012 that is directly retrieved from Kenneth French's Data Library.

	Excess Return	St. Dev.	Sharpe Ratio	N
Healthcare	13.02%	25.15%	0.518	444
Medical Equipment	10.74%	18.40%	0.584	444
Shipbuilding and Railroad	9.97%	26.42%	0.377	444
Gold	5.82%	37.09%	0.157	444
Coal	7.95%	36.41%	0.218	444
Utilities	10.92%	13.53%	0.807	444
Personal Services	9.26%	22.00%	0.421	444
Computer Software	14.25%	30.96%	0.460	444
Electronic Equipment	9.78%	27.46%	0.356	444

Appendix 15: Sharpe Ratios of all 9 Industry Portfolios. Results are based on monthly return data from January 1976 to December 2012 that is directly retrieved from Kenneth French's Data Library. The utility sector dominates in terms of Sharpe Ratio.



Appendix 16: Regression Analyses. The table presents the exposure to the four factors of all nine industry portfolios based on a four-factor (Carhart, 1997) regression model. The alpha return is annualized. The results are based on monthly return data for the complete data period (1976 – 2012).

Industry	α	β_M	S_{SMB}	H_{HML}	M_{MOM}	$Adj. R^2$
Healthcare	2.217%	0.978	0.629	0.259	0.201	0.454
	(0.70)	(16.54)	(7.39)	(2.82)	(3.65)	
Medical Equipment	4.311%	0.856	0.085	-0.149	0.058	0.577
	(2.11)	(22.47)	(1.56)	(-2.51)	(1.66)	
Shipbuilding and Railroad	-1.352%	1.137	0.459	0.438	-0.004	0.483
	(-0.42)	(18.80)	(5.27)	(4.66)	(-0.07)	
Gold	-1.912%	0.612	0.523	0.189	0.233	0.089
	(-0.32)	(5.43)	(3.22)	(1.08)	(2.22)	
Coal	-5.629%	1.257	0.468	0.382	0.223	0.301
	(-1.08)	(12.86)	(3.36)	(2.54)	(2.47)	
Utilities	4.404%	0.557	-0.025	0.349	0.071	0.385
	(2.43)	(16.50)	(-0.53)	(6.64)	(2.25)	
Personal Services	-0.622%	0.976	0.539	0.144	0.164	0.577
	(-0.25)	(21.43)	(8.22)	(2.03)	(3.88)	
Computer Software	7.245%	1.282	0.401	-0.816	0.097	0.649
	(2.31)	(21.95)	(4.77)	(-8.98)	(1.78)	
Electronic Software	2.841%	1.283	0.195	-0.544	-0.081	0.694
	(1.09)	(26.52)	(2.80)	(-7.24)	(-1.80)	

Appendix 17: "Regression Analyses – Market up States". The table presents the outperformance of the MV portfolio during the 100 months of highest MCW portfolio excess returns. The regression is conducted on the basis of the Four-factor regression model (Carhart, 1997). The alpha performance of the MV portfolio is annualized. The MV portfolio exhibits an insignificant alpha return.

α	4.574% (0.68)
β_M	0.584 (6.44)
S_{SMB}	0.079 (1.45)
H_{HML}	0.279 (4.35)
M_{MOM}	0.008 (0.29)
$Adj. R^2$	0.362

Appendix 18: Summary Statistics – Market up States". The excess return, standard deviation, and Sharpe Ratio of the Minimum Variance Portfolio, the Market Capitalization Weighted Portfolio, and the Equally Weighted Portfolio. The sample is based on 100 months of excess return data that are characterized by the highest excess return of the market portfolio.

	MV	MCW	EQW
Excess Return	3.624%	5.962%	6.201%
Std. Dev.	2.145%	1.973%	2.494%
Sharpe-Ratio	1.689	3.022	2.486