TILBURG UNIVERSITY

School of Economics and Management

**Do investors benefit from adding commodities to a portfolio of stocks, bonds and bills?**

Niels Ansems
s103911
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Supervisor : Martijn F. Boons

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# 1. Introduction

In this paper it will be discussed if institutional investors can benefit from adding commodities to their portfolio’s consistent out of stocks, bonds and bills. With benefitting described as making the portfolio less risky and or more profitable. Commodities have historically been regarded as extremely volatile and risky. But then Erb & Harvey (2006) and Gorton & Rouwenhorst (2006) showed that commodity portfolio's could generate equity like returns when they have a high enough diversification. They also stated that portfolio's consistent out of stocks and bonds could benefit from adding commodities because of the diversification effect, and that they could also be well used as an inflation hedge. Pension funds and long term investors are most certainly interested in opportunities to decrease the long term risk of their investment portfolio's, since their goal is to maintain a stable long term return. Therefore including commodities could be fairly interesting for them, but not so much for the private investors.

The historical returns of commodity futures have often been examined, Erb & Harvey (2006) show that the annual individual returns of commodities are close to zero. But as mentioned before they found that a commodity portfolio could generate equity like returns if they achieve a high enough diversification. Diversification is a good way to decrease the risk of your portfolio, but will only work when asset values will not move up and down at the same time. In other words when assets are highly uncorrelated. Gorton & Rouwenhorst (2006) showed that because of the negative correlation between most of the commodities and the various assets like bonds and stocks there are several diversification possibilities. They give 2 explanations for this effect, one of them is that commodity futures perform better in times of unexpected inflation, where stock and bonds mostly seem to have a hard time. As a second reason they mention the cyclical variance of the US Bonds and Stocks. They have determined that commodities could significantly diversify these. The fact that precious metals could be a safe investment during times of high stock market volatility was added by (Hiller et al. 2006).

In the past Anson (1999) and Ankrim & Hensel (1993) showed that including commodities will improve the risk/return trade off.  This is backed by the research of Erb & Harvey (2006) who found  a   -0.03 correlation between the commodity index GSCI and the S&P 500, this indicates that there are a lot of diversification possibilities. This could give an opportunity for a rebalanced combined portfolio with a lower variance or higher return. Then again they do warn for the fact that commodity returns in the future might not take the same path as they did in the past. This collaborates with Tang & Xiong (2009) who concluded that the historical correlation data might not be a good estimate for the future. So that institutional investors might have miss judged the diversification possibilities of commodities.  Another research in the field of adding commodities to portfolio's is Conover et al. (2010) and Bodie & Rosansky (1980). They stated that including commodities in your portfolios together with stocks, can definitely decrease risk without losing any return. But Conover et al. (2010) showed that for a significant decrease in risk in any of the 5 most popular /common investment techniques, namely momentum, value, small-cap, high-cap and growth. At least 10% of the portfolio had to exist out of commodities, again because of the diversification effect of commodities in the portfolio's. Marshall & Cahan & Cahan (2008) are also positive about adding commodities to portfolios. They state that commodity futures markets have a couple of advantages over stock markets. Since futures markets have less transactions costs then stock markets. Also according to them recent studies have showed that commodity futures are easily adaptable in several strategies.

According to Hong & Yogo (2011) the investments of institutional investors in commodities increased from 15 billion in the end of 2003 to 317 billion in July 2008. They also stated that in the same period the interest in commodity futures grew enormous from 103 billion to 509 billion. Because of this rapid growth in commodity investments, Tang & Xiong (2009) found in their research that the prices of non- energy commodities became far more correlated with the price of oil. Which could have big effect on commodity portfolio's like the GSCI, where according to Erb & Harvey (2006) oil has a big share. What's even more interesting for our paper is that Tang & Xiong (2009) concluded that during the rapid growth of commodity investments the historical low correlation between US Stocks & Bonds and Commodities probably will not be an accurate estimate for the future. They suggest that the correlation between them may indeed vary over time.

Bicchetti & Maystre (2012) provide some more insight about the change in correlation between commodities and US Stocks. They showed that prior to 2008 there was not much movement in correlation between these two, it was always close to zero. But after the collapse of Lehman brothers these correlations started to increase significantly until early 2010. According to them this has 3 main reasons, first of the technical innovations which gave the investors the opportunity to close deals way faster than before. Second investors became more active on the stock market, where they used to be kind of passive. And last but not least the active investing strategies started out good, as a consequence passive investing became less popular even during a lot of uncertainties. Another explanation of the increased correlation is given by Bicchetti & Maystre (2012), during the enormous growth in commodity markets commodities are starting to "behave" differently and are deviating from their roots. In such a way that in times of economic stress or crisis commodities aren't the safe haven as they used to be. Their findings of the correlation between the S&P 500 and the GSCI matched with the suggestion of Tang & Xiong (2009), that the correlation could indeed vary over time.  They found that these two are way more correlated with each other than 20 years ago. These findings are also equivalent to the research of Buyuksahin & Robe (2010) , they also find that the commodity - various asset correlations is significantly positively related to financial stress. Therefore in times of financial stress the diversification possibilities of adding commodities to portfolio's decreases.

So there is an enormous increase in commodity investments. This led to an increase in correlation between commodities and stocks & bonds. Therefore we are interested if there are still some diversifications opportunities left. So that a combined portfolio consistent out of three risky assets: US Stocks, Us Bonds and Commodities with the riskless asset t-bills, can still be more profitable or less risky than the US stocks & US Bonds & t-bills combined. To research this we will construct a portfolio consistent out of the three risky assets. We picked the S&P GSCI index for the commodities, the S&P 500 for the US stocks and the Barclays US Aggregate as the bonds. We will use the t-bills as a riskless asset, which will define our risk free rate. So that in our research we can use excess returns by subtracting the risk free rate from the returns of the 3 risky assets. In this research we will also design a test for two of the commodity sub - indexes namely Energy & Precious Metal added to a portfolio with stocks and bonds. We will do all of  this for the time frame January 1992 till December 2012. But because of the commodity "boom" since 2003, we will also do a separate test for the sub - sample 2003 till 2012.

To determine whether investors can benefit from including commodities in their portfolios, we will introduce Markowitz (1952) Mean-Variance theory. It's the theory of maximizing return for a given variance, or minimizing variance for a given mean return. We will use this theory in our research to test for the investors maximum utility, by finding the optimal portfolio. In order to find an optimal portfolio for a riskless asset with three risky assets, in such a way that we reach the optimal trade/off between risk and return. We'll need to maximize the Sharpe Ratio according to Markowitz (1952) Mean-Variance theory. Where the Sharpe ratio is nothing more than the amount of extra risk accompanies the extra return. With its formula as $S = \frac{\mu_p - R_f}{\sigma_p}$ , we can see that it's nothing more than the Portfolio Return - the risk free rate divided by its standard deviation. By maximizing the Sharpe ratio we can determine the optimal weights for the three risky assets for any given level of risk. And so make a graph like figure 1.1.

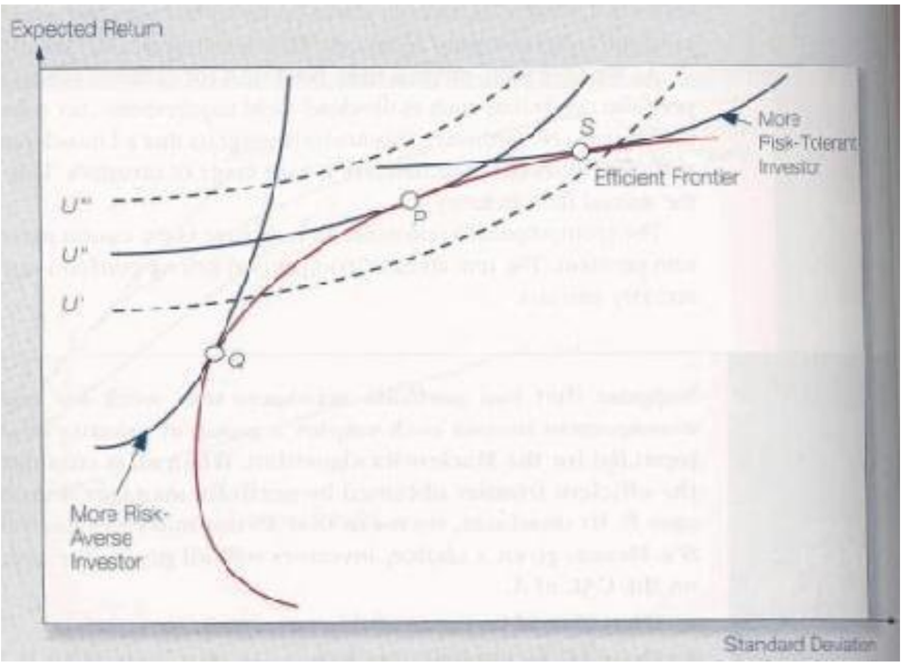



*Fig 1.1 . Source: Investment Analysis reader UVT*

In figure 1.1 the red line indicates the Efficient Frontier of risky assets. Every spot on this frontier has the same Sharpe ratio. While the place on this line for any single utility maximizing investor is determined by its risk aversion level. This gives us the opportunity to test whether adding commodities to our portfolio will increase the Sharpe ratio.

The remainder of this paper is organized as follows. Section 2 will explain the methods and techniques used to analyze if investing in commodities can increase the Sharpe ratio. Where section 3 will describe the dataset and give some summary statistics. Section 4 presents and discusses the research results found in the analysis. Last but not least in section 5 we will summarize and give our conclusions. To finish we will also give some recommendations for future research.

# 2. Methodology

The purpose of this paper is to identify whether investing in commodities can improve the performance of a portfolio consistent out of stocks and bonds. Improvement described as a matter of decreasing risk or increasing the return such that the Sharpe ratio is maximized. We will measure the performance of a portfolio by its Sharpe ratio, because as we have discussed before the optimal investment portfolio is found by maximizing the Sharpe ratio.

In this sections we first of discuss the mean-variance utility function and then we start with explaining the underlying assumptions who make sure we get to the optimal mean-variance trade-off. Next we will explain the equations derived out of the utility function needed to test our hypotheses. Last but not least we will show how we are going to calculate all of this.

## 2.1 Mean-Variance

Our research is based on the Markowitz (1952 mean-variance utility function. From now on we will assume that the investors want to maximize their utility. The mean-variance utility function is described as:

$$U(W|\mu,\Sigma) = w'\mu - \frac{1}{2}\lambda^{-1}w'\Sigma w$$

We have W as nominal wealth, and we implant $\mu$ as a vector of excess returns on the S&P 500 , GSCI and the Barclays US Aggregate. We will determine the excess returns by subtracting the risk free asset of the actual returns, the source and actual numbers will be discussed in section 3. We will take λ as the investors risk tolerance (λ > 0), and Σ stands for the variance and covariance matrix of these returns. The Mean-variance is efficient if it maximizes its expected return for a certain variance, and it minimizes it's variance for a certain expected return.

According to Jorion & Khoury (1995) this utility function is optimal if the following three assumptions are met. Namely:

- Capital markets are perfect.
- The investor has constant absolute risk aversion and maximizes expected utility.
- Asset returns are normally distributed.

The assumption that capital markets are perfect is not a real constraint, but still essentially for the optimization. It's there to ensure that transactions cost and constraints on positions in the S&P 500, the bonds and for the GSCI will not make a difference in determining the maximum utility. The constraint that investors must have a constant risk aversion and always want to maximize their expected utility, is their so that the outcome of the utility function (1) can be determined without any problems. If the risk aversion was a variable, it would still be a part of the equation when we are maximizing our utility. In contrary to a constant that will disappear when you take its first order condition. And if he was not to choose for the maximum utility he could as well not be on the efficient frontier, which would make our research useless since where testing for maximum utility. The last fact states that asset returns should be normally distributed, so that the whole distribution of asset returns can be described as a μ with an Σ and where kurtosis and skweness are 0.

Since we determined that in our test, investors will have a constant absolute risk aversion. We will assume in the remainder of this paper, that the investors have a risk tolerance of 1 (λ = 1). This will not have any effect on our research as we have showed with fig 1.1, since every optimal portfolio will have the same Sharpe ratio and thus will be on the efficient frontier line. For determining the optimal weights and there accompanying squared Sharpe ratios in our portfolio we need to derive a equation out of the utility function (1), by taking its first order condition.

$$w* = \Sigma^{-1}\mu \qquad (2)$$

$$\theta^2 = \mu'\Sigma^{-1}\mu \qquad (3)$$

With equation (2) we now can determine the optimal investment weights denoted by w* , and the maximum squared Sharpe ratio denoted by $\theta^2$ . As mentioned before in this paper we will take excess returns, on K stocks and bonds, $r_t^x$. For the commodities we will also take excess returns on N,$r_t^c$. This taken in account gives us the following Sharpe ratios:

$$\Theta_x^2 = \mu_x' \Sigma_x^{-1} \mu_x \quad (4)$$

$$\Theta_c^2 = \mu_c' \Sigma_c^{-1} \mu_c \quad (5)$$

Where in equation (4) $\mu_x$ is a vector containing the excess return of the stocks and bonds and the $\Sigma_x$ as the variance - covariance matrix for the returns of stocks and bonds. Where for equation (5) stands the same except this time for the commodity sub-indexes Energy & Precious Metal. Now we want to combine (4) and (5) for a optimal combined portfolio. This gives us:

$$\Theta_{xc}^2 = \mu_{xc}' \Sigma_{xc}^{-1} \mu_{xc} \quad (6)$$

$\mu_{xc}$ and $\Sigma_{xc}$ are of dimensions K+N. And so $\Theta_{xc}^2$ stands for the squared Sharpe ratio of the optimal combined portfolio. We will implant this test twice, the first where $\Theta_{xc}^2$ is the squared Sharpe ratio of portfolio consistent out of stocks, bonds and the GSCI. The second one will be the squared Sharpe ratio of the portfolio consistent out of stocks, bonds and the sub - indexes Energy and Precious Metal.

Since we want to test for the effect of adding the sub - indexes energy and precious metal to a portfolio combined out of stocks and bonds (K). We use the test described in Gibbons et al. (1989) they explain a simple spanning test in their paper, which allows us to examine the effect of adding N commodities to an existing portfolio of stocks and bonds. So we can determine if there is a significant improvement for the equity portfolio. This test can be rewritten for the difference in the squared Sharpe ratio as follows:

$$R_t^c = \alpha + \beta r_t^x + \varepsilon_t \quad (7)$$

$$\Theta_{xc}^2 - \Theta_x^2 = \alpha' \Sigma_\varepsilon^{-1} \alpha \quad (8)$$

To examine the effect of energy and precious metal on the portfolio, we can also run the regression equation (7) for both of the sub - indexes. Out of the regression we can obtain the intercept coefficients. So we can determine the $\alpha$ in equation (8), which stands for a 2*1 vector containing the intercept coefficients. Out of both of these regressions we can also determine the residuals (ε), and calculate their variances and accompanying co-variance. Such that we can construct a 2*2 variance-covariance matrix $\Sigma_\varepsilon$. We now need to determine a null hypothesis as, that adding any commodities will not result in increasing the Sharpe ratio, such that investors do not benefit from including the GSCI or the sub - indexes to a portfolio consistent out of stocks and bonds. If this is true it's obvious that no mean-variance investors will want to add the GSCI or the sub - indexes since they will not add any value to the portfolio. To test these hypotheses we use the Wald Test Statistics:

$$Wstandard = T * \left(\frac{\theta_{xc}^2 - \theta_x^2}{1+\theta_x^2}\right) \sim X^2 N \quad (9)$$

Where according to (Gibbons et al. 1989) T stands for the number of time series observations on returns, and $X^2 N$ stands for a Chi-square distribution with N degrees of freedom.

## 2.2 Calculations

We start off with turning the returns in to excess returns, and computing there standard deviations. So that we can determine and compare the Sharpe ratio's of the single assets. Next we are looking for the maximized Sharpe ratio of a combined portfolio. We will set up a table in excel containing the weights of the bonds, stocks and commodities, but we will also add the portfolio expected return denoted as $\mu_p = w' * \mu$ . The standard deviation as $\sigma_p = \sqrt{(w' * \sum p * w)}$ and the Sharpe ratio as $\theta_p = \frac{\mu_p}{\sigma_p}$ .

If this is in order we can use the Excel Solver function to maximize the Sharpe ratio by changing the portfolio weights. We will use the solver function twice, once with and once without a short sale constraint.  Where the short sale constraint is constructed such that all the weights of the S&P 500, Barclays Us aggregate and S&P GSCI are ≥0. We also take notice of the sum of the total weights for the portfolio. Note that in our research these weights do not have to add up to 1 because we are using Excess returns. But we will add a constraint such that they do add up to 1, but this will not change the outcome because of linearity .We will also maximize the Sharpe ratio for a portfolio consistent out of only stocks and bonds. Next we will perform a similar analysis but this time for a portfolio consistent out of stocks, bonds and the Sub - indexes. After we have determined the optimal portfolio's, we will use their Sharpe ratio's and those of the single assets to make a graph. The Sharpe ratio will be the slope of the lines.

Next we will determine whether the GSCI and or the sub - indexes Energy & Precious Metal add significant value to a portfolio consistent out of stocks and bonds, as said before we will use the Wald Test Statistic. But first we need to determine the maximum squared Sharpe ratio for the portfolio with stocks, bonds and GSCI by using equation (6), and the squared Sharpe ratio for the portfolio with only stocks and bonds with equation (4). For the combined portfolio with the sub indexes we will use two different calculations. The first will be to calculate the needed squared Sharpe ratio's by using equation (4) and (6). For the second option we will run two regressions. Where for the first one the y variable will be the monthly excess returns of Precious Metal and the accompanying x variable will be the monthly excess returns of the S&P500 and Barclays US Aggregate. For the second regression only the y variable will change from the sub index Precious Metal to Energy. As mentioned earlier we can now determine the $\alpha$ vector and the $\Sigma_{\varepsilon}$ matrix. So we can use equation (8) to get the desired result. Now we have all the missing variables for equation (9), so that we can perform a Wald T test twice. Once for the portfolio with the GSCI, stocks and bonds with N = 1, the other for a portfolio with stocks, bonds and the sub - indexes where N = 2. Note that we will use similar calculations for the sub -sample.

# 3. The Dataset

The empirical analysis in this paper will be based on a portfolio consistent out of Commodities, US Bonds and US Stocks. We compiled our database using DataStream to obtain the monthly total return indexes for these single assets. We have chosen the S&P GSCI index for the commodities in our portfolio, its former known as the Goldman Sachs Commodity Index. This index consists out of a large range of commodities with active future contracts. According to the (S&P GSCI commodity indices) there are 24 different commodities of such kind, and they're divided in to five sectors: Energy, Industrial Metals , Precious Metals, Agriculture and Livestock. According to Erb & Harvey (2006) and the S&P GSCI commodity indices the largest sector in the GSCI index is energy. These sectors are also called sub-indexes. As mentioned before we will also determine the effect of adding sub-indexes. So our dataset also contains the sub - indexes Energy and Precious metal.

For the US stocks in our portfolio we will take the S&P 500, which is maintained by a team of economists and analysts in service of Standards & Poors. The S&P 500 equity indices shows that the portfolio exist out of 500 major companies in the leading industries in the U.S. The final part of our portfolio consists out of US Bonds. For this position we chose the Barclays Capital Aggregate Bond Index. It's former known as the Lehman Aggregate Bond Index but now owned by Barclays Capital. To gather this data we used the bonds total return since inception.

For all of our assets we have downloaded the monthly data from January 1992 till December 2011 in US $. Since we have the intention to use excess returns in our research, we need a risk free rate. Therefore we downloaded the monthly risk free rate from the Kenneth R. French data library (Kenneth R. French website). So that we could subtract the risk free rate and obtain our excess returns. We will now show the descriptive statistics of our database in Table 3.1.

| Descriptive Statistics | Average excess return | Standard Deviation | Sharpe Ratio |
|---|---|---|---|
| S&P 500 | 0,47% | 4,55% | 0,10 |
| BARCLAYS US AGGREGATE | 0,34% | 1,23% | 0,28 |
| S&P GSCI Commodity | 0,26% | 6,36% | 0,04 |
| S&P GSCI Precious Metal | 0,54% | 4,80% | 0,11 |
| S&P GSCI Energy | 0,52% | 9,03% | 0,06 |

*Table 3.1 Reports the descriptive statistics for the various asset classes used in this study. The dataset is compiled from January 1992 till December 2012. It's a report of the monthly mean excess returns and its accompanying standard deviation and Sharpe ratio.*

We can see that the monthly excess return of the S&P GSCI is significantly lower than the return on the stocks and bonds. At the same time the standard deviation of the commodities is way higher than those two, which obviously leads to a Sharpe ratio that is way beneath the ratios of the S&P 500 and the Barclays US Aggregate. Our results match with Daskalaki & Skiadopoulos (2011) and Jensens et al. (2000) who both showed commodities performed less than the other asset classes. Another interesting fact to notice is the remarkably low standard deviation of the US Bonds, which results in the highest Sharpe Ratio for the single assets. That indicates that US Bonds will probably have a big part in the combined portfolio. The S&P 500 does have a higher return than the bonds but it has to deal with a higher standard deviation. According to Daskalaki & Skiadopoulos (2011) all of the individual commodities where outperformed by stocks except for crude-oil and gold. This is interesting since we have chosen for the sub - indexes Precious Metal and Energy who respectively exist a lot out of gold and crude-oil. Taken this in account we can see the consequences in our study, because of the slightly higher Sharpe ratio for the sub - index Precious metal versus the Stocks. But also the higher Sharpe ratio for the sub - index Energy compared to the GSCI Commodity index. We will now take a look at the correlations between the asset classes, given in table 3.2.

| Correlation | S&P 500 | BARCLAYS US AGGREGATE | S&P GSCI Commodity | S&P GSCI Precious Metal | S&P GSCI Energy |
|---|---|---|---|---|---|
| S&P 500 | 1,000 | | | | |
| BARCLAYS US AGGREGATE | 0,055 | 1,000 | | | |
| S&P GSCI Commodity | 0,318 | 0,003 | 1,000 | | |
| S&P GSCI Precious Metal | 0,065 | 0,141 | 0,273 | 1,000 | |
| S&P GSCI Energy | 0,256 | 0,015 | 0,967 | 0,189 | 1,000 |

*Table 3.2 reports the correlation coefficient between the various assets over January 1992 till December 2012.*

Out of table 3.2 we can see that the correlation between the GSCI and the bonds is remarkably low. This already indicates that there might be some diversification possibilities. We do find a fairly higher correlation between the commodities and stocks than Daskalaki & Skiadopoulos, (2011) did. This is also in line with the predictions of Bicchetti & Maystre (2012), who suspected a reasonable increase in correlation between commodities and US Stocks. Although we find a way lower correlation for the sub - index Precious metal and the S&P 500. This collaborates with Erb & Harvey (2006) who concluded that the individual commodities could behave fairly different from the total GSCI commodity index. So there might be a diversification opportunity for sub - indexes as well.

## 3.1 Sub - Sample

According to Hong & Yogo (2011) the institutional investments in commodities increased enormously since 2003. Therefore we chose the following sub - sample January 2003 till December 2011. Like before we will first show the descriptive statistics for the sub - sample.

| | Average excess return | Standard Deviation | Sharpe Ratio |
|---|---|---|---|
| S&P 500 | 0,48% | 5,02% | 0,09 |
| BARCLAYS US AGGREGATE | 0,32% | 1,18% | 0,27 |
| S&P GSCI Commodity | 0,41% | 7,75% | 0,05 |
| S&P GSCI Precious Metal | 1,38% | 5,95% | 0,23 |
| S&P GSCI Energy | 0,51% | 9,73% | 0,05 |

*Table 3.1.1 Reports the descriptive statistics for the various asset classes used in this study. The dataset is compiled from January 2003 till December 2012. It's a report of the monthly mean excess returns and its accompanying standard deviation and Sharpe ratio.*

Table 3.1.1 gives quite similar results to table 3.1, almost all the Sharpe ratios of the various asset classes show not much deviation in the sub - sample. Except for the sub - index precious metal, it's Sharpe ratio has increased enormously. Which is logic since the average excess return of precious metal is way higher in the sub - sample and the standard deviation didn't grew accordingly. This is enormous increase is most certainly because of the extreme years in the commodity boom, namely 2005-2008 described by Hong & Yogo (2011) and Daskalaki & Skiadopoulos (2011). But before we conclude that there are in fact new diversification opportunities we will take a look at the correlation matrix for the sub - sample in table 3.2.2.

| | S&P 500 | BARCLAYS US AGGREGATE | S&P GSCI Commodity | S&P GSCI Precious Metal | S&P GSCI Energy |
|---|---|---|---|---|---|
| S&P 500 | 1,000 | | | | |
| BARCLAYS US AGGREGATE | 0,051 | 1,000 | | | |
| S&P GSCI Commodity | 0,461 | -0,053 | 1,000 | | |
| S&P GSCI Precious Metal | 0,150 | 0,231 | 0,345 | 1,000 | |
| S&P GSCI Energy | 0,406 | -0,075 | 0,984 | 0,270 | 1,000 |

*Table 3.1.2 reports the correlation coefficient between the various assets over January 2003 till December 2012.*

Our findings in table 3.1.2 collaborate with Tang & Xiong (2009). We can most definitely see an increase in correlation between the S&P 500 and the GSCI and its sub - indexes. The rest of the correlations between the assets don’t seem to deviate much in the sub - sample. This is remarkable because of the enormous increase in Sharpe ratio for the sub - index precious metal. Therefore we expect for the sub - sample that Precious metal has a bigger part in the combined portfolio.

# 4. Test results

As discussed in section 2.2 we will start off with determining the optimal portfolio containing stocks, bonds and commodities with the solver function for the time span 1992 till 2011. The Variance - Covariance matrix used is found in Appendix A1. We show the results in table 4.1:

| **Portfolio optimization** | **Equal weights** | **Max Sharpe Ratio** | **Max Sharpe Ratio with short sale constraint** | **Max sharpe Ratio,without GSCI.** |
|---|---|---|---|---|
| S&P 500 weight | 0,33 | 0,08 | 0,08 | 0,08 |
| Barclays US Aggregate weight | 0,33 | 0,91 | 0,91 | 0,92 |
| S&P GSCI Commodity weight | 0,33 | 0,01 | 0,01 | 0,00 |
| Sum Weights | 1,00 | 1,00 | 1,00 | 1,00 |
| μp | 0,358% | 0,351% | 0,351% | 0,352% |
| σp | 3,012% | 1,205% | 1,205% | 1,210% |
| θ = μp/σp | **0,1190** | **0,2914** | **0,2914** | **0,2912** |

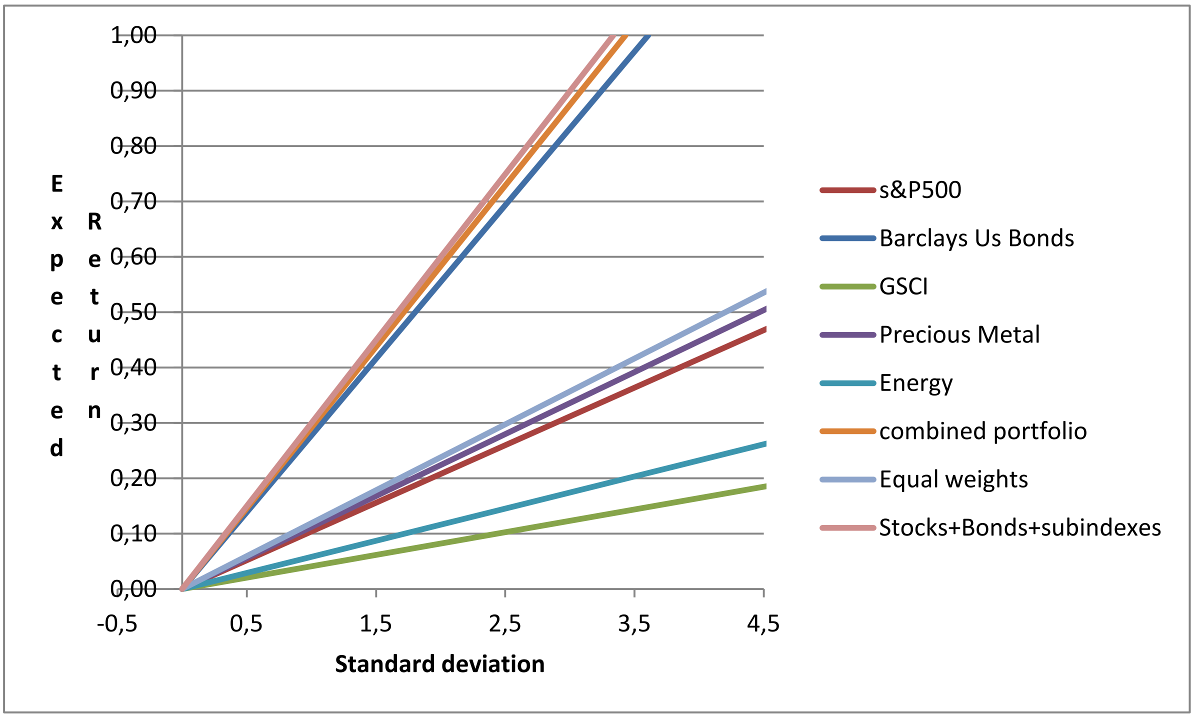
*table 4.1 Shows the weights in percentages of the various assets for each of the given portfolio optimizations in the columns. Also reports the accordingly portfolio return* $\mu_p$*, the portfolio standard deviation* $\sigma_p$ *and the portfolio's Sharpe ratio* $\theta_p$*for the given weights. It covers the data from January 1992 till December 2011.*

The optimal weights are given when we are maximizing the Sharpe ratio. As you can see the maximal Sharpe ratio differs a lot from the equal weighted portfolio, however not that much from the single asset Barclays US Bonds. Therefore it doesn't come as a surprise that in the optimal weights the US Bonds have a dominant share. As far as the commodities you can see that they do have a piece in the pie but is fairly small. This is also why the last column, the maximized Sharpe ratio of a portfolio without commodities almost has the same Sharpe ratio. We do see a slight use of diversification by the optimal portfolio, since it has a slightly lower standard deviation than that of the lowest single assets without having a lower return. We will now show in table 4.2 the maximized Sharpe ratios of a portfolio consistent out stocks, bonds and the sub - indexes Precious Metal and Energy.

| Portfolio optimization | **Max Sharpe Ratio** | **Max Sharpe Ratio without sub-indexes** |
|---|---|---|
| S&P 500 weight | 0,07 | 0,08 |
| Barclays US Aggregate weight | 0,86 | 0,92 |
| Precious metal | 0,06 | 0,00 |
| Energy | 0,01 | 0,00 |
| Sum Weights | 1,00 | 1,00 |
| µp | 0,36% | 0,35% |
| σp | 1,21% | 1,21% |
| θ = µp/σp | **0,300** | **0,291** |

*table 4.2 Shows the weights in percentages of the various assets for each of the given portfolio optimizations in the columns. Also reports the portfolio return* $\mu_p$*, the portfolio standard deviation* $\sigma_p$ *and the portfolio's Sharpe ratio* $\theta_p$*for the given weights. It covers the data from January 1992 till December 2011.*

In table 4.2 we determined the optimal weights again; the variance covariance matrix used is to be found in Appendix A.1.2. The optimal portfolio shows a 6% weight for the sub - index Precious metal which is significantly higher than the 1% of the GSCI in table 4.1. We also see a slightly higher Sharpe ratio for the portfolio with the sub - indexes. Now that we have compiled the Sharpe ratio of the optimal portfolio's we can present them graphically in graph 4.1.




*Graph 4.1 a graphical visualization of the Sharpe ratios, where the slope of the line is determined by its Sharpe ratio. It covers the data 1992 till 2011.*

We now consider Graph 4.1 which presents the trade-off between expected excess returns and their accompanying standard deviation for the various single assets and portfolio's.
The slope of the line defines the Sharpe ratio. Note that the lines start in the point of origin since we use excess returns. It also visualizes the small improvement for the optimal portfolio's which include the GSCI or the Sub - Indexes versus the single asset US bonds. As we can see in the graph the portfolio with stocks, bonds and the sub - indexes maintains the highest Sharpe ratio of them all. As we have discussed earlier it is also interesting to see how both energy and precious metal seem to have a much better trade-off than the GSCI. We will now determine if adding the GSCI or its sub - indexes to a portfolio with stocks and bonds will significantly improve the Sharpe ratio.

## 4.1 Test Statistics

### 4.1.1 GSCI

As mentioned in 2.2 we will first derive the squared Sharpe ratio of a portfolio consistent out of stocks, bonds and the GSCI by using equation (6). The variance covariance matrix used is shown in appendix A.1.

This gives us $\Theta_{xc}^2$= 0,08494, as the squared Sharpe ratio of the portfolio consistent out of stocks, bonds and the GSCI. In order to determine if the GSCI significantly improves the Sharpe ratio, we will also calculate the squared Sharpe ratio of the portfolio consistent out of stocks and bonds with equation (4). The variance covariance matrix used is shown in appendix A.2

This gives us $\Theta_{x}^2$= 0,08478, as the squared Sharpe ratio of the portfolio consistent out of stocks and bonds. The last variable missing for our test statistic is the T, earlier described as number of time series observations on returns. We can easily derive that out of our dataset namely: 240 (12months * 20 years).We can now perform a Wald test to determine whether the GSCI significantly improves the Sharpe ratio. With H0: $\Theta_{xc}^2 \leq \Theta_{x}^2$, H1: $\Theta_{xc}^2 > \Theta_{x}^2$ and H0 get's rejected if the in equation (9) obtained W standard is $\geq X^2N$.

As calculated W standard = 0,0349
The $X^2N$ at a level 0,05 significance and in this case 1 degree of freedom.

$X^2 0{,}05(1)$ =3,8414

So W standard < $X^2 0{,}05(1)$ , which means H0 doesn't get rejected. So we can assume for a 0,05 significance level that the GSCI doesn't improve the Sharpe ratio of the portfolio of stocks and bonds. The P-value for this test is , p-value= 0,85. What simply tells us that for a confidence level of < 0,85 H0 doesn't get rejected.

### 4.1.2 Sub-Indexes

Next we will determine whether the sub-indexes can significantly improve the Sharpe ratio of a portfolio consistent out of stocks and bonds. First we will start with calculating the squared Sharpe ratio of the portfolio consistent out of stocks, bonds and the sub indexes with equation (6). The variance covariance matrixes used are in appendix A.1.2 .

This gives us $\Theta_{xc}^2 = 0{,}0904$ , as the squared Sharpe ratio of the portfolio with stocks, bonds and the sub - indexes. The squared Sharpe ration of the portfolio of stocks and bonds is the same as in section 4.1.1 namely $\Theta_x^2$= 0,0847.

Now we can determine the outcome of equation (8):

$\Theta_{xc}^2 - \Theta_x^2$ =0,005

As mentioned in section 2.2, we will determine the outcome of equation (8) in 2 separate ways. The second one will be by performing two regressions. The regression output is stored in appendix B.1 and B.2. Out of these regressions we get the intercept coefficients for the 2*1 α vector:

$$\alpha = [0{,}0033\ 0{,}0029]^T$$

Together with the $\Sigma_\varepsilon$ calculated by residuals of the regression, which you can find in appendix B.3. We can now calculate the result of equation (8) for the second time.

$\alpha'^{\Sigma_\varepsilon^{-1}}\alpha$ =0,005. As you can see both results should and in fact do match. Next we will perform another Wald T test. With again H0: $\Theta_{xc}^2 \leq \Theta_x^2$, H1: $\Theta_{xc}^2 > \Theta_x^2$ and H0 get's rejected if the in equation (9) obtained W standard is ≥ $X^2 N$

As calculated W standard = 1,250

The $X^2 N$ at a level 0,05 significance and in this case 2 degrees of freedom.

$X^2 0{,}05(2)$= 5,992

So W standard < $X^2 0{,}05(2)$ , again this means that H0 doesn't get rejected. So we can assume that for a 0,05 significance level the sub-indexes don't improve the Sharpe ratio of the portfolio of stocks and bonds. The P-value for this test is , p-value= 0,54. Again this tells us that for a confidence level of < 0,54 H0 doesn't get rejected.

## 4.2 Sub Sample

For the sub sample we will choose a quite similar path, we will start with the portfolio optimization using the solver function but this time of the time span of 2003 till 2011. As shown in table 4.2.1

| Portfolio optimization | **Max Sharpe Ratio** | **Max sharpe Ratio, without commodities** |
|---|---|---|
| S&P 500 weight | 0,05 | 0,07 |
| Barclays US Aggregate weight | 0,93 | 0,93 |
| S&P GSCI Commodity weight | 0,02 | 0,00 |
| Sum Weights | 1 | 1 |
| Mp | 0,329% | 0,330% |
| Σp | 1,153% | 1,163% |
| θ = μp/σp | **0,286** | **0,284** |

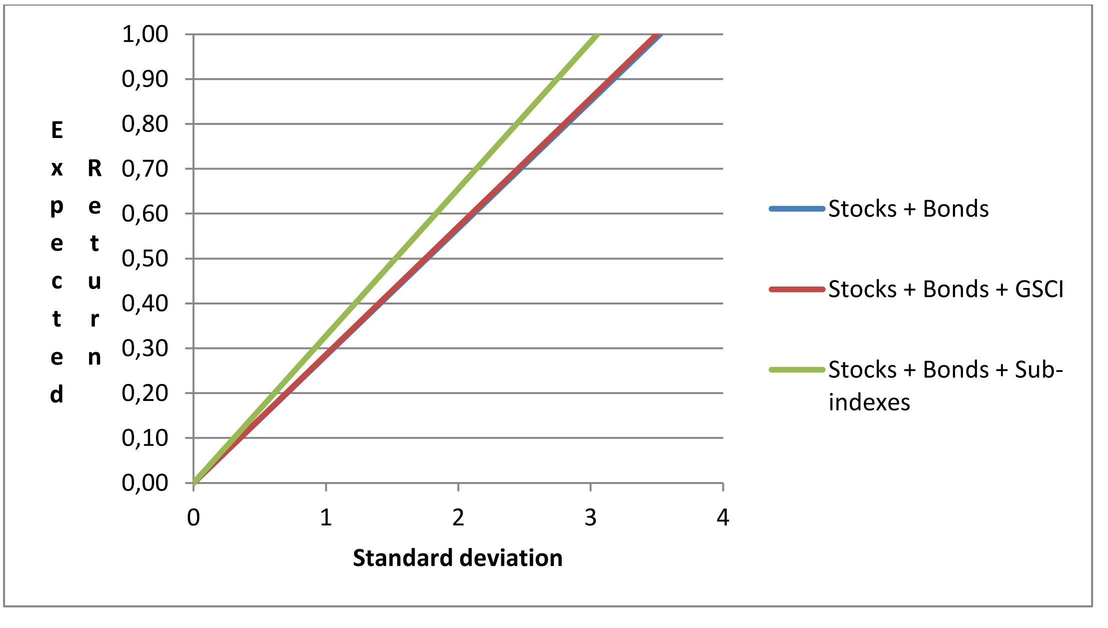
*table 4.2.1 Shows the weights in percentages of the various assets for each of the given portfolio optimizations in the columns. Also reports the accordingly portfolio return $\mu_p$, the portfolio standard deviation $\sigma_p$ and the portfolio's Sharpe ratio $\theta_p$ for the given weights. It covers the data from January 2003 till December 2011.*

The findings are in line with our descriptive statistics, as expected there isn't much difference between table 4.2.1 and table 4.1. But as we mentioned before there was an enormous increase in the Sharpe ratio for the sub - index precious metal. Therefore we will now take a look at table 4.2.2, where we show the optimal weights for a portfolio of stocks, bonds and the sub indexes.

| Portfolio optimization | **Max Sharpe Ratio** | **Max Sharpe Ratio without sub-indexes** |
|---|---|---|
| S&P 500 weight | 0,05 | 0,07 |
| Barclays US Aggregate weight | 0,83 | 0,93 |
| Precious metal | 0,12 | 0,00 |
| Energy | 0,00 | 0,00 |
| Sum Weights | 1,00 | 1,00 |
| Mp | 0,457% | 0,330% |
| Σp | 1,392% | 1,163% |
| θ = μp/σp | **0,328** | **0,284** |

*table 4.2.2 Shows the weights in percentages of the various assets for each of the given portfolio optimizations in the columns. Also reports the accordingly portfolio return $\mu_p$, the portfolio standard deviation $\sigma_p$ and the portfolio's Sharpe ratio $\theta_p$for the given weights. It covers the data from January 2003 till December 2011.*

Again our expectations are matched, since precious metal has a significant bigger part in the optimal combined portfolio. We can also determine that the sharp ratio of the portfolio including precious metal has a higher Sharpe ratio than the portfolio only consisting out of stocks and bonds. Once again we will show our results graphically in graph 4.2.




*Graph 4.2 a graphical visualization of the Sharpe ratios, where the slope of the line is determined by its Sharpe ratio. It covers the data 2003 till 2011.*

As we have discussed earlier with graph 4.1, the same accounts for graph 4.2. It's a graphical visualization of table 4.2.1 and 4.2.2. With the Sharpe ratio of the portfolio's as the slope of the line. The remarkable and interesting thing in graph 4.2 and table 4.2.2 is the Sharpe ratio of the portfolio consistent out of stocks, bonds and the sub - indexes. They show the largest increase in Sharpe ratio, therefore we will now determine if adding the sub-indexes give a significant improvement to the portfolio for our sub - sample.

### 4.2.1 Test Statistics sub - sample

To determine whether the sub - indexes improve the portfolio of stocks and bonds significantly in the sub - sample. We perform another Wald T-test, but first we will need to calculate the squared sharp ratio of the portfolio consistent out of stocks, bonds and the sub - indexes. We will use equation (6) again, the variance covariance matrix used is found in appendix A3.

This gives us $\theta_{xc}^2 =$ 0,1076, as the squared Sharpe ratio of the portfolio with stocks, bonds and the sub - indexes. In order to determine if the sub - indexes improve the Sharpe ratio; we will also calculate the squared Sharpe ratio of the portfolio consistent out of stocks and bonds with equation (4). The variance covariance matrix used is shown in appendix A.3.1

This gives us $\theta_x^2$= 0,0805, as the squared Sharpe ratio of the portfolio consistent out of stocks and bonds. This time we are testing over a smaller sample such that T = 108. We can run a Wald test one last time. With again H0: $\theta_{xc}^2 \leq \theta_x^2$, H1: $\theta_{xc}^2 > \theta_x^2$ and H0 get's rejected if the in equation (9) obtained W standard is ≥ $X^2N$.
As calculated W standard = 2,701

The $X^2N$ at a level 0,05 significance and in this case 2 degrees of freedom.
$X^2 0{,}05(2)$= 5,992

So W standard < $X^2 0{,}05(2)$ , we conclude that H0 doesn't get rejected. So that we can assume for a 0,05 significance level that the sub-indexes won't improve the Sharpe ratio of the portfolio of stocks and bonds. The P-value for this test is , p-value= 0,26. Therefore we conclude that for a confidence level of < 0,26 H0 doesn't get rejected.

# 5. Conclusion

This paper investigates whether an investor can benefit from adding commodities to a portfolio of stocks, bonds and bills. We therefore introduced mean variance theory, and used it to analyze our data. Previous literature showed that investors could indeed benefit from adding commodities. But our results show no significant improvement of the Sharpe ratio when commodities are added, which does collaborate with the paper of Daskalaki & Skiadopoulos (2011). Therefore we conclude out of our sample that mean variance investors will not benefit from adding commodities to their portfolios of stock, bonds and bills.

We did find some evidence of a small increase in the Sharpe ratio for the sub - sample, when precious metal was added to a portfolio of stocks and bonds. But we still don't think this is a reliable estimate for the future since the commodity boom of 2003 - 2008 has a too big of an influence on the sub - sample.

We do also have a critical note on our conclusion, the fact that the correlations between the stocks and commodities do significantly change over time, as we have showed with our sub sample, and also presented by Hong & Yogo(2011). This can surely influence the diversification possibilities of commodities in portfolio's consistent out of stocks and bonds. Therefore we recommend some future research in the correlations between the various assets, as well as the future Sharpe ratios of commodity portfolios.

# Appendix A

## A.1

| Variance - Covariance | S&P 500 | BARCLAYS US AGGREGATE | S&P GSCI Commodity |
|---|---|---|---|
| S&P 500 | 0,002068626 | 3,0967E-05 | 0,000918825 |
| BARCLAYS US AGGREGATE | 3,0967E-05 | 0,000151867 | 2,62053E-06 |
| S&P GSCI Commodity | 0,000918825 | 2,62053E-06 | 0,004041861 |

## A.1.2

| Variance - Covariance | S&P 500 | BARCLAYS US AGGREGATE | S&P GSCI Precious Metal | S&P GSCI Energy |
|---|---|---|---|---|
| S&P 500 | 0,002068626 | 3,0967E-05 | 0,000142901 | 0,001048881 |
| BARCLAYS US AGGREGATE | 3,0967E-05 | 0,000151867 | 8,35872E-05 | 1,67669E-05 |
| S&P GSCI Precious Metal | 0,000142901 | 8,35872E-05 | 0,002307152 | 0,000820573 |
| S&P GSCI Energy | 0,001048881 | 1,67669E-05 | 0,000820573 | 0,008146095 |

## A.2

| VCV | S&P 500 | BARCLAYS US AGGREGATE |
|---|---|---|
| S&P 500 | 0,002068626 | 3,0967E-05 |
| BARCLAYS US AGGREGATE | 3,0967E-05 | 0,000151867 |

## A.3

| VCV | S&P 500 | BARCLAYS US AGGREGATE | S&P GSCI Precious Metal | S&P GSCI Energy |
|---|---|---|---|---|
| S&P 500 | 0,00252 | 3,02426E-05 | 0,000447815 | 0,001981367 |
| BARCLAYS US AGGREGATE | 3E-05 | 0,000138093 | 0,000161579 | -8,62181E-05 |
| S&P GSCI Precious Metal | 0,00045 | 0,000161579 | 0,003535768 | 0,0015634 |
| S&P GSCI Energy | 0,00198 | -8,62181E-05 | 0,0015634 | 0,009469693 |

## A3.1

| VCV | S&P 500 | BARCLAYS US AGGREGATE | S&P GSCI Commodity |
|---|---|---|---|
| S&P 500 | 0,00252 | 3,02426E-05 | 0,001790707 |
| BARCLAYS US AGGREGATE | 3E-05 | 0,000138093 | -4,81564E-05 |
| S&P GSCI Commodity | 0,00179 | -4,81564E-05 | 0,006003628 |

# Appendix B

## B.1

Regression Precious Metal

| *Regression Statistics* | |
|---|---|
| Multiple R | 0,14926014 |
| R Square | 0,02227859 |
| Adjusted R Square | 0,01402778 |
| Standard Error | 0,04769473 |
| Observations | 240 |

| | *Coefficients* | *Standard Error* | *t Stat* | *P-value* |
|---|---|---|---|---|
| Intercept | 0,00329958 | 0,00320676 | 1,02895 | 0,30455 |
| X Variable 1 | 0,06006597 | 0,067951691 | 0,88395 | 0,37762 |
| X Variable 2 | 0,52484917 | 0,250789483 | 2,09279 | 0,03743 |

## B.2

Regression Energy

| *Regression Statistics* | |
|---|---|
| Multiple R | 0,25512 |
| R Square | 0,06509 |
| Adjusted R Square | 0,0572 |
| Standard Error | 0,08764 |
| Observations | 240 |

| | *Coefficients* | *Standard Error* | *t Stat* | *P-value* |
|---|---|---|---|---|
| Intercept | 0,00288 | 0,00589 | 0,48866 | 0,62553 |
| X Variable 1 | 0,50639 | 0,12486 | 4,05577 | 6,8E-05 |
| X Variable 2 | -0,0083 | 0,46081 | -0,018 | 0,98562 |

## B.3

| Variance covariance | Precious Metal | Energy |
|---|---|---|
| Precious Metal | 0,002255751 | 0,000747 |
| Energy | 0,000746637 | 0,007616 |

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S&P GSCI commodity indices
http://www.standardandpoors.com/servlet/BlobServer?blobheadername3=MDT-Type&blobcol=urldata&blobtable=MungoBlobs&blobheadervalue2=inline%3B+filename%3DFS_SP_GSCI_LTR.pdf&blobheadername2=Content-Disposition&blobheadervalue1=application%2Fpdf&blobkey=id&blobheadername1=content-type&blobwhere=1244056432335&blobheadervalue3=UTF-8

S&P 500 equity indices
(http://www.standardandpoors.com/servlet/BlobServer?blobheadername3=MDT-Type&blobcol=urldata&blobtable=MungoBlobs&blobheadervalue2=inline%3B+filename%3Dfs-sp-500-ltr.pdf&blobheadername2=Content-Disposition&blobheadervalue1=application%2Fpdf&blobkey=id&blobheadername1=content-type&blobwhere=1244088633364&blobheadervalue3=UTF-8)

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