Bachelor Thesis Finance

Is the 1/n asset allocation strategy undervalued?

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Abstract

Nowadays, the 1/n strategy is often placed in the list of naïve strategies by scholars. This model ignores the historical data and therefore does not come to the optimal portfolio by means of the modern portfolio theory. In this paper an examination is done to test whether the model really is naïve, or undervalued. Further different datasets are used to show a possible difference in results between the periods before the financial crisis and periods in which the crises is included. Results showed that in the each of the four datasets considered not one model outperforms the 1/n portfolio strategy. As a consequence the equal asset allocation strategy tends to be undervalued.

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1. Introduction and Literature Survey

1.1 Introduction

Since the invention of money a difficult, but important, decision for many people is how to invest their capital. This decision becomes even more difficult and important by a new worldwide trend. This new trend implies the upcoming saving plans, in which all the decisions have to be made by the consumers themselves (Employee Benefit Research Institute, 1997). Therefore a good portfolio strategy is essential.

In circa 400 A.D. Jewish Rabbi Issac Bar Aha recommended always to invest a third into land, a third into merchandise and to keep a third at hand[i]. This method later became well-known under the name “1/n asset allocation strategy”, “equal asset allocation strategy” or “naïve strategy” and is further defined by DeMiguel et al.(2009) as ”the one in which a segment 1/n of wealth is allocated to each of N assets available for investment at each rebalancing data.” The strategy requires investing an equal part of the capital in the different present assets. Nowadays this rule is often labelled as naïve and too simple, by McClatchy and VandenHul (2005) for example.

In the 20th century there have been invented more strategies for optimal investment. This was started by Markowitz (1952). He found a rule which optimizes the allocation of wealth across certain risky assets in a static setting, in a situation where investors only care about the mean and variance of the portfolio’s return. In 1958, Tobin proved that an optimal asset allocation portfolio would exist of two funds if one could hold a risk-free asset on top of the risky assets. Sharpe (1964) and Lintner (1965) showed that in the equilibrium the portfolio of risky assets would be the market portfolio. After that Samuelson (1969) and Merton (1969) explained that these proved rules for portfolios would be optimal, even in a setting which is multiperiod and when the set of investment opportunities is constant. In 1971, Merton optimized the portfolio conducts in a stochastic investment opportunity set.

For implementing the policies mentioned above, one needs to estimate the parameters of the methods. With those parameters the optimal portfolio weights can be estimated. The Bayesian approach is of great importance in the literature about the estimation error. The approach considers different implementations. One of the implementations is “shrinkage estimators”

(Jobson, Korkie, and Ratti, 1979; Jobson and Korkie, 1980; Jorion, 1985, 1986), which is used in this paper, under the name Bayes-Stein portfolios. Another approach requires constraints upon shortsales (Frost and Savarino (1988) and Chopra (1993)). Shortsales are in practice often difficult to implement and can be very costly. That is the reason for prohibiting short selling in this paper.

This Paper examines the possible outperforming of the 1/n asset allocation strategy by the other strategies considered. As mentioned earlier the 1/n asset allocation strategy often is placed into the list of naïve methods. That is why one would expect that other strategies will outperform the 1/n rule. The results of the models will be statistically tested for significance difference. Existing literature about the 1/n dates from the pre-crisis period and that is way it is very interesting to look whether the financial crisis has a major impact on the outcomes or not. Several investment periods will be considered to answer this question.

The search is organized in the following manner. First, a survey of existing literature will be present. After this, section 2 will describe each asset-allocation model considered. Section 3, called " empirical study", will explain the search and list up the results, followed by a robustness test. In section 4, the conclusion whether the 1/n asset-allocation is underestimated or not will be made. Finally limitations and recommendations for further search will be given.

1.2 Literature survey

Earlier research pointed out the use of naïve decision making, like the 1/n asset allocation method, for retirement investing by employees. (McClatchy and VandenHul (2005), Benartzi and Thaler (2001) and Liang and Weisbenner (2002)). As mentioned before there is a trend which implies more responsibility of individuals in making the decisions for their investments. There are a lot of positive aspects of this upcoming trend, like investing to personal risk aversion and circumstances. On the other hand many have expressed concerns about the quality of the decisions participants make. (Mitchell and Zeldes, 1996).

In 1995 John Hancock Financial Services showed that a majority thought that a diversified portfolio was less safe than their own company stock. Later, McClatchy and VandenHul (2005) explained that 90% of the respondents belief that investing in company stock is as least as secure as investing in a diversified fund. In 2001 Benartzi and Thaler determined that one-third of the investment portfolio of large retirement savings is invested in the own company equity stock. This implies that when a company goes bankrupt, employees not only lose their jobs but also their retirement savings. In this case the risk diversification is very little and therefore risk is very high.

Benartzi and Thaler (2001) experimented asset allocation of savings for retirement. They showed that the relative invested wealth in stocks is seriously influenced by the choice of employees for different options. The search was based on a comparison between retirement plans of pilots and teachers. One plan had a positive 18% deviation from the US mean; the other one caused a negative 23% deviation. Besides that Benartzi and Thaler find that the 1/n method is the most used one in nearly all combinations of present assets. This popularity of the 1/n rule is not influenced by the funds offered, while the final allocation of assets is. Simonson (1990), Read and Loewenstein (1995) and Benartzi and Thaler (2001) explained this popularity of the 1/n asset-allocation strategy by behavioural aspects. They all showed that individuals diversify much more when they face multiple choices at the same moment than when they can be made at several moments. In some situations this is very plausible. For instance eating dinner, one does not usually pick three courses of the same food. In 1984 Kahneman & Tversky explained that even when the returns are lower; individuals diversify their wealth, even when he repeated his experiment for 20 times. Individuals do so because they are not good in analysing complex situations, especially when future is very uncertain. Besides that, there is an aversion against losses. Kahneman & Tversky (1979) showed that individuals valued losses higher than an appreciation with the same value.

Benartzi and Thaler (2001) further found that when people have to choose one single asset for the whole asset allocation, one tends to choose for a 100% equity allocation instead of a bond or a combined fund. In this situation there is no diversification opportunity. They also conclude that there is a positive relationship between the relative number of stock and the relative part of investing in equities. Also the percentage of investing in stock increases, when investment horizon increases. After this they argued that the 1/n method is an underdog of the mean variance model. Later, in 2005, McClatchy and VandenHul showed the same. They claim that when the component of company stock in the 1/n strategy is large the mean-variance method will be the dominated strategy, especially when the number of present assets is small. After an experiment with only 6 diversified present assets, they showed that more than 80% of the time the mean-variance strategy outperforms the 1/n asset allocation rule.

In 1980, however, Jobson and Korkie argued that rules of thumb like the 1/n asset allocation rule can outperform the theory of Markowitz at large samples. Only at the large samples because the estimators used at this search, the mean and the variance, do not make inferences at small samples.

Therefore it looks like the size of the sample is of major impact. Michaud (1998) agreed and stated "because of estimation risk, an equally weighted portfolio may often be substantially closer to the true mean variance optimality than an optimized portfolio." Demiguel et al. (2009) determined that the 1/n asset allocation strategy is very efficient and normally outperforms the mean variance approach at the out-of-sample Sharp ratio. Moreover the implementation of the model is not difficult. This is because the strategy does not depend on asset returns or optimization.

Furthermore despite all models that are developed in the past century, investors still use more easy methods like the equal asset allocation. Also, the 1/n asset allocation strategy has a lower turnover than other strategies, which means that the composition of the portfolio does not change very often. The 1/n strategy even only is rebalanced if the present assets come or go. Therefore the model has ower transaction costs. Besides that the strategy has only positive weights in each asset, thus short selling does not exist. Some other strategies can have negative weights and therefore accept short selling. Large short positions can be very costly and can be difficult to implement. Strategies that prohibit short positions have some weights of zero, and then they are less balanced. In reality short selling constraints usually apply. Evstigneev, Hens and Schenk-Hopp (2004) determined in a search on evolutionary stable rules that the 1/n rule performs well. According to Bloomfield, Leftwich and Long (1977) the mean –variance portfolios do not perform better than the 1/n asset allocation strategy. Finally Carlson, Chapman, Kaniel and Yan (2004) explained that simple rules for portfolios, like the 1/n rule, outperform the optimal policies.

According to Windcliff and Boyle (2004) the 1/n asset allocation rule is only optimal when all assets considered are indistinguishable and uncorrelated. This implies that all the assets in the portfolio have the same mean and variance. Moreover all the correlation coefficients in respect to the other assets are zero. Not surprisingly, this is not true in the real world. Chan, Karceski and Lokonishok (1999) showed that in a sample of 500 American stocks, there were correlations with an average of 28%.

2. Theory

In this section, the various models for portfolio-choice will be considered. First the "naïve" strategy will be discussed, followed by the more sophisticated models. By comparing the models, one can make a conclusion if the 1/n asset allocation strategy really is a naïve strategy and thus whether it will be outperformed by the more sophisticated models. The conclusion of this paper can be used for asset allocation in all kinds of shapes and sizes, like pension funds, private investors, professional investors, banks and so on.

The 1/N strategy for asset allocation

Suppose there are *N* risky assets. The 1/N asset allocation rule divides the capital among all the assets *N*, which are present and available for investment. The method has no optimization or estimation and therefore ignores the historical data. This will be the equally-weighted portfolio. The weight of each asset within the portfolio is given by:

$$\boldsymbol{W_j = \frac{1}{N}} \qquad \textbf{(1)}$$

Sample-based mean-variance optimal portfolios

This model, which is invented by Markowitz (1952), is a strategy that optimizes the portfolio by the inputs of expected returns, variances of return and covariances among the returns. The mean-variance model obtains a framework that makes it possible to build an asset allocation portfolio with user-specific restrictions. In other words, each investor is confronted with the determination between risk and return. That describes the relationship between expected returns, variances and correlations (Steinbach, 2001). Delong and Gerrard (2007) referred that the purpose of the model is often used for selection problems by individuals.

The investor considers a portfolio w_t which maximizes (at each time t)

$$\boldsymbol{W_t^{\top}\mu_t - \frac{\Upsilon}{2}W_t^{\top}\sum\nolimits_t W_t} \qquad \textbf{(2)}$$

The μ_t represents the N vector with excess returns (reduced with the risk-free return). $\sum_t$ is the variance-covariance N x N matrix which belongs to it. γ will be the risk aversion the investor is willing to take.

The elements μ_t and $\sum_t$ are determined through the next sample formula, where M is the length of the sample.

$$\widehat{\boldsymbol{U}_t} = \frac{\mathbf{1}}{\boldsymbol{M}} \sum_{s=t-M+1}^{t} \boldsymbol{R_s} \tag{3}$$

$$\widehat{\sum_t} = \frac{\mathbf{1}}{\boldsymbol{M-N-2}} \sum_{s=t-M+1}^{t} (\boldsymbol{R_s} - \widehat{\boldsymbol{\mu}_t})(\boldsymbol{R_s} - \widehat{\boldsymbol{\mu}_t})^{\top} \tag{4}$$

Then the optimal mean variance portfolio is given by (at each time t):

$$\widehat{\boldsymbol{W}}_t^{MV} = \frac{\mathbf{1}}{\boldsymbol{\gamma}} \widehat{\sum_t^{-1}} \widehat{\boldsymbol{\mu}_t} \tag{5}$$

Minimum-variance portfolios

The minimum-variance portfolio does not optimize the asset-allocation, except in the situation where all the expected returns on the assets are assumed to be the same. This strategy is considered because it has had a lot of attention since Chan, Karceski, and Lakonishok (1999) and Jagannathan and Ma (2003) worked with it.

The main point of this strategy is to minimize the variance of the portfolio, so minimizing the risk. Thus one would to minimize the following (at any time t):

$$\boldsymbol{W_t}^{\top} \sum_t \boldsymbol{W_t} \tag{6}$$

With $W_t{}^{\top}\ 1_N = 1$ and the solution of the above equation leads the portfolio:

$$\widehat{\boldsymbol{W}}_t^{MIN} = \frac{\mathbf{1}}{\mathbf{1}_N^{\top} \sum_t^{-1} \mathbf{1}_N} \times \widehat{\sum_t^{-1}}\ \mathbf{1}_N \tag{7}$$

For this strategy, only the estimate of the covariance matrix of asset returns has to be used. The estimates of expected returns will be completely ignored.

Bayes-Stein portfolios

In earlier searches the Bayesian approach has been applied in many ways. In this paper the empirical Bayes-Stein portfolio will be used for dealing with the estimation error. This type of the Bayesian approach use empirical Bayes estimators. The method moves the weights of the portfolio more to the minimum-variance portfolio and shrinks the estimated returns to a more common value. This type is also used by Jobson and Korkie (1980), Jorion (1985, 1986), Frost and Savarino (1986), and Dumas and Jacquillat (1990).

For the implementation in this search the Bayesian interpretation of the shrinkage estimator will be used. The grand mean μ will be the mean of the minimum-variance portfolio, μ^{MIN}. Earlier, Jorion (1986) did this with the following estimators for expected return and covariance matrix

$$\hat{\mu}_t^{BS} = (1 - \widehat{\emptyset})\hat{\mu}_t + \widehat{\emptyset}\hat{\mu}_t^{MIN} \quad \textbf{(8)}$$

$$\widehat{\boldsymbol{\emptyset}_t} = \frac{\boldsymbol{N+2}}{(\boldsymbol{N+2}) + \boldsymbol{M}(\hat{\boldsymbol{\mu}}_t - \boldsymbol{\mu}_t^{min})^{\top} \widehat{\Sigma_t^{-1}} \{\hat{\boldsymbol{\mu}}_t - \boldsymbol{\mu}_t^{min}\}} \quad \textbf{(9)}$$

Where $\hat{\mu}_t^{min} \equiv \hat{\mu}_t^{\top} \widehat{W}_t^{min}$, $0 < \widehat{\emptyset_t} < 1$ and $\widehat{\Sigma_t} = \frac{\boldsymbol{1}}{\boldsymbol{M-N-2}} \sum_{\boldsymbol{s=t-M+1}}^{\boldsymbol{t}} (\boldsymbol{R_s} - \hat{\boldsymbol{\mu}}_t)(\boldsymbol{R_s} - \hat{\boldsymbol{\mu}}_t)^{\top}$

This is the average excess return on the sample minimum variance portfolio.

3. Empirical study

3.1 Methodology

For coming to well reflected results, there have to be comprehensive methods. First data which can show differences in periods, for showing a possible difference before crisis and during crisis have to be considered. Then the data have to be translated in a form which reflects the tradeoff between return and risk, which will be explained more in paragraph "measurements". Finally the outcomes have to be tested on statically significance. This will be clarified in paragraph "Hypotheses".

Datasets

The data used for this search come from the internet site of Kenneth French, under the file name "10 Industry Portfolios. " This Portfolio consists of 10 groups of industries, like the energy industry and the telecom industry. All collected data will be put in several different investment periods of 10 years. Besides that, facts of 10 international stock markets are needed. Benartzi and Thaler (2001) stated that the use of indices provide more probability of diversification. The different markets that are considered are listed below and are obtained from Thompson DataStream.

Name	***City of Trade***
DJC commodities index	New York
30 year treasury bond index	Chicago
Hang seng	Hongkong
Nikkei 225	Tokyo
SMI	Zurich
All Ordinaries	Sydney
Bovespa	Sao Paolo
FTSE100	London
MXX	Mexico
S&P500	New York

To look whether the recent credit crunch has an impact of the results, the two different sets of data are divided in two datasets each, one dataset in the pre-crisis period and one dataset with the crisis in it (Until the year 2010).

Name in this paper	***Sort of Data***	***Period***
Industry Portfolio Before Crisis	*10 International indices*	*Pre-crisis (1998-2007)*
Industry Portfolio Crisis Included	*10 International indices*	*Crisis included (2001 - 2010)*
International Indices Before Crisis	*10 Industry portfolios*	*Pre-crisis(1998-2007)*
International Indices Crisis Included	*10 Industry portfolios*	*Crisis included (2001 - 2010)*

The first 2 months of every $^1/_3$ year in the datasets will be in-sample data, the next 2 months of the $^1/_3$ year will be the out-of-sample ones.

Measurements

The different models considered will be tested at differences with the Sharpe ratio and the Certainty Equivalent (CEQ). The reasons of considering these two ratios come from the fact that both formulas include the mean as well as the volatility (variance) of each portfolio. So not only the excess return is included but also the corresponding risk. The ratios measure the relative performance of the portfolios, instead of the absolute ones like the mean and variance. Besides that the CEQ considers the risk aversion of the investor and thus makes the result more in the interest of the investor. The methods will be discussed individually below. The sample-based approach will be used to calculate the weights of the mean-variance, the minimum-variance and the Bayes-Stein portfolios. This approach uses the in-sample data to determine the (optimal) weights. Then the performances of the determined portfolios will be calculated by combining the weights to the out-of-sample returns. The 1/n strategy ignores parameters of the past and therefore does not make use of the in-sample data.

The Sharpe ratio

The mean, variance and Sharpe ratio are respectively given by:

$$\widehat{\mu^k} = \frac{1}{T-M}\sum_{s=1}^{T-M} R_s^k$$

$$\left(\delta^k\right)^2 = \frac{1}{T-M-1}\sum_{s=1}^{T-M}\left(\hat{R}_s^k - \hat{\mu}^k\right)^2$$

$$\widehat{SR}_{OS}^k = \frac{\hat{\mu}_k}{\hat{\delta}_k}$$

Where all T is the total length of the dataset, M is the length of the sample and $\hat{R}_s^k$is the excess return (on top of the risk free return).

To measure if the difference of the Sharpe ratio of one of the sophisticated strategies to that of the 1/n strategy is significant, the P-value of the Sharpe ratio relative to the 1/n asset allocation rule is calculated. The approach that Jobson and Korkie (1981) used earlier, with correction of Memmel (2003), will be applied here.

Certainty Equivalent (CEQ)

The certainty-equivalent (CEQ) return is earlier defined by DeMiguel et al. (2009) as "the riskfree rate that an investor is willing to accept rather than adopting a particular risky portfolio strategy." It is given by the following equation under strategy *k*:

$$CEQ_{static}^{k} = \hat{\mu}^{k} - \frac{\Upsilon}{2}\left(\hat{\delta}^{k}\right)^{2}$$

With:
- $\hat{\mu}^k$ = Mean of excess return
- $\left(\hat{\delta}^k\right)^2$ = Variance of excess return
- Υ = Risk aversion of investor

To determine if the difference of outcomes is statistical significant, the test values and the p-values of difference will be calculated.

<u>Hypotheses</u>

Because the Sharpe Ratio and the CEQ both have the mean and the variance (volatility) in their formulas, testing these variables give a good indication. First, therefore the variance will be tested on equality by using a F-test.

The F-test will have the following form:

$$H_0: \sigma_0^2 / \sigma_1^2 \leq 1 \quad \text{Vs.} \quad H_1: \sigma_0^2 / \sigma_1^2 > 1$$

Where

σ^2 = the variance of respectively the Sharpe Ratios and the CEQ's of the different models.

σ_0^2 = The highest variance of the two.

σ_1^2 = The lowest variance of the two.

After the F-test is done, but before testing the hypotheses of difference, one has to know the variances of the Sharpe ratios and the CEQs are being considered as equal or unequal.This will be done because the formulas for determination of the test values are different for equal and unequal variance. Therefore it can have an impact on the results. This impact will be tested in paragraph 3.3.

To determine the inequality of variance, the Levene test will be used in this paper.

The Levene test will have the following form:

$$H_0: \sigma_0^2 / \sigma_1^2 = 1 \quad \text{Vs.} \quad H_1: \sigma_0^2 / \sigma_1^2 \neq 1$$

Where

σ^2 = the variance of respectively the Sharpe Ratios and the CEQ's of the different models.

σ_0^2 = The highest variance of the two.

σ_1^2 = The lowest variance of the two.

For these tests the test values will be used. The p-values are given also, in the appendix. The difference is statically significant if the p-value is below 0,05.

After qualifying the variances as equal or unequal, the differences in means will be tested, for the means, the Sharpe ratios and CEQ ratios. They will be compared on the difference between the 1/n asset allocation strategy and the sample-based mean-variance optimal portfolio, the Minimum-variance portfolio and the Bayes-Stein portfolios. The T-test will be used to determine if the differences are statically significant, and therefore if sophisticated model will outperform the 1/n strategy.

The T-test will have the following form:

$$H_0: \mu_0 - \mu_1 \leq 0 \quad \text{Vs.} \quad H_1: \mu_0 - \mu_1 > 0$$

Where

μ = The mean of the Sharpe Ratio and the CEQ of the different models.

μ_0 = The mean of the model with the highest average of the two.

μ_1 = The mean of the model with the lowest average of the two.

The p-values will be determined as well. If the values of this indicator are below 0,05, the difference is statically significant.

3.2 Results

Indicators Mean and Variance

To give an indicator for the results later in this section, the mean of returns and the variances are given below in table 1 and 2. The test values, of respectively the t-test on differences and the f-test of equality on variance are listed up between the parentheses. Table 1 points out the decrease of excess return between the datasets before crisis and that with the crisis included. For the indices the excess returns are even negative in the period where the crisis is included, which is normal in times of recession. Besides that table 2 shows that the variance tends to increase between the two datasets in time. To reject the null hypothesis of the means, the test value has to be above 1,734. Since not one of the values meets the criteria, the null hypothesis is not rejected. Thus, there is no prove that one of the models outperforms the 1/n asset allocation strategy or vice versa. In appendix table A1 the p-values of these tests are given. These p-values are all above the level of 0,05 and therefore not one of the differences is significant. The test values in table 2 are those of the F-test. The critical value of these test is 3,179. The null hypothesis will be rejected if a value is below this number. There is no test value above this number, and therefore the null hypothesis will not be rejected. The mean variance model and the Bayes-Stein model in the first dataset have the highest values, but they do not come to the value of 3,179.

Table 1: Means of portfolios with test values

	Industry Portfolio Before Crisis	**Industry Portfolio Crisis Included**	**International Indices Before Crisis**	**International Indices Crisis Included**
1/n Strategy	0,014	0,006	0,007	0,001
Mean Variance	0,021 (0,669)	0,007 (0,171)	0,006 (0,228)	-0,000 (0,122)
Minimum Variance	0,009 (0,601)	0,005 (0,148)	0,003 (0,686)	-0,002 (0,310)
Bayes-Stein	0,021 (0,669)	0,007 (0,171)	0,006 (0,228)	-0,000 (0,122)

Table 2: Variances of portfolios, with test values

	Industry Portfolio Before Crisis	**Industry Portfolio Crisis Included**	**International Indices Before Crisis**	**International Indices Crisis Included**
1/n Strategy	0,001	0,001	0,000	0,001
Mean Variance	0,001 (1,417)	0,002 (2,426)	0,000 (1,001)	0,000 (1,765)
Minimum Variance	0,001 (1,260)	0,001 (1,051)	0,000 (1,326)	0,001 (1,127)
Bayes-Stein	0,001 (1,417)	0,002 (2,426)	0,000 (1,001)	0,000 (1,765)

Because of the choice of making an estimating window of two months, the covariance/variance matrix has very low values. Therefore the Ø at the Bayes-Stein model has a value so close to one that it behaves just like the Mean-variance portfolio. The reason for still considering the model in this paper, is that it can behave differently by changing some assumptions, which will be done in the robustness test in section 4.

Equal or unequal variance

In table 3 and 4, respectively for the Sharpe ratio and the CEQ, the variance of the means are listed. These values are not the same as the portfolio variances of the models, these are given in the section before this one. Besides that, between the parentheses the test values of the Levene's Test for equality of variance in respect to the 1/n strategy are given in the tables. It tends to be that \ the crisis has had influence on the variances. For both, the Sharpe Ratio and the CEQ, the variance of the most models increased in the crisis period. This is often a feature of crisis. In turbulent times returns have often more extreme values. The 1/n strategy has in 3 of the 4 datasets, the lowest variance by means of the Sharpe ratio. By means of the CEQ, the variances are so low that variances of the models are very close. Looking to the test values of the variances of the Sharpe ratio, one can see that the highest values are in the 10 industry portfolios dataset before the crisis. By means of the CEQ the height of the values are more divided. However, the values have to increase the 3,179 to adopt hypothesis one and thus reject the null hypothesis. No value meets this criterion. Looking to the p-values, in appendix tables A2 and A3, the differences are not significant. The values are rather high and not one of them is below 0,05. Therefore there is no evidence that the variances of the model can be considered as unequal. For further results an assumption has to be made. This assumption implies the equality of the variance. In section 3.3, there will be a test on the outcomes if assumption of inequality is made and if it has a major impact on it.

Table 3: Variances of Sharpe ratios with test values Levene test

	Industry Portfolio Before Crisis	**Industry Portfolio Crisis Included**	**International Indices Before Crisis**	**International Indices Crisis Included**
1/n Strategy	4,495	4,996	24,154	70,278
Mean Variance	14,833 (2,993)	14,731 (0,637)	48,858 (0,335)	105,676 (0,503)
Minimum Variance	16,653 (2,903)	34,835 (1,883)	36,082 (0,104)	35,082 (1,094)
Bayes-Stein	14,833 (2,993)	14,731 (0,637)	48,858 (0,335)	105,676 (0,503)

Table 4: Variances of CEQ with test values Levene test

	Industry Portfolio Before Crisis	**Industry Portfolio Crisis Included**	**International Indices Before Crisis**	**International Indices Crisis Included**
1/n Strategy	0,001	0,003	0,001	0,001
Mean Variance	0,001 (1,220)	0,003 (0,763)	0,001 (0,217)	0,001 (2,413)
Minimum Variance	0,001 (0,542)	0,001 (0,004)	0 (1,542)	0,001 (0,300)
Bayes-Stein	0,001 (1,220)	0,003 (0,763)	0,001 (0,217)	0,001 (0,278)

Tests for significant difference

In this section there will be a comparison of the Sharpe ratios and CEQ's of the various sophisticated models with those of the 1/n strategy. Of both ratios, the test values and the p-values in relation to the 1/n strategy are calculated. Conclusions on differences can then be made.

Sharpe ratio

In table 3 the average Sharpe ratio of each strategy is given. The test values of the models in relation to the 1/n strategy are below the Sharpe ratios of the Mean-Variance, Minimum-Variance and Bayes-Stein Model. Notable is that the average Sharpe ratio of the 1/n asset is higher than

Table 5: Sharpe ratios and test values

	Industry Portfolio Before Crisis	**Industry Portfolio Crisis Included**	**International Indices Before Crisis**	**International Indices Crisis Included**
1/n asset allocation	7,373	0,8046	0,135	-1,351
Mean Variance	1,349 (0,856)	1,470 (0,821)	0,036 (0,063)	-1,879 (0,218)
Minimum Variance	0,928 (0,915)	0,772 (0,027)	0,187 (0,227)	-0,278 (0,573)
Bayes-Stein	1,349 (0,856)	1,470 (0,821)	0,036 (0,063)	-1,879 (0,218)

those of the other models in the two before crisis datasets, but not in the other two datasets. However the differences are not proven by the test values, because no value in the table is above the 1,734-level and therefore there is no prove for any outperforming of the 1/n asset allocation strategy or another strategy. The only reason for a negative Sharpe ratio, like all the values in the last dataset, is a negative excess return. This is because a negative variance (and therefore volatility) is impossible and the only two variables in the Sharpe ratio are the mean of returns and the volatility. The main reason for the high Sharpe ratio-value of the 1/n asset allocation strategy in the first dataset, is the very low variance of it. However, the differences still are not significant. All values are positive, because the highest Sharpe ratio of the two models which are tested is always chosen as the μ_1 and the lowest as μ_2.

The p-values of the t-tests for differences by means of the Sharpe ratios are given in the appendix, in table A4. Because the significance level is set on 0,05, the values have to be over it to be statically significant. Since no value in the table meets this criterion, there is no evidence to state that one of the sophisticated models outperforms the 1/n strategy, or vice versa.

Certainty Equivalent

Below, the average CEQ's and their test values in respect to the 1/n asset allocation strategy are listed up. By interpreting this information, one can see that in all of the cases the CEQ decreases between the period before crisis and the one which includes the crisis.. This is in accordance with the findings of the Sharpe Ratios. However the comparison of the more sophisticated models with the 1/n strategy is not exactly in accordance to previous findings, because the naïve strategy has not the highest means in all the two datasets which are before the credit crunch. In the Industry Portfolio the mean variance slightly performs better. But, the difference is not significant. Like in the previous section, the ratios in the last dataset are negative. However the CEQ cannot only be negative by a negative return, but also by a small return and a large variance. Looking at the means and the variance, the first reason seems to be the case here. The test values, between the parentheses, prove that not one model outperforms one other. These values have to be above 1,734, to reject the H_0, like the test values of the Sharpe Ratios. The data all have a value below it, and therefore the H_0 has not been rejected once. Like the test values of the Sharpe ratios, in this table all values are positive.

Table 6: CEQ's and test values

	Industry Portfolio Before Crisis	Industry Portfolio Crisis Included	International Indices Before Crisis	International Indices Crisis Included
1/n asset allocation	0,013	0,006	0,007	0,000
Mean Variance	0,020 (0,631)	0,007 (0,150)	0,005 (0,229)	-0,001 (0,103)
Minimum Variance	0,009 (0,595)	0,004 (0,160)	0,003 (0,680)	-0,003 (0,314)
Bayes-Stein	0,020 (0,631)	0,007 (0,150)	0,005 (0,229)	-0,001 (0,103)

All the p-values, in appendix table A5, have a value of more than 0,05. The closest value is 0,250; but that is not enough for proving significance. Like at the Sharpe ratios, the p-values of the first dataset are the lowest and therefore the closest to the level of significance.

3.3 Robustness test

In this paper a couple of assumptions have been made to come to the results in the previous paragraph. The first assumption is an estimation window of two months, 30 times in the dataset, rather than one of half year for 10 times.. The second assumption that has been made is a holding period for two months, rather than half a year. Furthermore the risk aversion of the investor has put on a stable and constant level of 1 in the search. At least, as mentioned before, the assumption that variance is equal has been made. To test or the assumption that have been made are of great influence on the results, a robustness test is considered.

<u>Less numbers of estimation windows and holding periods</u>

To check whether the number estimation windows and holding period are of great impact on the results, these parameters are reset to 10, with periods of half a year. The results of the test values and p-values of the Sharpe ratio and the CEQ are listed up in the appendix, under table A6 to A9. The results show a difference with the results in the previous chapter. Overall the p-values in these tests tend to be higher. So the results with less estimation windows and less holding period seem to result in insignificant values which are higher. Besides that the test values seem to be lower and therefore do they come less close to the critical value.

Risk aversion

By changing the risk aversion, one can notice that the results are not influenced at all. This is because the risk aversion is only part of the mean variance model (and therefore the Bayes-Stein model) and the CEQ ratio. In de mean variance model the risk aversion is a part of the formula for the optimal weights. But all the weights change with a same level by changing the risk aversion, so the optimal weights have the same value as before changing. Besides that the risk aversion is also a part of the CEQ formula. However by changing the γ the CEQ will change relative the same in each case, so the results will not be affected.

Unequal variance

Because the tests of equality of variance did not reject the H_0 there is no prove to state that the variances of the Sharpe ratio or of the CEQ are unequal or equal. To come to results, one assumed the equality of variance. In this paragraph, the assumption will be changed in 'one assumes the inequality of variance.' The results are given in the appendix, tables A10 to A13. The change, results in very little change is the p-values. Besides that it does not at all effect the test values. Therefore the inequality or equality of variance does not have a major impact on the results.

4. Conclusion and further research

4.1 Conclusion

No evidence is found of the outperforming of a model by another model. This implies that the 1/n asset allocation strategy, often called the naïve strategy, is not outperformed by a more sophisticated model. Therefore the name of 'naïve strategy' for this model is not been proven, because it does not perform less well than the optimal portfolios. The 1/n asset allocation strategy is undervalued and should be considered more seriously by investors.

4.2 Limitations and Further research

In the robustness test of this paper the estimation window is tested. Besides that the estimation window, that is static in this search, can also be rolling. This rolling approach can have an affect the results. The same applies to the holding period.

The scale of this search is rather small and therefore can conclusions be different with a paper based on larger scales, like mentioned before in the first section. Moreover a larger scale is more realistic, because in the real world of investments the number of present assets is much higher than 10 and the number of historical periods are higher than 30, considered in this paper.

Since the 'naïve' strategy is proven as a good alternative for simple investments in this paper, pension funds or other investment advisors can advice to use this model. If this advice will be given to investors, it is very important to set a composition that accounts for the 1/n strategy. There have to be a good distribution between bonds and stocks, further research on that is recommended.

Moreover this paper does not include any trading costs, like transaction costs, valuate risks et cetera. Because the 1/n asset allocation divides the investor's capital among all present assets, the trading cost will be higher at inception. On the other hand, like mentioned before, at rebalancing the 1/n strategy will change little or not in most cases and therefore transaction costs will be smaller for this 'naïve method'. Moreover other strategies are optimal with (large) short positions which can be very costly. The 1/n strategy does not have such positions.

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6. Appendix

Table A1: p- values t-test Means

	Industry Portfolio Before Crisis	Industry Portfolio Crisis Included	International Indices Before Crisis	International Indices Crisis Included
Mean Variance	0,438	0,253	0,253	0,452
Minimum Variance	0,442	0,275	0,248	0,377
Bayes-Stein	0,438	0,253	0,253	0,452

Table A2: p- values Levene test Sharpe Ratio

	Industry Portfolio Before Crisis	Industry Portfolio Crisis Included	International Indices Before Crisis	International Indices Crisis Included
Mean Variance	0,089	0,428	0,565	0,481
Minimum Variance	0,094	0,175	0,748	0,300
Bayes-Stein	0,089	0,428	0,565	0,481

Table A3: p- values Levene test CEQ

	Industry Portfolio Before Crisis	Industry Portfolio Crisis Included	International Indices Before Crisis	International Indices Crisis Included
Mean Variance	0,089	0,428	0,565	0,481
Minimum Variance	0,094	0,175	0,748	0,300
Bayes-Stein	0,089	0,428	0,565	0,481

Table A4: p- values t- test Sharpe

	Industry Portfolio Before Crisis	Industry Portfolio Crisis Included	International Indices Before Crisis	International Indices Crisis Included
Mean Variance	0,185	0,208	0,475	0,414
Minimum Variance	0,182	0,489	0,411	0,285
Bayes-Stein	0,185	0,208	0,475	0,414

Table A5: p- values t- test CEQ

	Industry Portfolio Before Crisis	Industry Portfolio Crisis Included	International Indices Before Crisis	International Indices Crisis Included
Mean Variance	0,265	0,441	0,410	0,464
Minimum Variance	0,277	0,437	0,250	0,377
Bayes-Stein	0,265	0,441	0,410	0,464

Table A6: p- values t- test Sharpe (other estimation and holding periods)

	Industry Portfolio Before Crisis	Industry Portfolio Crisis Included	International Indices Before Crisis	International Indices Crisis Included
Mean Variance	0,321	0,303	0,357	0,286
Minimum Variance	0,495	0,373	0,354	0,492
Bayes-Stein	0,321	0,303	0,357	0,286

Table A7: p- values t- test CEQ (other estimation and holding periods)

Mean Variance	0,417	0,482	0,409	0,304
Minimum Variance	0,338	0,471	0,379	0,484
Bayes-Stein	0,417	0,482	0,409	0,304

Table A8: test values t- test Sharpe (other estimation and holding periods)

Mean Variance	0,475	0,526	0,374	0,575
Minimum Variance	0,013	0,328	0,381	0,003
Bayes-Stein	0,475	0,526	0,374	0,575

Table A9: test values t- test CEQ (other estimation and holding periods)

Mean Variance	0,0469	0,2158	0,2334	0,5223
Minimum Variance	0,4261	0,0734	0,3129	0,0420
Bayes-Stein	0,0469	0,2158	0,2334	0,5223

Table A10: p-values t- test Sharpe ratio (inequality assumed)

Mean Variance	0,200	0,208	0,475	0,414
Minimum Variance	0,184	0,489	0,411	0,285
Bayes-Stein	0,200	0,208	0,475	0,414

Table A11: p-values t- test CEQ (inequality assumed)

Mean Variance	0,441	0,266	0,410	0,459
Minimum Variance	0,437	0,277	0,250	0,377
Bayes-Stein	0,441	0,266	0,410	0,459

Table A12: test values t- test Sharpe ratio (inequality assumed)

Mean Variance	0,856	0,821	0,063	0,218
Minimum Variance	0,915	0,027	0,027	0,573
Bayes-Stein	0,856	0,821	0,063	0,218

Table A13: test values t- test CEQ (inequality assumed)

Mean Variance	0,150	0,631	0,229	0,103
Minimum Variance	0,160	0,595	0,250	0,314
Bayes-Stein	0,150	0,631	0,229	0,103