Regulatory Capital Requirements under FTK and Solvency II for Pension Funds

by
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Abstract

This thesis investigates the effect on the Regulatory Capital Requirement for a pension fund under the Financial Assessment Framework and under Solvency II. We also test whether the Regulatory Capital Requirements which we find under the Financial Assessment Framework and Solvency II are indeed sufficient according to models based on historical data. This means that the probability of a funding ratio below 100% should be 2.5% under the Financial Assessment Framework and 0.5% under Solvency II. The result of the investigation is that replacing the current legislation with Solvency II, leads to an increase in the Regulatory Capital Requirement of about 10% for our specific pension fund. In order to test the probabilities of a funding ratio below 100%, 10,000 possible developments of the required funding ratio have been simulated for two different ALM models: the Hoevenaars model and GAM model. After reducing the risks in the legislations to the risks which are present in the ALM models, we find that both the Financial Assessment Framework and Solvency II, underestimate the total risk according to both ALM models for our specific pension fund. However, the GAM model is more prudent than the Hoevenaars model. The results of the Hoevenaars model are more in line with both legislations. We conclude that according to the Hoevenaars model, the Financial Assessment Framework slightly overestimates the risk for stocks and slightly underestimates the risk for government bonds and corporate bonds. Finally, we find that according to the Hoevenaars model, the risk for commodities is underestimated under the Financial Assessment Framework and Solvency II.
Chapter 1

Introduction and problem motivation

1.1 Problem motivation

In the Netherlands, it is obligatory to participate in a pension scheme in case it is provided by the employer. Since employees are obliged to invest part of their salary in a pension scheme, it is important that there is good supervision. The legislation for pension funds is embedded in the Financial Assessment Framework. In the current situation pension funds need to keep in reserve a certain level of capital such that there is a certainty of 97.5% that after one year the funding ratio will be at least 100%. This level of capital is the so called Regulatory Own Funds.

Lately, there has been a lot of discussion about the Financial Assessment Framework. Due to ageing of the population, the fact that people live longer and the interest rate environment, the costs of pensions have increased significantly. Besides, the returns on the financial market have been lower than expected. People argue that the legislation for pension funds should therefore be tightened. Possibly, the legislation for pension funds will move towards Solvency II.

Solvency II is legislation mainly for insurers in which insurers have similar requirements. This legislation will be introduced on January 1, 2014. There are some differences between the Financial Assessment Framework and Solvency II. One important difference is that insurers need a security margin of 99.5% instead of 97.5%. The level of capital which is needed in order to achieve the security margin of 99.5% is called the Solvency Capital Requirement. Besides the difference in security margins, the methodology is somewhat different as well.

For reading simplicity, we will hereafter refer to the Regulatory Own Funds.
as 'Regulatory Capital Requirement under FTK' and consistently refer to the Solvency Capital Requirement as 'Regulatory Capital Requirement under Solvency II'.

In this thesis we will investigate the differences in the Regulatory Capital Requirements under Solvency II and the Financial Assessment Framework. In this way we can investigate the effect on the capital requirements in case Solvency II will be applied to pension funds. Besides this, we will test whether the Regulatory Capital Requirements under FTK and Solvency II indeed lead to the required probabilities of underfunding.

Summarizing, in this thesis the following research question will be investigated:

"What is the Regulatory Capital Requirement under the current Financial Assessment Framework in comparison to the Regulatory Capital Requirement obtained by the standard model of Solvency II and are the obtained levels of Regulatory Capital Requirement indeed sufficient when tested by simulating under different ALM models for different kinds of pension profiles?"

1.2 Company profile

This study has been conducted at Towers Watson. Towers Watson is a global consultancy company that advises organizations in order to improve their performance in the areas of financial- and risk management and Human Resources. In January 2010, Towers Watson was formed by the merger of Towers Perrin and Watson Wyatt. In the Netherlands, Towers Watson has 450 employees divided over six offices. Towers Watson works for a lot of different clients, like pension funds, insurance companies and multinationals.

1.3 Structure of this thesis

In the next chapter we will give an introduction to the Financial Assessment Framework. First, this chapter treats how assets and liabilities should be valued. Then we will discuss the model which is commonly used to determine the Regulatory Capital Requirement under FTK in more detail. At the end of the chapter we will discuss the future perspective of the Financial Assessment Framework.

In chapter 3, the Solvency II legislation will be introduced. The standard model which can be used in order to determine the Regulatory Capital Requirement under Solvency II will be explained in more detail. The focus of chapter 3 is on the two largest subcomponents of the total Regulatory Capital Requirement under Solvency II. These are the Regulatory Capital Requirement for market risk and the Regulatory Capital Requirement for life underwriting risk.
Chapter 4 first describes the characteristics of the pension fund we will use throughout this thesis. This pension fund is constructed using an existing pension fund and deleting some random participants in order to anonymize the fund. Chapter 4 also describes the determination of the Regulatory Capital Requirements under FTK and Solvency II. The chapter ends with a comparison of the Financial Assessment Framework and Solvency II.

Chapter 5 concerns the Hoevenaars model. The first section contains a description of the ALM model. Afterwards, the data which we will use in order to be able to simulate the Hoevenaars model is presented. Finally, we will describe the process of simulating possible funding ratios and obviously end with some conclusions.

The focus of chapter 6 is on the Global Asset Model (GAM model). This model is the ALM model which has been developed by Towers Watson. First, we describe the GAM model. Second, we present the data which is used for simulation. The chapter continues with a description of the simulation process and finally ends with some conclusions.

In chapter 7, we treat the performance of both ALM models versus the Financial Assessment Framework and Solvency II. In order to make the comparisons fair, we only take risks into account which are also present in the ALM models.

Since the Financial Assessment Framework and Solvency II should be applicable to all kinds of pension funds, we also determine the Regulatory Capital Requirements for a second pension fund in chapter 8. Again, we investigate the performance of both ALM models versus Solvency II and the Financial Assessment Framework.

Chapter 9 consists of a sensitivity analysis. In this analysis we will treat the sensitivity of the model for changes in the value of the assets, liabilities and asset mix.

Of course, we will end with a conclusion and we will do some recommendations for further research.
Chapter 2

Financial Assessment Framework (FTK)

In this chapter, we will discuss the Financial Assessment Framework. Since the FTK is legislation for pension funds, we first start with a short introduction to the Dutch pension system. Afterwards, we will give an introduction to the FTK in which the main topics will be discussed. Thereafter, we discuss how assets and liabilities should be valued. In section 4, we will discuss the solvency test in more detail. This includes the different models that can be used in order to determine the Regulatory Capital Requirement under FTK and in particular the contents of the standard model. Finally, we will discuss the future developments of the FTK.

2.1 Introduction to the Dutch pension system

The pension system in the Netherlands consists of three pillars. The first pillar is called the 'AOW' (Algemene Ouderdomswet) and was introduced after the Second World War. The aim of the ‘AOW’ is to avoid poverty among elderly people. Every inhabitant in the Netherlands receives the same level of ‘AOW’ after retirement under the condition that the inhabitant lived in the Netherlands from age 15 until retirement.

The level of pension income in pillar 2 is in general different for every inhabitant. It depends for example on the pension scheme which was provided by the employer, the salary of the employee and the number of years in which the employee was active on the labor market. There are two types of pension schemes: Defined Benefit (DB) and Defined Contribution (DC) schemes. In DB schemes, the pension promise after retirement is known in advance. Contrary, in DC schemes not the benefits are known in advance but the contribution which has to be paid. Since in DC schemes risks are with the participants, DC schemes are beyond the scope of this thesis. The level of pension income of the second pillar...
is complementary to the 'AOW' income. The Dutch pension system is based on solidarity between different generations as well as solidarity within a generation. It is obligated for participants to participate in this pillar since otherwise the solidarity effect could disappear.

The last pillar is voluntary and consists of individual savings. One can participate in this pillar in case someone wants a higher level of income after retirement.

In order to obtain a reasonable level of pension income, pension premiums need partly to be invested in risky assets. In case the returns of the assets are insufficient all participants carry the responsibility together. If available, a sponsor could also carry some responsibilities of the risks. A pension fund has several steering instruments in case of a deteriorating financial position. Examples of steering instruments are: increase premiums, change the asset mix, change the indexation policy or reduce pension rights. We already mentioned that supervision is important since employees are obliged to invest in pension schemes. This supervision should ensure that the pension fund acts in the interest of the participants. It should also contribute to creating confidence in the system. In the Netherlands, the supervision is conducted in the Financial Assessment Framework\footnote{In Dutch: Financieel Toetsingskader (FTK)} (hereafter: FTK).

\subsection*{2.2 Introduction to FTK}

The Financial Assessment Framework is the legislation for pension funds. It has its focus on the financial position and policy of the fund. The FTK was introduced on January 1, 2007 and is embedded in the Pension Act. The goal is to enable pension funds to absorb financial losses without detriment of the participants. Its principle is to stimulate pension funds to have a responsible business practice and proper risk management.

The legislation prescribes that pension funds need to keep in reserve a certain level of capital in order to be able to absorb financial losses, the so called Regulatory Capital Requirement. This Regulatory Capital Requirement under FTK should be established in such a way that, with a confidence level of 97.5\%, after one year the value of the assets will exceed the value of the liabilities. The regulator of the FTK is De Nederlandsche Bank (hereafter: DNB).

In summary, the Regulatory Capital Requirement is set in such a way that for 9 different predetermined risks, a certain capital has to be kept. These risks are interest rate risk, equity and real estate risk, currency risk, commodity risk, credit risk, technical insurance risk, liquidity risk, concentration risk and operational risk. The capital requirements can be determined by applying one worst case scenario to the level of assets and liabilities in equilibrium state. The amount of capital according to the worst case scenario is determined such that
the chance of underfunding is 2.5% in total. This means that statistically once every 40 years we are dealing with a funding ratio which is less than 100%. It is desirable that the probability of underfunding in one year from now is independent of the current funding ratio of the pension fund. Otherwise, the lower the current funding ratio, the lower the Regulatory Capital Requirement due to a lower value of the assets. The legislation only prescribes how much the Regulatory Capital Requirement should be in order to have the probability of an event in which the value of the liabilities exceeds the value of the asset equal to 2.5%. This is why an iterative process is used in order to find the equilibrium point. The worst case scenario is embedded in the FTK. This scenario is uniform for both small and large pension funds. By applying a standardized model, the same requirements apply for all pension funds. This makes the financial position of the pension funds more comparable and more transparent.

It is important to note that the FTK is only applicable to the unconditional rights and not to the conditional rights. The reasoning behind this is that unconditional rights are only granted in case the pension fund has a good financial position. The FTK has its focus on a worst case scenario and in this scenario pension funds are not obliged to grant conditional rights. For this reason, conditional rights are assumed to be no risk for a pension fund.

### 2.3 Valuation

The Pension Act ([Pensioenwet, 2010](#)) states in article 126 that assets have to be valued according to market valuation. This is consistent with the fair value as stated in IAS 39 (International Accounting Standards). According to IAS 39, the fair value is defined as the "amount for which an asset could be exchanged, between knowledgeable, willing parties in an arm’s length transaction.

Regarding the liabilities, market valuation has to be applied as well. The reasoning behind market valuation is that the effects on changes in the interest rates are more transparent and as a result can be controlled in a better way. Since liabilities are not frequently traded and often long term, the liabilities are harder to value. As a consequence, the market value for liabilities is defined as the net present value of the expected future cash flows. This implies that the value of the liabilities strongly depends on the term structure which is used for discounting. Since the liabilities have to be discounted according to market valuation, we use the swap curve for discounting.

After assets and liabilities are valuated, we are able to determine the funding ratio of the pension fund. The funding ratio is defined as:

\[
\text{Funding ratio} = \frac{\text{Value of the assets}}{\text{Value of the liabilities}}
\]  

(2.1)

In this thesis we define a pension fund to be underfunded in case the funding ratio is less than 100%. We use this assumption, since the definition of the
Regulatory Capital Requirement under FTK states that the value of the assets should exceed the value of the liabilities in one year from now with a confidence level of 97.5%.

2.4 Solvency test

The FTK requires two things. First, it requires the pension fund at this moment to have a funding ratio which is at least in accordance to the Minimal Regulatory Capital Required (MCR). The Minimal Regulatory Capital Required is defined as (source: [BesluitFTK, 2006]):

1. In case of investment risk:

   \[
   \text{MCR} = 4\% \max \left( \frac{\text{Net technical provision}}{\text{Gross technical provision} \times 0.85} \right) \text{Gross technical provision}
   \]  
   (2.2)

   where the net technical provision\(^2\), is equal to the gross technical provision minus the reinsured part of the technical provision.

2. In case there is no investment risk and management expenses\(^3\) are set for at least 5 years:

   \[
   \text{MCR} = 1\% \ \text{Gross technical provision}
   \]  
   (2.3)

3. In case there is no investment risk and management expenses are set for less than 5 years:

   \[
   \text{MCR} = 25\% \ \text{Net management expenses of the previous fiscal year}
   \]  
   (2.4)

In practice, the Minimal Regulatory Capital Required is commonly about 4-5% of the technical provision. In case the capital of the pension fund is less than the MCR, the pension fund has a funding shortfall\(^4\). When this is the case, the pension fund has to recover within three years.

Second, it requires the pension fund to meet the legal requirements for the Regulatory Capital Requirement under FTK. A fund meets the latter condition, in case the funding ratio after one year will be at least 100% with a confidence level of 97.5%. In case the pension fund does not meet the latter condition, it has a reserve shortfall\(^5\). In this case, recovery within 15 years is required.

There are three models which can be used in order to determine the Regulatory Capital Requirement under FTK. Commonly, pension funds use the so

\(^2\)in Dutch: voorziening pensioenverplichtingen
\(^3\)in Dutch: beheerskosten
\(^4\)in Dutch: dekkingstekort
\(^5\)in Dutch: reservetekort
called "standard model". In case the risk profile of the pension fund differs too much from the outcomes of the standard model, a fund can try to get permission from DNB to use either a simplified model or an internal model.

In order to get permission from DNB to use the simplified model, some requirements have to be met. The most important requirements are:

- a funding ratio which is at least 130%
- a pension scheme which is not too complicated
- a risk averse investment strategy is in place

In case a pension fund prefers to apply the simplified method, the Regulatory Capital Requirement under FTK is equal to 30% of the technical provisions. In practice, only small pension funds will be able to meet the required conditions.

In the internal model, the model has to be fit to the actual risk profile of the fund. Since it takes all relevant risks into account, this model is supposed to be more precise than the standard model. The pension fund itself can develop the internal model. However, it has to be approved by DNB. In addition to the internal model, the standard model has to be applied once every three years. The results of the standard model serve as a benchmark.

Since pension funds commonly use the standard model, we will focus on this model in this thesis. In the next section, we will explain the standard model in more detail.

### 2.4.1 Standard Model

The value (in euros) of the total Regulatory Capital Requirement under FTK can be calculated by the following formula:

\[
S = \sqrt{S_1^2 + S_2^2 + 2\rho S_1 S_2 + S_3^2 + S_4^2 + S_5^2 + S_6^2 + S_7^2 + S_8^2 + S_9^2}
\]  

(2.5)

where,

- \( S \) = total Regulatory Capital Requirement under FTK
- \( S_1 \) = component for interest rate risk
- \( S_2 \) = component for equity and real estate risk
- \( S_3 \) = component for currency risk
- \( S_4 \) = component for commodity risk
- \( S_5 \) = component for credit risk
- \( S_6 \) = component for technical insurance risk
- \( S_7 \) = component for liquidity risk
Dutch National Bank (DNB) prescribes that the correlation coefficient between the component for interest rate risk and the component for market risk is equal to 0.5. The correlation coefficients between the other risks are zero by assumption. (source: [DNB, 2007])

The Dutch legislation called "Regeling Pensioenwet en wet verplichte beroepspensioenregeling" ([FTK, 2006]), defines the shocks which need to be calculated in order to determine the Regulatory Capital Requirement under FTK.

### 2.4.1.1 $S_1$: Interest rate risk

Interest rate risk is the risk that the value of the investments will change due to a change in the level of interest. In order to determine the component for interest rate risk, the effects of a shock of the interest rate should be calculated. The FTK prescribes that the effects of two shocks have to be calculated, namely both a decrease as well as an increase of the interest rate. The size of the shock depends on the duration. The sizes of the shocks can be found in table 2.1. The effect of the change in interest rate which has the most negative effect will be the component for the interest rate risk.

For example, when the current interest rate with duration 15 is equal to 2.93%, then in the scenario in which the interest rate will decrease, the interest rate will drop from 2.93% to 2.26% (0.77 * 2.93%). Moreover, in the scenario in which the interest rate will increase, the interest rate will rise from 2.93% to 3.78% (1.29 * 2.93%). In this way, two shifted term structures can be determined relative to the current term structure.

Commonly, a decrease in the interest rate will have the most negative effect. In case the interest rate decreases, the current value of the liabilities will increase since we have to discount with a lower interest rate. However, in case there are assets in the portfolio which are negatively correlated to a change in the interest rate, for example government bonds, the value of the assets increases as well in the event of a decrease in the interest rate. Since the duration for liabilities is in general longer than the duration for assets, and since not all assets are sensitive to changes in the interest rate, the value of the liabilities will commonly increase more than the value of the assets. As a result, the funding ratio will decrease.

### 2.4.1.2 $S_2$: Equity and real estate risk

Equity risk is the risk that the pension fund can have a loss due to fluctuations in share prices. The component for equity and real estate risk distinguishes four categories: shares listed on mature markets, shares on emerging markets, private

---

6source: zero-coupon yield curve at 31-12-2011 (published by DNB)
Table 2.1: Interest factors (source: [FTK, 2006])

<table>
<thead>
<tr>
<th>Duration (year)</th>
<th>Factor decrease interest rate</th>
<th>Factor increase interest rate</th>
<th>Duration (year)</th>
<th>Factor decrease interest rate</th>
<th>Factor increase interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.63</td>
<td>1.60</td>
<td>14</td>
<td>0.77</td>
<td>1.31</td>
</tr>
<tr>
<td>2</td>
<td>0.66</td>
<td>1.51</td>
<td>15</td>
<td>0.77</td>
<td>1.29</td>
</tr>
<tr>
<td>3</td>
<td>0.69</td>
<td>1.45</td>
<td>16</td>
<td>0.77</td>
<td>1.29</td>
</tr>
<tr>
<td>4</td>
<td>0.71</td>
<td>1.41</td>
<td>17</td>
<td>0.77</td>
<td>1.29</td>
</tr>
<tr>
<td>5</td>
<td>0.73</td>
<td>1.37</td>
<td>18</td>
<td>0.77</td>
<td>1.29</td>
</tr>
<tr>
<td>6</td>
<td>0.74</td>
<td>1.35</td>
<td>19</td>
<td>0.78</td>
<td>1.28</td>
</tr>
<tr>
<td>7</td>
<td>0.75</td>
<td>1.34</td>
<td>20</td>
<td>0.78</td>
<td>1.28</td>
</tr>
<tr>
<td>8</td>
<td>0.75</td>
<td>1.33</td>
<td>21</td>
<td>0.78</td>
<td>1.28</td>
</tr>
<tr>
<td>9</td>
<td>0.75</td>
<td>1.33</td>
<td>22</td>
<td>0.78</td>
<td>1.28</td>
</tr>
<tr>
<td>10</td>
<td>0.76</td>
<td>1.32</td>
<td>23</td>
<td>0.78</td>
<td>1.28</td>
</tr>
<tr>
<td>11</td>
<td>0.76</td>
<td>1.32</td>
<td>24</td>
<td>0.78</td>
<td>1.28</td>
</tr>
<tr>
<td>12</td>
<td>0.77</td>
<td>1.31</td>
<td>25</td>
<td>0.79</td>
<td>1.27</td>
</tr>
<tr>
<td>13</td>
<td>0.77</td>
<td>1.31</td>
<td>&gt;25</td>
<td>0.79</td>
<td>1.27</td>
</tr>
</tbody>
</table>

equity and real estate. The sizes of the shocks in the value of the investments in the above mentioned categories can be found in table 2.2. The correlation between the four categories is equal to 0.75.

2.4.1.3 $S_3$: Currency risk

When a pension fund has investments denominated in foreign currencies, they face the risk that the price of the foreign currency against the euro will change. This risk is the currency risk. The size of the component for the currency risk is the effect of a decrease in the value of all relevant foreign currencies with respect to the euro of 20%.

2.4.1.4 $S_4$: Commodity risk

The component for commodity risk is the effect of a decrease in the value of the commodities of 30%.

2.4.1.5 $S_5$: Credit risk

In order to determine the size of the component for credit risk, a pension fund needs to determine the effect of an increase in the credit spread of 40%. The definition of credit spread is the difference in yield between corporate bonds and government bonds.
<table>
<thead>
<tr>
<th>Category</th>
<th>Size of the fall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shares listed on mature markets</td>
<td>25%</td>
</tr>
<tr>
<td>Shares on emerging markets</td>
<td>35%</td>
</tr>
<tr>
<td>Private equity</td>
<td>30%</td>
</tr>
<tr>
<td>Real estate</td>
<td>15%</td>
</tr>
</tbody>
</table>

Table 2.2: Equity and real estate risk factors

2.4.1.6 $S_6$: Technical insurance risk

The component for technical insurance risk consists of three parts: process risk, $TSO^7$ and $NSA^8$. The TSO is a risk surcharge for uncertainties in the trend of mortality rates. The NSA is a surcharge for negative stochastic deviations with respect to the expected mortality rates. Thus, it is the sensitivity of the capital of a pension fund in case the average age of death is higher than expected during the determination of the technical provision. The component for technical insurance risk can be determined by the following formula:

$$\left( \text{process risk} + \sqrt{\text{TSO}^2 + \text{NSA}^2} \right) \times \text{technical provision} \quad (2.6)$$

The size of this component depends on:

1. the number of participants $n$
2. the average age of the participants
3. whether or not there is widowers pension
4. the parameters prescribed by DNB

The process risk is defined as:

$$\text{Process risk} = \frac{c_1}{\sqrt{n}} + \frac{c_2}{n} \quad (2.7)$$

where,

$c_1 = \text{basic percentage}$
$c_2 = \text{correction term for skewness}$
$n = \text{number of participants}$

The values for $c_1$ and $c_2$ depend on both the average age of the participants and the type of pension plan. The percentages can be found in table 2.3 and 2.4 respectively.

---

7 in Dutch: Trendsterfte onzekerheid
8 in Dutch: Negatieve stochastische afwijkingen
<table>
<thead>
<tr>
<th>age (year)</th>
<th>old age pension (OP)</th>
<th>old age pension (OP) and widowers pension (NP)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OP (to achieve), capital-based</td>
<td>OP (achieved), capital-based</td>
</tr>
<tr>
<td>30</td>
<td>6% 208%</td>
<td>23%</td>
</tr>
<tr>
<td>35</td>
<td>7% 93%</td>
<td>19%</td>
</tr>
<tr>
<td>40</td>
<td>8% 58%</td>
<td>18%</td>
</tr>
<tr>
<td>45</td>
<td>10% 39%</td>
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<td>50</td>
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<td>21% 5%</td>
<td>4%</td>
</tr>
<tr>
<td>65</td>
<td>28% 10%</td>
<td>10%</td>
</tr>
<tr>
<td>70</td>
<td>37% 14%</td>
<td>14%</td>
</tr>
<tr>
<td>75</td>
<td>48% 19%</td>
<td>19%</td>
</tr>
<tr>
<td>80</td>
<td>63% 27%</td>
<td>27%</td>
</tr>
<tr>
<td>85</td>
<td>81% 37%</td>
<td>37%</td>
</tr>
<tr>
<td>90</td>
<td>104% 53%</td>
<td>53%</td>
</tr>
</tbody>
</table>

Table 2.3: $c_1$: basis percentage (source: DNB, 2006)

<table>
<thead>
<tr>
<th>age (year)</th>
<th>old age pension (OP)</th>
<th>old age pension (OP) and widowers pension (NP)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OP (to achieve), capital-based</td>
<td>OP (achieved), capital-based</td>
</tr>
<tr>
<td>30</td>
<td>0% 1872%</td>
<td>199%</td>
</tr>
<tr>
<td>35</td>
<td>0% 727%</td>
<td>148%</td>
</tr>
<tr>
<td>40</td>
<td>0% 362%</td>
<td>106%</td>
</tr>
<tr>
<td>45</td>
<td>0% 190%</td>
<td>71%</td>
</tr>
<tr>
<td>50</td>
<td>0% 95%</td>
<td>40%</td>
</tr>
<tr>
<td>55</td>
<td>0% 38%</td>
<td>13%</td>
</tr>
<tr>
<td>60</td>
<td>0% 0%</td>
<td>0%</td>
</tr>
<tr>
<td>65</td>
<td>0% 0%</td>
<td>0%</td>
</tr>
<tr>
<td>70</td>
<td>0% 0%</td>
<td>0%</td>
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<tr>
<td>75</td>
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<tr>
<td>80</td>
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<tr>
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<td>0% 0%</td>
<td>0%</td>
</tr>
<tr>
<td>90</td>
<td>0% 0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Table 2.4: $c_2$: correction terms (source: DNB, 2006)
Obviously, the component for technical insurance risk decreases in case the number of participants increases. This makes sense since the idiosyncratic risk disappears in case the number of participants is large.

The TSO is dependent on the average age of the participants and the type of pension scheme. The percentages for TSO can be found in table 2.5. This table shows that the lower the average ages, the higher the percentages are. This makes sense since there is more uncertainty for younger people. In the TSO, distinctions can be made between different risk groups $i$. For example, active members, early leavers and retired members. For each risk group, the average age and the corresponding TSO percentage has to be determined. Afterwards, the TSO percentage has to be multiplied by the technical provision for that category. Finally, the sum over all risk groups has to be calculated and divided by the total value of the technical provisions. In formula this means:

$$T SO = \frac{\sum_{i} \text{technical provision}_i \times TSO_i}{\text{Total value technical provisions}}$$  \hfill (2.8)

The NSA depends on the average age of the participants, the type of pension scheme and the number of participants. The percentages can be found in table 2.6. Just like in the TSO, different risk groups can be distinguished. The NSA

<table>
<thead>
<tr>
<th>age</th>
<th>OP (to achieve), capital-based</th>
<th>OP (achieved), capital-based</th>
<th>OP (to achieve), risk-based</th>
<th>OP (achieved), risk-based</th>
</tr>
</thead>
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<tr>
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<tr>
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<td>2%</td>
<td>2%</td>
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<td>2%</td>
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<tr>
<td>90</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
</tr>
</tbody>
</table>

Table 2.5: TSO percentage (source: DNB, 2006)
Table 2.6: NSA percentage (source: DNB, 2006)

<table>
<thead>
<tr>
<th>age</th>
<th>old age pension (OP)</th>
<th>old age pension (OP) and widowers pension (NP)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OP (to achieve),</td>
<td>OP (achieved),</td>
</tr>
<tr>
<td></td>
<td>capital-based</td>
<td>capital-based, risk-based</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OP (to achieve), risk-based</td>
</tr>
<tr>
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<td>40%</td>
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<td>40%</td>
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<td>40%</td>
<td>50%</td>
</tr>
</tbody>
</table>

is defined as:

\[ NSA^* = \frac{\sum_i \text{Technical provision}_i \times NSA_i}{\text{Total value technical provisions}} \]  

(2.9)

\[ NSA = \frac{NSA^*}{\sqrt{n}} \]  

(2.10)

2.4.1.7 \textit{S7: Liquidity risk}

Liquidity is a measure which indicates how difficult it is to trade an asset. Since the difficulty of trading an asset is a risk, there needs to be a compensation for the additional risk in the price of the asset. The price of an illiquid asset is in general lower than the price for an identical asset which is liquid. In case a pension fund holds illiquid assets, liquidity risk could be present. Liquidity risk is the risk that an asset cannot be traded immediately in case cash is needed. Liquidity risk could for example occur in real estate.

In FTK, the shock for illiquidity risk is 0\% by assumption. The reasoning behind this assumption is that under normal circumstances, there is only a small liquidity premium on assets. However, we think that the FTK describes shocks in order to simulate a worst case scenario and in the worst case scenario assets
could become illiquid. So, from this point of view it seems that the 0% assumption might lead to an underestimation of the Regulatory Capital Requirement. However, we do have the opinion that the assumption is reasonable. Pension funds are able to predict the cash flows of the liabilities reasonably well. This means that a pension fund knows up front at which moment in time about how much money has to be paid to the participants. Since a pension fund does not need to pay 100% of the liabilities at one moment in time, it is not necessary to have invested 100% in assets which are liquid. In summary, as long as a pension fund has enough liquid assets in order to be able to meet its short term obligations, liquidity risk is negligible.

2.4.1.8 $S_8$: Concentration risk

Concentration risk occurs in case assets and liabilities are not diversified in a proper manner. There are different kinds of concentration risk. For example, a disproportionately high exposure to a particular stock, asset class, firm, market, sector or region.

In FTK, the shock for concentration risk is 0% by assumption. This is a result from the assumption that pension funds have well diversified portfolios.

According to Article 135 of the Pension Act (Pensioenwet, 2010), a pension fund needs to invest according to the prudent-person principle. Briefly, this means that investments should be in the interest of the participants, investments in sponsoring firms should be at most 5% and the investments should be valued according to market valuation. Article 135 is not applicable to government bonds. The prudent-person rule is judged by a certified actuary in an actuary report once every year. In case the risk profile of a pension fund differs too much from the assumptions in the standard model, the pension fund should undertake action in accordance to DNB.

We think that in practice, portfolios are not always diversified in a proper manner. As a result, the Regulatory Capital Requirement will be underestimated. Since there is supervision on a regular basis, we think that the pension funds where concentration risk is underestimated will be highlighted by means of the actuarial reports and appropriate steps should be taken. However, we think that the fact that investments in government bonds are beyond the scope of the prudent-person rule is not very prudent. This would for example mean that it is a prudent investment to invest 100% in government bonds from a financially unstable country.

2.4.1.9 $S_9$: Operational risk

Operational risk is the risk of a loss due to incorrect acting of humans, failing systems, internal processes or external events. In FTK, it is assumed that the shock for operational risk is 0%. Operational risk can be reduced by a proper
separation of functions, a good administration and supervision. Besides, various checks are performed by the accountant as well as the actuary. However, we acknowledge that some operational risk will always exist. The exact size of this risk is hard to quantify. Therefore, there probably is some estimation error. In case the expected size of the risk is small, one should question whether or not it is useful to try to determine the exact size of the impact. Besides this, we have the opinion that controlling operational risk is beyond the aim of the FTK.

2.5 Future perspective of FTK

Until the crisis in 2008, participants did not enough realize that there is uncertainty in pensions. Two committees were formed in order to investigate the sustainability of the current pension system. Both committee Frijns ([Frijns et al., 2010]) and committee Goudswaard ([Goudswaard et al., 2010]) agree that the current pension system is under pressure. Three main threats are the ageing of the population, the fact that people live longer on average and the exposure to financial risks. The committee Goudswaard conducted a study and concluded that in order to maintain the current pension ambition, the premium should increase from the current 13% to 17% in 2025. This would have an enormous impact. Both committees state that the current premiums have already reached their maximum. As a consequence, a new balance between ambitions, guarantees and costs have to be established. This can, according to the committees, for example be achieved by reducing the pension ambition, linking the age of retirement and/or the amount of the payment to the life expectancy or move part of the risk to the participants (‘soft’ rights).

Taking into account the recommendations of the committee Frijns and Goudswaard, the government and social partners introduced on June 2011 a new pension agreement. This agreement states that the pensionable age increases in 2020 from 65 years to 66 years, and in 2025 to 67 years. However, in May 2012 the government proposed to increase the pensionable age in 2013 until 2015 every year by 1 month and in 2016 until 2019 every year by 2 months such that the pensionable age already equals 66 years in 2019. Besides, the pensionable age will already be increased from 66 years to 67 years in 2023.

Following the pension agreement, both committees advise to introduce a new legislation for pension funds. The new legislation will consist of two parts: Financial Assessment Framework 1 (FTK1) and Financial Assessment Framework 2 (FTK2). FTK1 will be a stricter version of the current FTK and will be applied to the unconditional ‘hard’ rights. FTK2 will be applied to the ‘soft’ conditional rights.
Chapter 3

Solvency II

In this chapter, we will discuss the Solvency II legislation. Since this legislation applies to insurers, we will first give an introduction to insurance contracts. Afterwards, we will discuss the main topics of the Solvency II legislation. Thereafter, we will focus on the Regulatory Capital Requirement and give an overview of the construction of the Regulatory Capital Requirement under Solvency II. Afterwards, we will explain the technical specifications of the seven submodules of the market module: interest rate risk, equity risk, property risk, credit spread risk, currency risk, concentration risk and illiquidity risk. Finally, we will explain the technical specifications of the seven submodules of the life module. These submodules are: mortality risk, longevity risk, disability risk, lapse risk, expense risk, revision risk and catastrophe risk.

3.1 Introduction to insurance contracts

An insurance contract is a contract between an insured and an insurer. The insured pays a premium to the insurer in return for the promise that the insurer covers financial losses in some predefined cases. This could for example be in case of fire, a car accident or a recurring payment from the retirement date until death.

Contrary to pension funds, insurance companies are allowed to make a profit. On the other hand, in case the premiums an insurance company receives are insufficient, the shareholders of the insurance company also have to take the losses.

Supervision on insurance contracts is important since it should ensure that the interest of the insured is protected. Another advantage is that it creates confidence in the system. Finally, it reduces the risk of losses for shareholders. At this moment, the legislation for insurers is conducted in the Solvency I directive. However, Solvency II is expected to be introduced on 1 January 2014. Solvency I has its focus mainly on technical insurance risk. As a result, it does not give
a good view of the real risks for the insurance company. Solvency II takes more risks into account and is dependent on the real risks that the insurance company faces. This implies market valuation among other things. Another difference is that Solvency II leads to more consistent rules for different EMEA (Europe, the Middle East and Africa) countries.

3.2 Introduction to Solvency II

Solvency II ([SolvencyII, 2009]) is a directive for EMEA insurance companies. Similar to the FTK, the goal of Solvency II is to enable insurers to absorb financial losses without detriment of the participants. The aim of the directive is to introduce a framework for risk management which includes solvency requirements for all insurers which are operating within EMEA. The motivation for an EMEA framework is to make the sector more transparent and have equal requirements concerning insurance companies within EMEA. The directive prescribes that insurance companies need to keep in reserve a certain level of capital in order to be able to absorb financial losses. This capital should be established in such a way that, with a confidence level of 99.5%, after one year the value of the assets will still exceed the value of the liabilities.

The Solvency II framework consists of three pillars. The first pillar contains the quantitative requirements. For example, it sets out the requirements for calculating the Solvency Capital Requirement (SCR) and Minimum Capital Requirement (MCR). Recall that we refer to the Solvency Capital Requirement as ‘Regulatory Capital Requirement under Solvency II’. In case an insurance company has a funding ratio which is below the MCR, the recovery period is three months. In case the funding ratio is less than the SCR, the recovery period is six months ([Joseph et al., 2012]). The second pillar has its focus on the supervision of insurers. The last pillar focuses on transparency and disclosure requirements. Since the focus of this thesis is on Regulatory Capital Requirements, we will focus on the first pillar of Solvency II.

In summary, in order to be able to determine the Regulatory Capital Requirement under Solvency II, capital requirements for several modules have to be calculated. Since we are interested in the application of Solvency II for pension funds, we focus on the market module and the life module. Both modules consist of seven submodules. For each of the submodules, the Solvency II directive prescribes a worst case scenario. This worst case scenario is consistent with a total probability of underfunding which is equal to 0.5%. This means that statistically once every 200 years underfunding occurs. Just as in FTK, the worst case scenario under Solvency II is uniform for all insurance companies. This makes the financial position of the insurance companies internationally consistent and more transparent.

Under Solvency II, the assets have to be valued according to market valua-
tion. This is consistent with FTK. However, the discounting of the liabilities under Solvency II is inconsistent with FTK. Recall that under FTK, the liabilities have to be discounted by the swap curve. Under Solvency II, the liabilities have to be discounted by a swap curve which is corrected for illiquidity risk. In section 3.3.1.7 we will explain this in more detail.

3.3 Regulatory Capital Requirement under Solvency II

According to the Solvency II Directive ([SolvencyII, 2009]) the Regulatory Capital Requirement under Solvency II has to be determined in such a way that the funding ratio after one year will be at least 100% with a confidence level of 99.5%. In other words, underfunding should occur at most once every 200 years. The Regulatory Capital Requirement under Solvency II can be calculated by either an internal model (which is developed by the insurance company itself) or by the standard model (which is provided by the regulator). In this thesis, we will discuss the standard model.

The Regulatory Capital Requirement under Solvency II can be divided into six modules. Figure 3.3 contains a graphical representation of the modules. The Regulatory Capital Requirement under Solvency II can be determined by means of the following formula:

\[
SCR = BSCR + Adj + SCR\text{Op} \tag{3.1}
\]

where,

- \(BSCR\) = Basic Regulatory Capital Requirement under Solvency II
- \(Adj\) = Adjustment for risk absorbing effect
- \(SCR\text{Op}\) = Regulatory Capital Requirement under Solvency II for Operational risk

The BSCR can be determined by the following formula:

\[
BSCR = \sqrt{\sum_{i,j} \text{Corr}_{ij} \text{SCR}_i \text{SCR}_j + \text{SCR}_{\text{intang}}} \tag{3.2}
\]

where,

- \(\text{Corr}_{ij}\) = The correlation between i and j according to the correlation matrix (see Figure 3.1)
- \(\text{SCR}_i\) = Regulatory Capital Requirement under Solvency II of component i
- \(\text{SCR}_j\) = Regulatory Capital Requirement under Solvency II of component j
- \(\text{SCR}_{\text{intang}}\) = Regulatory Capital Requirement under Solvency II for intangible risk
- \(i, j\) = Market risk, default risk, life underwriting risk, non-life underwriting risk and health underwriting risk
Figure 3.1: Overview of the construction of the Regulatory Capital Requirement under Solvency II (source: [QIS5, 2010])
In this thesis, we define the Regulatory Capital Requirement under Solvency II as the BSCR taking into account only the market module and life module. This means that we assume the risk absorbing effect to be zero. Besides, the Regulatory Capital Requirement for operational risk is as well assumed to be zero. These assumptions are in line with the assumptions under FTK.

### 3.3.1 Regulatory Capital Requirement under Solvency II for market risk

The Regulatory Capital Requirement for market risk is specified in the technical specifications of the Solvency II directive ([QIS5, 2010](#)) and is equal to:

$$SCR_{market} = \max\left(\sqrt{\sum_{r,x,c} CorrMktUp_{r,c} MKT_{up,r} MKT_{up,c}}, \sqrt{\sum_{r,x,c} CorrMktDown_{r,c} MKT_{down,r} MKT_{down,c}}\right)$$

(3.3)

where,

$CorrMktDown_{r,c} = \text{The correlation between} \ r \ \text{and} \ c \ \text{according to the correlation matrix in case the interest rate decreases (see table 3.2)}$

$CorrMktUp_{r,c} = \text{The correlation between} \ r \ \text{and} \ c \ \text{according to the correlation matrix in case the interest rate increases (see table 3.3)}$

and $MKT_{r}$ and $MKT_{c}$ have to be replaced by:

- $Mkt_{interest}$: component for interest rate risk
- $Mkt_{equity}$: component for equity risk
- $Mkt_{property}$: component for property risk
- $Mkt_{credit}$: component for credit spread risk
- $Mkt_{currency}$: component for currency risk
### Table 3.2: Correlation matrix market risk under interest rate down stress

<table>
<thead>
<tr>
<th>CorrMktDown</th>
<th>Interest</th>
<th>Equity</th>
<th>Property</th>
<th>Credit</th>
<th>Currency</th>
<th>Concentration</th>
<th>Illiquidity risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest</td>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.25</td>
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<td>0</td>
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<td>Equity</td>
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<td>1</td>
<td>0.75</td>
<td>0.75</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Property</td>
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<td>0.75</td>
<td>1</td>
<td>0.5</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Credit</td>
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<td>0.75</td>
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<td>1</td>
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<td>0</td>
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<tr>
<td>Currency</td>
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<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Concentration</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Illiquidity risk</td>
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<td>0</td>
<td>0</td>
<td>-0.5</td>
<td>0</td>
<td>0</td>
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### Table 3.3: Correlation matrix market risk under interest rate up stress

<table>
<thead>
<tr>
<th>CorrMktUp</th>
<th>Interest</th>
<th>Equity</th>
<th>Property</th>
<th>Credit</th>
<th>Currency</th>
<th>Concentration</th>
<th>Illiquidity risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Equity</td>
<td>0</td>
<td>1</td>
<td>0.75</td>
<td>0.75</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Property</td>
<td>0</td>
<td>0.75</td>
<td>1</td>
<td>0.5</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Credit</td>
<td>0</td>
<td>0.75</td>
<td>0.5</td>
<td>1</td>
<td>0.25</td>
<td>0</td>
<td>-0.5</td>
</tr>
<tr>
<td>Currency</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Concentration</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Illiquidity risk</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.5</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

- \( \hat{Mkt}_{concentration} \): component for concentration risk
- \( \hat{Mkt}_{illiquidity} \): component for illiquidity risk

#### 3.3.1.1 Interest rate risk

In case of the interest rate risk, \( \hat{Mkt}_{int} \), we distinguish two cases:

- \( \hat{Mkt}^{Up}_{int} \) = The component for the interest risk in market risk in case of an upward term structure
- \( \hat{Mkt}^{Down}_{int} \) = The component for the interest risk in market risk in case of a downward term structure

\( \hat{Mkt}^{Up}_{int} \) is by definition the change in the NAV (Net value of the assets - liabilities) in case of an upward altered term structure. The upward altered term structure depends on the maturity and is defined as \((1 + s^{up})\) multiplied by the current term structure. The values for \( s^{up} \) can be found in table 3.4.
<table>
<thead>
<tr>
<th>Maturity</th>
<th>Relative change $s^{up}$</th>
<th>Relative change $s^{down}$</th>
<th>Maturity</th>
<th>Relative change $s^{up}$</th>
<th>Relative change $s^{down}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>70%</td>
<td>−75%</td>
<td>14</td>
<td>34%</td>
<td>−28%</td>
</tr>
<tr>
<td>0.5</td>
<td>70%</td>
<td>−75%</td>
<td>15</td>
<td>33%</td>
<td>−27%</td>
</tr>
<tr>
<td>1</td>
<td>70%</td>
<td>−75%</td>
<td>16</td>
<td>31%</td>
<td>−28%</td>
</tr>
<tr>
<td>2</td>
<td>70%</td>
<td>−65%</td>
<td>17</td>
<td>30%</td>
<td>−28%</td>
</tr>
<tr>
<td>3</td>
<td>64%</td>
<td>−56%</td>
<td>18</td>
<td>29%</td>
<td>−28%</td>
</tr>
<tr>
<td>4</td>
<td>59%</td>
<td>−50%</td>
<td>19</td>
<td>27%</td>
<td>−29%</td>
</tr>
<tr>
<td>5</td>
<td>55%</td>
<td>−46%</td>
<td>20</td>
<td>26%</td>
<td>−29%</td>
</tr>
<tr>
<td>6</td>
<td>52%</td>
<td>−42%</td>
<td>21</td>
<td>26%</td>
<td>−29%</td>
</tr>
<tr>
<td>7</td>
<td>49%</td>
<td>−39%</td>
<td>22</td>
<td>26%</td>
<td>−30%</td>
</tr>
<tr>
<td>8</td>
<td>47%</td>
<td>−36%</td>
<td>23</td>
<td>26%</td>
<td>−30%</td>
</tr>
<tr>
<td>9</td>
<td>44%</td>
<td>−33%</td>
<td>24</td>
<td>26%</td>
<td>−30%</td>
</tr>
<tr>
<td>10</td>
<td>42%</td>
<td>−31%</td>
<td>25</td>
<td>26%</td>
<td>−30%</td>
</tr>
<tr>
<td>11</td>
<td>39%</td>
<td>−30%</td>
<td>30</td>
<td>25%</td>
<td>−30%</td>
</tr>
<tr>
<td>12</td>
<td>37%</td>
<td>−29%</td>
<td>&gt;30</td>
<td>25%</td>
<td>−30%</td>
</tr>
<tr>
<td>13</td>
<td>35%</td>
<td>−28%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4: Interest factors (source: [QIS5, 2010])

Consistently, $Mkt^{Down}_{int}$ is by definition the change in the NAV in case of a downward altered term structure. The downward altered term structure also depends on the maturity and is defined as $(1 + s^{down})$ multiplied by the current interest rate. The values for $s^{down}$ can as well be found in table 3.4.

For example, in case the current interest rate with a maturity of 15 years, $R_0(15)$, is 2.93% we have:

- The interest rate in case of the upward scenario:
  \[
  R^{up}_1(15) = R_0(15) * (1 + s^{up}) = 2.93% * (1 + 0.33) = 3.90%
  \]

- The interest rate in case of the downward scenario:
  \[
  R^{down}_1(15) = R_0(15) * (1 + s^{down}) = 2.93% * (1 - 0.27) = 2.14%
  \]

Summarizing, in formulas the components for the interest rate risk are defined as:

\[
Mkt^{Up}_{int} = \Delta NAV|_{up} \tag{3.4}
\]

\[
Mkt^{Down}_{int} = \Delta NAV|_{down} \tag{3.5}
\]
The component for interest risk is equal to:

\[ Mkt_{int} = \max \left( Mkt_{int}^{\text{Down}}, Mkt_{int}^{\text{Up}} \right)_+ \]  

(3.6)

It is important to note that \( \Delta NAV \) is positive in case the scenario leads to a decrease of NAV.

### 3.3.1.2 Equity risk

Equity risk is the risk of changes in the value of equity prices. In order to determine the component for equity risk, a split between two categories has to be considered. The first category is the "global equity" category. This category consists of equities which are listed in regulated markets in EEA (European Economic Area) or OECD (Organization for Economic Cooperation and Development) countries. The second category is the "Other equity" category and consists of the equities which do not belong to the first category. For example, hedge funds, equities which are listed on emerging markets and non-listed equities and investments which are not included somewhere else in the market module.

In order to determine the component for equity risk for the "global equity" category, the scenario of a drop in the equity value of 30\% has to be calculated. Then the component is defined as:

\[ Mkt_{eq}^1 = \max(\Delta NAV \mid \text{decrease of 30\% in value of equities}, 0) \]  

(3.7)

The component for "other equity" can be determined by calculating the result of the scenario in which the value of the equities decrease by 40\%. This component is defined as:

\[ Mkt_{eq}^2 = \max(\Delta NAV \mid \text{decrease of 40\% in value of equities}, 0) \]  

(3.8)

Note that in these formulas \( Mkt_{eq} \) has to be a non-negative number since a negative value for \( \Delta NAV \) would lead to an increase of the NAV. An increase of the NAV is obviously not defined as risk and therefore is not considered in the solvency test.

Now, the total component for market risk can be determined in the following way:

\[ Mkt_{equity} = \sqrt{\sum_{r,c} \text{Corr}_{r,c} Mkt_r Mkt_c} \]  

(3.9)

where,

\( \text{Corr}_{r,c} \) = correlation coefficient which can be found in table

\( Mkt_r, Mkt_c \) = component for the two categories
Table 3.5: Correlation coefficients equity risk

<table>
<thead>
<tr>
<th></th>
<th>Global</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global</td>
<td>1</td>
<td>0.75</td>
</tr>
<tr>
<td>Other</td>
<td>0.75</td>
<td>1</td>
</tr>
</tbody>
</table>

3.3.1.3 Property risk
The component for property risk can be calculated by determining the effect on the NAV for the scenario in which the value of the real estate decreases with 25%. Expressed in formulas this means:

\[ Mkt_{\text{property}} = \max(\Delta NAV \mid \text{decrease in value of real estate of 25\%, 0}) \] (3.10)

3.3.1.4 Credit spread risk
Credit spread risk is the result of the sensitivity of the value of assets and liabilities to a change in the level of credit spread over the risk free term structure. The capital requirement for the credit spread risk consists of three subcomponents:

\[ Mkt_{\text{credit}} = Mkt_{\text{bond}}^{sp} + Mkt_{\text{sc}}^{sp} + Mkt_{\text{cd}}^{sp} \] (3.11)

where,
- \( Mkt_{\text{bond}}^{sp} \) = component for the credit spread risk of bonds
- \( Mkt_{\text{sc}}^{sp} \) = component for the credit spread risk of structured credit products
- \( Mkt_{\text{cd}}^{sp} \) = component for the credit spread risk of credit derivatives

Bonds
The component for the credit spread risk of bonds can be determined as the effect of a shock on the NAV. The size of the shock is dependent on the rating class. The formula for \( Mkt_{sp}^{bond} \) is given by:

\[ Mkt_{sp}^{bond} = \max \left( \sum_i MV_i \ast Duration_i \ast F^{up}(\text{rating}_i), 0 \right) \] (3.12)

where,
- \( MV_i \) = the market value of risk exposure
- \( Duration_i \) = duration of the risk exposure
- \( F^{up} \) = level of credit risk exposure which delivers a shock which is consistent
Table 3.6: Credit spread risk factors for bonds (source: [QIS5, 2010])

<table>
<thead>
<tr>
<th>Rating</th>
<th>Spread</th>
<th>Duration Floor</th>
<th>Duration Cap</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.9%</td>
<td>1</td>
<td>36</td>
</tr>
<tr>
<td>AA</td>
<td>1.1%</td>
<td>1</td>
<td>29</td>
</tr>
<tr>
<td>A</td>
<td>1.4%</td>
<td>1</td>
<td>23</td>
</tr>
<tr>
<td>BBB</td>
<td>2.5%</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>BB</td>
<td>4.5%</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>B or lower</td>
<td>7.5%</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Unrated</td>
<td>3.0%</td>
<td>1</td>
<td>12</td>
</tr>
</tbody>
</table>

with the 99.5% Value-at-risk. The credit spread risk factors for bonds can be found in table 3.6

**Structured credit products**

In order to determine the component for the credit spread risk of structured products two subcomponents have to be calculated. The first component is the component for the underlying assets and the second is the component for the direct spread shock. The component for the underlying assets is defined as:

\[ Mkt_{sp,\text{underlying}} = \max (\Delta NAV|\text{underlying spread shock}, 0) \]  \hspace{1cm} (3.13)

The component for the spread shock in the value of the underlying assets of structured products is defined as the effect on the NAV in case the value of the assets decreases immediately by:

\[ \sum_i MV_i \frac{(G(rating_i, tenure_i) - attach_i)}{detach_i - attach_i} \]  \hspace{1cm} (3.14)

where,

- \( MV_i \) = the market value of the risk exposure
- \( G(rating_i, tenure_i) \) = function of the rating of the institution and the tenure of the risk exposure. The function delivers a shock which is consistent with 99.5% VaR. The values can be found in table 3.7

- \( attach_i \) = the attachment point
- \( detach_i \) = the detachment point

The component for direct spread shock is given by:

\[ Mkt_{sp,\text{direct}} = \max (\Delta NAV|\text{direct spread shock}, 0) \]  \hspace{1cm} (3.15)

31
Table 3.7: Values for the function G (source: [QIS5, 2010])

<table>
<thead>
<tr>
<th>G(rating&lt;sub&gt;i&lt;/sub&gt;, tenure&lt;sub&gt;i&lt;/sub&gt;)</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC or lower</th>
<th>Unrated</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0-2 years]</td>
<td>0.4%</td>
<td>0.9%</td>
<td>2.8%</td>
<td>5.3%</td>
<td>14.6%</td>
<td>31.1%</td>
<td>52.7%</td>
<td>6.3%</td>
</tr>
<tr>
<td>[2-4 years]</td>
<td>0.8%</td>
<td>1.7%</td>
<td>4.9%</td>
<td>9.6%</td>
<td>23.9%</td>
<td>44.8%</td>
<td>66.6%</td>
<td>11.4%</td>
</tr>
<tr>
<td>[4-6 years]</td>
<td>1.2%</td>
<td>2.8%</td>
<td>6.5%</td>
<td>13.1%</td>
<td>30.1%</td>
<td>51.2%</td>
<td>70.7%</td>
<td>15.7%</td>
</tr>
<tr>
<td>[6-8 years]</td>
<td>1.8%</td>
<td>4.1%</td>
<td>8.4%</td>
<td>16.4%</td>
<td>35.3%</td>
<td>55.0%</td>
<td>72.6%</td>
<td>19.6%</td>
</tr>
<tr>
<td>[8+ years]</td>
<td>2.4%</td>
<td>5.3%</td>
<td>10.3%</td>
<td>19.6%</td>
<td>39.3%</td>
<td>57.8%</td>
<td>73.5%</td>
<td>23.5%</td>
</tr>
</tbody>
</table>

Table 3.8: Credit spread risk factors for structured products (source: [QIS5, 2010])

<table>
<thead>
<tr>
<th></th>
<th>F&lt;sup&gt;sc,up&lt;/sup&gt;</th>
<th>Duration Floor</th>
<th>Duration Cap</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.9%</td>
<td>1</td>
<td>36</td>
</tr>
<tr>
<td>AA</td>
<td>1.1%</td>
<td>1</td>
<td>29</td>
</tr>
<tr>
<td>A</td>
<td>1.4%</td>
<td>1</td>
<td>23</td>
</tr>
<tr>
<td>BBB</td>
<td>2.5%</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>BB</td>
<td>6.75%</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>B or lower</td>
<td>11.25%</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Unrated</td>
<td>3.0%</td>
<td>1</td>
<td>12</td>
</tr>
</tbody>
</table>

The component for the direct spread shock on the value of structured products is defined as the effect on the NAV in case the value of the products decreases immediately by:

\[
\sum_i MV_i \times Duration_i \times F_{sc,up}^{\text{rating}_i}
\]

where,

- \(MV_i\) = the market value of risk exposure
- \(Duration_i\) = duration of the risk exposure
- \(F_{sc,up}^{\text{rating}_i}\) = function of the rating of the risk exposure that delivers a shock which is consistent with 99.5% VaR. The values can be found in table 3.8

Finally, the component for the credit spread risk of structured products is given by the maximum of the two subcomponents:

\[
Mkt_{sp}^{sc} = \max\left(Mkt_{sp,underlying}^{sc}, Mkt_{sp,direct}^{sc}\right)
\]
Credit derivatives

Credit derivatives include:

- Credit default swaps (CDS)
- Total return swaps (TRS)
- Credit linked notes (CLN)

In order to determine the component for the credit spread risk on credit derivatives two subcomponents need to be calculated:

\[ Mkt_{sp,up}^{cd} = \max (\Delta NAV|\text{upward shock}, 0) \]  \hspace{2cm} (3.18)

\[ Mkt_{sp,down}^{cd} = \max (\Delta NAV|\text{downward shock}, 0) \]  \hspace{2cm} (3.19)

The component for the spread shock for the value of credit derivatives is defined as the effect on the NAV in case the value of the products increases or decreases immediately by the factors which can be found in table 3.9. Finally, the credit spread risk for credit derivatives is defined as:

\[ Mkt_{sp}^{cd} = \max (Mkt_{sp,up}^{cd}, Mkt_{sp,down}^{cd}) \]  \hspace{2cm} (3.20)

3.3.1.5 Currency risk

For all relevant foreign currencies, \( i \), the capital requirement has to be determined. Both the effect on the NAV of an upward shock and a downward shock have to be calculated. In the upward scenario, the value of the currency \( i \) against the local currency increases by 25%. In the downward scenario, the value of the currency \( i \) against the local currency decreases by 25%. However, there are some exceptions:

- Danish krone against Euro, Litas or Estonian krone: +27.5%/-27.5%
- Latvian lats against Euro, Litas or Estonian krone: +26%/-26%

<table>
<thead>
<tr>
<th></th>
<th>Increase of the spreads (absolute)</th>
<th>Decrease of the spreads (relative)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>1.3%</td>
<td>-75%</td>
</tr>
<tr>
<td>AA</td>
<td>1.5%</td>
<td>-75%</td>
</tr>
<tr>
<td>A</td>
<td>2.6%</td>
<td>-75%</td>
</tr>
<tr>
<td>BBB</td>
<td>4.5%</td>
<td>-75%</td>
</tr>
<tr>
<td>BB</td>
<td>8.4%</td>
<td>-75%</td>
</tr>
<tr>
<td>B or lower</td>
<td>16.2%</td>
<td>-75%</td>
</tr>
<tr>
<td>Unrated</td>
<td>5.0%</td>
<td>-75%</td>
</tr>
</tbody>
</table>

Table 3.9: Factors for credit derivatives (source: \[QIS5, 2010\])
- Latvian lats against Danish krone: +28.5%/−28.5%

In formulas this means:

\[ Mkt_{cr,i}^{up} = \max (\Delta NAV|\text{upward shock}, 0) \] (3.21)

\[ Mkt_{cr,i}^{down} = \max (\Delta NAV|\text{downward shock}, 0) \] (3.22)

For each currency \( i \), the capital requirement is defined as:

\[ Mkt_{cr,i} = \max (Mkt_{cr,i}^{up}, Mkt_{cr,i}^{down}) \] (3.23)

The total capital requirement for currency risk is as follows:

\[ Mkt_{cr} = \sum_i Mkt_{cr,i} \] (3.24)

3.3.1.6 Concentration risk

In concentration risk, the only risk considered is the risk that arises in case the exposure to a particular counterparty is disproportionately high. In Solvency II, a disproportionately high exposure to particular geographical areas or particular industry sectors is disregarded.

In order to determine the component in market risk for concentration risk, three things have to be calculated:

- the excess exposure to each counterparty \( i \)
- the capital requirement for each counterparty \( i \)
- the total capital requirement for concentration risk

The excess exposure to each counterparty can be calculated by the following formula:

\[ XS_i = \max \left( 0; \frac{E_i}{Assets_{x,i}} - CT_i \right) \] (3.25)

where,

\( E_i \) = exposure to counterparty \( i \)

\( Assets_{x,i} \) = total value of all assets considered

\( CT_i \) = concentration threshold which depends on the rating of \( i \) (see table 3.10)
The capital requirement for each counterparty is defined as the effect on the NAV in case the value of the products decreases immediately by $XS_i \times g_i$. The values for $g_i$ depend on the rating of the counterparty and are stated in table 3.10. For unrated counterparties, $g_i$ depends on the funding ratio. These values for $g_i$ can be found in table 3.11. In formula, the capital requirement for each counterparty is given by:

\[ Mkt_{conc,i} = (\Delta NAV | \text{decrease of value of } XS_i \times g_i) \quad (3.26) \]

Finally, the total capital requirement for concentration risk is defined as:

\[ Mkt_{conc} = \sqrt{\sum_i (Conc^2_i)} \quad (3.27) \]

In FTK, the concentration risk is assumed to be equal to zero. For consistency, we also assume the concentration risk under Solvency II to be equal to zero.

### 3.3.1.7 Illiquidity risk

As we already discussed in chapter 2, illiquidity is a measure which indicates how difficult it is to trade an asset. In Solvency II, liquidity for the holder of an asset is defined as ‘the ability to sell or cash in this the asset at any time at a price equal to the present value of the future cash flows discounted at the risk free interest rate, but adjusted for expected credit risk and credit risk uncertainty’ ([CEIOPS, 2010](#)). Illiquidity could for example occur in case an asset is not immediately saleable due to the absence of a market where the asset is regularly traded or in case the value of the asset is uncertain. Since the difficulty of trading an asset is a risk, there needs to be a compensation for the additional
risk in the price of the asset. The price of an illiquid asset is in general lower than the price for a similar liquid asset. The liquidity premium is defined as the difference between the prices of the illiquid asset and the liquid asset. In case the liquidity premium decreases, the risk is less rewarded and as a result it is less attractive to invest in illiquid assets. This is a risk for the holder of the illiquid asset.

In Solvency II, illiquidity risk also applies to the liabilities. This is caused by the way liabilities have to be discounted. In FTK, the liabilities have to be discounted by the swap curve. In Solvency II, the swap curve has to be modified slightly in order to achieve a discount curve which is consistent with a risk-free interest rate curve. The curve has to be decreased as a compensation for the credit risk and afterwards increased to compensate for the illiquidity premium. The reasoning behind the latter is that there is an absence of a market where liabilities could be regularly traded. This means that liabilities are illiquid.

The effect on the Regulatory Capital Requirement of an increase in the credit spread is captured in the spread risk module. The effect on the Regulatory Capital Requirement of a decrease of the illiquidity premium is captured in this illiquidity risk module.

In the paper of the CFO Forum and CRO Forum ([Forum, 2010]) it is proposed to reduce the inter-bank swap curve by 10bps in order to reflect the impact of credit risk. Afterwards, the curve should be increased in order to compensate for the illiquidity premium. The illiquidity premium per annum relative to the swap curve is equal to 43bps in the first 10 years and then linearly reduces to 0bps within the next 5 years.

The capital requirement for illiquidity risk is by definition the effect on the NAV in case the illiquidity premium decreases by 65%. In formula this means:

$$Mkt_{illiquidity} = \max(\Delta NAV | \text{decrease of illiquidity premium by 65\%}, 0)$$

(3.28)

It makes sense that a decrease of the illiquidity premium is a risk for the pension fund. Namely, in case the illiquidity premium decreases, the liabilities should be discounted by a lower curve and as a result the present value of the technical provisions will increase.

In the paper of the Forum ([Forum, 2010]), the results of the proxy measure of the liquidity premium are stated. The proxy is derived from market data. The results can be found in figure 3.2. From figure 3.2 we can conclude that in times of crisis, the illiquidity premium increases. An increase of the illiquidity premium leads to a decrease of the present value of the technical provision.

Solvency II prescribes shocks in order to generate a worst case scenario. How-
ever, in figure 3.2 we see that in a worst case scenario the illiquidity premium increases instead of decreases. Thus, in a worst case scenario there is no illiquidity risk. It even has a positive effect on the funding ratio. So in a worst case scenario, the increase of the illiquidity premium can partly neutralize the negative effects of the other risks. For that reason, we think that in the formula which determines the Regulatory Capital Requirement, illiquidity risk should be negatively correlated with the other risks.

3.3.2 Regulatory Capital Requirement under Solvency II for life underwriting risk

The Regulatory Capital Requirement for life underwriting risk is specified in the technical specifications of the Solvency II directive (QIS5, 2010) and is equal to:

$$SCR_{life} = \sqrt{\sum_{r,c} Corr_{LIFE_{r,c}} \cdot LIFE_r \cdot LIFE_c}$$

(3.29)

where,

$$Corr_{LIFE_{r,c}} =$$ The correlation between subcomponent r and c according to the correlation matrix (see table 3.12)

and $LIFE_r$ and $LIFE_c$ have to be replaced by the following seven subcomponents:

- $LIFE_{mortality}$: component for mortality risk
- $LIFE_{longevity}$: component for longevity risk
- $LIFE_{disability}$: component for disability risk
3.3.2.1 Mortality risk

Mortality risk is the risk that an insurer faces in case it makes policy agreements in which the insurer has to make one or more payments in case of death of the insured. So, this risk applies to latent widowers pension. Obviously, in case the mortality rates increase, the technical provisions will increase as well. As a result, the funding ratio will decrease.

The component for mortality risk is by definition the effect on the NAV in case all mortality rates increase by 15%. In formula:

\[ LIFE_{\text{mortality}} = (\Delta NAV \mid \text{increase in mortality rates of 15\%}) \]  (3.30)

3.3.2.2 Longevity risk

Longevity risk is the risk that an insurer faces in case it makes policy agreements in which the insurer has to make recurring payments to the insured until the policy holder dies. So, this risk applies to elderly pension as well as widowers pension which is already started. An example of such a policy agreement is an annuity contract. In case the mortality rates decrease, the technical provisions will increase. This makes sense since the period in which payments have to be made to the insured becomes longer. As a result, the funding ratio will decrease.

The component for longevity risk is the effect on the NAV in case all mortality rates decrease by 20% by definition. In formula:

\[ LIFE_{\text{longevity}} = (\Delta NAV \mid \text{decrease in mortality rates of 20\%}) \]  (3.31)
3.3.2.3 Disability and morbidity risk

Disability and morbidity risk is the risk that the liabilities will increase due to changes in the trend, volatility or level of morbidity- and disability rates. These risks are present in policy agreements which are constructed in such a way that the insurer has to make recurring payments in case of disability or illness. The date of the final payment depends on the contract. This can for example be in case of death, recovery or on a predefined date.

The component for disability and morbidity risk consists of two parts. The first part is the effect on the NAV in case the disability rates increase by 35% in the next year. For the following years, an increase of the disability rates by 25% should be applied. The second part is the effect on the NAV in case the recovery rates decrease by 20%. In formula:

\[ \text{LIFE}_{\text{disability}} = (\Delta \text{NAV} | \text{increase in disability rates}) \]

\[ + (\Delta \text{NAV} | \text{decrease in recovery rates of 20\%}) \]  

In formula (3.32)

In this thesis, we assume no recovery of disabled participants. This is a very prudent assumption.

3.3.2.4 Lapse risk

Lapse risk is the risk that insurers face since policy holders have the right to surrender the insurance policy. Since it is not possible to surrender a pension policy, lapse risk is beyond the scope of this thesis.

3.3.2.5 Expense risk

Expense risk is the risk that the expenses of the insurance company are higher than expected at the time the premium was set. The component for expense risk is the effect on the NAV in case the value of the expected expenses increases by 10%. In addition, the expense inflation rate increases by 1% per year. In short notation this is:

\[ \text{LIFE}_{\text{expense}} = (\Delta \text{NAV} | \text{increase of expenses and inflation rate}) \]  

In formula (3.33)

In this thesis, we determine the component for expense risk as the effect on the NAV in case the cost surcharge increases by 10%.

3.3.2.6 Revision risk

The intention of revision risk is to "capture the risk of adverse variation of an annuity’s amount, as a result of an unanticipated revision of the claims process" ([CEIOPS, 2009](#)).

The component for revision risk is the effect on the NAV in case the annual
amount which has to be paid for annuities increases by 3%. In short notation this is:

\[ \text{LIFE}_{\text{revision}} = (\Delta \text{NAV} | \text{increase of annual amount to be paid of 3\%}) \] (3.34)

Since this risk should only be applied to non-life claims, we do not take this risk into account in this thesis.

### 3.3.2.7 CAT risk

Catastrophe risk is the risk that an extreme event occurs. This could for example be a pandemic or nuclear explosion. This risk is mainly relevant for contracts in which the insurer has to make one or more payments in case of death of the policyholder. The component for catastrophe risk is the effect on the NAV in case the number of policyholders dying over the coming year increases by 1,5 per mille in absolute numbers. In short notation this is:

\[ \text{LIFE}_{\text{CAT}} = (\Delta \text{NAV} | \text{increase of number of people dying by 1,5 per mille}) \] (3.35)

Catastrophe risk is reinsured by our pension fund. This means the risk component for CAT risk is equal to zero.
Chapter 4

Modeling Regulatory Capital Requirements under FTK and Solvency II

In this chapter, we will apply the FTK and Solvency II legislation to a specific pension fund. The characteristics of the pension fund are described in section 4.1. In section 4.2 and 4.3 we will determine the Regulatory Capital Requirements under the conditions of the FTK and Solvency II legislation respectively. Finally, we will compare both legislations.

4.1 Description of the pension fund

Since we would like to perform this study as realistically as possible we use the data of an existing pension fund. Due to privacy reasons, we have to reduce the data such that the pension fund is not recognizable anymore. In the pension fund there are about 1,500 unique participants. We would like to reduce this number by about 15%. First, we delete the participants who have deviating pension types. After this reduction, there are 1467 participants left. Thereupon, we randomly delete participants in order to maintain the characteristics of the pension fund as much as possible. After the complete reduction, 1275 participants are left.

In the data, participants are divided into groups. The groups which are present in the reduced data are: active participants, disabled participants, deferred participants, retired participants and widowers. Elderly pension and widower’s pension are the only two types of pension which are present in the data. The pensionable age is 65 years. The average age of the participants is 54.9 years and 61% of the participants is male.
<table>
<thead>
<tr>
<th>Group</th>
<th>Elderly Pension</th>
<th>Widower’s Pension</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active</td>
<td>3,821,865</td>
<td>1,443,532</td>
<td>5,265,397</td>
</tr>
<tr>
<td>Disabled</td>
<td>162,873</td>
<td>79,100</td>
<td>241,973</td>
</tr>
<tr>
<td>Deferred</td>
<td>2,394,126</td>
<td>1,418,491</td>
<td>3,812,617</td>
</tr>
<tr>
<td>Retired participants</td>
<td>4,216,313</td>
<td>2,508,442</td>
<td>6,724,755</td>
</tr>
<tr>
<td>Widowers</td>
<td>-</td>
<td>1,201,684</td>
<td>1,201,684</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>10,595,178</strong></td>
<td><strong>6,651,249</strong></td>
<td><strong>17,246,427</strong></td>
</tr>
</tbody>
</table>

Table 4.1: Pension entitlements disaggregated per pension type and pension group

The total value of the annual pension entitlements\(^1\) is 17.2 million euros. The disaggregation of the pension entitlements per pension type and per group are stated in table 4.1.

The technical provision is defined as the amount of money which the pension fund needs to have at this moment in time in order to be able to meet its future obligations. The calculation of the technical provision is based on the discounted value of the expected future payments to the participants. Discounting can be performed by either a fixed discount rate, nominal term structure or real term structure.

When a fixed discount rate is used for discounting, the classical technical provision is determined. In table 4.2 the classical technical provision is disaggregated per pension type and per group. The provision is calculated by using a fixed discount rate of 4%. The nominal technical provision is determined using the nominal term structure of 31-12-2011 published by DNB. The disaggregated nominal technical provision is stated in table 4.3. Obviously, the real technical provision is determined using the real term structure of 31-12-2011 which is as well published by DNB. The disaggregated real technical provision can be found in table 4.4.

When we compare the three different technical provisions, we see that the classical technical provision has the lowest value and the real technical provision the highest. It makes sense that the value for the nominal technical provision is higher than the value for the classical technical provision, since the nominal term structure is completely below 4%. In case the interest rate is lower, the technical provision becomes higher since there is less discounting. The real interest rate is even lower than the nominal interest rate, since it is corrected for inflation. As a result, the real technical provision will be higher than the nominal technical provision.

Note that all technical provisions which are stated in the tables are gross techni-

\(^1\)in Dutch: pensioenaanspraken
Classical technical provisions (gross)

<table>
<thead>
<tr>
<th>Group</th>
<th>Elderly Pension</th>
<th>Widower’s Pension</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active</td>
<td>36,269,115</td>
<td>3,077,901</td>
<td>39,347,016</td>
</tr>
<tr>
<td>Disabled</td>
<td>1,810,647</td>
<td>179,901</td>
<td>1,990,548</td>
</tr>
<tr>
<td>Deferred</td>
<td>22,410,751</td>
<td>2,742,727</td>
<td>25,153,479</td>
</tr>
<tr>
<td>Retired participants</td>
<td>44,404,578</td>
<td>8,166,231</td>
<td>52,570,809</td>
</tr>
<tr>
<td>Widowers</td>
<td>-</td>
<td>13,704,798</td>
<td>13,704,798</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>104,895,091</strong></td>
<td><strong>27,871,558</strong></td>
<td><strong>132,766,649</strong></td>
</tr>
</tbody>
</table>

Table 4.2: Classical technical provisions (gross) disaggregated per pension type and pension group in Euros

Nominal technical provisions (gross)

<table>
<thead>
<tr>
<th>Group</th>
<th>Elderly Pension</th>
<th>Widower’s Pension</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active</td>
<td>45,959,238</td>
<td>4,259,676</td>
<td>50,218,914</td>
</tr>
<tr>
<td>Disabled</td>
<td>2,202,311</td>
<td>241,588</td>
<td>2,443,899</td>
</tr>
<tr>
<td>Deferred</td>
<td>28,596,963</td>
<td>3,843,737</td>
<td>32,440,699</td>
</tr>
<tr>
<td>Retired participants</td>
<td>49,618,652</td>
<td>9,781,527</td>
<td>59,400,179</td>
</tr>
<tr>
<td>Widowers</td>
<td>-</td>
<td>15,586,606</td>
<td>15,586,606</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>126,377,163</strong></td>
<td><strong>33,713,134</strong></td>
<td><strong>160,090,297</strong></td>
</tr>
</tbody>
</table>

Table 4.3: Nominal technical provisions (gross) disaggregated per pension type and pension group in Euros

Cal provisions. This means that the costs for executing the pension scheme are also included. The gross technical provision is defined as the value for the net technical provision multiplied by (1 + costs surcharge). For our pension fund, the costs surcharge is equal to 1.4%.

### 4.2 Regulatory Capital Requirement under FTK

In this thesis, we determine the Regulatory Capital Requirement at the calculation date 31-12-2011. As we already mentioned in the previous section, we have removed some random participants of the fund for privacy reasons. This will have consequences for the liabilities. The original funding ratio of the pension fund is equal to 83.7%. Since we would like to keep the characteristics of the fund, we scale the total value of the assets such that the initial funding ratio remains 83.7%. In order to determine the Regulatory Capital Requirement, we use the strategic asset mix of the pension fund. We use the strategic asset mix instead of the actual asset mix since the actual asset mix is just the asset mix at one particular moment in time while the strategic asset mix is the target asset mix of the fund. The strategic asset mix is stated in table 4.5. In order to determine the Regulatory Capital Requirement under FTK, we have
Table 4.4: Real technical provisions (gross) disaggregated per pension type and pension group in Euros

<table>
<thead>
<tr>
<th>Group</th>
<th>Elderly Pension</th>
<th>Widower’s Pension</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active</td>
<td>62,261,406</td>
<td>6,503,261</td>
<td>68,764,667</td>
</tr>
<tr>
<td>Disabled</td>
<td>2,787,664</td>
<td>346,994</td>
<td>3,134,658</td>
</tr>
<tr>
<td>Deferred</td>
<td>39,050,713</td>
<td>6,004,920</td>
<td>45,055,634</td>
</tr>
<tr>
<td>Retired participants</td>
<td>55,583,916</td>
<td>12,083,525</td>
<td>67,667,441</td>
</tr>
<tr>
<td>Widowers</td>
<td>-</td>
<td>17,979,973</td>
<td>17,979,973</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>159,683,699</strong></td>
<td><strong>42,918,673</strong></td>
<td><strong>202,602,372</strong></td>
</tr>
</tbody>
</table>

Table 4.5: Strategic asset mix of the pension fund

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government Bonds</td>
<td>36%</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>24%</td>
</tr>
<tr>
<td>Stocks</td>
<td>40%</td>
</tr>
</tbody>
</table>

Table 4.5: Strategic asset mix of the pension fund

to do some iterations. First, we start with the total current value of the assets. This total value is divided into the subcategories of the assets using the strategic asset mix. Afterwards, the shocks prescribed by the FTK legislation are applied. As a result, we find the first value of the Regulatory Capital Requirement. Since we would like the capital requirement to be independent of the current value of the assets, we have to find an equilibrium. In order to achieve this, the new total current value of the assets is equal to the old total current value plus the value of the technical provision. Again, the total value is divided according to the asset mix and the shocks are applied to the new values. This time we find the second value of the Regulatory Capital Requirement. This process is repeated until the capital requirement remains stable. In practice, this is achieved after about 5 iterations. The capital requirement we find in the last iteration is called the Regulatory Capital Requirement.

When we apply the iterative process to our pension fund, we find that the Regulatory Capital Requirement under FTK is equal to 25,386,031 euro. This is consistent with a required funding ratio of 115.9%. The general results of the calculation are stated in table 4.6. In table 4.7, we show the partition of the Regulatory Capital Requirement under FTK into the subcomponents $S_1$ until $S_9$.

Figure 4.1 is a graphical representation of the Regulatory Capital Requirement under FTK. The percentages are defined as the capital requirement per sub-component divided by the discounted value of the liabilities. From this figure, we can conclude that interest rate risk and equity risk are the largest risks for
<table>
<thead>
<tr>
<th>Subcomponent</th>
<th>Capital Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$ Interest rate risk</td>
<td>8,844,624</td>
</tr>
<tr>
<td>$S_2$ Equity risk</td>
<td>18,547,633</td>
</tr>
<tr>
<td>$S_3$ Currency risk</td>
<td>5,675,576</td>
</tr>
<tr>
<td>$S_4$ Commodity risk</td>
<td>0</td>
</tr>
<tr>
<td>$S_5$ Credit risk</td>
<td>1,749,343</td>
</tr>
<tr>
<td>$S_6$ Technical Insurance risk</td>
<td>4,784,276</td>
</tr>
<tr>
<td>$S_7$ Liquidity risk</td>
<td>0</td>
</tr>
<tr>
<td>$S_8$ Concentration risk</td>
<td>0</td>
</tr>
<tr>
<td>$S_9$ Operational risk</td>
<td>0</td>
</tr>
</tbody>
</table>

| $S$ Total Regulatory Capital Requirement | 25,386,031 |

Table 4.7: Subcomponents of the Regulatory Capital Requirement under FTK

### 4.3 Regulatory Capital Requirement under Solvency II

Just as we did for the Regulatory Capital Requirement under FTK, we determine the Regulatory Capital Requirement under Solvency II at the calculation date 31-12-2011. In table 4.8 it can be seen that the discounted value of the liabilities under Solvency II differs from the discounted value of the liabilities which we found under FTK. This is caused by a difference in the discount curve. As we already mentioned, the liabilities under Solvency II have to be discounted by the swap curve which is altered to correct for the illiquidity premium. The discounted value of the liabilities under Solvency II is equal to 158,225,013. Since we would like to have the same initial funding ratio under FTK and Solvency II, we scale the value of the assets such that the initial funding ratio equals 83.7%. We again use the strategic asset mix which is stated in table 4.5.
Table 4.8: General results under Solvency II

Recall that under Solvency II, the capital requirement for credit risk is defined as:

\[ Mkt_{b,sp}^{bond} = \max \left( \sum_i MV_i \times Duration_i \times F_{up}(rating_i), 0 \right) \]  

(4.1)

where,

- \( MV_i \) = the market value of risk exposure
- \( Duration_i \) = modified duration of the risk exposure
- \( F_{up} \) = spread risk factors which can be found in table 3.6

In order to be able to determine the component for credit risk, we determine the
market value, duration and rating class for every corporate bond. The duration is defined as:

\[
D = \frac{\sum_{t=1}^{n} t C_t (1+i_t)^t}{\sum_{t=1}^{n} C_t (1+i_t)^t}
\]  

(4.2)

where \( n \) is the maturity of the bond, \( C_t \) is the cashflow at time \( t \) and \( i \) the discount curve. We found that the total capital requirement for credit risk is equal to 5,450,400.

When we apply the shocks according to the rules of Solvency II to our pension fund, we find that the Regulatory Capital Requirement under Solvency II is equal to 41,767,990 which is consistent with a required funding ratio of 126.4%. Table 4.9 states the partition of the Regulatory Capital Requirement under Solvency II into the subcomponents.

<table>
<thead>
<tr>
<th>Subcomponent</th>
<th>Capital Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate risk</td>
<td>11,685,734</td>
</tr>
<tr>
<td>Equity risk</td>
<td>23,999,160</td>
</tr>
<tr>
<td>Property risk</td>
<td>0</td>
</tr>
<tr>
<td>Credit spread risk</td>
<td>5,450,400</td>
</tr>
<tr>
<td>Currency risk</td>
<td>7,649,732</td>
</tr>
<tr>
<td>Concentration risk</td>
<td>0</td>
</tr>
<tr>
<td>Illiquidity risk</td>
<td>645,987</td>
</tr>
<tr>
<td>Mortality risk</td>
<td>177,707</td>
</tr>
<tr>
<td>Longevity risk</td>
<td>8,828,201</td>
</tr>
<tr>
<td>Disability risk</td>
<td>0</td>
</tr>
<tr>
<td>Lapse risk</td>
<td>0</td>
</tr>
<tr>
<td>Expense risk</td>
<td>221,515</td>
</tr>
<tr>
<td>Revision risk</td>
<td>0</td>
</tr>
<tr>
<td>CAT risk</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total Regulatory Capital Requirement</strong></td>
<td><strong>41,767,990</strong></td>
</tr>
</tbody>
</table>

Table 4.9: Subcomponents of the Regulatory Capital Requirement under Solvency II

Figure 4.2 presents a graphical representation of the Capital Requirement under Solvency II. Just as in the graphical representation of the Regulatory Capital Requirement under FTK, the percentages are defined as the Capital Requirement per subcomponent divided by the discounted value of the liabilities. From figure 4.2 it can be concluded that equity risk, interest rate risk and longevity risk are the largest three risks to our pension fund.
4.4 Comparison Regulatory Capital Requirement
FTK and Solvency II

The largest difference between FTK and Solvency II is the difference in security level. Since the security level under Solvency II is higher, the shocks which have to be applied to the assets and liabilities are tougher as well. In figure 4.10 we give an overview of the shocks which have to be applied according to both legislations.

Another important difference between both legislations is that Solvency II applies to insurance companies in EMEA countries while FTK only applies to Dutch pension funds.
<table>
<thead>
<tr>
<th></th>
<th>FTK</th>
<th>Solvency II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate risk</td>
<td>Max effect shock table 2.1</td>
<td>Max effect shock table 3.4</td>
</tr>
<tr>
<td>Equity risk</td>
<td>Depreciation shares on mature markets 25%, emerging markets 35%, private equity 30%</td>
<td>Depreciation global equity of 30%, other equity 40%</td>
</tr>
<tr>
<td></td>
<td>Depreciation of 15%</td>
<td>Depreciation of 25%</td>
</tr>
<tr>
<td>Real Estate risk</td>
<td>Depreciation currency of 20% wrt euro</td>
<td>Depreciation currency of 25% wrt euro</td>
</tr>
<tr>
<td>Real Estate risk</td>
<td>Depreciation of 30%</td>
<td>Depreciation of 40%</td>
</tr>
<tr>
<td>Commodity risk</td>
<td>Increase credit spread 40%</td>
<td>Size depends on rating asset (see table 3.6)</td>
</tr>
<tr>
<td>Currency risk</td>
<td>(process risk + $\sqrt{TSO^2 + NSA^2}$) * technical provision</td>
<td>Mortality risk: increase in mortality rates of 15%</td>
</tr>
<tr>
<td></td>
<td>Mortality risk: increase in mortality rates of 15%</td>
<td>Longevity risk: decrease in mortality rates of 20%</td>
</tr>
<tr>
<td></td>
<td>FTK</td>
<td>Solvency II</td>
</tr>
<tr>
<td>Credit risk</td>
<td>0% by assumption</td>
<td>Decrease in illiquidity premium of 65%</td>
</tr>
<tr>
<td>Credit risk</td>
<td>0% by assumption</td>
<td>Excess risk * shock table 3.10</td>
</tr>
<tr>
<td>Credit risk</td>
<td>0% by assumption</td>
<td>-</td>
</tr>
<tr>
<td>Liquidity risk</td>
<td>0% by assumption</td>
<td>-</td>
</tr>
<tr>
<td>Concentration risk</td>
<td>0% by assumption</td>
<td>-</td>
</tr>
<tr>
<td>Concentration risk</td>
<td>0% by assumption</td>
<td>-</td>
</tr>
<tr>
<td>Operational risk</td>
<td>0% by assumption</td>
<td>-</td>
</tr>
<tr>
<td>Disability risk</td>
<td>Increase in disability rates: next year 35%, following years: 25%</td>
<td>Increase in disability rates: next year 35%, following years: 25%</td>
</tr>
<tr>
<td></td>
<td>Decrease in recovery rates of 20%</td>
<td>Decrease in recovery rates of 20%</td>
</tr>
<tr>
<td></td>
<td>We assume disability risk to be reinsured so we assume 0%</td>
<td>We assume disability risk to be reinsured so we assume 0%</td>
</tr>
<tr>
<td>Lapse risk</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Expense risk</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CAT risk</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Discounting liabilities</td>
<td>Swap curve</td>
<td>Swap curve corrected for credit risk and illiquidity premium</td>
</tr>
</tbody>
</table>

Table 4.10: Overview of size of shocks in FTK and Solvency II
In this chapter, we calculated that the required funding ratio for our pension fund is equal to 115.9% under FTK and 126.4% under Solvency II. This means that increasing the security level from 97.5% to 99.5%, leads to an increase in the required capital of about 10% for this specific example. In figure 4.3, we give an overview of the difference in sizes of subcomponents under FTK and Solvency II.

Figure 4.3: Graphical representation Regulatory Capital Requirement under FTK and Solvency II
Chapter 5

ALM model Hoevenaars

The aim of this chapter is to model the dynamics of assets and liabilities in order to be able to model possible developments of the funding ratio by simulating scenarios over a one year horizon. When we have the funding ratio for each scenario, we will determine the number of scenarios in which the value of the liabilities exceeds the value of the assets. In this chapter we use the model developed by Hoevenaars et al. ([Hoevenaars et al., 2007]) in order to model the dynamics of the assets and the liabilities. Hereafter we will refer to this model simply as the Hoevenaars model.

5.1 Description of the Hoevenaars model

The model developed by Hoevenaars is a vector autoregression model (VAR model). A VAR model is a statistical model which is used for the analysis of multiple time series. VAR models are commonly used in order to describe dynamic behaviour. Besides, it is also useful for forecasting. A VAR model consists of equations for each endogenous variable $i$. These equations describe the evolution of the variables over time. The equations are constructed as linear functions of the past evolution of the variables. A $p$-th order vector autoregression model, $VAR(p)$, is defined as:

$$y_t = c + B_1 y_{t-1} + B_2 y_{t-2} + \ldots + B_p y_{t-p} + \epsilon_t$$

(5.1)

where,

- $c$ = a vector of length $i$ which consists of constants
- $B_j$ = a matrix of size $i \times i$, for each $j = 1 \ldots p$
- $\epsilon_t$ = a vector of length $i$ which consists of error terms

In this model, we use the data of $p$ periods before time $t$ in order to determine $y_t$. This is why this $VAR(p)$ model is sometimes also referred to as a vector autoregression model with $p$ lags. The Hoevenaars model is constructed as a
VAR model with 1 lag.

The return vector $z_t$ contains the following 12 asset classes: return on bonds, the nominal 3-month US treasury bill, the real 3-month US treasury bill yield, the credit spread, the dividend-price ratio, return on stocks, the yield spread, return on commodities, credits, hedge funds, return on the liabilities and the real-estate. The historical data of the first 7 asset classes are available for a much longer period than for the last five asset classes. Since we would like to use as much data as possible, we apply the same approach as Hoevenaars to estimate the VAR model.

Hoevenaars solves the data problem by dividing the return vector into two groups of asset classes. The first 7 asset classes remain in the return vector $y_t$. For the last 5 asset classes we introduce a return vector $x_t$. The first group of asset classes still follows the unrestricted VAR(1) model:

$$y_t = c + By_{t-1} + \epsilon_t$$ \hspace{1cm} (5.2)

where,

- $y_t$ = a vector of length 7 which consists of the returns of the first asset group
- $B$ = a matrix of size $7 \times 7$
- $\epsilon_t$ = a vector of length 7 which consists of error terms

An unrestricted VAR model means that the assets affect both their own expected returns and the expected returns of the other assets. Note that in this VAR model the error terms are normally distributed with mean zero and covariance matrix $\Sigma_{\epsilon\epsilon}$.

The second group of asset classes follows a restricted VAR model:

$$x_t = d + D_0y_t + D_1y_{t-1} + Hx_{t-1} + \eta_t$$ \hspace{1cm} (5.3)

where,

- $x_t$ = a vector of length 5 which consists of the returns of the second asset group
- $d$ = a vector of length 5 which consists of constants
- $D_0, D_1$ = matrices of size $7 \times 5$
- $y_t, y_{t-1}$ = vectors of length 7 which both consist of the returns of the assets in the first group
- $H$ = a diagonal matrix of size $5 \times 5$
- $\eta_t$ = a vector of length 5 which consists of error terms
The error terms of the restricted model are normally distributed with mean 0 and covariance matrix \( \Omega \). The diagonal matrix \( H \) is the matrix which causes the restriction in the model. In words, it implies that the return of asset \( i \) in the second asset group, only affects its own expected return. Hence, it does not affect the expected returns of the other asset classes in the second group. The restricted model is based on the assumption that the assets in the second asset group have no dynamic feedback to the other assets. Note that the asset groups are all dependent on the variables in the first asset group \( y_t \). This can be concluded since \( D_0 \) and \( D_1 \) are unrestricted matrices.

When we want to combine both VAR models and write it in matrix notation, just as in [Hoevenaars et al., 2007], we first have to rewrite \( x_t \) such that \( y_t \) is not present in the formula anymore:

\[
x_t = d + D_0 y_t + D_1 y_{t-1} + H x_{t-1} + \eta_t
\]

\[
= d + D_0 (c + B y_{t-1} + \epsilon_t) + D_1 y_{t-1} + H x_{t-1} + \eta_t
\]

\[
= d + D_0 c + D_0 B y_{t-1} + D_0 \epsilon_t + D_1 y_{t-1} + H x_{t-1} + \eta_t
\]  
\( (5.4) \)

When we combine formula (5.2) and (5.4) and write it in matrix notation we find:

\[
z_t = \Phi_0 + \Phi_1 z_{t-1} + u_t
\]  
\( (5.5) \)

where,

\[
z_t = \begin{bmatrix} y_t \\ x_t \end{bmatrix}, \quad \Phi_0 = \begin{bmatrix} c \\ d + D_0 c \end{bmatrix}, \quad \Phi_1 = \begin{bmatrix} B \\ D_0 B + D_1 H \end{bmatrix}, \quad u_t = \begin{bmatrix} \epsilon_t \\ D_0 \epsilon_t + \eta_t \end{bmatrix}
\]

In this model, we follow [Hoevenaars et al., 2007] and assume the covariance of \( \epsilon_t \) and \( \eta_t \) to be equal to zero. Since the expectations of \( \epsilon_t \) and \( \eta_t \) both equal zero, the covariance matrix of \( u_t \) can be constructed in the following way:

\[
\Sigma = \begin{bmatrix} Cov(\epsilon_t, \epsilon_t) & Cov(\epsilon_t, (D_0 \epsilon_t + \eta_t)) \\ Cov((D_0 \epsilon_t + \eta_t), \epsilon_t) & Cov((D_0 \epsilon_t + \eta_t), (D_0 \epsilon_t + \eta_t)) \end{bmatrix}
\]  
\( (5.6) \)

where,

- \( Cov(\epsilon_t, \epsilon_t) = Var(\epsilon_t) = \Sigma_{ee} \)
- \( Cov(\epsilon_t, (D_0 \epsilon_t + \eta_t)) = E(\epsilon_t (D_0 \epsilon_t + \eta_t)) = E(\epsilon_t D_0 \epsilon_t + \epsilon_t \eta_t) = E(\epsilon_t D_0 \epsilon_t) + E(\epsilon_t \eta_t) = E(\epsilon_t \epsilon_t) D_0 + \Sigma_{ee} D_0 \)
- \( Cov((D_0 \epsilon_t + \eta_t), \epsilon_t) = E((D_0 \epsilon_t + \eta_t) \epsilon_t) = E(\epsilon_t D_0 \epsilon_t + \epsilon_t \eta_t) = E(\epsilon_t D_0 \epsilon_t) + E(\epsilon_t \eta_t) = E(\epsilon_t \epsilon_t) D_0 + \Sigma_{ee} D_0 \)
- \( Cov(D_0 \epsilon_t + \eta_t, D_0 \epsilon_t + \eta_t) = E((D_0 \epsilon_t + \eta_t)^2) = E(D_0 \epsilon_t D_0 + \eta_t^2 + 2D_0 \epsilon_t \eta_t) = D_0^2 E(\epsilon_t \epsilon_t) D_0 + E(\eta_t^2) = D_0^2 Var(\epsilon_t) D_0 + Var(\eta_t) = D_0^2 \Sigma_{ee} D_0 + \Omega \)
As a result, the covariance matrix of \( u_t \) is defined as:

\[
\Sigma = \begin{bmatrix}
\Sigma_{cc} & \Sigma_{ce} D_0 \\
\Sigma_{ec} D_0 & D_0' \Sigma_{ee} D_0 + \Omega
\end{bmatrix}
\]  
(5.7)

5.2 Data

The choice of asset classes in this thesis is based on the datasets used by [Hoevenaars et al., 2007] and [Campbell and Viceira, 2001]. All returns are defined as logreturns. For all assets, we use quarterly data.

5.2.1 Description of the data

5.2.1.1 Asset group 1

Return on bonds

The return on bonds is constructed using the 20-Year Treasury Constant Maturity Rate which can be obtained from the website of FRED ([Federal Reserve Bank of St. Louis, 2012]). For some reason, this data is not available from January 1987 until September 1993. We estimate the missing values by performing a least squares regression in Eviews. In this regression we use the 20-Year Treasury Constant Maturity Rate as the dependent variable and the regressors are a constant, the 10-Year Treasury Constant Maturity Rate and the 30-Year Treasury Constant Maturity Rate. The latter two data series can as well be obtained from the website of FRED ([Federal Reserve Bank of St. Louis, 2012]). Since the data of the 30-Year Treasury Constant Maturity Rate starts at February 1977 and is not available from March 2002 until January 2006, we use the period February 1977 until February 2002 as the sample period for the regression.

Now we have the complete data series, we can approximate the return by the following formula which is proposed by [Hoevenaars et al., 2007]:

\[
br_{n,t+1} = \frac{1}{4} y_{n,t+1} - D_{n,t} (y_{n,t+1} - y_{n,t})
\]  
(5.8)

where,

- \( n \) = the maturity of the bond
- \( D_{n,t} \) = the duration of the bond
- \( y_{n,t} = \ln(1 + Y_{n,t}) \)
- \( Y_{n,t} \) = the yield of the bond

The duration of the bond is approximately given by:

\[
D_{n,t} = \frac{1 - (1 + Y_{n,t})^{-n}}{1 - (1 + Y_{n,t})^{-1}}
\]  
(5.9)
Since we approximate the bond return using the 20-Year Treasury Constant Maturity Rate, we set $n$ equal to 20.

**Nominal US 3-Month Treasury Bill**
The data of the Nominal US 3-Month Treasury Bill is obtained from the website of FRED \(\text{[Federal Reserve Bank of St.Louis, 2012]}\). A T-bill is a short-term government loan since the maturity is at most one year. It is often used in models as the risk free rate.

**Real US 3-Month Treasury Bill**
In order to determine the Real US 3-Month Treasury Bill, we first have to obtain the inflation rates. In this thesis, the Consumer Price Index for All Urban Consumers All Items (CPI) is used for inflation. These rates are also obtained from the website of FRED \(\text{[Federal Reserve Bank of St.Louis, 2012]}\). Afterwards, the Real US 3-Month Treasury Bill can be determined by correcting the Nominal US 3-Month Treasury Bill for inflation.

**Credit spread**
We define the credit spread as the difference between the credit yield and the Nominal US 3-Month Treasury Bill. This gives an indication to what extent risk taking is rewarded. For the credit yield we use the Moody’s Seasoned Baa Corporate Bond Yield, which can be obtained from the website of FRED \(\text{[Federal Reserve Bank of St.Louis, 2012]}\).

**Dividend-price ratio**
The dividend-price ratio is constructed using the S&P Composite and Dividend which we obtain from the website of Shiller \(\text{[Shiller, 2012]}\).

**Return on stocks**
The return on stocks is defined as the logreturn on the S&P Composite.

**Yield spread**
We define the yield spread as the difference between the 10-Year Treasury Constant Maturity Rate and the Nominal US 3-Month Treasury Bill.

### 5.2.1.2 Asset group 2

**Commodities**
For the return on commodities we use the Standard and Poors Goldman Sachs Commodity Index Commodity Spot (GSCI). The data of the return on commodities is obtained from Datastream.

**Credit returns**
The credit returns are based on the US Long Term Corporate Bond Index and can be found at Ibbotsen via the website of Duke University \(\text{[Ibbotson, 2005]}\).
Hedge funds
We use the Conservative Hedge Fund Research Indices (HFRI index) as data for the hedge fund. This data can be obtained from Datastream. According to [Hoevenaars et al., 2007], this "return index is broadly diversified across different style factors" and as a consequence the data "exhibits different risk and return properties".

Return on the Liabilities
The liability return can be determined by the following formula which is developed by [Hoevenaars et al., 2007]:

\[
\begin{align*}
r_{L,t+1} &= \frac{1}{4}rr_{n,t+1} - D_{n,t} (rr_{n,t+1} - rr_{n,t}) + \pi_{t+1} \\
&= \text{The return on the liabilities} \\
rr_{n,t} &= \text{The n-period real yield (based on the 10-Year Treasury Constant Maturity Rate)} \\
D_{n,t} &= \text{Modified duration of the liabilities (assumed to be equal to 15.5} \\
&\quad \text{in order to be consistent with our pension fund}) \\
\pi_{t+1} &= \text{Inflation rates (CPI)}
\end{align*}
\]

In equation (5.10), unconditional indexation is assumed. This explains why the discounting of the liabilities should be performed by the real yield.

Real-estate
For real-estate we use the FTSE NAREIT US Real Estate Index Series. This data can be obtained from the website of REIT ([REIT, 2012]).

For the summary statistics of the data we refer to Appendix A.

5.3 Simulation
Now we have the data we need, we can estimate the parameters of the Hoevenaars model and afterwards simulate the possible developments of all assets and liabilities.

5.3.1 VAR estimation
When we perform a Vector Autoregression model with one lag in Eviews on the assets in the first asset group with a constant included, we get the estimates which are stated in table 5.1.

In this table, we see that the nominal T-bill also shows some predictive power for the stock returns. The nominal T-bill is negatively correlated with the stock returns. This is in line with the results found by [Fama and Schwert, 1977] and
A possible explanation for the negative correlation could be that in case the nominal T-bill rises, it becomes more attractive to save money. As a result, there is less demand for stocks which results in lower stock returns.

The $R^2$ of stock returns is equal to 0.07. This shows that the stock returns are hard to explain by our model. [Campbell and Thompson, 2007] demonstrate that variables which have a low $R^2$ can still be economically meaningful. We also expect to find a low $R^2$ for stock returns since stock returns are hard to predict.

Table 5.1 shows that both the nominal T-bill as well as the yield spread have much predictive power for bond returns. The difference between the nominal T-bill and the bond return has to do with the length until maturity. Nominal T-bills are short term investments and bonds are usually long-term investments. In our case we constructed the bond returns using the 10-year Treasury Constant Maturity Rate.

The high level of predictive power of the yield spread can be explained by the fact that the yield spread is defined as the difference between the 10-year Treasury Constant Maturity Rate and the nominal T-bill. The yield spread indicates the trends in the growth of the economy. In general, the yield spread will decrease in time of expansion and increase during times of recession. In the paper of [Hoevenaars et al., 2007], data was used until 2005, and the mean level of yield spread was equal to 1.24. When we include the data of the last 6 years (in which we had a period of recession), the mean level of yield spread is equal to 1.39. Thus, our historical data supports the statement that the yield spread will increase during times of recession.

The $R^2$ of bond returns is equal to 0.11. This means that the explanatory power for bond returns is slightly higher than for stock returns. However, it remains hard to explain.

A striking point in the results of the VAR(1) estimation is the difference in the value of R-squared for the real T-bill and the nominal T-bill. The nominal T-Bill can be pretty well predicted by our model since the value of R-squared equals 0.93. However, the R-squared for the real T-bill equals 0.32. Since the real T-Bill is defined as the difference between the nominal T-Bill and the Consumer Price Index, the difference must be caused by the difficulty of predicting the Consumer Price Index.

### 5.3.2 Determining parameters Hoevenaars model

In order to be able to determine the necessary parameters, we first have to perform Least Square Regressions on the assets in the second asset group. Equation 5.3 shows that assets in the second group depend on all assets in the first group.
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<th>DP</th>
<th>YS</th>
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<td>0.96</td>
<td>0.78</td>
<td>0.84</td>
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Table 5.1: Vector Autoregression Estimates with associated T-statistics in [ ]
as well as on itself. We emphasize again that the assets in the second group do not depend on the other assets in the second asset group. As a consequence, we perform different Least Square regressions for each asset in the second group.

The procedure which we follow in order to get the estimates we need is as follows. First we perform a Least Square analysis of the asset on itself as well as on all assets in the first asset group. We would like to explain the dependent variable by as little assets as possible since we do not have many datapoints of the dependent variable available. As a consequence, we delete the least significant variable each time the Least Squares regression is performed and rerun the regression. The regression in Eviews tests whether the variable is significant different from zero or not thus it is a two-sided regression. As a result, we repeat the procedure of deleting variables until all variables are significant at 5%.

After we have performed all Least Square regressions, we are able to calculate all matrices we need for the simulation. The matrices for $\Phi_0$, $\Phi_1$ and $\Sigma$ can be found in appendix B in tables B.1, B.2 and B.3 respectively.

### 5.3.3 Simulation of asset returns and associated funding ratios

Since all matrices are defined, we only need to generate values for $\epsilon_t$ in order to be able simulate the VAR(1) model. We already mentioned that the error terms are normally distributed with mean zero and covariance matrix $\Sigma$. Thus, we want to generate random numbers which have mean zero and follow covariance matrix $\Sigma$. This can be achieved by a method proposed by Cholesky. One first has to determine the Cholesky factor $L$. The Cholesky factor $L$ is a lower triangular matrix and is defined as:

$$LL^T = \Sigma \quad (5.11)$$

Afterwards, $n$ numbers have to be drawn from the standard normal distribution. As a result, the vector $Ln$ will be a vector of error terms which are normally distributed with mean zero and covariance matrix $\Sigma$.

In order to simulate the possible developments of all assets, we use Monte Carlo simulation and run the model 10,000 times over 60 quarters. As starting values, we use the most recent observation in the data. The Matlab code we use for simulation is stated in appendix C.

The mfile simulates 10,000 times the total return for all assets over a period of both 1 year and 15 years. When we multiply these returns by the asset mix, we find 10,000 possible developments of the funding ratio. The asset mix of our pension fund can be found in table 4.5. When the possible developments of the funding ratio are multiplied by the starting value of the funding ratio, we find 10,000 possible funding ratios in 1 year and 15 years from now. Now, we would
like to test whether it is indeed the case that in 2.5% or 0.5% of the cases under FTK and Solvency II respectively we have underfunding.

5.4 Results

5.4.1 Results based on requirements FTK

In chapter 4 we determined that, according to the FTK, the minimal funding ratio of our pension fund should be 115.9%. We will use this funding ratio as the starting value of the funding ratio.

After simulating the Hoevenaars model, we find that the funding ratio one year from now is in the worst case only 86.26%. On the other hand, in the most optimistic scenario the funding ratio rises from 115.9% at the start to 198.86% in one year from now. The average funding ratio equals 126.81%. This means that on average the funding ratio will increase in the coming year. However, the standard deviation of the distribution of the funding ratios equals 14.80%. We find that in 2.05% of the scenarios we are dealing with an underfunding in one year from now. The histogram of the funding ratios in one year from now can be found in figure 5.1.

![Figure 5.1: Histogram of possible funding ratios 1 year from now](image)

Since we would like to know what happens to the funding ratio in the long run in case no intervention takes place, we also determined the funding ratios 15 years from now. It turns out that in the worst case scenario the funding ratio even drops to 52.47%. On the other hand, in the most favorable scenario we find that the funding ratio rises to 466.56%. Just like in case of a one year horizon, the average funding ratio in 15 years from now is increased with respect to the current funding ratio. On average, the funding ratio over a 15 year horizon will increase to 137.35%. The standard deviation is much larger in case of the
extended horizon and is equal to 36.70%. We find that in 13.18% of the sce-
narios we have a funding ratio in 15 years from now which will be below 100%.
The histogram of the funding ratios in 15 years from now is stated in figure 5.2.

As we already mentioned in chapter 2, the FTK prescribes that the Regu-
latory Capital Requirement under FTK should be established in such a way that,
with a confidence level of 97.5%, after one year the value of the assets will exceed
the value of the liabilities. In other words, the probability of underfunding in
one year from now should be at most 2.5%. Using the Hoevenaars model, we
find that in 2.05% of the scenarios we have underfunding with a 95% confidence
interval of (1.77%,2.33%). So when we compare our ALM model to the FTK,
we can reject the hypothesis that FTK estimates the risk correctly at a 5% level
assuming that the Hoevenaars model describes the world correctly. In this case,
FTK overestimates the number of scenarios in which underfunding takes place.
In other words, the Hoevenaars model is less prudent. When we would like to
have the probability of a funding ratio below 100% in one year from now to
be 2.5%, we need a current funding ratio of 114.53%. As a consequence, the
associated Regulatory Capital Requirement under FTK will be lower in case it
will be based on the outcomes of the Hoevenaars model.

We determined the distribution of the funding ratios in 15 years from now in
order to see whether intervention in case of underfunding is necessary. From the
results we can conclude that in case no intervention takes place, the probability
of underfunding increases significantly. It seems that the market is not able to
cover extreme losses itself within 15 years. Recently, it was announced that on
April 1, 2013 many Dutch pension funds need to cut the pension entitlements.
The conclusion that intervention is needed for the funding ratios to recover, is
in line with the results from our long term analysis.
5.4.2 Results based on requirements Solvency II

In section 4.3 we found that the required funding ratio is equal to 126.4%. In order to test whether it is indeed the case that in 0.5% of the cases we have underfunding, we use the required 126.4% as the starting value for the funding ratio.

After simulating the Hoevenaars model 10,000 times, we find that in the worst case scenario we have a funding ratio of 94.08% in one year from now. In the most optimistic scenario we find a funding ratio of 216.87%. On average, the funding ratio will increase in the coming year from 126.4% to 138.29%. However, there is a standard deviation of 16.14%. We find that in 0.13% of the cases, we have a funding ratio which is less than 100%. This percentage is less than the required 0.5%. According to the Hoevenaars model, the initial funding ratio should be equal to 121.50% in order to have a funding ratio which is below 100% in 0.5% of the cases. A histogram of the possible funding ratios in one year from now can be found in figure 5.3.

![Histogram of possible funding ratios 1 year from now](image)

Since we are also interested in the development of the funding ratios in the long term, we also simulate the model 10,000 times over a 15 year horizon. We find that the minimum funding ratio is equal to 57.22%. The highest funding ratio in 15 years from now is equal to 508.83%. On average, the funding ratio increases from 126.4% to 149.80%. However, we find a standard deviation of 40.02%. In case the initial funding ratio equals 126.40%, the probability of underfunding in 15 years from now is equal to 6.43%. Figure 5.4 contains a histogram of the possible funding ratios 15 years from now.

In chapter 3 we already mentioned that according to the Solvency II Directive, the Regulatory Capital Requirement under Solvency II should be determined in
such a way that the funding ratio in one year from now will be at least 100% with a confidence level of 99.5%. This means that underfunding should occur at most once every 200 years which is consistent with a probability of underfunding in one year from now of 0.5%. By simulating the Hoevenaars model we find that in 0.13% of the cases we have a funding ratio which is below the 100% with a 95% confidence interval of (0.06%, 0.20%). So when we compare the Hoevenaars model to Solvency II, we can reject the hypothesis that Solvency II estimates the risk correctly at a 5% level assuming that the Hoevenaars model is correct. In this case, the probability of underfunding of 0.5% is overestimated. This means that the Solvency II Directive is more prudent than the ALM model of Hoevenaars. When we would like the probability of underfunding in one year from now to be 0.5% under the Hoevenaars model, we need a current funding ratio of 121.50% instead of 126.4%. As a result, the Regulatory Capital Requirement under Solvency II will be lower in case it will be based on the Hoevenaars model.
Chapter 6

ALM model Towers Watson

In this chapter we will model the dynamics of assets and liabilities using the ALM model which has been developed by Towers Watson. This ALM model is called the Global Asset Model (hereafter: GAM model). We again simulate 10,000 scenarios and determine the possible developments of the funding ratios. Afterwards, we will check whether the probabilities of underfunding which we find are in line with the requirements of FTK and Solvency II.

6.1 Description GAM model

The underlying philosophy of the GAM model can be summarized in a few statements. The model is based on a combination of history, theory and current market conditions together with judgment based on current and forward looking yields. An overview of the structure of the input for the GAM model is stated in figure 6.1. Towers Watson takes the view that the past can be very helpful in judging uncertainty. On the other hand, they believe that history can only be of limited used for predicting the future. Towers Watson chooses to develop a global model, since they have the view that global uniformity can be helpful in reducing sampling errors. For example, global inflation influences local inflation so global inflation can be very useful in predicting local inflation.

Basically, the model works as follows. First, for different economic regions around the world the model calculates the series separately. Second, the series are combined in order to form a global one. Finally, the ALM experts from each country can modify the Global Model in order to adapt it to the local conditions. The Global Investment Committee (GIC) determines the assumptions for each country or region on a quarterly basis. The assumptions are based on the judgment of the committee itself as well as on the opinions of other ALM experts. Via a survey, these opinions are collected. We are aware of the fact that the GAM model is very subjective due to judgments of ALM experts. However, every model is based on assumptions which are debatable. For example, the
Hoevenaars model is built on the assumption that the past is representative for the future. Towers Watson believes that all approaches can lead to valuable insights. Since we have no insight into how the input is exactly converted to the output, we treat the model as a black box model.

### 6.2 Data description

Since we cannot simulate the GAM model ourselves, we are dependent on the data which Towers Watson puts into the model. As a result, we cannot use the exact same data as input for the model as we did in the Hoevenaars model. In the GAM model, 29 assets are available. We try to use similar data as much as possible.

We define the Nominal T-Bill as the Europe Cash Return. These returns are the returns on a short-term investment. Using the Nominal T-Bill, we again construct the Real T-Bill by correcting the Nominal T-Bill for inflation. The Real T-Bill is defined as the difference between the Europe Cash Return and the Europe CPI Rate.

As stock return we use the Europe Equity Return and we define the bond return as the Europe Long All Government Bond Return with a duration of 20. For commodities, we take the Commodity Hedged Return and the Europe Property Return is used as the return on Real Estate. We set the Credit Returns equal to the High Yield Hedged Return and for Hedge funds we use the Fund of Hedge Fund Hedged Return.
For consistency between the Hoevenaars model and the GAM model, we use the same formula for liabilities. Recall that the return on the liabilities is defined by:

\[ r_{L,t+1} = rr_{n-1,t+1} - D_{n,t} (rr_{n-1,t+1} - rr_{n,t}) + \pi_{t+1} \]  

(6.1)

where,

\[ r_{L,t+1} = \text{the return on the liabilities} \]
\[ rr_{n,t} = \text{the n-period real yield} \]
\[ D_{n,t} = \text{duration of the liabilities} \]
\[ \pi_{t+1} = \text{inflation rates} \]

We defined the \( rr_{n,t} \) as the Europe Core Real Yield which has a duration of 15. Just as in the Hoevenaars model, we assume \( D_{n,t} \) to be equal to 15.5 in order to match the duration with the duration of the liabilities of our pension fund. For \( \pi_{t+1} \) we again use inflation.

6.3 Simulation

From the Investment Department of Towers Watson we received 10,000 simulations for all assets over a 15 year horizon. Since we want to determine the possible developments of the funding ratio in one year from now, we multiply the returns by the asset mix. We use the same asset mix as we did in the Hoevenaars model. The asset mix is stated in table 4.5.

We first use the GAM model in order to test whether or not we have underfunding in at most 2.5% of the scenarios in case we use the required funding ratio from FTK as input. Afterwards, we will test whether or not the probability of underfunding is at most 0.5% in case we use the required funding ratio from Solvency II as the current funding ratio.

We determine the possible funding ratios for three different cases. First, we determine the funding ratios in one year from now using the returns of the first year. Second, we repeat this procedure using the one-year returns of year 10. In the model of Towers Watson, short-term expectations are included which makes the first few years less stable. Since we want to circumvent the short-term expectations and prefer a stable model, we also determined the funding ratios using the one-year returns of year 10. Finally, we calculate the possible funding ratios 15 years from now. The latter can be interesting in order to see whether or not the funding ratio recovers by itself.
6.4 Results

6.4.1 Results based on requirements FTK

Recall that the required funding ratio according to the FTK legislation is equal to 115.9% for our pension fund. Again, we use this funding ratio as value for the current funding ratio. After determining the possible funding ratios in one year from now using the returns of the first year, we find that the worst case scenario is a funding ratio of 71.09%. The most optimistic scenario will result in a funding ratio of 233.58%. On average, the funding ratio in one year will be equal to 124.63%. This means that on average the funding ratio will increase in the coming year. However, the standard deviation of the distribution equals 18.82%. The histogram of the possible funding ratios in one year from now using the returns of the first year is stated in figure 6.2. In this thesis, the focus is on the number of cases in which we are dealing with underfunding. We find that in 8.06% of the scenarios with a 95% confidence interval of (7.53%, 8.59%) we are dealing with a funding ratio which is below 100%. This interval is above the 2.5% which is required according to the FTK. The starting value of the funding ratio should be equal to 126.24% in order to achieve the required 2.5% for scenarios in which we have underfunding.

In case we determine the possible funding ratios in one year from now using the returns in year 10, we find that for the scenario which leads to the lowest funding ratio, we have a funding ratio of 72.14%. On the other hand, for the scenario which leads to the highest funding ratio we find a ratio of 172.32%. On average, the funding ratio will increase slightly in the first year from 115.9% to 118.0%. However, we have a standard deviation of 12.11%. The histogram of the possible funding ratios in one year from now using the returns of year 10
Figure 6.3: Histogram of possible funding ratios 1 year from now using returns year 10

We find that in 6.42% of the scenarios we have a funding ratio which is less than 100%. This probability of underfunding has a 95% confidence interval which is equal to (5.94%, 6.90%). This means that this does not satisfy the requirements as defined in the FTK. The minimum required starting value of the funding ratio should be equal to 122.1% according to the GAM model.

When we compare the funding ratios in one year from now determined by the returns in the first year and the returns in year 10, we see that this makes a large difference. The standard deviation in case the 10th year return is used, is 6.71% less than in case of the return of the first year. This leads to a reduction of the percentage of underfunding of 1.64%. Since there are fewer positive extreme values, the average funding ratio as well decreases by 6.63%. In case of the first year returns, the average funding ratio is 124.63%. This can lead to a much more positive view on how the assets will develop in the coming year than in case of the 118.0% we find in the 10th year scenarios.

In order to be able to see what happens in the long run in case no intervention takes place, we determine the funding ratios 15 years from now. In the worst case scenario we find a funding ratio which is equal to 47.27%. In the most favorable scenario we find a funding ratio of 870.87%. On average, the funding ratios will increase from 115.9% to 177.0%. However, the distribution of the funding ratios has a standard deviation of 80.91%. The high standard deviation is caused by a few extreme values. These outliers also cause the average funding ratio to be high. The histogram of the possible values of the funding ratios in 15 years from now are stated in figure 6.4. We find that in 8.62% of the scenarios we have underfunding with a 95% confidence interval of (8.07%, 9.17%).

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When we compare the possible funding ratios in one year from now with the possible funding ratios 15 years from now, we see that the standard deviation increases significantly. However, the fraction of cases in which we have a funding ratio which is lower than 100% increases slightly. Given the fact that we have underfunding, the average funding ratio in case of a horizon of 1 year (94.3%) is higher than the average funding ratio in case of a 15 year horizon (89.24%). So, we can conclude that the number of cases in which we have underfunding slightly increases in case the horizon increases. However, if we have underfunding, the funding ratio will on average be less than we would have in the case of a shorter horizon.

Comparing the results of the one year horizon using the returns in year 10 with the results of a horizon of 15 years, we again conclude that the standard deviation increases significantly. However, in this case we see that the number of cases in which we have underfunding increases more over time (from 6.42% to 8.62%). This could imply that intervention is needed.

### 6.4.2 Results based on requirements Solvency II

Recall that we found in section 4.3 that the required funding ratio under Solvency II is equal to 126.4%. When we determine the possible funding ratios in one year from now using the returns of the first year and 126.4% as the starting value of the funding ratio, we find that in the worst case scenario the funding ratio equals 77.53%. In the most optimistic scenario the ratio is equal to 254.74%. On average the funding ratio will increase in the coming year from 126.4% to 135.92%. The standard deviation is equal to 20.52%. We find that in 2.43% of the scenarios we have a funding ratio which is less than 100% with a 95% confidence interval of (2.13%, 2.73%). This is more than the required 0.5%. In figure 6.5 we state the histogram of the possible funding ratios in one year.
We also determined the possible funding ratios in one year from now using the returns in year 10. In that case we find that in the worst case scenario the funding ratio is equal to 78.68%. The highest funding ratio which we found equals 187.94%. The funding ratio will on average increase in the coming year from 126.4% to 128.69%. The standard deviation is at 13.21%, 7.31% less than the standard deviation which we found in case we use the returns of the first year. We find that in 1.20% (with (0.99%, 1.41%) as 95% confidence interval) of the scenarios we have a situation in which the value of the assets is less than the value of the liabilities. This is more than the required 0.5% which means that we can reject the hypothesis that Solvency II estimates the risk correctly at a 5% level. The histogram of the possible funding ratios in one year from now using the returns of year 10 is stated in figure 6.6.

Comparing the results we found using the returns in the first year and the returns in year 10, we can conclude that this has much effect on the outcomes. The standard deviation is reduced by 7.31% in case the returns in year 10 are used. Logically, the number of scenarios in which we have underfunding also decreases from 2.43% to 1.20%. This means that using the return from year 10 leads to a reduction in the percentage of underfunding of more than 50%. The average return in case we use the first year return is equal to 135.92%. This is caused by some positive extreme values. Since the standard deviation in year 10 is much less, the average return is also less (128.69%).

In order to perform a long term analysis we determine the possible funding ratios in 15 years from now. The lowest funding ratio we find is equal to 51.55%. On the other hand, we find a funding ratio of 949.77% in the most favorable
scenario. On average, the funding ratio will increase in the next 15 years to 193.04%. The distribution of the possible funding ratios in 15 years from now, has a very large standard deviation of 88.28%. The probability of underfunding is equal to 4.60% with a 95% confidence interval of (4.19%, 5.01%). Figure 6.7 contains the histogram of the possible funding ratios in 15 years from now.

When we compare the results of the possible funding ratios in one year from now with the results obtained using a 15 year horizon we see that the standard deviation increases significantly. The standard deviation over a 15 year horizon is very large (88.28%). This means that there is a lot of uncertainty. The number of scenarios in which we have underfunding increases over time with from
When we compare the results over a horizon of one year using the returns of year 10 with the results over a 15 year horizon, we again conclude that the uncertainty increases significantly over time. The probability of underfunding increases by about a factor 4 over time. This could imply that financial losses cannot be recovered by the market over a long term horizon. This is an argument in favor of intervention.
Chapter 7

Performance ALM models versus FTK and Solvency II

In this chapter, we will test the performance of the standard models versus the ALM models. We also address the differences. Obviously, one major difference between the legislations and the ALM models is that both legislations describe just one worst case scenario, while the ALM models develop 10,000 scenarios. We end this chapter with a comparison between the Hoevenaars model and the GAM model.

7.1 Hoevenaars model versus FTK

In section 5.4.1 we found that in case we use the required funding ratio according to the FTK legislation (115.9%) as the starting value of the funding ratio, we have a probability of underfunding in one year from now which is equal to 2.05% with a 95% confidence interval of (1.77%, 2.33%). This means that we can reject the hypothesis that FTK estimates the risk correctly at a 5% level assuming that the Hoevenaars model describes the world correctly. In this case we find that the Hoevenaars model is less prudent than the FTK legislation. Since we made some assumptions in our Hoevenaars model, the comparison between the ALM model and FTK is not completely fair. Since the FTK model takes more risks into account, we have to determine the required funding ratio under FTK again, using only the risks which the Hoevenaars model also takes into account. Since we remove some risks from the standard model, the required funding ratio will decrease. As a result, the probability of underfunding which we find in the Hoevenaars model will increase.

One risk which we do take into account in the FTK model and which we do not take into account in the Hoevenaars model is the currency risk. For comparison the performance of the two models, we set the exposure to currency risk in the FTK model from 15.3% to 0%. Reducing the currency risk to 0% leads to a
Another difference between the models is the technical insurance risk. In the Hoevenaars model we use a stationary fund. We only project the possible movements of the assets and liabilities in one year from now regarding mortality risk or longevity risk. Thus, we assume that someone who dies will be replaced with someone who has the same characteristics. Therefore we reduce the component for technical insurance risk in the standard model to 0. This leads to another reduction in the required funding ratio from 115.4% to 115.1%.

The final difference is that under FTK the risks for the liabilities, government bonds and corporate bonds are determined using the cashflows. Under the Hoevenaars model we have no cashflows available, so the risks are estimated using the durations. In order to be able to compare the Hoevenaars model with the ALM model, we also determine the risks under FTK using the durations. For consistency, we set the duration of the liabilities equal to 15.5 since this is the duration of the cashflow of the liabilities of the pension fund. The duration of the government bonds is set equal to 20 since under the ALM model we use the 20-Year Treasury Constant Maturity Rate. Under the Hoevenaars model, the corporate bonds are estimated using the US Long Term Corporate Bond Index. Since we do not know the duration of this index, we estimate the duration by the mean of the corporate bonds in which our pension fund has invested. This duration is about 15 years. We set the credit spread equal to 3.14% since this is the starting value of the spread for the Hoevenaars model. Applying the FTK standard model after adapting the model leads to a required funding ratio of 113.5%.

When we use 113.5% as the initial funding ratio in the Hoevenaars model, we find that the probability of underfunding in one year from now is equal to 2.97% with a 95% confidence interval of (2.64%, 3.30%). In order to have the probability of underfunding equal to 2.5%, we need an initial funding ratio in the Hoevenaars model which is equal to 114.53%. We find that the FTK legislation is a little less prudent than the Hoevenaars model. Another way stated, based on the historical data of this thesis applied to this specific pension fund, according to the Hoevenaars model, we reject the hypothesis that FTK legislation estimates the risk correctly at a 5% level.

### 7.2 GAM model versus FTK

In section 6.4.1 we found that using the initial funding ratio of 115.9%, we have a probability of underfunding which is equal to 8.06% with a 95% confidence interval of (7.53%, 8.59%). This is significantly more than the required 2.5%. However, this comparison is not completely fair. Just as in the comparison with the Hoevenaars model, the FTK legislation takes some risks into account which are not included in the GAM model. Again, eliminating risks from the FTK
legislation will lead to a decrease of the required funding ratio and as a result will increase the probability of underfunding even more.

For the same reasons as in the previous section, currency risk and technical insurance risk have to be eliminated. Again, we use the duration approach instead of the cash flow approach. This means that the duration of the liabilities is equal to 15.5 for this fund. Also, the duration of the government bonds equals 20 years. However, the duration for the corporate bonds is just 4 years since we only have data available of corporate bonds with this duration. This duration is less than the duration based on the cash flow. Since the reduction in duration has much influence on the component for interest rate risk and the latter is the largest risk for the pension fund, we expect that this has much influence on the required funding ratio. After adapting the durations, we find a required funding ratio which is equal to 114.4%.

As described above, the Regulatory Capital Requirement under FTK is larger in case we use the assumptions of the GAM model than in case of the Hoevenaars model. This should be caused by the difference in duration of the corporate bonds. Corporate bonds have effect on the component for interest rate risk as well as on the component for credit risk. In figure 7.1, both capital requirements are split into the subcomponents.

![Figure 7.1: Graphical representation Regulatory Capital Requirement under FTK for Hoevenaars and GAM](image)

From figure 7.1 we can indeed conclude that the difference in the required funding ratio is caused by the corporate bonds. Under the Hoevenaars model, the component for interest rate risk is less than under the GAM model. Since bonds can be used for hedging against interest rate risk, it makes sense that
the component is less using the Hoevenaars model because the duration of the corporate bonds is larger in case of the Hoevenaars model. We also see that the component for credit risk is more under the Hoevenaars model. This makes sense since credit risk increases in case the duration increases. Since the interest rate risk has more effect on the total Regulatory Capital Requirement due to the correlation between interest rate risk and equity risk, the increase in interest rate has more weight and as a result, the required funding ratio under the GAM model is higher.

When we use the 114.4% as input for the GAM model, we find that the probability of underfunding increases to 9.51%. In order to have the probability of the event in which the value of the assets is less than the value of the liabilities equal to 2.5% under the GAM model, we need an initial funding ratio which is equal to 126.24%. This means that the GAM model is way more prudent than the rules according to the FTK. Concluding, based on the GAM model applied to this specific pension fund using the historical data which we stated in section 5.2, the FTK underestimates the risk significantly.

### 7.3 Hoevenaars model versus Solvency II

Section 5.4.2 states that given the initial required funding ratio of 126.4%, the probability of underfunding in one year from now is equal to 0.13% with a 95% confidence interval of (0.06%,0.20%). Since this is less than the 0.5% which is required, this could imply that Solvency II overestimates the risk for our pension fund. However, in order to make a good comparison between the models, we have to adjust the Solvency II risks to the risks which are included in the Hoevenaars model. This will cause the required funding ratio to decrease and as a result will increase the probability of underfunding in one year from now. This could lead to another conclusion regarding whether or not Solvency II underestimates the risk for our pension fund according to the Hoevenaars model.

Several risks are present in the standard model of Solvency II which are not included in the GAM model. The first risk is currency risk. In case we change the exposure to foreign currencies from 15.3% to 0% we find that the required funding ratio decreases from 126.4% to 124.5%. Since in the GAM model we assume that everyone survives the coming year, we also set the values for mortality risk and longevity risk equal to 0. Eliminating mortality risk in the standard model hardly leads to a reduction in the required funding ratio, since the component for mortality risk is very small for our pension fund. However, eliminating longevity risk does have a large impact on the required funding ratio. The latter will decrease from 124.5% to 122.4%. Expense risk is another risk which is not included in the GAM model. Since the component for expense risk is very small, the required funding ratio will only decrease by 0.1% to 122.3%. The final risk which we eliminate is illiquidity risk. This will have a large impact on the required funding ratio since we also have to eliminate the illiquidity premium.
This means that the liabilities have to be discounted by a discount curve which is lower and as a result, the discounted value of the liabilities will increase. This will lead to a reduction of the required funding ratio to 121.7%.

For the same reasons as in the first section, we change the cash flow approach to the duration approach. Again, the duration of the liabilities is equal to 15.5 for this fund. Also, the duration of the government bonds equals 20 years and the duration of the corporate bonds is equal to 15. We find that the required funding ratio decreases from 121.7% to 120.3%.

When we use 120.3% as the initial funding ratio for the Hoevenaars model, we find that the probability of an event in which the value of the assets is less than the value of the liabilities in one year from now, is equal to 0.66% with a 95% confidence interval of (0.50%, 0.82%). Thus, under the assumption that the Hoevenaars model is correct, we cannot reject the hypothesis that Solvency II estimates the risk correctly.

7.4 GAM model versus Solvency II

In section 6.4.2 we found that using the initial funding ratio of 126.4%, the probability of underfunding in one year from now is equal to 2.43% with a 95% confidence interval of (2.13%, 2.73%). This is significantly more than the 0.5% which is required according to the Solvency II legislation. However, in order to be able to compare the GAM model with the Solvency II directive, we have to eliminate some risks from the Solvency II directive. Probably this will increase the probability of underfunding in one year from now even more.

In the previous section we already discussed that currency risk, mortality risk, longevity risk, expense risk and illiquidity risk should be eliminated from the standard model in order to take the same risks into account under both GAM and Solvency II. Also, we adapt the durations in the standard model to the duration of the data used under the GAM model. As a result, the required funding ratio which we have to use for comparison is equal to 118.0%.

Recall that under FTK we found that the required funding ratio under the Hoevenaars model is less than under the GAM model. Under Solvency II, we find the opposite. In figure 7.2 we show the capital requirement for the ALM model per subcomponent. We only show the results for the capital requirements which are different from 0%.

From this figure we can conclude that again the capital requirement for interest rate risk is less under the Hoevenaars model. As we already mentioned, this is caused by the difference in duration. The little difference in capital requirement for equities has to do with the fact that the Regulatory Capital Requirement is determined in an equilibrium state (as we already explained
Figure 7.2: Graphical representation Regulatory Capital Requirement under Solvency II for Hoevenaars and GAM

In case we would determine the Regulatory Capital Requirement in the current situation, the capital requirements for equity risk would be equal. The final difference is the difference in capital requirement for credit risk. The component under the Hoevenaars model is much higher due to the longer duration of the corporate bonds. Since under Solvency II, the difference between the capital requirements for credit risk is much larger than under FTK, the total Regulatory Capital Requirement under the Hoevenaars model is larger.

Using the new required funding ratio as initial funding ratio under the GAM model leads to a probability of underfunding in one year from now of 6.59%. This is more than 10 times as much as the required probability of underfunding under Solvency II. In order to satisfy the capital requirements under Solvency II, the required funding ratio should be equal to 137.8% according to the GAM model. This means that the GAM model is much more prudent than the standard model of Solvency II. Otherwise stated, when we test the Solvency II capital requirements using simulations based on historical data under the GAM model, we have strong evidence that the capital requirement under Solvency II is not sufficient under the GAM model.

7.5 Hoevenaars versus GAM

In the previous sections we interpreted the results of both ALM models using as input the required funding ratios which we found after adapting the standard models under FTK and Solvency II such that all models take the same risks...
into account. In figure 7.3 we give a graphical overview of the results.

Both the first and third bar in figure 7.3 under 'FTK funding ratio' and 'SII funding ratio' show the results of the standard models under FTK and Solvency II respectively after excluding the risks from the standard model which are not included in the ALM models. The second bar states how much the initial funding ratio should be using the Hoevenaars model under FTK and Solvency II respectively. The fourth bar states the required level of the initial funding ratio using the GAM model. The columns 'FTK underfunding' and 'SII underfunding' reflect the associated probabilities of underfunding after one year using the funding ratios which are stated under the columns 'FTK funding ratio' and 'SII funding ratio' respectively.

From figure 7.3 we can conclude that the GAM model consistently produces the highest probabilities of underfunding after one year. This means that the number of bad scenarios under the GAM model is larger than under the Hoevenaars model. In order to be able to see which assets cause the bad scenarios, we produce histograms of all three assets which are present in the asset mix of our pension fund under both Hoevenaars and GAM, a histogram of the development of the asset return and liabilities and finally the histogram of the possible funding ratios in one year from now. In order to be able to compare the returns under Hoevenaars and GAM, we plot the histograms on top of each other. The histograms are stated in figure 7.4. Some associated summary statistics can be found in figure 7.5.
Figure 7.3: Overview of the results after adapting FTK and Solvency II such that the same risks as in ALM models are included
The first three histograms in figure 7.4 show that the variance in the returns under GAM is larger than under Hoevenaars. This means that the GAM model is more insecure about the future. On average, the returns under GAM and Hoevenaars differ not very much for stock returns and corporate bond returns. Since there is more spread under the GAM model, we have more negative as well as positive scenarios. Since FTK and Solvency II only focus on the negative scenarios, the GAM model has more negative scenarios in comparison to the Hoevenaars model.

Since it could be possible that a negative result of a particular asset in one scenario is compensated by a positive return of another asset in the same scenario we plot the histogram of the total asset return per scenario as the fourth figure in figure 7.4. The fifth histogram shows that the variance of the liabilities under

Figure 7.4: Development of assets and liabilities under FTK
Hoevenaars is larger than the variance under GAM. The Hoevenaars model has more scenarios in which the value of the liabilities is lower. Obviously, a lower value of the liabilities also leads to a higher value of the funding ratio which thus reduces the probability of underfunding. The final histogram in figure 7.4 shows that there are indeed more states of underfunding under GAM.

Summarizing, the fact that we have more scenarios of underfunding under GAM is caused by more insecurity in the asset returns as well as fewer scenarios in which the value of liabilities decreases.

Figure 7.5: Summary statistics of the simulations

Figure 7.5 shows some remarkable results. First, the histograms under GAM are more skewed than the histograms under the Hoevenaars model. This shows the short term opinion of the experts. The stocks and governments bonds are positively skewed which means that the distribution is slightly skewed to the left. However, this is partly compensated by the corporate bonds which follow a histogram which is negatively skewed and thus is slightly skewed to the right. Since 24% of the total value is invested in corporate bonds, the total asset return is slightly positively skewed as well. So, under GAM the tail on the left side of the distribution of the total returns fatter than the right tail.

Another striking result is that the variance of corporate bonds is less than the variance of government bonds. This is counterintuitive since corporate bonds are in general more risky since credit risk is present. However, we get this result due to the differences in the duration of the bonds. The duration of the government under both GAM and Hoevenaars is equal to 20. The duration of the corporate bond under Hoevenaars is based on a long term corporate bond while the duration of the corporate bonds under GAM is just 4. This difference in
duration explains why the standard deviation of both bonds under GAM differ much more from each other than under Hoevenaars.
Chapter 8

Pension fund 2

The results so far in this thesis are based on one particular pension fund. Since both the FTK and Solvency II legislation should be applicable to all kinds of pension funds, we test the performances of both legislations as well using another pension fund. We end this chapter with a comparison between pension fund 1 and pension fund 2.

8.1 Description of the pension fund

The second pension fund is a pension fund which has about 2600 participants. The modified duration of the previous pension fund was equal to about 15.5 years. The modified duration of our new pension fund is equal to 19.5 years. So this means that the average age of our new pension fund is less than the duration of the first. The discounted value of the liabilities is equal to 560.28 million euro. Since the total value of the assets amounts to 556.43 million euro, the current funding ratio of the pension fund equals 99.3%. The asset mix of the pension fund is stated in table 8.1.

<table>
<thead>
<tr>
<th>Asset Type</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government Bonds</td>
<td>42.5%</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>11.3%</td>
</tr>
<tr>
<td>Stocks</td>
<td>43.7%</td>
</tr>
<tr>
<td>Commodities</td>
<td>2.5%</td>
</tr>
</tbody>
</table>

Table 8.1: Strategic asset mix of pension fund 2

Again, we determine the Regulatory Capital Requirements at the calculation date 31-12-2011. We also make use of the same underlying data.
8.2 Regulatory Capital Requirements

In this section, we determine the Regulatory Capital Requirements under FTK and Solvency II. Since we would like to test whether the probability of underfunding using the required funding ratio is sufficient, we only take the risks into account which are present in the ALM models (see chapter 7).

8.2.1 Regulatory Capital Requirement under FTK

When we apply the shocks which are prescribed by the FTK to our new pension fund, we find that the Regulatory Capital Requirement is equal to 90.56 million euro under the Hoevenaars model and 94.43 million euro under the GAM model. Since the value of the liabilities is equal to 560.28 million euro, the Regulatory Capital Requirement is consistent with a required funding ratio of 116.2% under Hoevenaars and 116.9% under the GAM model. There is a difference in the required funding ratios since the regulatory capital requirements are determined using other durations. The duration of the corporate bonds under the GAM model is 4 years while the duration under the Hoevenaars model is equal to 15 years. This has both an effect on the component for interest rate as well as the component for credit risk. A tabular representation of the results is stated in table 8.2.

<table>
<thead>
<tr>
<th>Available Capital</th>
<th>556,426</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liabilities</td>
<td>560,276</td>
</tr>
<tr>
<td>Current Funding Ratio</td>
<td>99.3%</td>
</tr>
<tr>
<td>Regulatory Capital Requirement Hoevenaars</td>
<td>90,559</td>
</tr>
<tr>
<td>Total Required Capital Hoevenaars (Liabilities+RCR)</td>
<td>650,835</td>
</tr>
<tr>
<td>Required Funding Ratio Hoevenaars</td>
<td>116.2%</td>
</tr>
<tr>
<td>Regulatory Capital Requirement GAM</td>
<td>94,434</td>
</tr>
<tr>
<td>Total Required Capital GAM (Liabilities+RCR)</td>
<td>654,710</td>
</tr>
<tr>
<td>Required Funding Ratio GAM</td>
<td>116.9%</td>
</tr>
</tbody>
</table>

Table 8.2: General results under FTK for pension fund 2

Figure 8.1 states a graphical representation of both Regulatory Capital Requirements under FTK divided into the subcomponents $S_1$ until $S_6$. The percentages of the capital requirements per subcomponent are defined as the capital requirement per subcomponent divided by the current discounted value of the liabilities. This figure shows that interest rate risk and equity risk are again by far the two largest risks for the pension fund. It also shows that the correlation between the components has large impact on the capital requirement. In this example it reduces the requirement by about 5% under Hoevenaars and by about 4% under GAM. The differences in requirements per subcomponent between Hoevenaars...
and GAM occur for the same reasons as in section 7.2.

Figure 8.1: Graphical representation Regulatory Capital Requirement under FTK for pension fund 2

8.2.2 Regulatory Capital Requirement under Solvency II

In case we apply the shocks according to Solvency II to the pension fund, we see that the Regulatory Capital Requirement under Solvency II is equal to 131.50 million euro under Hoevenaars and 126.51 under the GAM model. Since the discounted value of the liabilities is equal to 552.37 million euro, the required funding ratio is equal to 123.5% under the Hoevenaars assumptions while it is equal to 122.6% under the assumptions of the GAM model. Note that in this chapter the discounted value of the liabilities under Solvency II and FTK are equal while there was a difference in chapter 4. Since in this chapter we only take risks into account which are also present in the ALM models, liquidity risk is disregarded. As a result, the liabilities under Solvency II and FTK have to be discounted by the same curve. Table 8.3 contains a tabular representation of the general results.

In order to be able to see which risks are the largest risks to the pension fund, we split the Regulatory Capital Requirement under Solvency II into subrequirements per risk. Again, the percentages of the capital requirements per risk are defined as the capital requirement per subcomponent divided by the current discounted value of the liabilities. The result is graphically stated in figure 8.2. Just as in the previous example, the components for interest rate risk and equity
| Available Capital | 556,426 |
| Liabilities       | 560,276 |
| Current Funding Ratio | 99.3% |
| Regulatory Capital Requirement Hoevenaars | 131,497 |
| Total Required Capital (Liabilities+RCR) Hoevenaars | 691,772 |
| Required Funding Ratio Hoevenaars | 123.5% |
| Regulatory Capital Requirement GAM | 126,506 |
| Total Required Capital (Liabilities+RCR) GAM | 686,782 |
| Required Funding Ratio GAM | 122.6% |

Table 8.3: General results under Solvency II for pension fund 2

risk are the two risks which have the largest impact on the capital requirement. The differences in requirements per subcomponent between Hoevenaars and GAM occur for the same reasons as in section 7.4.

Figure 8.2: Graphical representation Regulatory Capital Requirement under Solvency II for pension fund 2

8.3 Regulatory Capital Requirements versus ALM models

In this section, we will use the required funding ratios which we found in the first part of this chapter, as input for the Hoevenaars and GAM model and test whether or not the required funding ratios are sufficient.
8.3.1 Regulatory Capital Requirements versus Hoevenaars model

In section 8.2.1, we established that the required funding ratio according to the FTK legislation should be 116.2% in order to have the probability of underfunding in one year from now equal to 2.5%. Now we would like to test whether or not an initial funding ratio of 116.2% indeed leads to a probability of underfunding of 2.5% in one year from now. In order to achieve this we simulate 10,000 possible funding ratios using the asset mix which is stated in table 8.1 and data which is explained in section 5.2. We find that in 5.33% of the scenarios we have a funding ratio which is below 100% with a 95% confidence interval of (4.89%, 5.77%). In order to meet the FTK requirements we need an initial funding ratio of 122.7%. This means that with at least 95% confidence, the FTK underestimates the risk.

We found in section 8.2.2 that the required funding ratio under Solvency II for this specific pension fund equals 123.5%. This funding ratio is determined in such a way that the probability of underfunding in one year from now should be equal to 0.5%. In order to test this statement, we simulate 10,000 scenarios using the Hoevenaars model. Obviously, we take 123.5% as the initial value of the funding ratio. Then we find that the probability that the value of the liabilities exceeds the value of the assets in one year from now is equal to 2.25% with a 95% confidence interval of (1.96%, 2.54%). In summary, for this particular pension fund, we do not reject the hypothesis that Solvency II correctly estimates the risk.

8.3.2 Regulatory Capital Requirements versus GAM model

We already verified that the required funding ratio under FTK under the assumptions of the GAM model is equal to 116.9%. This time, we simulate 10,000 possible scenarios for the funding ratio using the GAM model of Towers Watson. We find that the probability of underfunding is equal to 8.61% with a 95% confidence interval of (8.06%, 9.16%). This is more than the required 2.5%. According to the GAM model, the initial funding ratio should be equal to 128.75% in order to have the probability of underfunding in one year from now equal to 2.5%. This means that the FTK underestimates the risk according to the GAM model.

In order to test the Regulatory Capital Requirement under Solvency II we repeat the procedure again. However, this time we take 122.6% as the initial value of the funding ratio. We find that the probability of a funding ratio which is less than 100% in one year from now is equal to 5.13% with a 95% confidence interval of (4.70%, 5.56%). This is more than the required 0.5%. A probability of underfunding of 0.5% is consistent with an initial funding ratio of 141.7%. This means that the Solvency II legislation underestimates the risks in case we compare them to the results we found using the GAM model for this particular
pension fund.

Figure 8.3 contains a graphical overview of the results of this chapter. From this figure, we again conclude that the GAM model consistently has the largest probability of underfunding in one year from now. Since the results for pension fund 2 are based on the same underlying data as for our first pension fund, this is again caused by the fact that under GAM there is more insecurity in the asset returns as well as fewer scenarios in which the value of the liabilities decrease.
Figure 8.3: Overview of the results for pension fund 2
8.4 Pension fund 1 versus Pension fund 2

In this chapter, we determined the Regulatory Capital Requirements under FTK and Solvency II for our new pension fund. In this section, we will compare the results which we found for pension fund 1 and pension fund 2.

8.4.1 Differences in characteristics of the pension funds

The first difference between pension fund 1 and 2 is the age of the participants of the funds. The duration of the liabilities in pension fund 1 is equal to 15.5 years, while the duration of the liabilities under the second equals 19.5 years. Besides this, the total value of the pension entitlements of pension fund 1 is larger than the value for pension fund 2. However, this should not have any influence on the required funding ratio. Another difference is the difference in asset mixes. We recall the asset mixes of both pension funds in table 8.4.

<table>
<thead>
<tr>
<th></th>
<th>Pension fund 1</th>
<th>Pension fund 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government Bonds</td>
<td>36%</td>
<td>42.5%</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>24%</td>
<td>11.3%</td>
</tr>
<tr>
<td>Stocks</td>
<td>40%</td>
<td>43.7%</td>
</tr>
<tr>
<td>Commodities</td>
<td>0%</td>
<td>2.5%</td>
</tr>
</tbody>
</table>

Table 8.4: Strategic asset mix of the pension fund 1 and pension fund 2

8.4.2 Comparison of the Regulatory Capital Requirements

8.4.2.1 FTK

We already verified that the Regulatory Capital Requirement for the second pension fund equals 116.2% under FTK using the assumption of the Hoevenaar model. This is more than the 113.5% which we found for our first pension fund. In order to invest what causes this difference, we state the sizes of the subrequirements for both pension funds in figure 8.4.

In figure 8.4 we see that the component for interest rate risk is less for the first pension fund. This is caused by the fact that the duration of the liabilities for the second pension fund is much larger. A change in the interest rate will have more effect for pension fund 2 since there is more discounting in that pension fund. Since the second pension fund has a little less invested in bonds, interest rate risk is also a little less hedged. This also has little effect on the difference in requirement for interest rate risk. However, the difference in duration of the liabilities has the most influence.

We also see that the component for equity risk is less for pension fund 1. Obviously, since pension fund 2 has invested a little more in stocks, the capital
Figure 8.4: Comparison capital requirements pension fund 1 and pension fund 2 under FTK

Figure 8.5: Required funding ratio and associated probability of underfunding for pension fund 1 and 2 under FTK
requirement for equity risk will be a little larger as well. Only pension fund 2 has a requirement for commodity risk, since only this fund invests in commodities.

Finally, figure 8.4 shows that, contrary to the other subrequirements, the component for credit risk is larger for the first pension fund. This is caused by the fact that pension fund 1 invests more than twice as much in corporate bonds.

In figure 8.3 we give an overview of the results which we find using the Hoevenaars model in case we set the initial funding ratio equal to the required funding ratio. From this we can conclude that the probability of underfunding for pension fund 2 is higher than for pension fund 1.

In order to investigate what causes the probability of underfunding to be higher in comparison to the first pension fund, we plot the histograms of the liabilities and commodities for both pension funds, since these components are different for the pension funds. The result is stated in figure 8.6.

In the left histogram of figure 8.6 we see that the histograms of pension fund 1 and pension fund 2 are the same. This makes sense since the underlying data is equal. In order to test whether the FTK model underestimates the risk of commodities, we determine the 2.5%-quantile. We find that this is consistent with a shock of 53.99%. Since the FTK legislation prescribes a shock of 30% for commodities, the FTK underestimates the risk in comparison to the Hoevenaars model. This causes the probability of underfunding to be more than the required 2.5%. The right histogram shows that an increase in duration of the liabilities leads to more variance in the histogram. This is in line with what we expected to find.
8.4.2.2 Solvency II

In this chapter, we determined that the Regulatory Capital Requirement under Solvency II using the assumptions of the Hoevenaars model for pension fund 2 equals 123.5%. For the first pension fund, we found a required funding ratio of 120.3%. In figure 8.7 we give an overview of the subrequirements for both pension funds. The differences in subrequirements are again caused by the difference in duration of the liabilities and the difference in asset mix between pension fund 1 and 2.

![Figure 8.7: Comparison capital requirements pension fund 1 and pension fund 2 under Solvency II](image)

In case we use the required funding ratios which we found as input for the Hoevenaars model, we find that the probability of underfunding is equal to 2.25% for the second pension fund. Recall that the probability of underfunding for pension 1 equals 0.66%. This means that the probability of underfunding for pension fund 2 deviates more from the required 0.5%.
In order to test whether Solvency II underestimates the risk for commodities, we determine the 0.5%-quantile. We find that this quantile is associated with a shock of 68.6%. This is more than the shock of 40% which has to be applied according to the Solvency II legislation. This means that according to the Hoevenaars model, Solvency II underestimates the risk of commodities.
Chapter 9

Sensitivity Analysis

In this chapter, we will investigate the sensitivity of the model. First, we will investigate the impact of some characteristics of the pension fund, like the current value of the assets, asset mix and cash flow of the liabilities, on the Regulatory Capital Requirement. Finally, we will test the sensitivity of the parameters prescribed by the FTK legislation.

9.1 Assets

As we already mentioned in this thesis, the Regulatory Capital Requirement is independent of the current value of the assets. In our opinion this makes sense since in case this would not be true, the Regulatory Capital Requirement will decrease in case of a decrease in the asset value and consistently increase in case of an increase in the asset value. The legislations only describe how much the required funding ratio should be in order to have the probability of underfunding equal to 2.5% under FTK or 0.5% under Solvency II. The current funding ratio can be useful in judging whether a pension fund meets the requirements. Recall that the asset mix does have influence.

9.2 Asset mix

The asset mix of the pension fund does have influence on the Regulatory Capital Requirements since the asset mix determines the subdivision of the total value of the assets into the asset classes. Since the shock which has to be applied according to the legislation differs per asset class, the Regulatory Capital Requirement will change as well.

In this section we will determine the Regulatory Capital Requirements and associated probabilities of underfunding using different asset mixes for both FTK and Solvency II. The division in the asset mix which we used throughout this thesis for the first pension fund was 36% government bonds, 24% corporate
bonds and 40% stocks. So the mix between bonds and stocks is 60%/40%. All asset mixes which we will investigate are stated in Table 9.1.

<table>
<thead>
<tr>
<th>Asset</th>
<th>mix 1</th>
<th>mix 2</th>
<th>mix 3</th>
<th>mix 4 (mix of fund 1)</th>
<th>mix 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government bond</td>
<td>12</td>
<td>24</td>
<td>30</td>
<td>36</td>
<td>48</td>
</tr>
<tr>
<td>Corporate bond</td>
<td>8</td>
<td>16</td>
<td>20</td>
<td>24</td>
<td>32</td>
</tr>
<tr>
<td>Stocks</td>
<td>80</td>
<td>60</td>
<td>50</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>Bonds/Stocks</td>
<td>20/80</td>
<td>40/60</td>
<td>50/50</td>
<td>60/40</td>
<td>80/20</td>
</tr>
</tbody>
</table>

Table 9.1: Asset mixes for sensitivity analysis

9.2.1 FTK

Figure 9.1 shows the results of the different asset mixes under FTK. From this figure we can conclude that the GAM model produces probabilities of underfunding which are for all asset mixes significantly above the required 2.5%. This means that for all asset mixes, the GAM model is much more prudent than the standard model of FTK. From the figure we can also conclude that the probabilities of underfunding using the Hoevenaars model are much closer to the required probabilities. For the first three asset mixes (in which the percentage invested in stocks is at least the percentage invested in bonds) the probabilities of underfunding are below the required 2.5%, while the last two asset mixes lead to probabilities of underfunding which are above the 2.5%. This could imply that the FTK model overestimates the risk of stocks in comparison to the Hoevenaars model. In order to test this statement we determine the 2.5%-quantile of the scenarios of stocks constructed by the Hoevenaars model. The associated shock which we find at this quantile is equal to 21.32%. Recall that the shock which has to be applied according to the FTK model is 25%. So we can conclude that the FTK model indeed overestimates the risk of stocks in comparison to the Hoevenaars model.

The finding that the probability of underfunding under the Hoevenaars model in asset mix 4 and 5 is above the required 2.5% could imply that the risks of bonds are underestimated. In order to test this statement we first determine the shocks under FTK. Government bonds only have effect on interest rate risk. We find that the shock for interest rate risk which is prescribed by FTK is consistent with an increase in the value of the government bonds by 13.40%. Since the shock prescribed by the legislation is associated with an increase in the value of governments bonds, we determine the 97.5%-quantile of the scenarios in the Hoevenaars model. We find that this quantile is associated with a shock of 17.21%. This means that the FTK model indeed underestimates the risk of government bonds in comparison to the Hoevenaars model.

Corporate bonds have effect on both interest rate risk and credit risk. In FTK,
Figure 9.1: Results of the different asset mixes under FTK
we find that the prescribed shock for interest rate is consistent with an increase of 10.05% in the value of the corporate bonds. We also find that the shock on credit risk is consistent with an increase in the value of corporate bonds of 16.19%. In the Hoevenaars model the two effects are aggregated. We can aggregate these two risks by the square root formula of FTK:

$$\text{Aggregated shock} = \sqrt{S_1^2 + S_5^2} = \sqrt{10.05\%^2 + 16.19\%^2} = 19.05\% \quad (9.1)$$

The 97.5%-quantile of the scenarios for corporate bonds in the Hoevenaars model is equal to 23.12%. This means that the FTK model underestimates the risk for corporate bonds in comparison to the Hoevenaars model. Summarizing, in comparison to the Hoevenaars model, the FTK overestimates the risk for stocks and underestimates the risks for bonds. In table 9.2 we give an overview of the shocks according to the Hoevenaars model and the FTK legislation. We also quantify the differences.

<table>
<thead>
<tr>
<th></th>
<th>FTK</th>
<th>Hoevenaars</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size of shock in value stock</td>
<td>-25%</td>
<td>-21.32%</td>
<td>-3.68%</td>
</tr>
<tr>
<td>Increase in value of government bonds</td>
<td>13.40%</td>
<td>10.05%</td>
<td>3.31%</td>
</tr>
<tr>
<td>Increase in value of corporate bonds</td>
<td>19.05%</td>
<td>23.12%</td>
<td>4.07%</td>
</tr>
</tbody>
</table>

Table 9.2: Increase in values according to FTK and Hoevenaars

In order to see for which asset mixes the FTK overestimates the risk and for which assets mixes it underestimates the risk, we multiply the difference in shocks by the asset mixes. The results are stated in figure 9.2. In case the FTK model overestimates the risk in comparison to the Hoevenaars model, we have a negative number. Contrary, a positive number means that the FTK model underestimates the risk in comparison to the Hoevenaars model. We see that the results of figure 9.2 are in line with the findings of figure 9.1.

![Figure 9.2: Difference in shocks FTK and Hoevenaars multiplied by asset mix](image-url)
9.2.2 Solvency II

Figure 9.3 contains the results under Solvency II. Again, we can conclude that the GAM model produces probabilities of underfunding which are much higher than the required 0.5%. This means that in comparison to Solvency II, the GAM model is much more prudent than the standard model. The probabilities of underfunding which we found using the Hoevenaars model under asset mix 1 until 4 are very close to the required 0.5%. The results under asset mix 5 could differ a little bit more. This could imply that Solvency II underestimates the risk of the bonds in comparison to the Hoevenaars model. In order to test this statement we compare the shocks which we have to apply according to Solvency II with the shocks which we found under the Hoevenaars model.

We find that for government bonds the shock which is prescribed by Solvency II is associated with an increase in the value of government bonds of 18.05%. We compare this with the 99.5%-quantile under the Hoevenaars model which is equal to 22.51%. This means that according to the Hoevenaars model, the legislation underestimates the risk for government bonds. For corporate bonds we have to determine the effect on both interest rate risk and credit risk. The prescribed shock in Solvency II is consistent with an increase in the value of corporate bonds of 12.12% and 21.00% for interest rate risk and credit risk respectively. Again, we have to aggregate these two risks in order to be able to compare the risk with the risk in the Hoevenaars model. This can be done by the following square root formula:

\[
\text{Shock} = \sqrt{\text{interest risk}^2 + \text{credit risk}^2 + 2 \cdot \rho \cdot \text{interest risk} \cdot \text{credit risk}}
\]

\[
= \sqrt{12.12\%^2 + 21.00\%^2 + 2 \cdot 0.5 \cdot 12.12\% \cdot 21.00\%} = 29.02\% \quad (9.2)
\]

Note that the correlation coefficient between interest rate risk and credit risk equals 50% under Solvency II.

In the Hoevenaars model, we find a 99.5%-quantile for corporate bonds of 27.77%. This means that according to the Hoevenaars model, the component for credit risk is slightly overestimated by Solvency II. Summarizing, according to the Hoevenaars model, the risk of government bonds is underestimated by 4.46% and corporate bonds are overestimated by 1.25%. In the asset mixes which we tested in this sensitivity analysis, the percentage invested in government bonds is always larger than the percentage invested in corporate bonds. This is the case since we constructed the asset mixes in such a way that the ratio between the two bonds remains equal. However, as a result, the absolute value of the difference in investments in government bonds and corporate bonds does not remain stable. In asset mix 5, the absolute value between the two types of bonds is the largest. As a result, the compensating effect of corporate bonds on the underestimation of the risk in government bonds is the least for this asset mix. This is the cause of the fact that the risk for asset mix 5 is underestimated the most.
Figure 9.3: Results of the different asset mixes under Solvency II
9.3 Liabilities

The required funding ratio under FTK is defined as:

\[
\text{Required funding ratio} = \frac{\text{RCR} + \text{Liabilities}}{\text{Liabilities}} \times 100\% \quad (9.3)
\]

A change in the cash flow of the liabilities does influence the capital requirement. The explanation for this is that a change in the cash flows of the liabilities (in general) changes the duration. The shock which has to be applied for the component for interest rate risk depends on the duration of the pension fund. As a result, the numerator in formula (9.3) changes by another factor than the denominator and as a consequence, the required funding ratio will change as well.

9.4 Correlation $S_1$ and $S_2$

The FTK legislation states that the correlation coefficient between interest rate risk and equity risk should be set equal to 50%. In this section, we will test the sensitivity of the required funding ratio. In figure 9.4 we graphically give the required funding ratios for different correlation coefficients between $S_1$ and $S_2$. Note that the required funding ratio does not linearly depend on the correlation coefficient since the Regulatory Capital Requirement is determined according to a square root formula. In table 9.3 we present the results numerically. In this table we indeed see that the relationship between the correlation coefficient and the required funding ratio is not linear.
Table 9.3: Results of sensitivity analysis for correlation coefficient between \( S_1 \) and \( S_2 \)

<table>
<thead>
<tr>
<th>Correlation coefficient</th>
<th>Required funding ratio</th>
<th>Increase required funding ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>112.33%</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>112.58%</td>
<td>0.25%</td>
</tr>
<tr>
<td>0.2</td>
<td>112.83%</td>
<td>0.25%</td>
</tr>
<tr>
<td>0.3</td>
<td>113.07%</td>
<td>0.24%</td>
</tr>
<tr>
<td>0.4</td>
<td>113.30%</td>
<td>0.23%</td>
</tr>
<tr>
<td>0.5</td>
<td>113.53%</td>
<td>0.23%</td>
</tr>
<tr>
<td>0.6</td>
<td>113.75%</td>
<td>0.22%</td>
</tr>
<tr>
<td>0.7</td>
<td>113.97%</td>
<td>0.22%</td>
</tr>
<tr>
<td>0.8</td>
<td>114.18%</td>
<td>0.21%</td>
</tr>
<tr>
<td>0.9</td>
<td>114.38%</td>
<td>0.20%</td>
</tr>
<tr>
<td>1</td>
<td>114.58%</td>
<td>0.20%</td>
</tr>
</tbody>
</table>

These results show the importance of the right value of the correlation coefficient in the FTK legislation. Namely, a required funding ratio of 112.33% leads to a probability of underfunding of 3.59% according to the Hoevenaars model. In this case, accordance to the Hoevenaars model, FTK underestimates the risk. However, in case a perfect correlation is assumed, the required funding ratio of 114.58% leads to a probability of underfunding of 2.48%. In the latter, Hoevenaars would agree with the FTK legislation. We suggest that further research is necessary in order to investigate the value of the correlation coefficient.
Chapter 10

Conclusions

In this chapter, we will give a summary of the results which we found in this thesis. We also answer the research question. Finally, we end with a discussion about introducing Solvency II for pension funds.

10.1 Summary of the results

In this thesis we investigated the following research question:

"What is the Regulatory Capital Requirement under the current Financial Assessment Framework in comparison to the Regulatory Capital Requirement obtained by the standard model of Solvency II and are the obtained levels of Regulatory Capital Requirement indeed sufficient when tested by simulating under different ALM models for different kinds of pension profiles?"

Regulatory Capital Requirements

In order to be able to answer this question, we first determined the Regulatory Capital Requirements under FTK and Solvency II for our pension fund. We found that the required funding ratio under FTK is equal to 115.9%. It seems that the largest risks for our pension fund are by far equity risk and interest rate risk. Applying the standard model of Solvency II, we found that the required funding ratio is equal to 126.4%. Again, the components for equity risk and interest rate risk are the largest risks for our pension fund. This means that for the setting in this thesis, the implementation of Solvency II for pension funds will lead to an increase in the required capital of about 10%.

Probabilities of underfunding

Afterwards, we tested whether the required funding ratios which we found using the standard models, indeed lead to the required probabilities of underfunding of 2.5% under FTK and 0.5% under Solvency II. In order to test this, we used
two models: the Hoevenaars model and the GAM model.

Using the Hoevenaars model, we found that the probability of underfunding in one year from now under FTK is equal to 2.05% with a 95% confidence interval of (1.77%, 2.33%). This is less than the required 2.5%. Using the GAM model, we found that the probability of underfunding equals 8.06% with a 95% confidence interval of (7.53%, 8.59%). This means that with 95% confidence, the Hoevenaars model is less prudent than the FTK legislation, while the GAM model is much more prudent than the FTK legislation. Stated otherwise, comparing the Hoevenaars model with FTK, we conclude with 95% confidence that the FTK overestimates the risk and comparing the GAM model with FTK, we have strong evidence that the FTK underestimates the risk.

The probability of underfunding under Solvency II is 0.13% with a 95% confidence interval of (0.06%, 0.20%) according to the Hoevenaars model and 2.43% with a 95% confidence interval of (2.13%, 2.73%) according to the GAM model. Similar to the results we found for the FTK model, Solvency II overestimates the risk according to the Hoevenaars model and it underestimates the risk according to the GAM model with at least 95% confidence.

![Figure 10.1: Overview required funding ratio and probability of underfunding for FTK and Solvency II](image)

From figure 10.1 we can conclude that under the Hoevenaars model, FTK and Solvency II overestimate the risk, while under the assumptions of the GAM model, we can conclude that FTK and Solvency II underestimate the risk. However, the comparison which we made was not completely fair. In the standard models of both legislations, we took more risks into account than in the ALM models. In order to be able to make a fair comparison, we deleted the risks that are not present in the ALM models from the standard models. Also, we adjusted the durations of the bonds and liabilities in the standard model such
that the durations under the standard model and the ALM models are equal. Since the duration for the corporate bonds are different under the Hoevenaars model and the GAM model, we also found different required funding ratios.

Adapting the standard models

After adapting the standard models, we found that the required funding under FTK using the assumptions of the Hoevenaars model, equals 113.5%. This funding ratio is associated with a probability of underfunding of 2.97%. This means that according to the Hoevenaars model, the FTK does not overestimate, but underestimate the risk.

Under the assumptions of the GAM model, we found that the Regulatory Capital Requirement under FTK is associated to a funding ratio of 114.4%. Using this funding ratio as input for the GAM model leads to a probability of underfunding of 9.51%. Since this is much more than the required 2.5%, we can conclude that according to the GAM model, the FTK underestimates the risk. This is in line with what we found using the Hoevenaars model.

In case of Solvency II, we found that the new required funding ratio under the assumption of the Hoevenaars model is equal to 120.3%. This leads to a probability of underfunding of 0.66% with a 95% confidence interval of (0.50%,0.82%). The required probability of underfunding of 0.5% is within this interval. Under the assumption of the GAM model, the funding ratio which we found equals 118%. This is associated with a probability for the liabilities to exceed the assets of 6.59% with a 95% confidence interval of (6.10%,7.08%). From these results we can conclude that according to the Hoevenaars model, we do not reject the hypothesis that Solvency II correctly estimates the risk. Besides this, we can conclude that according the GAM model, Solvency II underestimates the risk significantly. In figure 10.2 we give an overview of the results.

From the results which we found, it is clear that the GAM model consistently produces much higher probabilities of underfunding than the Hoevenaars model. We found that this is caused by much more insecurity under the GAM model as well as fewer scenarios in which the values of the liabilities decrease.

Pension fund 2

Since both legislations should be applicable to all kinds of pension funds, we also performed an analysis on a second pension fund. This pension fund is somewhat smaller and the average age of the participants is lower. Besides this, the asset mix also differs.

For this pension fund, the required funding ratio under FTK equals 116.2% in case we use the assumptions of the Hoevenaars model. This is consistent with a probability of underfunding of 5.33% with a 95% confidence interval of
Figure 10.2: Overview required funding ratio and probability of underfunding for FTK and Solvency II after adapting the standard models

(4.89%, 5.77%). This is more than the required 2.5%. Under the assumption of the GAM model we found a funding ratio of 116.9%. This leads to a probability of underfunding of 8.61% with a 95% confidence interval of (8.06%, 9.16%). So also for the second pension fund, we can conclude with at least 95% confidence that the FTK underestimates the total risk according to the Hoevenaars and GAM model.

Under Solvency II, the required funding ratio is equal to 123.5% under the Hoevenaars assumption and 122.6% under the GAM assumptions. We found that these funding ratios are in accordance with probabilities of underfunding of 2.25% with a 95% confidence interval of (1.96%, 2.54%) and 5.13% with a 95% confidence interval of (4.70%, 5.56%) respectively. This means that according to our ALM models, we can conclude with 95% confidence that Solvency II underestimates the risk. In figure 10.3 we give an overview of the results for pension fund 2. We also include the results for pension fund 1 for comparison.

In section 8.4 we found that the difference in required funding ratio between pension fund 1 and 2 is caused by the difference in asset mix and the longer duration of the liabilities for pension fund 2. We also found that according to the Hoevenaars model, the risk for commodities are underestimated by both the FTK legislation and Solvency II legislation.

Sensitivity analysis

In the sensitivity analysis, we analyzed the impact of the asset mix on the required funding ratios and associated probabilities of underfunding in one year from now. We found that the FTK overestimates the risks of the stocks in comparison to the Hoevenaars model. We also found that the risk for both
government bonds and corporate bonds are underestimated in comparison to the Hoevenaars model. As a result, the FTK overestimates the risk for asset mixes in which the percentage invested in stocks is at least the percentage invested in bonds. Similarly, it underestimates the risk for asset mixes in which the percentage invested in stocks is less than the percentage invested in bonds. In figure 10.4 we state the result of the sensitivity analysis.

The results of the sensitivity analysis under Solvency II are stated in figure 10.5. We found that according to the Hoevenaars model, the risk for government bonds is underestimated. On the other side, the risk for corporate bonds is slightly overestimated according to the Hoevenaars model. In case there is invested in both government bonds and corporate bonds, the estimating errors partly compensate each other.

Concluding, the results of the Hoevenaars model are much more in line with both legislations than the GAM model. The fact that the GAM model produces more pessimistic scenarios is probably caused by expert’s opinions. For a pension fund which invests 50% in stocks and 50% in bonds, the probabilities of underfunding are the closest to the required probabilities. For the other asset mixes, the probabilities of underfunding which we found are above or below the required probabilities of underfunding. However, one should keep in mind that the required probabilities of underfunding are established for an average pension fund. So, in case the probabilities of underfunding differ a little bit from the required funding ratios, one should not immediately reject FTK or Solvency II.
Figure 10.4: Results of the different asset mixes under FTK

Figure 10.5: Results of the different asset mixes under Solvency II
10.2 Discussion

The answer to the question what the effect is on the Regulatory Capital Requirement in case the FTK will be replaced with Solvency II is that the required capital for our specific pension fund will increase by 11.3%. We find that theoretically, it is possible to apply Solvency II to pension funds. FTK and Solvency II are similar legislations, with the main difference that the Solvency II legislation is much more prudent.

However, we believe that introducing Solvency II for pension funds will have a major disadvantage. Namely, there is less money left which can be used for investing in assets in case the Regulatory Capital Requirement increases. Investing less in risky assets, will probably lead to a lower expected return. As a result, the costs for pensions will increase. As we already mentioned in chapter 2, committee Frijns ([Frijns et al., 2010]) and committee Goudswaard ([Goudswaard et al., 2010]) both state that the current level of premiums already have reached their maximum. Another option would be to reduce the Regulatory Capital Requirement. This can be achieved by proper risk management or by investing in less risky assets. However, the consequence of the last option is again that the expected return will decrease. Another option would be to reduce the pension ambition. Of course, this is also not desirable.

We also question whether it is necessary to have a security margin of 99.5% for pension funds. There are some reasons in favor of a lower security margin for pension funds than for insurance companies. First of all, contrary to an insurance company, pension funds have some steering instruments. In case it is necessary, the board of a pension fund could decide for example to increase premiums, reduce the pension ambition, cut rights etcetera.

Another difference is that for pension funds, losses have to be carried by the participants. Since there is solidarity within generations as well as between generations, the burden of the loss can be shared. Contrary, insurance contacts are often individual contracts. In case of losses, the shareholders of the insurance company have to take the loss. Since the losses of insurance contracts have to be carried by the shareholders, while the losses of pension contracts have to be carried by the participants, shareholders prefer a higher security margin.

We also think that the moment in which Solvency II will be introduced is important. In case the requirements are tightened when the pension fund is in the state of a shortfall, the rights probably have to be cut even more. If the rights of all participants are reduced by the same percentage, people who are the closest to the age of retirement in general carry more of the burden since they have more rights and they have statistically the smallest probability of taking advantage from the higher required funding ratio. Since there is a shift from the older participants to the younger participants, this can be seen as reversed intergenerational solidarity. Besides this, in case the rights have to be cut by a
higher percentage in order to satisfy the required funding ratio under Solvency II, this can lead to distrust in the pension system.

Finally, we think it is a challenge to introduce one legislation which is applicable to all pension funds within Europe. Since there are many differences between pension systems within Europe, the systems are hard to compare.

Another point of discussion is the assumption about the correlation coefficient in the FTK legislation. Recall that the correlation coefficient between interest rate risk and equity risk is assumed to be 50% and the other correlations are assumed to be 0%. In the authors opinion it is doubtful whether it is realistic to set the other correlations equal to 0%. Besides, it seems that during periods of crisis, assets are even more correlated than during normal market conditions. Since FTK and Solvency II are legislations which prescribe worst case scenarios, we argue that correlation coefficients should be based on the market during periods of crisis. However, we acknowledge that it is hard to estimate the correct correlation coefficients.
Chapter 11

Recommendations for further research

In this thesis, we made some assumptions which could be relaxed in order to make things more realistic.

First of all, we assumed a closed pension fund. Obviously, this is not realistic. In reality, the composition of the participants in the funds changes over time. In order to achieve this, mortality rates should be included in the ALM models.

Second, we assumed indexation to be unconditional. This means that we see indexation as a part of the entitlements. In practice, indexation is often conditional. This implies that indexation is only provided in case the funding ratio is sufficient. In order to achieve this, the total value of the entitlements should be modeled in such a way that it depends on the funding ratio. However, since both legislations take a horizon of one year into account, this will probably have not very much effect on the results.

Third, the underlying data of the GAM model is European data while the Hoevenaars is based on US data. Since Solvency II applies to insurance companies within EMEA countries and FTK only to Dutch pension funds, it could be investigated what the effect on the results would be in case the Hoevenaars model is based on European data. We also want to emphasize the importance of good data for ALM models. In case the input is not correct, the output will not be correct either.

Another point is that, in practice, some pension funds have the goal in their investment strategy to hedge a fixed percentage of the interest rate risk. If this would be the case, the asset mix differs over time. So in order to achieve this, the asset mix should be dependent on the value of the governments bonds, cor-
porate bonds, liabilities and discount rate.

In the sensitivity analysis we showed that the correlation coefficient has much effect on the required funding ratio. Therefore we recommend investigating the value of the correlation coefficients. In case one wants to test the current correlation coefficient between $S_1$ and $S_2$ under the assumption that all other correlations are equal to zero, it can be done as follows. First, one has to determine the required funding ratio which is associated with a probability of underfunding of 2.5% according to the ALM model. Afterwards, one has to find multiple scenarios which are consistent with a funding ratio of 100%. Then the shocks which were applied in these scenarios should be determined. The value of these shocks should be entered in sheet which determines the required funding ratio according to the legislation. The value of the correlation coefficient which is necessary in order to find the same required funding ratio under the legislation should be the correlation in the Hoeveanaars model between interest rate risk and equity risk. This procedure should be repeated for multiple scenarios which are consistent with a funding ratio of 100% in the ALM model, since in this way we find a confidence interval for the value of $\rho$. Note that this procedure is based on the assumption that the correlations between the other risks are equal to zero. However, as we already mentioned, we question whether this assumption is realistic.

Recently, there are some plans to introduce the Ultimate Forward Rate (UFR) to Solvency II. This UFR is based on both the long term expectation for inflation and the long term expectation for the short rate. It turns out that these are at this moment estimated for Europe as 2.0% and 2.2% respectively. This means that the UFR in total equals 4.2%. ([van der Westen, 2012](#)). However, there is still discussion about the UFR. The European Committee proposes to construct the curve such that the market curve should be applied until a so called Last Liquid Point (LLP) of 20 years and afterwards, the forward rate will converge in 40 years to 4.2%. The European Parliament also proposes to set the LLP equal to 20 years, however they propose the forward rate to converge to 4.2% within 10 years after LLP ([van der Westen, 2012](#)). It is expected that the final decision about the UFR will be presented in September 2012. After the announcement, the UFR could be included in the model which determines the Regulatory Capital Requirement and the effects could be quantified.
Appendix A

Summary statistics data

In this appendix we will give the summary statistics of the data used in the ALM which is based on the Hoevenaars model.

<table>
<thead>
<tr>
<th></th>
<th>Commodities</th>
<th>Credit return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0111</td>
<td>0.0773</td>
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<tr>
<td>Standard Error</td>
<td>0.0083</td>
<td>0.0018</td>
</tr>
<tr>
<td>Standard Deviation</td>
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<td>0.0270</td>
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<tr>
<td>Sample Variance</td>
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<td>0.0007</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.0648</td>
<td>0.4506</td>
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<tr>
<td>Skewness</td>
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<td>0.7563</td>
</tr>
<tr>
<td>Minimum</td>
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<td>0.0339</td>
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<tr>
<td>Maximum</td>
<td>0.3460</td>
<td>0.1579</td>
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<table>
<thead>
<tr>
<th></th>
<th>Hedge Funds</th>
<th>Liabilities</th>
<th>Real Estate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0155</td>
<td>0.0247</td>
<td>0.0226</td>
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<tr>
<td>Standard Error</td>
<td>0.0029</td>
<td>0.0048</td>
<td>0.0078</td>
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<tr>
<td>Standard Deviation</td>
<td>0.0272</td>
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<td>0.0991</td>
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<tr>
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<td>-1.0410</td>
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<tr>
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<td>-0.1527</td>
<td>-0.1566</td>
<td>-0.4548</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.0734</td>
<td>0.4512</td>
<td>0.3071</td>
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Table A.1: Summary statistics second asset group data
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<th>Bond return</th>
<th>T-bill</th>
<th>Real T-bill</th>
</tr>
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<tbody>
<tr>
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<td>0.0030</td>
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<td>0.0004</td>
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<td>Standard Deviation</td>
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<td>Sample Variance</td>
<td>0.0021</td>
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<td>0.0000</td>
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<td>Kurtosis</td>
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<td>Maximum</td>
<td>0.2242</td>
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<table>
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<tr>
<th></th>
<th>Credit spread</th>
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<th>Stock return</th>
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<tr>
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<td>0.0022</td>
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<td>Standard Deviation</td>
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<td>Sample Variance</td>
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<td>0.1601</td>
<td>0.0011</td>
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<td>-0.9198</td>
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<td>Minimum</td>
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<td>-4.498074</td>
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<td>Maximum</td>
<td>0.0534</td>
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<table>
<thead>
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<td>Standard Deviation</td>
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<tr>
<td>Sample Variance</td>
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<td>Kurtosis</td>
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<tr>
<td>Skewness</td>
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<tr>
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</tr>
<tr>
<td>Period available</td>
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Table A.2: Summary statistics first asset group data
Appendix B

Estimated matrices
Hoevenaars model

In this appendix, the matrices $\Phi_0$, $\Phi_1$ and $\Sigma$ can be found in tables B.1, B.2 and B.3 respectively. These matrices have been estimated in Eviews.

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<th>Variable</th>
<th>Value</th>
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<td>Constant</td>
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<tr>
<td>Nominal T-bill</td>
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<tr>
<td>Dividend/Price</td>
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</tr>
<tr>
<td>Yield Spread</td>
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<tr>
<td>Credit Spread</td>
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<tr>
<td>Bond Return</td>
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<tr>
<td>Commodities</td>
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<tr>
<td>Real Estate</td>
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<tr>
<td>Credit Returns</td>
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<td>Hedge Funds</td>
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Table B.1: Values for the matrix $\Phi_0$
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<th>RTB(-1)</th>
<th>TB(-1)</th>
<th>Stockr(-1)</th>
<th>D/P(-1)</th>
<th>YS(-1)</th>
<th>CS(-1)</th>
<th>Bondr(-1)</th>
<th>Comm(-1)</th>
<th>Realest(-1)</th>
<th>CR(-1)</th>
<th>Hedgefd(-1)</th>
<th>Liab(-1)</th>
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<tr>
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<td>0.9830</td>
<td>0.0030</td>
<td>-0.0001</td>
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<tr>
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Table B.2: Values for the matrix $\Phi_1$
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<th>CS</th>
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Table B.3: Values for the covariance matrix \( \Sigma \)
Appendix C

Matlab code simulation

Hoevenaars model

The following m-file is used in order to simulate the possible developments of the assets.

```matlab
clear all
teller=0;
teller2=0;

% import data
load phi0.txt
load phi1.txt
load startwaarden.txt
load l.txt
load assetmix.txt

% set seed in order to get the same results each time the mfile runs
seed=34567891;

% percentages -> values
phi0=phi0/100;
startwaarden=startwaarden/100;
l=l/100;
assetmix=assetmix/100;

% input
aantalsim=10000;
nquarters=60; %15 years -> continuity analysis
assets=length(phi0);
fundingratio0=1.154;

% define matrices
z=zeros(assets,nquarters);
totall15=zeros(assets,aantalsim);
total1=zeros(assets,aantalsim);
```
```matlab
fundingratio1=zeros(1,aantalsim);
fundingratio15=zeros(1,aantalsim);

% simulating total asset return per asset over 60 quarters
for simulatienummer=1:aantalsim
    randn('seed',simulatienummer);
    z(:,1)=phi0+phi1*startwaarden+l*randn(nassets,1);
    for i=2:nquarters
        z(:,i)=phi0+phi1*z(:,i-1)+l*randn(nassets,1);
    end
    for i=1:4
        if z(2,i)<0;
            z(2,i)=0;
        end
    end
    if simulatienummer>=0
        for i=1:nassets
            total1(i,simulatienummer)=sum(z(i,:));
            total15(i,simulatienummer)=sum(z(i,1:4));
        end
        fundingratio1(1,simulatienummer)=transpose(assetmix(1:11))*(exp(total1(1:11,simulatienummer)))/(exp(total1(12,simulatienummer)))*fundingratio0;
        fundingratio15(1,simulatienummer)=transpose(assetmix(1:11))*(exp(total15(1:11,simulatienummer)))/(exp(total15(12,simulatienummer)))*fundingratio0;
        stijgingassets(1,simulatienummer)=transpose(assetmix(1:11))*(exp(total1(1:11,simulatienummer)));
        stijgingliab(1,simulatienummer)=(exp(total1(12,simulatienummer)));
    end
end
sorteren1=sort(fundingratio1);
sorteren12=sort(fundingratio15);
valueatrisk95=sorteren1(1,0.95*aantalsim);
valueatrisk99=sorteren1(1,0.99*aantalsim);
for i=1:aantalsim
    if fundingratio1(1,i)<1.00;
        teller=teller+1;
    end
    if fundingratio15(1,i)<1.00;
        teller2=teller2+1;
    end
end
percentageonderdekking=teller/(aantalsim)*100
percentageonderdekking15=teller2/(aantalsim)*100
```
Appendix D

Matlab code GAM model

clear all

% starting values
F0=1.154;
count=0;
count10=0;
count15=0;

% import data
load datasimulatie.txt
load datasimulatie10.txt
load datasimulatie15.txt
load assetmix.txt

assetmix=assetmix/100;

assets=(datasimulatie(:,1:8)*assetmix)+1;
liabilities=datasimulatie(:,9)+1;
ΔFR=assets./liabilities;
FR=ΔFR*F0;

assets10=(datasimulatie10(:,1:8)*assetmix)+1;
liabilities10=datasimulatie10(:,9)+1;
ΔFR10=assets10./liabilities10;
FR10=ΔFR10*F0;

assets15=(datasimulatie15(:,1:8)*assetmix);
liabilities15=datasimulatie15(:,9);
ΔFR15=assets15./liabilities15;
FR15=ΔFR15*F0;

for i=1:length(assets)
    if FR(i)<1
        count=count+1;
    end
end
for i=1:length(assets)
    if FR10(i)<1
        count10=count10+1;
    end
end
for i=1:length(assets)
    if FR15(i)<1
        count15=count15+1;
    end
end
average1=mean(FR);
average10=mean(FR10);
average15=mean(FR15);
stdev1=std(FR);
stdev10=std(FR10);
stdev15=std(FR15);
Percentage underfunding=count/length(assets)*100;
Percentage underfunding10=count10/length(assets)*100;
Percentage underfunding15=count15/length(assets)*100;
Bibliography


