

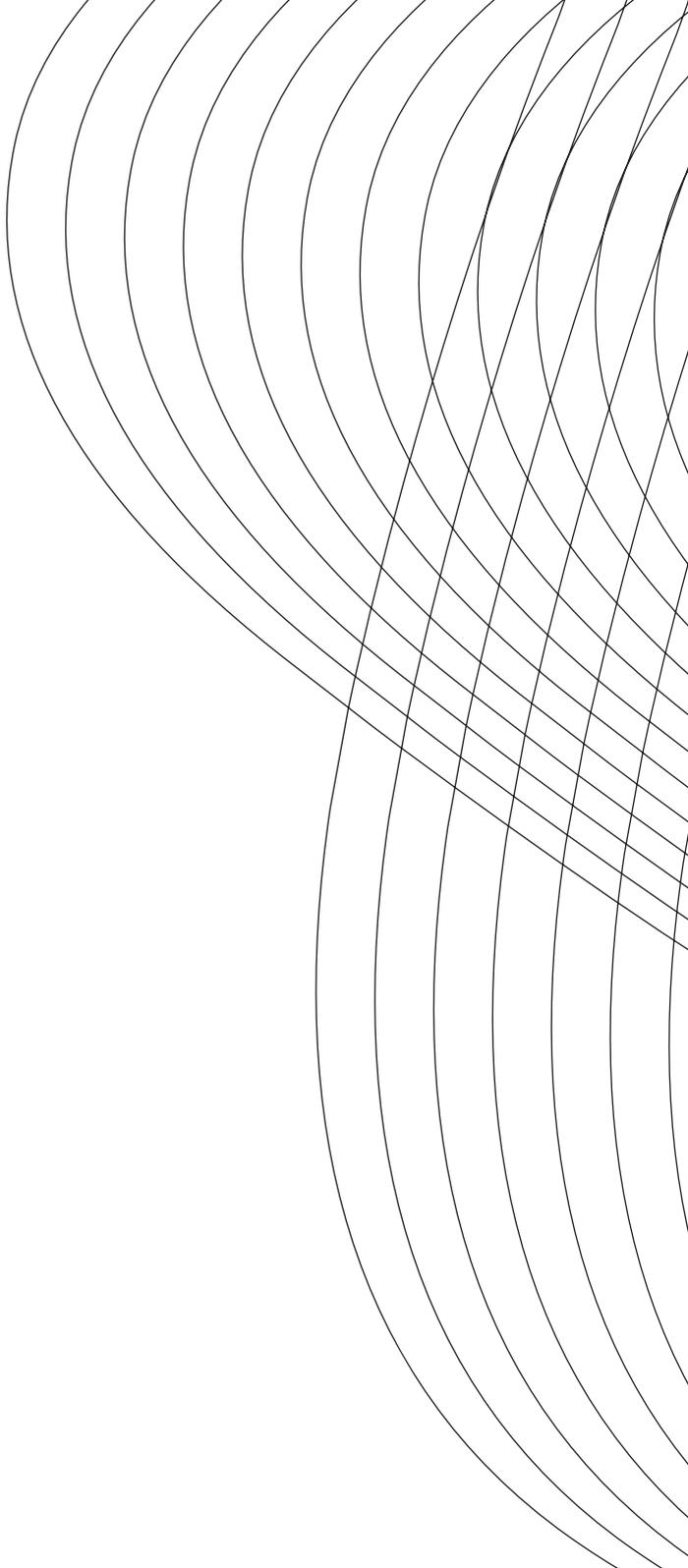


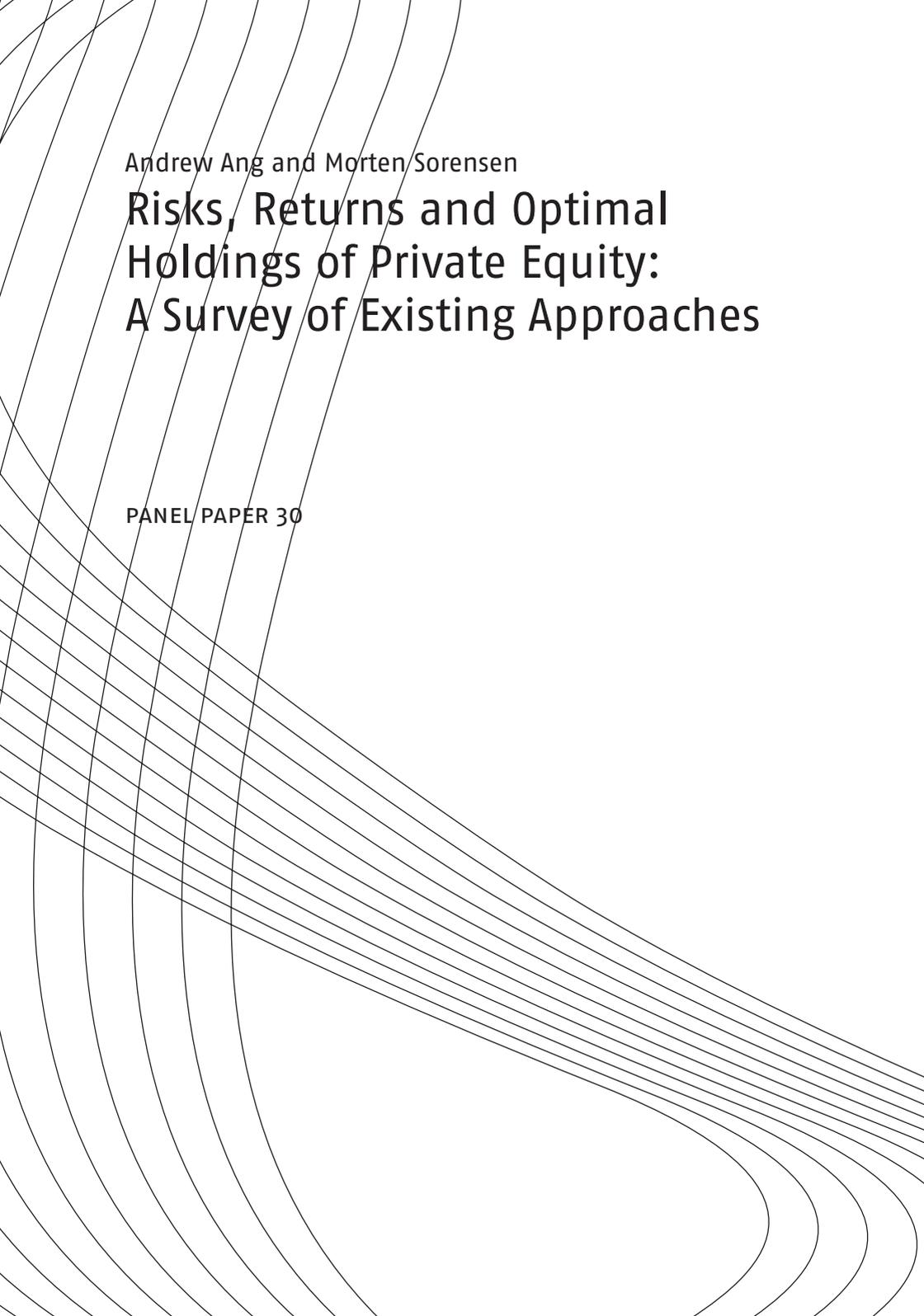
Network for Studies on Pensions, Aging and Retirement

Netspar PANEL PAPERS

Andrew Ang and Morten Sorensen

Risks, Returns and Optimal Holdings of Private Equity: A Survey of Existing Approaches





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Editorial address

Netspar, Tilburg University
PO Box 90153, 5000 LE Tilburg
info@netspar.nl

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PREFACE

Netspar stimulates debate and fundamental research in the field of pensions, aging and retirement. The aging of the population is front-page news, as many baby boomers are now moving into retirement. More generally, people live longer and in better health while at the same time families choose to have fewer children. Although the aging of the population often gets negative attention, with bleak pictures painted of the doubling of the ratio of the number of people aged 65 and older to the number of the working population during the next decades, it must, at the same time, be a boon to society that so many people are living longer and healthier lives. Can the falling number of working young afford to pay the pensions for a growing number of pensioners? Do people have to work a longer working week and postpone retirement? Or should the pensions be cut or the premiums paid by the working population be raised to afford social security for a growing group of pensioners? Should people be encouraged to take more responsibility for their own pension? What is the changing role of employers associations and trade unions in the organization of pensions? Can and are people prepared to undertake investment for their own pension, or are they happy to leave this to the pension funds? Who takes responsibility for the pension funds? How can a transparent and level playing field for pension funds and insurance companies be ensured? How should an acceptable trade-off be struck between social goals such as solidarity between young and old, or rich and poor, and

individual freedom? But most important of all: how can the benefits of living longer and healthier be harnessed for a happier and more prosperous society?

The Netspar Panel Papers aim to meet the demand for understanding the ever-expanding academic literature on the consequences of aging populations. They also aim to help give a better scientific underpinning of policy advice. They attempt to provide a survey of the latest and most relevant research, try to explain this in a non-technical manner and outline the implications for policy questions faced by Netspar's partners. Let there be no mistake. In many ways, formulating such a position paper is a tougher task than writing an academic paper or an op-ed piece. The authors have benefitted from the comments of the Editorial Board on various drafts and also from the discussions during the presentation of their paper at a Netspar Panel Meeting.

I hope the result helps reaching Netspar's aim to stimulate social innovation in addressing the challenges and opportunities raised by aging in an efficient and equitable manner and in an international setting.

Roel Beetsma

Chairman of the Netspar Editorial Board

Affiliations

Andrew Ang – Columbia Business School

Morten Sorensen – Columbia Business School

RISKS, RETURNS AND OPTIMAL HOLDINGS OF PRIVATE EQUITY: A SURVEY OF EXISTING APPROACHES

Policy Recommendations

Our findings and recommendations for investments in private equity, which we define as investments in privately held companies (thus, involving direct trading between investors instead of via organized exchanges), may be summarized as follows:

1. Empirical approaches commonly used to estimate the risk and return of standard publicly traded securities are difficult to apply. Complicating features of private equity investments include the limited data, the irregular nature of such investments, and the sample selection problems that typically arise in reported private equity data. Adjusting for these difficulties requires sophisticated econometric techniques. Without appropriate adjustment, naïve analyses tend to understate the risk and volatility, and may exaggerate performance estimates.

Recommendation: Interpret reported estimates of private equity risk and return with caution. Simple standard methodologies fail to consider all of the nuances that must be explored in any thorough and accurate evaluation of a private equity investment. Studies that develop methodologies to perform these adjustments are still in a preliminary phase, and a consensus on the appropriate adjustments has yet to emerge.

2. Commonly used fund performance measures, such as the internal rate of return (IRR), the total value to paid-in (TVPI) multiple, and public market equivalent (PME), are problematic. There is substantial variation in the estimates of these measures across studies and data sources. The measures can, to some extent, be manipulated by the timing and magnitude of the individual investments. Moreover, these fund performance measures use only rough risk adjustments. Fundamentally, these measures are not derived from underlying financial theories of risk and return, which renders them difficult to interpret consistently.

Recommendation: Interpret commonly reported performance measures with caution. They are not return measures as commonly understood.

3. Asset-allocation models that account for transaction costs (which are high for private equity) and illiquidity risk (which is substantial for private equity) recommend modest holdings of private equity. In these models, rebalancing will be infrequent, which implies that wide swings in the holdings of private equity can be expected. Also, the holdings of illiquid private equity will be much lower than predicted by asset-allocation models, assuming that all assets can be rebalanced when desired.

Recommendation: When determining optimal private equity allocations, asset-allocation models must account for the inability to rebalance private equity positions. Allocations to illiquid private equity investments should generally be modest.

4. Current private equity vehicles have substantial agency issues, whereas public equity vehicles do not. While there is

heterogeneity in private equity contracts, private equity fees are high, consuming at least one-fifth of gross private equity returns. Incentive fees account for less than one-third of general partner compensation.

Recommendation: Undertake, where possible, to bring back in-house (to the institutional asset owners) some part of the fees paid to externally managed private equity funds with general partners; if the quality of the private equity investments can be maintained, this move would lead to substantial savings for the asset owners.

Abstract

This paper surveys the academic literature that examines the risks and returns of private equity investments, optimal private equity allocation, and compensation contracts for private equity firms. Empirical evidence shows that the irregular nature of private equity investments complicates the estimation and interpretation of standard risk and return measures. These complications have led to substantial disparity in performance estimates reported across studies. Moreover, studies suggest that the illiquidity and transaction costs inherent in private equity investments have substantial implications for optimal holdings of these assets. Finally, studies of contracts governing the relationships between investors, private equity funds and underlying portfolio companies suggest that these contracts address both moral hazard and information frictions, which typically results in substantial management and performance fees earned by the private equity firms.

I. Introduction

Private equity investments are investments in privately held companies, which trade directly between investors instead of via organized exchanges. The investments are typically made through a private equity fund organized as a limited partnership, with the investor as a limited partner and the private equity firm as the general partner who oversees and manages the investments in the individual companies. Depending on the type of companies invested in, private equity funds are typically classified as buyout, venture capital, or some other type of fund specializing in illiquid non-listed investments. *Buyout funds* invest in mature established companies, using substantial amounts of leverage to finance the transactions. *Venture capital funds* invest in high-growth start-ups, using little or no leverage. Finally, it is not uncommon for limited partners to also invest directly in individual companies. These investments are often structured as co-investments in the portfolio companies, alongside the investments made through the private equity fund.

Private equity is often considered as a distinct asset class, differing in fundamental ways from investments in public equity. Since no active market exists for private equity positions, these investments are illiquid and difficult to value. The investments are for the long term. Private equity funds typically have horizons of 10–13 years, during which the invested capital cannot be redeemed. Moreover, partnership agreements specifying the governance of funds are complex, specifying the general partner's compensation as a combination of ongoing fees (management fees), a profit share (carried interest), transaction fees and other fees.

This paper surveys the academic research concerning the risks and returns of private equity investments, as well as the optimal holdings of private equity in an investment portfolio. It also contains a review of private equity contracts. It should be noted that researchers typically have had limited access to information about the nature and performance of private equity investments, and research in this area is preliminary and often inconclusive. Research into many important aspects of these investments, such as the performance of private equity during the recent recession, the secondary market for limited partner positions, and co-investments by limited partners, has only recently begun. Moreover, our survey only covers studies of private equity defined as companies owned by private equity funds. We thus do not consider the substantial number of privately held and independently owned companies, ranging from independent grocery stores and dry cleaners to large family-owned businesses (see Moskowitz and Vissing-Jorgensen 2002; Kartashova 2011; and Faccio, Marchica, McConnell and Mura 2012).

Section II introduces two problems that researchers have encountered in measuring private equity risk and returns. The first of these is the statistical problem that arises because private equity returns are observed infrequently, typically with well-performing funds being overrepresented in the data. This makes it difficult to estimate standard measures of risk and return, such as the capital asset-pricing model's alphas and betas. The second problem is how to interpret the resulting estimates. Standard asset-pricing models are established under assumptions that are appropriate for traditional financial markets, with transparent, liquid and low-friction transactions. These assumptions are problematic for private equity investments, and the estimated alphas and betas may need to be adjusted in order to provide

meaningful measures of risk and return in the private equity context. One way of interpreting the risks and returns of private equity investments, especially illiquidity risk, is for an investor to consider private equity from an investor-specific asset-allocation perspective.

Section III summarizes the literature on the optimal allocation of private equity in portfolios consisting of liquid public equity and illiquid private equity. A new generation of asset-allocation models considers these issues, since the first generation of asset-allocation approaches assumed that assets can be rebalanced without cost at any time. The literatures on asset allocation incorporating transaction costs (which are very high for private equity investments) and search frictions (due to counterparties often being hard to find for transfer of private equity investments) give strong recommendations on optimal holdings of illiquid private equity assets.

Section IV surveys the literature on agency issues and private equity contracts, with special emphasis on fees and the lack of transparency. Most private equity investments are made through intermediaries. Current private equity investment vehicles cannot disentangle factor returns that are unique to the private equity asset class from those that can be attributed to managerial skill. Furthermore, commonly-used contracts may exacerbate rather than alleviate agency issues.

II. Estimating Private Equity Risk and Return

II A. Defining Risk and Returns

To establish notation and terminology, it is useful to begin with the standard model for risk and return. For traded financial assets, risk and return are usually measured in the context of the capital asset-pricing model as the alpha and beta coefficients estimated in the one-factor linear regression (the *expected return regression*),¹

$$R_i(t) - R_f(t) = \alpha + \beta [R_m(t) - R_f(t)] + \epsilon_i.$$

In this equation, $R_i(t)$ is the return earned by the investor from period $t-1$ to period t , $R_f(t)$ is the risk-free rate over the period from $t-1$ to t , and $R_m(t)$ is the return on the market portfolio. The return earned on a financial asset from time $t-1$ to t is defined as

$$R(t) = \frac{P(t) + CF(t)}{P(t-1)} - 1,$$

where $CF(t)$ is the cash flow paid out at time t , and $P(t)$ is the market price quoted at time t , immediately after payment of the cash flow. For traded assets, the expected return regression is straightforward to estimate, namely by regressing (for example, on a weekly basis) the asset's observed returns on the corresponding market returns over the same periods.

Under appropriate assumptions about investor preferences [that is, constant relative risk aversion (CRRA) or mean-variance

1 This specification assumes that alpha and beta are constant over the duration of the deal. While it would be interesting to investigate the term structure of the risk and return, the data limitations and other complications described here have hindered empirical studies of these dynamics. Substantial evidence suggests that alphas and betas vary over time for listed equity, as Ang and Kristensen (2012) show.

utility], along with assumptions about the market environment (that is, the absence of transaction costs, short-sales constraints, and the ability of investors to continuously trade and rebalance their portfolios], the capital asset-pricing model specifies that each asset's expected return is determined by the expected return regression with an alpha equal to zero. This important result has several implications. First, it implies that beta is the appropriate measure of risk, as it measures the correlation between the return on the asset and the return on the overall market (*systematic risk*). In the capital asset-pricing model, systematic risk is the only risk that is priced; idiosyncratic risk is not priced because it can be diversified. Second, the expected return regression implies that an asset's expected return increases linearly in beta. Finally, it implies that in equilibrium, alpha should be zero. A positive alpha can be interpreted as an abnormal positive return.

Following this logic, the standard approach to evaluating the risks and returns of financial assets proceeds in two steps. Firstly, alpha and beta are estimated using the expected return regression. Secondly, invoking the capital asset-pricing model, the estimated alpha is interpreted as an abnormal risk-adjusted return, and the beta is interpreted as the systematic risk.

For private equity investments, problems arise during both steps. In the first step, privately held companies by definition do not have regularly observed market values, and the returns earned from investing in these companies are only observed at exit. Hence, period-by-period returns are unavailable, making it difficult to estimate the expected return regression directly. Better-performing privately held companies may also be overrepresented in the data, creating sample-selection problems that would cause the alpha coefficient to be overestimated and the beta coefficient to be underestimated. In the second

step, after estimating alpha and beta, it is unclear whether or not these coefficients appropriately measure risks and returns. The assumptions of liquid and transparent markets underlying the capital asset-pricing model are far from the realities of private equity investing. To reflect the actual risks and returns facing limited partner investors, the estimated parameters may require various adjustments to account for the cost of illiquidity, idiosyncratic risk, persistence, funding risk, and so forth.

The lack of regularly quoted market prices and returns presents a fundamental challenge for empirical studies of the risk and return of private equity investments. Alternative approaches have either used company-level performance data or fund-level data with the cash-flow streams between the limited partners and general partners. The benefits and drawbacks of these approaches are discussed next.

II B. Estimates Using Company-level Data

Company-level data contain information about investments by buyout or venture capital funds in individual companies. For these investments, the data typically contain the name of the company, the amount invested, the investment date, the exit date, and the exit amount. Such data are confidential and proprietary, so that researchers have had to obtain data through direct contact with limited partners and professional data providers.

Franzoni, Nowak and Phalippou (2012) analyze company-level data for buyout investments. Cochrane (2005) and Korteweg and Sorensen (2010) use company-level data for venture capital investments in start-ups. The application to venture capital investing is more challenging, because the sample-selection problem is particularly severe for these investments.

Compared to fund-level data, company-level data have two advantages. First, there are many more companies than funds, which improves the statistical power of the analysis. Companies can be classified in terms of industries and types, allowing for a more nuanced differentiation of the risks and returns across industries and types and over time. Second, investments in individual companies have well-defined returns. If there are no intermediate cash flows, the return as defined above can be calculated directly from the initial investment and the distribution of the proceeds at exit. As long as intermediate cash flows are few and small, as for buyout investments, this calculation provides a reasonable return measure. With more intermediate cash flows, such as for venture capital investments, the calculation may be performed separately for each investment round.

A disadvantage of company-level data is that the return figures typically do not exclude management fees and carried interest paid by the limited partners to the general partners. The estimated risks and returns reflect the total risks and returns of the investments (before fees), not those earned by a limited partner (net of fees). Translating between net-of-fee and before-fee returns typically requires additional assumptions and numerical simulations (for two approaches, see Metrick and Yasuda 2010, and Franzoni, Nowak and Phalippou 2012).

Continuous-time Specifications. A technical disadvantage of company-level data is that the returns are measured over different periods. Returns are measured from the time of the initial investment to the time of exit, and the duration varies substantially across investments. The standard (discrete-time) capital asset-pricing model is a one-period model, where the period may be a day, a month, or a quarter. The model does not

compound, however, and the returns must all be calculated over periods of the same duration.

A standard solution is to use the continuous-time version of the capital asset-pricing model. This version does compound, which allows for a comparison of the risks and returns of investments of different durations. Campbell, Lo and MacKinlay (1997) discuss extensively the underpinnings of this model. In the continuous-time capital asset-pricing model, the expected return regression is restated in log-returns (continuously-compounded returns) as

$$\ln[1+R_i(t)]-\ln[1+R_f(t)]=\delta+\beta\left(\ln[1+R_m(t)]-\ln[1+R_f(t)]\right)+\epsilon_i.$$

One complication with the continuous-time capital asset-pricing model is that the estimated intercept of the expected return equation cannot be interpreted as an abnormal return, as in the standard discrete-time capital asset-pricing model. Under specific assumptions about the way volatility increases with the duration of an investment, the abnormal returns can be calculated using the following adjustment:

$$\alpha=\delta+\frac{1}{2}\sigma^2.$$

This non-linear adjustment leads to high alphas when the volatility of individual deals is high (see Cochrane 2005, and Kortweg and Sorensen 2011, for details about the derivation and implementation of the adjustment). For example, Cochrane (2005) reports an annual volatility around 90%, resulting in an estimated alpha of 32% annually. This appears unreasonably high compared to studies using fund-level data, raising doubts about the appropriateness of the assumptions about the growth of volatility with the duration of the investments.

Franzoni, Nowak and Phalippou (2012) sidestep this problem by estimating the capital asset-pricing model after forming

portfolios of deals, rather than focusing on individual deals. This substantially lowers volatility and reduces the magnitude of the adjustment. It does, however, reduce the other advantages of using individual deals: in particular, it reduces statistical power, and the analysis must use a modified internal rate of return approximation of returns.

Selection Bias. Another problem with company-level data is sample selection. To illustrate, venture capital investments are structured over multiple financing rounds, and better-performing companies tend to raise more such rounds. Hence, datasets with valuations of individual venture capital rounds are dominated by these better-performing companies. Moreover, distressed companies are usually not formally liquidated, and are often left as shell companies without economic value ("zombies"). This introduces another selection problem for the empirical analysis. When observing old companies without new financing rounds or exits, these companies may be alive and well or they may be zombies, in which case it is unclear when the write-off of the company's value should be recorded. This latter problem is less severe for buyout investments, because they mostly result in a well-defined exit (acquisition or IPO) or a well-defined liquidation.

The selection problem is illustrated in Figure 1 (from Korteweg and Sorensen 2010). The universe of returns is illustrated by all of the dots. The data, however, only contain the observed good returns above the x-axis (in black). Worse returns (shaded gray) are unobserved. Since only the black dots are observed, a simple estimation of the expected return regression gives an estimate of alpha that is biased upwards, an estimate of beta that is biased downwards, and a total volatility that is too low.

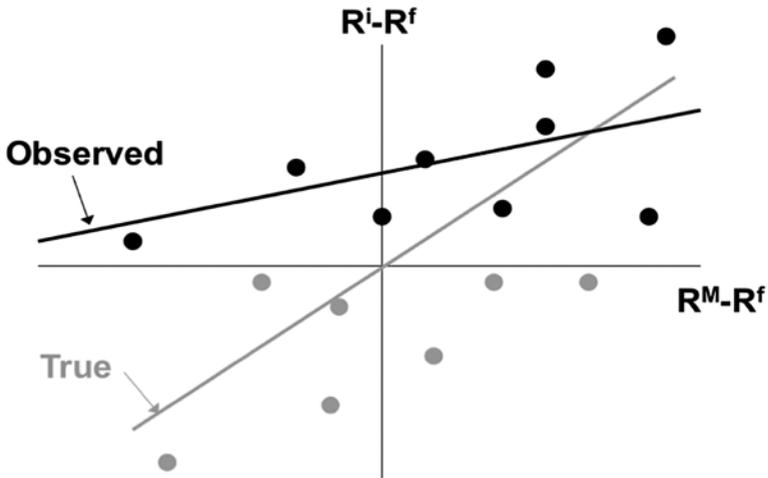


Figure 1: Illustration of Selection Bias

Hence, an analysis that does not correct for these biases will be overly optimistic about the risk and return performance of these investments.

The statistical methodology for addressing such selection biases was first introduced by Heckman (1979). Cochrane (2005) estimates the first dynamic selection model using venture capital data, and finds that the effect of selection bias is indeed large. Cochrane finds that the selection correction reduces the intercept of the log-market model, denoted δ above, from 92% to -7.1%. Cochrane also highlights the difficulty of translating this intercept into an abnormal return. Korteweg and Sorensen (2010) estimate an extended version of Cochrane's model. They also find that selection bias overstates the risk-return tradeoff of venture capital investments. Without selection bias, the estimate of the intercept, δ , is -19% annually; selection bias reduces this estimate to -68% (note, again, these intercepts cannot be interpreted as returns).

In the continuous-time capital asset-pricing model, the estimated beta coefficient can be interpreted as systematic risk, without adjustments. Cochrane (2005) finds a slope of 0.6–1.9 for the systematic risk—although this seems low. It includes estimates at the individual industry levels of, for example, -0.1 for retail investments.

Korteweg and Sorensen (2010) report substantially higher beta estimates of 2.6–2.8 in the continuous-time capital asset-pricing model, which may be more reasonable for young startups funded by venture capital investors. They also find substantial time variation as venture capital investing has matured. They estimate alphas over the 1987–1993, 1994–2000 and 2001–2005 periods, and find that the alphas in the early period were positive but modest, the alphas in the late 1990s were very high, but the alphas in the 2000s were negative, consistent with patterns found by studies using fund-level data.

II C. Estimates Using Fund-level Data

Fund-level data are typically obtained from limited partners with investments across many private equity funds. Each observation represents the performance of an entire portfolio of investments. In addition to information about the fund, such as its type and vintage year, these data may contain the cash flow stream between the limited partner and the fund or a performance measure calculated from this cash flow stream (for example, the internal rate of return, total value to paid-in, and public market equivalent). When individual cash flows are available, however, they are typically not tied to individual portfolio companies.

There are several advantages to fund-level data. First, they reflect actual limited partner returns, net of fees, resulting in estimates of the risks and returns actually realized by the limited

partners. The sample selection problem is smaller, since the performance of companies that ultimately never produce any returns for the investing funds (zombies) is eventually reflected in the fund-level cash flows. Other sample selection problems may arise, however. Fund-level performance is typically self-reported, and better-performing funds may be more likely to report their performance (as suggested by Phalippou and Gottschalg 2009, although Stucke 2011 argues that returns reported by Venture Economics understate actual performance).² Still, these selection problems are likely smaller than the problems that arise with company-level data. Finally, since funds have similar lifetimes (typically ten years), the expected return equation can be estimated directly, thereby avoiding the problems associated with the continuous-time log-return specification used for company-level data.

Fund-level Performance Measures. The main disadvantage of fund-level data is accurately measuring the “return.” Calculating period-by-period returns, as previously defined, requires assessing the market value of the private equity investment [$P(t)$ in the return calculation] at regular periods. Reported net asset values (NAVs) are noisy substitutes for these values (for example, it has been customary to value a company at cost until it experiences a material change in the circumstances, which does not capture smaller ongoing changes in its prospects or market values). Given the absence of regularly quoted returns, several alternative measures have been proposed. However, none of these measures

2 Anecdotal evidence from Harris, Jenkinson and Kaplan (2011) suggests that this bias made Venture Economics more attractive for benchmarking general partner performance.

is a return, as previously defined, and their relationships to asset-pricing models are somewhat tenuous.

Internal Rate of Return. A natural starting point is to interpret the internal rate of return (IRR) of the cash flows between the limited partner and general partner as a return earned over the life of the fund. Let the cash flow at time t be $CF(t)$. It is useful to separate these cash flows into the capital calls paid by the limited partner to the general partner, denoted $Call(t)$, and the distributions of capital from the general partner back to the limited partner, denoted $Dist(t)$. The internal rate of return is then defined as

$$PV = \sum \frac{CF(t)}{(1+IRR)^t} = \sum \frac{Dist(t) - Call(t)}{(1+IRR)^t} = 0$$

$$\Rightarrow \frac{\left(\sum \frac{Dist(t)}{(1+IRR)^t} \right)}{\left(\sum \frac{Call(t)}{(1+IRR)^t} \right)} = 1$$

Ljungqvist and Richardson (2003) investigate cash-flow data from a large limited partner investing in funds raised in 1981–1993 (19 venture capital funds and 54 buyout funds). They report average internal rates of return (net of fees), combining private equity and venture capital investments, for 1981–1993, of 19.81% for these funds, while the average S&P 500 return is 14.1%, suggesting that private equity investments outperform the market.

Kaplan and Schoar (2005) use fund-level quarterly performance measures from Venture Economics that cover 1,090 venture capital and buyout funds, of which 746 funds were fully or mostly

liquidated at the time of the study. They find that venture capital and buyout funds generate returns that are slightly below those of the S&P 500 Index on an equal-weighted basis (value-weighted venture capital funds perform slightly better than the Index), using their sample of fully-liquidated funds; the value-weighted internal rate of return is 13%.³ Extending the sample to mature (but not liquidated) funds raises the internal rate of return for venture capital funds to 30%, but leaves it unchanged at 13% for buyout funds, resulting in an overall average internal rate of return of 18%.⁴

Focusing on venture capital investments, Bygrave and Timmons (1992) find an average IRR of 13.5% over 1974–1989. Gompers and Lerner (1997), using investments of a single venture capital firm, report an IRR of 30.5% over 1972–1997.

A recent survey by Harris, Jenkinson and Kaplan (2011) summarizes the academic studies using fund-level data from various data providers.⁵ For buyout funds, they report weighted average IRRs of 12.3%–16.9%; the weighted average IRRs for venture capital funds are 11.7%–19.3%. Across time periods, buyout funds have had more stable performance, with weighted average IRRs of 15.1%–22.0% in the 1980s, 11.8%–19.3% in the

3 Phalippou and Gottschalg (2009) point out that it is difficult to value-weight private equity funds. One possibility is to weight by total committed capital, but funds vary in their investment speed, and poorer-performing funds may invest more slowly, introducing a downward bias in value-weighted performance estimates.

4 The final reported NAV of funds that are not fully liquidated is treated as a final cash flow in the calculation. Phalippou and Gottschalg (2009) argue that interim NAVs may exaggerate the actual values, leading to upward-biased performance estimates. In contrast, Stucke (2011) argues that the NAVs are substantially below actual economic value, using Venture Economics data. Kaplan and Schoar (2005) and Harris, Jenkinson and Kaplan (2011) use reported NAVs as stated.

5 These studies include Ljungqvist and Richardson (2003), Kaplan and Schoar (2005), Phalippou and Gottschalg (2008) and Robinson and Sensoy (2011).

1990s, and 5.8%–12.8% in the 2000s. Venture capital fund performance has become more volatile over time, with weighted average internal rates of return ranging from 8.6% to 18.7% in the 1980s, 22.9% to 38.6% in the 1990s, and –4.9% to 1.6% in the 2000s.

Overall, these figures reveal substantial variation in internal rates of return across studies and data sources. Moreover, the IRR is a problematic measure of economic performance. It is an absolute performance measure that does not calculate performance relative to a benchmark or market return. Moreover, the IRR calculation implicitly assumes that invested and returned capital can be reinvested at the internal rate of return rate. If a fund makes an early small investment with a large quick return, the investment can largely define the IRR for the entire fund, regardless of the performance of subsequent investments. Indeed, Phalippou (2011) suggests that general partners may actively manage their investments to inflate fund internal rates of return.

Total Value to Paid-in Capital Multiple. An alternative performance measure that is less susceptible to manipulation than the internal rate of return is the total value to paid-in capital multiple (TVPI). This multiple is calculated as the total amount of capital returned to the limited partner investors (net of fees) divided by the total amount invested (including fees). Formally, the total value to paid-in multiple is defined as

$$TVPI = \frac{\sum Dist(t)}{\sum Call(t)}$$

This calculation is performed without adjusting for the time value of money. The IRR is calculated under the implicit assumption that capital can be reinvested at the internal rate of return rate,

whereas the total value to paid-in multiple is calculated under the implicit assumption that capital can be reinvested at a zero rate. Harris, Jenkinson and Kaplan (2011) report weighted average total value to paid-ins of 1.76–2.30 for buyout investors and 2.19–2.46 for venture capital investors. This multiple varies substantially over time, however. For buyout funds, they report a multiple of 2.72–4.05 for the 1980s, 1.61–2.07 for the 1990s, and 1.29–1.51 for the 2000s; for venture capital funds, they report a multiple of 2.31–2.58 for the 1980s, 3.13–3.38 for the 1990s, and 1.06–1.09 for the 2000s.

Public Market Equivalent. Both the internal rate of return and total value to paid-in measures are absolute performance measures. The public market equivalent measure (PME) is used to evaluate performance relative to the market. It is calculated as the ratio of the discounted value of the limited partner's inflows divided by the discounted value of outflows— with the discounting performed using realized market returns:

$$PME = \frac{\left(\sum \frac{Dist(t)}{\Pi(1+R_m(t))} \right)}{\left(\sum \frac{Call(t)}{\Pi(1+R_m(t))} \right)}$$

Kaplan and Schoar (2005) argue that when private equity investments have the same risk as the general market (a beta equal to one), a public market equivalent greater than one is equivalent to a positive economic return for the limited partners. This interpretation may be misleading when the risk of distributions (the numerator in the public market equivalent) is greater than the risk of capital calls (including management fees,

which are largely a risk-free liability). Using a lower discount rate for capital calls would inflate the denominator and reduce the public market equivalent. Hence, more carefully accounting for different risks would suggest that the public market equivalent may have to exceed one by some margin before limited partners earn a positive economic return.⁶

Kaplan and Schoar (2005) find average equal-weighted public market equivalents of 0.96. Value-weighted, the public market equivalent for venture capital funds is 1.21, and the public market equivalent for buyout funds is 0.93. Phalippou and Gottschalg (2009) use data for 852 funds to calculate a public market equivalent of 1.01 (they call this measure the profitability index). The public market equivalent decreases to 0.88 after various adjustments.

Comparing different studies and data sources, Harris, Jenkinson and Kaplan (2011) report weighted-average public market equivalents of 1.16–1.27 for buyout funds and 1.02–1.45 for venture capital funds; public market equivalents for buyout funds varied from 1.03–1.11 in the 1980s, to 1.17–1.34 in the 1990s, and 1.25–1.29 in the 2000s. For venture capital funds, they report public market equivalents of 0.90–1.08 in the 1980s, 1.99–2.12 in the 1990s, and 0.84–0.95 in the 2000s. The 1990s was the venture capital decade, and the 2000s was the buyout decade.

Risk Measures. Fund-level data are poorly suited for estimating the risk of private equity investing. Thus, few (if any) academic studies attempt to use fund-level data. Instead, Ljungqvist

6 Additionally, as a technical point, the capital asset-pricing model prescribes that the discounting should be performed using expected returns—and not realized returns, as in the public market equivalent. Using the realized returns distorts the calculation (according to Jensen's inequality). The magnitude of this distortion is unclear, but most likely modest.

and Richardson (2003) estimate risk by assigning each portfolio company to one of 48 broad industry groups and use the corresponding average beta for publicly traded companies in the same industry; it is 1.08 for buyout investments and 1.12 for venture capital investments. Note that these betas do not adjust for the higher leverage used in buyout investments relative to venture capital investments. Assigning betas, they find a 5%–6% premium, which they interpret as the illiquidity premium of venture capital investments.

Kaplan and Schoar (2005) state that they “believe it is possible that the systematic risk of LBO funds exceeds 1 because these funds invest in highly levered companies.” They regress internal rates of return on S&P 500 returns, and find a coefficient of 1.23 for venture capital funds and 0.41 for buyout funds. A levered beta of 0.41 seems unreasonably low.

Persistence and Predictability. Several studies, including Kaplan and Schoar (2005), Phalippou and Gottschlag (2009), Hochberg, Ljungqvist and Vissing-Jorgensen (2010), find evidence of performance persistence for private equity funds. The performance of an early fund predicts the performance of subsequent funds managed by the same general partner. This persistence is interpreted as evidence that general partners vary in their skills and abilities to pick investments and manage the portfolio companies. Estimates suggest that a performance increase of 1.0% for a fund is associated with around 0.5% greater performance for the general partner’s next fund, measured either in terms of public market equivalent or internal rate of return. For more distant funds, persistence declines.

Due to data limitations, studies that document the predictability in private equity returns conduct statistical

in-sample analysis, rather than out-of-sample analysis. In Kaplan and Schoar (2005), for example, private equity funds in the “top quartile” do well, but these funds are identified ex post. Within a fund family, funds often have lifetimes of ten years (but overlap to some extent). In-sample analysis uses the ultimate performance of a previous fund to predict the performance of a subsequent fund—even if this fund is initiated before the ultimate performance of the previous fund is fully realized. To mitigate this concern, the studies employ various robustness checks, such as using intermediate NAVs instead of ultimate performance or using the performance of funds several generations ago, to predict future performance. Still, some recent research, such as Hochberg, Ljungvist and Vissing-Jorgensen (2010), find weaker evidence of persistence using only information available when the new fund is raised.

II D. Summary of Empirical Evidence

Based on the existing evidence from studies using fund-level data, it seems premature to make a precise assessment of how the risk of private equity investing compares to the risk of investing in publicly traded equities—even in terms of these basic metrics. Measuring private equity risk and returns is difficult because of the infrequent observations of fund or company values and selection bias. Studies using company-level data that account for selection bias find high alphas for private equity investments only during the late 1990s, but negative alphas post-2000. The positive alpha estimates are hard to interpret in terms of arithmetic returns, however, because of the very high volatility. Estimates of betas vary substantially, ranging as high as 3.6 for venture capital investments; generally, however, private equity betas are well above one. Studies using fund-level data have fewer selection

problems, yet still suffer from the fact that no direct private equity returns are observed. Unlike standard return measures, fund-level IRR, total value to paid-in, and public market equivalent measures can be misleading and should be interpreted with caution to infer private equity performance. In terms of raw performance, in the words of Harris, Jenkinson and Kaplan (2011), "it seems likely that buyout funds have outperformed public markets in the 1980s, 1990s, and 2000s." However, due to the uncertainty about the risk of private equity investments, it is not yet possible to say whether this outperformance is sufficient to compensate investors for their risk and whether the investments outperform on a risk-adjusted basis. Finally, there is evidence of persistence of private equity fund returns and some, albeit weaker and less consistent, evidence that characteristics such as fund size and past capital raisings predict private equity fund returns.

III. Asset Allocations to Private Equity

Having discussed the measurement of private equity returns, we now consider optimal allocations to private equity. This requires, of course, a suitable risk–return tradeoff for private equity investments, as well as correlation of private equity returns with other assets in the investor’s opportunity set. As pointed out in Section II, measuring these inputs for private equity for use in an optimization problem requires special considerations. We take as given these inputs, and focus on the illiquidity risk of private equity and how to incorporate it into an optimal asset-allocation framework. There have been several approaches to handling illiquidity risk in asset allocation, all of which have relevance. To put into context these contributions, we start with the case of asset allocation without frictions.

III A. Frictionless Asset Allocation

The seminal contributions of Merton (1969, 1971) characterize the optimal asset allocation of an investor with constant relative risk aversion (CRRA) utility investing in a risk-free asset (with constant risk-free rate) and a set of risky assets. The constant relative risk aversion utility function with risk aversion γ is given by

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}$$

The constant relative risk aversion utility is homogeneous of degree one, which means that exactly the same portfolio weights arise, regardless of whether \$10 million of wealth is being managed or \$1 billion. This makes the constant relative risk aversion utility function ideal for institutional asset management.

Assume the risky assets are jointly log-normally distributed. Under the case of independent and identically distributed (i.i.d.)

returns, the vector of optimal holdings, w , of the risky assets is given by

$$w = \frac{1}{\gamma} \Sigma^{-1} (\mu - r_f)$$

where Σ is the covariance matrix of the risky asset returns, μ is the vector of expected returns of the risky assets, and r_f is the risk-free rate. This is also the portfolio held by an investor with mean-variance utility optimizing over a discrete, one-period horizon.

There are two key features of this solution that bear further comment. First, the Merton model is dynamic and involves continuous rebalancing. That is, although the portfolio weights, w , are constant, the investor's policy is always to continuously sell assets that have risen in value and to buy assets that have fallen in value in such a way as to maintain constant weights. Clearly, the discrete nature of private equity investment and the inability to trade frequently mean that allocations to private equity should not be evaluated with the standard Merton model.

Second, the cost of employing a non-optimal strategy (for example, not holding a particular asset that should be held in an optimal portfolio) can be compared to the optimal strategy, and the cost of holding the non-optimal portfolio depends on the investor's risk aversion. That is, the cost of bearing non-optimal weights is dependent on the investor's risk preferences. The costs are computed using utility-certainty equivalents: the certainty-equivalent cost is how much an investor must be compensated in dollars per initial wealth to take a non-optimal strategy but have the same utility as the optimal strategy. A relevant cost, which the subsequent literature explores, is how much an investor should be compensated for the inability to trade assets like private equity for

certain periods of time or to be compensated for being forced to pay a cost whenever an asset is traded.

III B. Asset Allocation with Transactions Costs

Investing in private equity incurs substantial transactions costs in finding an appropriate private equity manager and conducting appropriate due diligence. Then, there are potentially large discounts to the recorded asset values that may be taken in transferring ownership of a private equity stake in illiquid secondary markets. Since Constantinides (1986), a large literature has extended the Merton model to incorporate transaction costs.

Constantinides (1986) considers the case of one risk-free and one risky asset. When there are proportional transaction costs, so that whenever the holdings of the risky asset increase (or decrease) by v , the holding of the riskless asset decreases by $(1+k)v$. When there are trading costs, the investor trades infrequently. Constantinides shows that the optimal trading strategy is to trade whenever the risky asset position hits upper and lower bounds, \underline{w} and \bar{w} , respectively. These bounds straddle the optimal Merton model in which there are no frictions. The holdings of risky to risk-free assets, y/x , satisfy the following:

$$\underline{w} \leq \frac{y}{x} \leq \bar{w}$$

so that when y/x lies within the interval $[\underline{w}, \bar{w}]$ there is no trade, and when y/x hits the boundaries on either side, the investor buys and sells appropriate amounts of the risky asset to bring the portfolio back to the optimum Merton model.

The no-trade interval, $\bar{w} - \underline{w}$, increases with the transactions costs, k , and the volatility of the risky asset. The transactions costs to sell private equity portfolios in secondary markets can be extremely steep. When the Harvard endowment tried to sell

its private equity investments in 2008, potential buyers were requiring discounts to book value of more than 50%.⁷ Even for transaction costs of 10%, Constantinides (1986) computes no-trade intervals greater than 0.25 around an optimal holding of 0.26 for a risky asset with a volatility of 35% per annum. Thus, private equity investors should expect to rebalance private equity holdings very infrequently.

The certainty-equivalent cost to holding a risky asset with large transaction costs is small for modest transaction costs (approximately 0.2% for proportional transaction costs of 1%), but can be substantial for large transaction costs, which is the more relevant range for private equity investments. For transaction costs of 15% or more, the required premium to bring the investor to the same level of utility as the frictionless Merton model is more than 5% per annum.

The literature has extended this framework to multiple assets (for example, Liu 2004) and different types of rebalancing bands. Leland (1996) and Donohue and Yip (2003) suggest rebalancing to the edge of a band rather than to a target within a band. Others, like Pliska and Suzuki (2004) and Brown, Ozik and Scholtz (2007), advocate extensions to two sets of bands, where different forms of trading are done at the inner band with more drastic rebalancing done at the outer band. In all these extensions, the intuition is the same: private equity investments should be expected to be rebalanced very infrequently, and the rebalancing bands will be very wide. The case of transaction costs when returns are predictable is considered by Garleanu and Pedersen (2010). A related study is Longstaff (2001), who allows investors to trade continuously, but only with bounded variation— so there are upper and lower bounds on the number of shares that can be

7 See “Liquidating Harvard” Columbia CaseWorks ID#100312, 2010.

traded every period. This makes Longstaff's model similar to the case of a time-varying transaction cost.

A major shortcoming of this literature is that it assumes that trade in assets is always possible, albeit at a cost, which is not true for private equity; over a short horizon, there may be no opportunity to find a buyer—and even if a buyer is found, there is not enough time, relative to the investor's desired short horizon, to raise capital to go through legal and accounting procedures to transfer ownership. An important friction for private equity investors in secondary markets is the search process in finding an appropriate buyer. There may be no opportunity to trade, even if desired, at considerable discounts. This case is considered by the next literature we examine.

III C. Asset Allocation with Search Frictions

As private equity investments do not trade on a centralized exchange, an important part of rebalancing a private equity portfolio is finding a counterparty in over-the-counter markets. Alternatively, if money is spun off from existing private equity investments, new or existing private equity funds in which to invest must be found. This entails a search process, incurring opportunity and search costs, as well as a bargaining process, which reflects investors' needs for immediate trade. The former requires a trading process that captures the discrete nature of trading opportunities. The latter is captured by a transaction cost, as modeled in the previous section.

Since Diamond (1982), search-based frictions have been modeled by Poisson arrival processes. Agents find counterparties with an intensity λ , and conditional on the arrival of the Poisson process, agents can trade and rebalance. This produces intervals where no rebalancing is possible for illiquid assets and the times

when rebalancing are possible are stochastic. This notion of illiquidity is that there are times where it is not possible to trade, at any price, an illiquid asset. These particular types of stochastic rebalancing opportunities are attractive for modeling private equity in another way: the exit in private equity vehicles is often uncertain. Although a private equity vehicle may have a stated horizon, say of ten years, the return of cash from the underlying deals may cause large amounts of capital to be returned before the stated horizon—or in many cases the horizon is extended to maximize the profitability of the underlying investments (or to maximize the collection of fees by general partners).

Several authors have used this search technology to consider the impact of illiquidity (search) frictions in various over-the-counter markets (such as Duffie, Garleanu and Pedersen 2005, 2007). While these are important advances for showing the effect of illiquidity risk on asset prices, they are less useful for deriving asset allocation advice on optimal private equity holdings. Duffie, Garleanu and Pedersen (2005, 2007) consider only risk-neutral and CARA utility cases, and restrict asset holdings to be 0 or 1. Garleanu (2009) and Lagos and Rocheteau (2009) allow for unrestricted portfolio choice, but Garleanu considers only CARA utility, and Lagos and Rocheteau focus on showing the existence of equilibrium with search frictions rather than on any practical calibrations. Neither study considers asset allocation with both liquid and illiquid assets.

III D. Asset Allocation with Stochastic Non-Traded Periods

Ang, Papanikolaou and Westerfield (2011) [APW] solve an asset-allocation problem with liquid securities, corresponding to equities that can be traded at any time, and illiquid securities, which can be interpreted as a private equity portfolio. The investor

has CRRA utility with an infinite horizon and can only trade the illiquid security when a liquidity event occurs, which is the arrival of a Poisson process with intensity λ . In this framework, the Merton model with continuous rebalancing is given by $\lambda \rightarrow \infty$. As λ decreases to zero, the opportunities to rebalance the illiquid asset become more and more infrequent. The mean time between rebalancing opportunities is $1/\lambda$. Thus, λ indexes a range of illiquidity outcomes.

The inability to trade for stochastic periods introduces a new source of risk that the investor cannot hedge. This illiquidity risk induces large effects on optimal allocation relative to the Merton model. APW show that illiquidity risk affects the mix of liquid and illiquid securities even when the liquid and illiquid returns are uncorrelated and the investor has log utility.

The most important result derived by APW is that the presence of illiquidity risk induces time-varying, endogenous risk aversion. The intuition is that there are two levels of wealth that are relevant for the investor: (1) total wealth, which is the same effect as the standard Merton problem where the risk is that if total wealth goes to zero, the agent cannot consume, and (2) liquid wealth. The agent can only consume liquid wealth. Thus, with illiquid and liquid assets, the investor also cares about the risk of liquid wealth going to zero. This can be interpreted as a solvency condition: an agent could be wealthy, but if this wealth is tied up entirely in illiquid assets, the agent cannot consume. Although the CRRA agent has constant relative risk aversion, the effective risk aversion—the local curvature of how the agent trades off liquid and illiquid risk in the portfolio—is affected by the solvency ratio of the ratio of liquid to illiquid wealth. This solvency ratio also becomes a state variable that determines optimal asset allocation and consumption. This illiquidity risk causes the optimal holdings

of even the liquid asset to be lower than the optimal holding of liquid assets in a pure Merton setting.

APW derive five findings that are important considerations for investing in private equity:

1. Illiquidity risk induces marked reductions in the optimal holdings of assets compared to the Merton model. Their calculations for the same risk aversion as a 60% risky asset holding (and 40% risk-free holding) in the Merton model, introducing an average rebalancing period of once a year, reduce the risky asset holding to 37%. When the average rebalancing period is once every five years, the optimal allocation is just 11%. Thus, private equity, which is highly illiquid, should be held in modest amounts.
2. In the presence of infrequent trading, the fraction of wealth held in the illiquid asset can vary substantially and is very right-skewed. That is, suppose that the optimal holding to illiquid assets is 0.2 when rebalancing can take place. Then the investor should expect the range of illiquid holdings to vary from 0.15 to 0.35 during non-rebalancing periods. Because of the skew, the average holdings to the illiquid asset will be higher than the optimal rebalancing point, at say 0.25. Thus, when an illiquid private equity portfolio is rebalanced, the optimal rebalancing point is much lower than for an average holding.
3. The consumption policy (or payout policy) with illiquid assets must be lower than the Merton payout policy with only liquid assets. Intuitively, holding illiquid assets means that there is additional solvency risk that liquid wealth goes to zero and consumption cannot be funded. Thus, payouts of funds holding illiquid assets should be lower than the case when these assets are all fully traded.

4. The presence of illiquidity risk means that an investor will not fully take advantage of opportunities that might look like close to an “arbitrage” – for example, where correlations to the liquid and illiquid returns are nearly plus or minus one. Traditional mean–variance optimizers without constraints would produce weights close to plus or minus infinity in these two assets. This does not happen when one asset is illiquid, because taking advantage of this apparent arbitrage involves a strategy that causes the investor’s liquid wealth to drop to zero with positive probability. Thus, near–arbitrage conditions when there is illiquidity risk are not exploited in APW in contrast to the Merton model.
5. Finally, the certainty–equivalent reward required for bearing illiquidity risk is large. APW report that when the liquid and illiquid returns are poorly correlated and the illiquid portfolio can be rebalanced, on average, once every five years (which is a typical turnover of many private equity portfolios), the liquidity premium is over 4%. For rebalancing once a year, on average, the illiquidity premium is approximately 1%. These numbers can be used as hurdle rates for investors considering investing in private equity.

A number of authors, including Dai, Li and Liu (2008), Longstaff (2009), De Roon, Guo and Ter Horst (2009) and Ang and Bollen (2010), also consider asset allocation where the illiquid asset cannot be traded over certain periods. However, in these studies the period of non–trading is deterministic. In contrast, the APW framework has stochastic and recurring periods of illiquidity. Deterministic non–trading periods are probably more appropriate for hedge fund investments, where lock–ups have known

expirations. Private equity investing may have random, and infrequent, opportunities to rebalance.

APW still miss a number of practical considerations that should be addressed by the future literature. The most important one has to do with the Merton setting into which APW introduce illiquidity, where there are no cash distributions; all risky asset returns (both liquid and illiquid) are capital gains. Private equity investments require cash flow management of capital calls and distributions. Some ad hoc simulations have been conducted by some industry analysts on this issue, like Siegel (2008) and Leibowitz and Bova (2009), but without explicitly solving for optimal portfolios with illiquidity risk. An extension of APW to incorporate cash flow streams could address this.

III E. Summary

The inability to continuously rebalance private equity positions, potentially even by paying transaction costs, makes optimal holdings of illiquid private equity investments very different from the standard Merton model, which assumes no illiquidity risk. Since transaction costs in rebalancing private equity portfolios are very large, in both entering new private equity positions and selling existing private equity positions, private equity positions should be expected to be rebalanced very infrequently, and investors should set very wide rebalancing bands. In asset-allocation models where illiquid assets such as private equity can only be traded upon the arrival of a (stochastically occurring) liquidity event, illiquidity risk markedly reduces the holdings of illiquid assets compared to the standard Merton model. For example, an asset that could be traded continuously in the Merton setting that is held with a 60% optimal weight would have an optimal holding of less than 10% if it could be rebalanced only

once every ten years, on average. The certainty-equivalent reward (or equivalently the hurdle rate) for bearing illiquidity risk is large. For a typical private equity investment that can be traded only once in ten years, on average, the illiquidity premium is well above 4%.

IV. Intermediary Issues in Private Equity

Most commonly, asset owners make private equity investments as a limited partner in a fund where investment decisions are made by fund managers acting as general partners. This arrangement raises potential agency issues. One characteristic of private equity investment is that the investment decisions arising from such management considerations and the related agency issues become intrinsically intertwined with private equity performance. In public equity markets, factor returns and active management can mostly be separated, due to the existence of investable index strategies.

IV A. Agency Issues

While the agency problem is central for private equity investments, there are few studies evaluating the optimal delegated portfolio management (see the good surveys by the Bank of International Settlements 2003, and Stracca 2006). There are, however, many studies on agency issues in standard corporate finance settings (for example, Salanie 1997, and Bolton and Dewatripont 2005). Delegated portfolio management is different from standard agency problems because the “action” chosen is generally observed (the investments made by the general partner), but the set of actions is unknown (the full set of deals available to the general partner). In contrast, in standard moral hazard problems the “action” is unobservable, but the set of potential actions is usually known. Thus, little is known about the optimal delegated portfolio contract, and the literature has few, if any, specific conclusions or prescriptions about what form the optimal private equity contract between limited partners and general partners should take.

Private equity investing is further complicated by having two levels of principal-agent relations rather than just a single one: a level between the limited partners (principal) and general partners (agent), and another level between the general partners as fund managers (principal) and the fund's underlying portfolio of companies (agent). Both levels rely on strong direct monetary incentives. Apart from these monetary incentives, however, the relation between limited partners and general partners is one with limited information, poor monitoring, rigid fee structures and the inability to withdraw capital or directly control managers. On the one hand, these features tend to heighten tensions between the limited partners and general partners and exacerbate, rather than alleviate, agency issues. On the other hand, the distance between the limited partner and general partner may allow general partners to invest and manage companies more freely.

The other principal-agent relation between the fund and its portfolio companies is one with strong governance, transparent information flows, good incentives for monitoring and a high alignment of interests between owners and management (see Jensen 1989). There is strong evidence that private equity funds add significant value, on average, to the companies in their portfolio. This literature is surveyed by Kaplan and Stromberg (2009).

The interactions between these two layers of principal-agent problems have not been fully explored. It is not inconceivable, though, that mitigating the principal-agent problems at the limited partner-general partner level would come at the cost of increasing the problems at the fund-company level. For example, greater transparency about the management of individual portfolio companies may in turn lead general partners to manage

these companies with an eye towards managing short-term earnings expectations and satisfying public expectations more broadly—a concern for publicly traded companies—rather than simply managing them to maximize their total value.

IV B. Private Equity Contracts

Because private equity is, by its nature, private, it is difficult to perform systematic large-sample studies of contractual features and see how they relate to performance. Gompers and Lerner (1999), Litvak (2009) and Metrick and Yasuda (2010) examine small samples of various private equity contracts. Several tentative conclusions emerge:

1. Private equity contracts are largely standardized. An often-quoted fee arrangement is a management fee of 2% and a carry of 20%. There is some variation in the numbers (for example, management fees tend to vary from 1% to 2.5%, and carried interest varies from 20% to 35%), but the general structure is widely used. Additionally, a substantial part of the general partner's compensation may be in the form of transaction fees. Private equity fees are high.
2. There is some variation in the specific provisions governing the calculation and timing of the fees and carried interest. For example, a management fee could be flat (on committed capital), declining over the life of the fund, a (time-varying but deterministic) combination of committed and managed capital, or even an absolute amount.
3. Fixed fee and performance components are not substitutes but complements. That is, funds tend to raise both the fixed and variable fee components, as well as the other compensation components. Fund size tends to be positively correlated with fees, and Kaplan and Schoar (2005), among others, find that

size is negatively correlated with performance. More recently, however, Robinson and Sensoy (2011a) investigate an extended sample with contract terms and performance, and find no relation between net-of-fee performance and the size of the fund or the fees.

4. There is a debate about the performance sensitivity of private equity compensation. Metrick and Yasuda (2010) find that close to one-half the present value of general partner compensation is from management fees rather than carried interest— and find this to be true for both venture capital and buyout funds. However, Chung et al. (2011) point out that a substantial amount of general partners' performance pay arises through the continuation value of raising future funds, which are highly sensitive to current performance.
5. Private equity contracts are complex documents. Litvak (2009), however, finds little relation between opaqueness and total compensation.

The management fees charged by private equity and venture capital funds are high. According to Metrick and Yasuda (2010), such fees consume at least one-fifth of gross private equity returns. They find that out of every \$100 invested with a venture capital fund, an average of \$23 is paid to the general partners in the form of carry and management fees. For buyout funds, the mean of the carry and management fees comes to \$18 per \$100. The high fees charged by general partners point to the fact that if an institutional investor wishing to allocate to private equity can do this in-house, then there are substantial savings available. Of course, attracting talent and running an in-house private equity shop presents a different set of agency issues than out-sourcing to private equity funds with general partners. Despite the pessimistic

view of returns of private equity investments to limited partners in Section II, the high private equity fees imply that if asset owners can come close to capturing gross returns, then private equity becomes much more attractive.

While opacity per se does not seem to be related to total compensation and returns, it has other important add-on effects for other aspects of an asset owner's larger portfolio. Complexity and non-transparency can increase agency problems and make risk management more difficult. The leverage involved in many buyout funds can be more expensive, and is often harder to monitor, than leverage done directly by the asset owner.

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Risks, Returns and Optimal Holdings of Private Equity: A Survey of Existing Approaches

In this paper, Morten Sorensen en Andrew Ang (both Columbia Business School) survey the academic literature about the risks and returns of private equity (PE) investing and optimal PE allocations. Empirically, the irregular nature of PE investments complicates the estimation and interpretation of standard risk and return measures. These complications have lead to substantial disparity in performance estimates reported across studies.