Dispersion Trading in German Option Market

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Abstract

There has been an increasing variety of volatility related trading strategies developed since the publication of Black-Scholes-Merton study. In this paper we study one of dispersion trading strategies, which attempts to profit from mispricing of the implied volatility of the index compared to implied volatilities of its individual constituents. Although the primary goal of this study is to find whether there were any profitable trading opportunities from November 3, 2008 through May 10, 2010 in the German option market, it is also interesting to check whether broadly documented stylized fact that implied volatility of the index on average tends to be larger than theoretical volatility of the index calculated using implied volatilities of its components (Driessen, Maenhout and Vilkov (2006) and others) still holds in times of extreme volatility and correlation that we could observe in the study period. Also we touch the issue of what is (or was) causing this discrepancy.

1. Introduction.

The dispersion trading strategy that we examine in this paper attempts to profit from trading call options of the index (in this occasion, the German stock index DAX 30) and the index itself against the call options of the index constituents and the stocks. The index is a weighted average of stocks, therefore, it should be theoretically possible to nearly perfectly hedge a position in call options of the index with the opposing positions in call options of the shares (i.e. long index call option should be hedged by a combination of short individual stocks' call options). Since the absolute values of call options are incomparable due different prices of the underlying asset (even if all those options are at-the-money, hence, the strike price equals the spot price, have the same time to maturity and the common interest rate as in this occasion), the methodology of the strategy approaches the implied volatilities, which are reverse engineered from the quoted option prices, assuming that they are determined by the famous Black-Scholes options pricing formula, rather than the values of the derivatives themselves. One could, therefore, expect that similarly as the index is a weighed average of the shares, the implied volatility of the index is also a weighted average of the implied volatilities of the individual stocks. However, in practice one can observe that these two volatilities do not always coincide. Firstly, it is a stylized fact that implied volatility of the index on average tends to be larger than theoretical volatility of the index calculated using implied volatilities of its components. Secondly, the mismatch can also arise from the market inefficiency.

To test whether there were trading opportunities caused by this mismatch we collected the German option data from November 3, 2008 to May 10, 2010. We compare the index option implied volatility with the theoretical index volatility, calculated using the traditional Markowitz model for the variance of the portfolio. If the difference between them is large enough, we initiate the trade by taking opposite positions in index call options and individual stocks call options and delta-hedge not to be exposed to movements of the market.

We find that the difference between index option implied volatility and the theoretical implied volatility largely depends on the time period used to calculate the correlation between stocks. Nevertheless, there still were 29 days when the market closed with a difference between implied index volatility and any of the theoretical volatilities big enough to enter a trade. We conclude that the number of opportunities is larger when the market is more volatile and slightly larger when the market correlation increases. As for trading results, we find that the dispersion trading strategy is actually very efficient before transaction costs are implemented. After they are, however, introduced, the profitability is dramatically cut. This implies that strategy should be mostly interesting for market makers, who do not pay the transaction costs, such as bid-ask spread or commissions.

Review of the literature.

Several studies on dispersion trading have been made. Marshall (2008), who is the base of research made in this paper, shows that S&P 500 "index-option-implied volatility" (both for calls and puts) tends to exceed "Markowitz-implied volatility" (MIV). MIV is calculated by plugging the implied volatilities of individual options, weights of the stocks (both observable) and historical correlation (estimated in pre-study period and assumed to be a good predictor of realized correlation) into a standard Markowitz formula for the variance of the portfolio:

$$\sigma_p^2 = \sum_{i=1}^{500} \sum_{j=1}^{500} w_i \, w_j \sigma_i \sigma_j \rho_{i,j}, \qquad (1)$$

where σ_p^2 denotes the variance of the portfolio, w_i - the weight of stock i within index, σ_i - the standard deviation of the stock i and $\rho_{i,j}$ represents the correlation between stocks i and j.

Marshall finds that for calls IOIV exceeded MIV on 312 days (compared to 193 days when it was the opposite). Moreover, on average the difference between IOIV and MIV for calls was 1.21 volatility points (which accounts for 5.03% of the IOIV) and 0.76 volatility points (or 1.4% of IOIV) for puts. He uses the Kolmogorov – Smirnov test to show the difference (IOIV-MIV) is normally distributed and rejects the null hypothesis of average difference to be equal to 0 with significance level of 0.000001. The conclusion achieved is that index option implied volatility is indeed rich (larger than theoretical one) and component implied volatility is on average cheap. Marshall advocates the theory that the difference comes from the correlation risk premium. Since MIV relates on historical return correlations, it is sensitive to the market cycles. It is well known that correlation tends to rise in stressful times so a seller of the option requires extra premium for this risk.

Hence, the direct strategy to exploit this mismatch is to buy options on individual stocks and sell options for indexes whenever the difference between IOIV and MIV is large enough to overcome bid-ask and other transaction costs (we will discuss transaction costs separately). The author concludes that between October 31, 2005 and November 1, 2007, there were 84 end-of-the-day (there should be much more within the days) trading opportunities for calls and 91 for puts after taking into account the costs. Marshall suggests that opportunities still exist because of the complexity of the strategy. First, it requires access to real-time data of thousands of options and second, it requires an ability to execute the trades extremely rapidly as the implied volatilities are continuously changing.

Lozovaia and Hizhniakova in their dispersion article (2009) discuss the possible techniques of the strategy. They point that the best timing to execute the direct strategy (short index, long constituents) is when implied index volatility exceeds the realized (or historical) one and when the implied index correlation is close to its maximum registered value since the strategy works better if implied volatility of the index is highly correlated with implied volatilities of its components. The other important issue is the selection of stock options for the offsetting position. BNP Paribas in their article on Option Strategies (2005) stresses that the strategy requires a lot of time and expertise to manage the greeks due to the complexity of the strategy as well as a large number of options traded. In addition, trading all components of the index might be too costly. Therefore BNP suggests buying 10-50 options of the largest weights and largest correlation with the index (also depending on available strikes and liquidity). Hence, a method of selecting individual stocks is needed. Deng (2008) uses Principal Component Analysis (PCA) to pick the best component stocks. He first constructs a Variance – Covariance matrix of weighted stock returns. Step 2 is to decompose the covariance matrix into the eigenvalue vector ordered by importance and the corresponding eigenvectors and to choose the first n principal components such that the explained variance is above 90%. Finally, he selects stocks according to principal components chosen in Step 2.

Several papers also focus on the cause of the mismatch between realized and implied correlation of index components. Driessen, Maenhout and Vilkov (2006) argue that index options are "rich" in volatility (meaning that the implied volatility is relatively high in these options) because correlation risk is priced. Index options are expensive as they allow investors to hedge against the market-wide correlation shocks and due to transaction costs and margin requirements this difference between volatility of index and its components cannot be fully arbitraged away. On the other hand, Deng (2008) claims that the profitability of dispersion trading almost vanished in 2000, when some institutional changes, which made arbitraging cheaper, were introduced. He finds it as evidence of inefficiency in the option markets as institutional changes should have not affected the fundamental structure of correlation risk premium. However, both these studies estimates the efficiency of dispersion trading as an every day rolled strategy. To goal of our study is to "wait" for profitable opportunities which are most likely caused by market inefficiency.

Approach.

The technique employed of the dispersion trading strategy we examine in this paper involves taking the opposite positions of the index call options and the individual shares' call options. At the initiation of the trade, the value of the index calls is EUR 1m, so is the combined value of the individual calls (the values of stocks' calls reflect the weight of the stock in the index). Not to be exposed to the movements in the market (but only to the movements of volatility) stock and index trading is also used to delta hedge. The number of options, bought or sold at the beginning, is stable throughout the trade, so the value of options' position depends solely on the development of options' prices. On the other hand, the stock position is adjusted daily according to the delta of the options. We assume that we can borrow or lend (depending on the value of the stocks and the index, needed to buy or short to delta-hedge) at a risk free rate. Hence, in total the profit or loss is a combination of a change in the value of the options, a change in the value of the stocks and the index owned and the interest earned or paid.

Other types of Dispersion Trading.

Although our primary goal is to examine whether there were any opportunities of trading index volatility versus its components' volatility during the given period (after employing the strategy described by Marshall), numerous other dispersion trading strategies also exist. One possible strategy consists of trading volatilities of the least volatile stocks (such as utilities) against the most volatile ones (such as financials). The trade is usually based on the historic difference between volatilities and is executed whenever a substantial (or large enough to overcome transaction costs) difference between the historic one and current one occur. The direction of the trade is such that an investor is betting that the current difference will converge to historic one. Also there are other ways than option trading to enter a position in dispersion trade.

However, the underlying idea whatever derivatives are traded is the same: the goal is to leave exposure of the instrument to volatility as this is one input that a trader has an opinion about and to hedge the other exposures (to the movement of underlying or interest rate) as much as possible. One frequently used derivative to execute dispersion trade is the variance swap. As we know, the payoff of a long position in variance swap is:

$$\mathrm{N}_{\mathrm{var}}(\sigma^2_{\mathrm{realized}} - \sigma^2_{\mathrm{strike}}),$$

Where $\mathrm{N}_{\mathrm{var}}$ is the variance notional, $\sigma^2_{\mathrm{realised}}$ - realized variance and $\sigma^2_{\mathrm{strike}}$- strike variance. Hence, the direct dispersion strategy consists from going long in variance of swaps of individual constituents and shorting the swap of the index. The variance swaps provide a relatively straightforward way to implement volatility trading and are often used in more complex strategies. Nevertheless, they do not help to avoid the traditional difficulties of so sophisticated strategies as they too require a lot of financial expertise and effort to manage the exposures and do the dynamic delta hedging, especially when trading large indexes. In addition, liquidity issues also arise as variance swaps on some equities are still not actively traded or the market is not deep enough, hence, a selection of stocks is also necessary.

2. Data.

All the necessary data was taken from the DataStream. Since the position in a trade is never hold more than a month, 1-month Euribor is assumed to be a risk free rate. The volatilities of the options provided in the DataStream are interpolated to be the implied volatilities of 1 month at-the-money option (more on this in Appendix).

The weights are taken from the official Deutsche Boerse website. The DAX is a capital weighted performance index, which is adjusted according to capital changes. By construction the dividends are assumed to be reinvested and all options written on the index or shares are European.

3. Methodology.

The calculation of Markowitz Implied Volatility (MIV) for the index requires a massive amount of correlations. Marshall, who studied the opportunities of dispersion trading in S&P 500 index, had to use the approximation of standard Markowitz formula (1) as it is technically almost impossible to obtain time series of 124750 correlations. He therefore obtains the volatility of the index replicating portfolio by plugging the correlation between each equity and index to the formula $\sigma_m = \sum_{i=1}^{500} w_i \, \sigma_i \rho_{i,m}$, where $\rho_{i,m}$ is the correlation mentioned above. Marshall argues that since the portfolio structured to replicate the index will have zero unsystematic risk relative to the index, the correlation between the replicating portfolio and the index will be 1.0. However, DAX consists only of 30 equities so it is not that complicated to calculate the time series of correlations between each asset. We therefore could achieve higher accuracy as the approximation becomes less reliable when sudden changes in weights (such as the decision of Detsche Boerse to cap the weights at 10% after Wolkswagen reached 27% in October 29, 2008) or substitutions of the constituents occur. In such a way the correlations between implied volatilities for 3 different time horizons (3 months, 6 months and 1 year) were calculated. Hence, the correlation between any two stocks is changing every day corresponding to the correlation between these two stocks on a given time horizon. MIV for each day is obtained by plugging in the weights, the implied volatilities and the obtained correlations to the standard Markowitz formula and then compared to the index option implied volatility (IOIV). The trade is initiated if MIV, calculated by using all three correlations of different time horizons, is larger or smaller than IOIV by 1 volatility point or more in each case. Such restrictions dramatically decrease the number of opportunities. However, they are necessary to decrease the volatility of the payoff of the strategy (and hence increase the Sharpe ratio) and also to prevent trying to capitalize on small profit opportunities as they would not offset the transaction costs.

The options which are closest to maturity (the 3rd Friday of the month) are usually traded. However, if maturity of the options is closer than 2 weeks at the day the trade is initiated, we trade options with the maturity at the next month. It is necessary to deal with longer term derivatives in order to be hedged against movements in the markets. Since the gamma of the at-the-money options rockets up when maturity is approaching, delta hedging would be not enough to secure the position against massive shocks in the P&L account.

4. Empirical results.

Having the weighting of DAX, implied call options volatilities for each equity from November 3, 2008 to May 10, 2010, and stock returns from May 11, 2008 (to calculate the longest term correlation between stocks of 1 year) we calculated the Markowitz Implied Volatility for a given period and compared it to Index Option Implied Volatility. Below we present the graph of implied correlation MIV vs IOIV and the statistical features of different volatilities.

As we can see, there is no systemic pattern regarding the difference between IOIV and MIV. Most of the time the IOIV lies between the 3 lines of different MIV's, occasionally jumping out of boundaries in either side (these are the days we test as potential statistical arbitrage opportunities). The only conclusions are that MIV, calculated using longer term correlations, at a given period on average slightly exceeds MIV by shorter term correlations. Also we can observe that volatility of the volatility increases if shorter term correlations are used, which could have been anticipated. Additionally, we can clearly spot a downwards sloping trend of all the volatilities, which was also very much expected as the initial peak can be related to post-Lehman turmoil in the markets and the consistent decreasing of volatility from March, 2009, is directly related to stock rally after the first quarter of 2009.

Table 1. Statistical Properties of Implied Volatilities.

	3 months	6 months	1 y	IOIV
Average	29.18%	29.84%	31.65%	30.91%
St.dev	10.83%	10.65%	10.31%	10.73%
Median	26.79%	27.68%	28.12%	27.80%
cases MIV>IOV	154	120	244	
cases IOIV>MIV	242	276	152	
correlation with IOIV	0.94	0.98	0.97	

Given the current literature one would expect IOIV on average to exceed synthetically created MIV. Indeed, we can see that IOIV is greater than MIV calculated using historical 3 months correlation between stocks by 1.73% (accounting for 5.59% of IOIV), and greater than 6-month MIV by 1.07% (3.45% of IOIV). Moreover, there were substantially larger amount of days when IOIV exceeded MIV than the other way around. On the other hand, MIV, calculated using the 1 year correlations, on average was greater than IOIV by all fundamental statistical measures (0.74% on average). Since we use the same implied volatility it can only mean that the historical 1 year correlation tended to be higher than the implied correlation of the index option. This could have been anticipated as longer term correlation tends to exceed short term correlation (systematic factors dominates idiosyncratic ones on the longer run). It might be the case that market believes 1 year is too long period to calculate correlation for options with maturity under 1 month and more relies on shorter term correlations. Hence, in our strategy 1-year MIV works rather as an additional protector from risky trades than an indicator.

To test this formally we examine the time series of difference between IOIV and the respected MIVs. Because of the autocorrelation of the differences it was not possible to appeal the Central Limit Theorem to ground the normality of the differences. To make the observation independent we therefore used bootstrapping approach to generate 1000 simulations of the average difference between IOIV and respective MIV. Based on the mean and standard deviation of the bootstrapped average differences we could strictly reject the null hypothesis that the average difference is 0 in all three time series (t-value must be greater than 2.33 to reject the null at 1% significance level).

Table 2. Differences between bootstrapped MIV and IOIV

	IOIV – 3m MIV	IOIV – 6m MIV	IOIV - 12m MIV
Average	1.72%	1.07%	-0.74%
Standard deviation	0.191%	0.116%	0.124%
T value	8.991	9.211	6.018

These findings are only partially in accordance with the theory of dispersion traders that index options tend to be rich in volatility comparing to individual components. It is clear that the difference between MIV and IOIV strongly depends on the period taken to calculate correlation. The tendency in this particular time period in DAX index reveals that the longer period for estimating correlation is used, the greater correlation is obtained. As a result MIV also increases if long term correlation is used and even exceeds IOIV if 1 year correlation is used. It can be interpreted as a sign that the traditional dispersion strategy, in which long position in individual options and short position in index option is constantly rolled, lost its credibility, or, as traders claim, it “was arbed to death”. Having quickly developing abilities to arbitrage (meaning both, more technical abilities to rapidly exploit any opportunity as well as more financial knowledge as the strategy became well documented) the profitability of the strategy vanished. This statement is in line with Volatility Dispersion Trading article of Deng (2008) who finds that the profitability of the strategy actually turned to be negative in 2001-2005 period (for S&P 500 index) when structural changes of option trading were introduced in 2000.

As mentioned before, in order to prevent relying on correlations estimated for 1 particular time period we initiated the trades only when IOIV was greater or smaller than all three different MIV. In addition, we required the difference to be larger than 1 volatility percent in order to minimize risk of the strategy. All in all, there were 80 days in the given period when these conditions were satisfied. In most occasions (57 out of 80) it was IOIV that exceeded volatility of the index option by more than 1 percent.

However, due to clustering of opportunities only 29 trades were initiated (new position is not opened if the previous one is not yet closed). In order to increase Sharpe ratio and manage risk profit take away and stop loss limits are used. The trades are closed if the profit reaches 10% or the loss exceeds 3%. Additionally, the position is not hold for more than two weeks even if S/L or T/P are not hit in order to avoid using capital too long for a non-performing trade. Once again, we only use the closing parameters of the day so both, the profit and loss, often exceed the limits. If we had access to intraday data this drawback could have been minimized or removed completely. To be neutral movements in the markets we are using delta hedging (the position in equities and the index is daily adjusted according to the delta of options). However, we do not hedge against movements in the interest rates as it would require trading additional derivatives. That would make the strategy more complex to initiate and manage as well as it would increase the transaction costs. Therefore the possible gains of rho hedging does not offset all the costs and effort needed to neutralize movements of interest rate.

The results of the strategy are presented below:

Table 3. Results in absence of transaction costs

Average returns	Volatility	Skewness	Kurtosis	Sharpe
10.22%	8.62%	-0.05	0.18	1.18

It is tempting to conclude that the dispersion trading strategy was very efficient in a given year. 10.22% average returns per 1 trade, relatively modest volatility of 8.62% and low interest rates produce Sharpe ratio equal to 1.18. As expected, the measure of asymmetry shows that the results are more skewed to the right and have longer left tail, and kurtosis is 0.18 (in comparison, normal distribution have 0 skewness and kurtosis equal to 3). Of course, it does not make sense to test the normality of distribution (or any other distribution) as it can not be normal by construction of stop loss and profit take away (returns will be clustered just above

10% and just below 3%); in addition, there is too small number of observations to make any definite conclusions about the distribution.

In comparison, DAX index gained 11.9% on annualized basis at a given period. However, the market was much more volatile than returns of the strategy (30.07% versus 8.62%), so overall risk/return tradeoff produced Sharpe ratio of only 0.36 comparing to 1.18 of the dispersion trading strategy.

It is also worth noting that the trades on average took only 3 days to either reach the profit target or to hit the stop loss. It is a favorable feature for the strategy as it requires less effort to manage it and decreases the opportunity cost.

However, the results must be treated with caution. Firstly, we can spot that the average returns are actually higher than our profit take away limit (10.22% versus 10%). Once again it could be solved with the access to continuous intraday data. In this occasion ability to close the position when the target is reached would decrease the average returns, but it would also substantially decrease the volatility. Since there were only 6 trades out of 29 which ended up in losses, the volatility would be minimized and as a result the Sharpe ratio would boost even with lower average returns.

Secondly and more importantly, no transaction costs, which are the main reason why different kind of discrepancies in prices occur, were taken into account. The dispersion strategy involves transaction costs of first setting up the portfolio of options and then closing it down in the end as well as costs of daily rebalancing stock portfolio in order to delta hedge. Marshall distinguishes three types of transaction costs the trader faces: bid-ask spread, the commission and the market impact cost. Dealing with all of them requires a set of assumptions. Bid-ask spread varies depending on the liquidity of the equity or derivative as well as the momentum in the markets. Most actively traded stocks have a very narrow spread while the least liquid ones have such a sizeable difference between the prices that a trader has to pay to buy and gets for selling it that any statistical arbitrage strategy becomes virtually impossible. A trader might also need to pay a commission, which is a fixed fee per contract or time period. However, for a couple of reasons we are going to assume that this strategy is free of commission. Firstly, the size of commission depends on agreement between a trader and a broker so it varies for different traders. Secondly, some professional traders who use this strategy (such as market makers) do not need to pay the commission at all, so we are going to neglect this type of costs in our calculations.

A trader might also incur market impact costs which are faced when the trader's position alone moves the price of the security to any direction. Naturally, it happens when either the position is very large in size, or the depth of the market is not sufficient to absorb the trade. Once again this type of costs is very difficult to estimate and varies substantially amongst different securities. We are therefore going to assume that DAX index is liquid and large enough that these costs would be neglected. Obviously, it is a questionable assumption as even in such an index as DAX there are some securities that are not so actively traded, hence, the strategy would incur more costs. One common way to avoid this problem is to trade only the largest stocks (the ones which have the biggest impact to the index) and derivatives on them instead of trading the whole index. Even though in such a way some theoretical accuracy of the strategy is lost, savings of transaction costs (commission as well as the market impact costs) make the trade more efficient (not to mention that it becomes more easily manageable).

Hence, we are therefore examining only the impact of bid - ask spread. Unfortunately, it is virtually impossible to exactly estimate these costs. Most of the data sets provide only the middle price between bid and ask as a real option price (however, Beygelman (2005) shows that the real option price should be not an arithmetical average between bid and offer but closer to the bid price). In addition, the spread varies not only amongst different underlyings, but also amongst time to maturity and the spot/strike price ratio. The liquidity of the option tends to decrease (hence, the spread increases) when the maturity is approaching or the option becomes deeper in-the-money or deeper out-of-the-money (ATM are the most liquid). Furthermore, the spread of even the same option fluctuates throughout the day. Moreover, in German market there are no minimum boundaries of a spread as in the US (where 5 cents is the minimum spread for option cheaper than $3 and 10 cents for more expensive ones), so we cannot assume that options of DAX constituents are liquid enough to be traded with a minimum spread. Due to these circumstances we base our estimations on US option market, which is better documented. We assume that the spreads of DAX constituents are as tight as they are in the most liquid US options (hence, the results will be upwards biased). Therefore we consider that the spread of the option traded for less than 1 euro is 5 cents, the spread of the option between 1 and 5 euro is 10 cents, 20 cents for the option traded under 10 euro, and 30 cents for the options over 10 euro. These rough assumptions, offered by some option traders (1option.com to mention one) just as any other attempts to estimate bid/ask spread offer only approximate transaction costs and provide a general intuition of whether the strategy could be efficient in the real market conditions. Moreover to assuming narrow spread for options, at this moment we will also neglect any transaction costs for trading stocks and the DAX index. It is once again a rough assumption since the portfolio of stocks has to be rebalanced every day in order to delta hedge. So even though the spreads on stocks are more tight as the underlyings are more liquid than options on them, these costs could easily destroy the profitability of the strategy due to high volume of trading (especially when the market is more volatile, requiring more active delta hedging). However, as mentioned above, we are testing if the costs of trading options alone could eat out the profitability of the strategy. The results after including the assumed bid/ask spread are the following:

Table 4. Results after introducing transaction costs

Returns	Volatility	Skewness	Kurtosis	Sharpe
3.34%	8.44%	0.38	0.81	0.4

We can see that even when being very optimistic about the transaction costs the profitability of the strategy drops dramatically from 10.22% to 3.34%. Naturally the Sharpe ratio was dragged down from 1.18 to 0.40 even though volatility slightly decreased. Moreover, it is important to keep in mind that delta-hedging costs were not taken into the account. If we assume a once again narrow bid/ask spread of 0.5% (of the stock price or index value) the strategy becomes unprofitable, and, in fact, generates losses. One possible way to reduce these costs could be to adjust the delta hedging in such a way that the stock portfolio would be rebalanced not daily but when delta hits a certain limit.

There is also another way to calculate the bid/ask spread. Instead of monetary terms the spread is sometimes expressed in volatility terms (comparing the implied volatilities of the option bid and offer prices). The spread expressed in such a way is very convenient to use as it directly shows what difference must occur between MIV and MOIV in the market that it would be a potentially profitable statistical arbitrage opportunity. However, the main problem is once again to calculate that spread. Just like in previous case, it also requires a huge set of assumptions - in fact, even a bigger one as the volatility spread is translated from the monetary one by at first converting it to each individual option and then obtaining the average (so the length of estimation period and also the method of averaging must be chosen).

However, no matter of the spread estimation method used we can conclude that at the given period the strategy was not profitable when even optimistic transaction costs were included. This statement goes in line with the current common opinion that arbitrageurs made the market too efficient for dispersion trading and that only the traders who do not need to pay bid/ask spread (such as market makers) can still employ the strategy.

Results in presence of volatility smile.

As mentioned earlier, the initial results might be biased because the options which are not at the money are mispriced. The model assumes the flat volatility with respect to strike/spot ratio, hence, it estimates the value of the option by plugging in the implied ATM volatility into the Black–Scholes formula. However, we know that in the option world the implied volatility decreases as the strike price increases, so, in other words, it makes a downwards sloping skew with respect to strike/spot price ratio. In our case (dealing with calls) it means that in-the-money options are undervalued by the model and the out-of-the-money ones are overpriced. Although in any one particular trade such mispricing could increase or decrease the real profit, the risk is that overall the expected returns might be too optimistic when the volatility smile is not taken into account.

In order to examine the impact of the skew we synthetically create the volatility smile and compare the results. Unfortunately, although there are numerous studies addressing the cause and the existence of the volatility smile (starting with the work of Rubinstein in 1994), the approximations of slope of the skew itself are not well documented. Since the behavior of the smile is dependable on situation in the market (i.e. demand for downward protection) as well as liquidity issues (the lack of liquidity for the option of a particular strike pushes the price and, hence, the implied volatility down), the shape of the skew is individual for every asset and also time varying. Due to these reasons practitioners use approximations of the smile which cannot provide exactness but rather a reasonable assumption of how the profitability will be affected by the implied volatility curve. With the absence of real market data for different strike options we are going to rely on Goldman Sachs studies on quantitative strategies where they plot that the implied volatility of the stock index is on average increasing by 4 percentage points when the strike/spot ratio decreases by 0.1, and remains constant at ATM - 6% level if the ratio increases above 1.15. Hence, we assume that the volatility skew is almost linear which is of course not accurate but it will provide some intuition about the strategy's sensitivity to this factor. All in all, results after including volatility skew into the model (but in absence of any transaction costs) are presented in the following table.

Table 5. Results after introducing volatility smile

Returns	Volatility	Skewness	Kurtosis	Sharpe
9.60%	7.56%	-0.59	0.1	1.27

The presence of volatility smile indeed altered the results. The returns dropped by 0.62% to 9.60% but the volatility also decreased by 0.88%. Consequently, the Sharpe ratio improved to 1.27 from 1.18. Also it is worth noting that in a few of occasions it took more days for the strategy to reach the profit target of 10% and a number of trades decreased as a result (the new trade is not initiated if there is another still open not to double the risk). In addition, the longer period of positions kept open also means more effort required to serve the trade as well as a rise in opportunity costs (opportunity to invest the capital elsewhere).

On the other hand, with the absence of transaction costs dispersion trading strategy still looks efficient after introducing the volatility skew. Nevertheless, there is couple of risk factors a trader should be aware of. Firstly, as previously mentioned the volatility smile is individual for each asset, so the behavior of options on less liquid stocks might be hardly predictable (as usual, one possible solution could be to try to avoid including such options to the portfolio at all and concentrate on more tradable ones). Secondly, aside from a volatility skew with respect to strike/spot ratio there is also a volatility smile with respect to time to maturity, which was not taken into account in the calculations. The reason is an almost unpredictable behavior of implied volatility curve when the maturity is approaching. Although in general implied volatility increases as time to maturity increases (as more events might happen), there are factors that can affect this tendency. The skew might be downwards sloping with respect to time in case the short term volatility exceeds the historical volatility. The time curve might be also influenced by the announcements which are certain to take place, such as quarterly reports. Hence, all the attempts to involve time skew would hardly add any accuracy. All in all, in order not to make too cumbersome assumptions, we decided that 1 month is an OK proxy for options with the maturity from 1 week to a month. However, traders who have an access to a larger data set could try to estimate the potential risk of the time skew (although in this particular strategy it should have only marginal effect as both positions, the long and the short, consists of the options with the same maturity).

Cluster Analysis.

Since the existence of alpha opportunities depends on the difference between IOIV and synthetically created MIV we can expect that more opportunities arise when the IOIV is larger or, in other words, these two variables are positively correlated. To test this relationship we used Spearman's rank order correlation as it does not require assumptions about the distribution of variables. The given period was divided into 18 months of 22 trading days and ranked according to the average IOIV and the number of alpha opportunities.

Table 6. Monthly Implied Volatility and Numbers of Trading Opportunities

Month	MIV(3m)	MIV(6m)	MIV(1y)	IOIV	Rank (IOIV)	Nr of Opportunities	Rank of Opportunities
1	0.5972	0.5739	0.5893	0.5797	1	14	2
2	0.4268	0.4288	0.4368	0.4456	2	12	3
3	0.3497	0.4160	0.4128	0.4254	4	10	4
4	0.3548	0.4160	0.4175	0.4261	3	9	5
5	0.4163	0.3973	0.4038	0.4085	5	4	7
6	0.3614	0.3346	0.4063	0.3556	6	2	9.5
7	0.3276	0.2927	0.3495	0.3167	7	0	16.5
8	0.2642	0.2722	0.3021	0.2993	8	2	9.5
9	0.2330	0.2666	0.2990	0.2772	9	0	16.5
10	0.2737	0.2799	0.2765	0.2657	10	15	1
11	0.2581	0.2527	0.2453	0.2531	12	1	12.5
12	0.2631	0.2788	0.2506	0.2566	11	1	12.5
13	0.2158	0.2667	0.2486	0.2521	13	1	12.5
14	0.1485	0.2147	0.2095	0.2171	16	4	7
15	0.1920	0.2045	0.2356	0.2242	15	1	12.5
16	0.2148	0.1869	0.2493	0.2249	14	0	16.5
17	0.1708	0.1416	0.1904	0.1584	18	0	16.5
18	0.2145	0.1843	0.2136	0.1849	17	4	7

Spearman’s rho showed strong positive link of 0.535. The t-value was 2.534, so the null hypothesis that correlation is not greater than 0 was rejected at 1% significance level (the Student distribution with 16 degrees of freedom was used to compare). We therefore can conclude that implied volatility of the index and a number of substantial discrepancies between IOIV and MIV are indeed positively correlated. As intuitively anticipated, the trader can expect more profitable opportunities when the markets are more volatile than in times when they are calm. From a sequence of monthly profitable opportunities we can see that the number gradually decreased from 14 (out of 22 days) in the first month after the turmoil caused by Lehman Brothers collapse to 0 in the middle of 2009 when it became clear that stock market recovery was gaining momentum.

Since the strategy deals not only with the volatility but also with the correlation of the stocks, it is also interesting to check whether the number of profitable opportunities is influenced by the implied correlation of the index.

As we can see from the graph, the movements of implied correlation had no systemic pattern in the given period. To test whether it affects the number of trading opportunities we used the same methodology as in the previous case.

Table 7. Monthly Implied Correlation and Number of Opportunities

Month	Implied Correlation (IC)	rank (IC)	Nr of Opportunities	Rank of Opportunities
1	0.5597	16	14	2
2	0.6329	7	12	3
3	0.6278	9	10	4
4	0.6491	5	9	5
5	0.6903	2	4	7
6	0.5713	14	2	9.5
7	0.5861	12	0	16.5
8	0.6381	6	2	9.5
9	0.5612	15	0	16.5
10	0.5782	13	15	1
11	0.5942	11	1	12.5
12	0.6186	10	1	12.5
13	0.6776	4	1	12.5
14	0.7932	1	4	7
15	0.6822	3	1	12.5
16	0.6293	8	0	16.5
17	0.5092	18	0	16.5
18	0.5131	17	4	7

Spearson's rho this time was 0.131, showing only weak positive link between implied correlation and the number of opportunities. Moreover, the null that the correlation is not greater than 0 could not be rejected even at 10% significance level. We can conclude that there is only weak relationship between the implied correlation and the number of trading opportunities.

5. Conclusions.

The widely discussed believe of dispersion traders that index options tend to be "rich" in volatility comparing to the volatility of its individual components was partially confirmed in this study on DAX index from November 3, 2008 to May 10, 2010. Index Option Implied Volatility exceeded Markowitz Implied Volatility, calculated using 3-month and 6-month correlations by 1.73% and 1.07% respectively. However, the average difference is considerably lower than the ones estimated in previous studies. Moreover, MIV, calculated using 1-year correlation, actually exceeded IOIV. This implies that the difference between MIV and IOIV strongly depends on the period taken to calculate correlation. Hence, the method of calculating correlation is critically important to the strategy. All in all, there were only 37 trading opportunities which matched our criteria for the difference between IOIV and every estimate of MIV to be larger than 1%. Comparing to the results of the previous studies it can be interpreted as a sign that markets are becoming more efficient, hence, opportunities of statistical arbitrage are decreasing. The small discrepancy which is left could be attributed to correlation risk premium and still not perfectly efficient market, although opportunities practically disappear when real market restrictions are imposed.

The initial theoretical model, which used ATM implied volatility for all the options and did not take into account the transaction costs, yielded very promising results. The average returns of the strategy were 11.44% and the Sharpe ratio reached 1.54. However, the situation completely turned around when even the most optimistic transaction costs, primarily option bid/ask spread, were introduced. After assuming that all the options are liquid enough to have spreads close to the tightest possible, the average returns dropped to only 3.35% and Sharpe to 0.47. Introduction of bid/ask spread for the stocks needed to delta-hedge, again even if assumed to be extremely tight, destroyed the profitability of the strategy completely. Moreover, a couple of types of other transaction costs, such as commission and market impact cost ("slippage") were not taken into account. This implies that only the traders who do not need to pay transaction costs (or as least as possible), such as market makers, could execute this plain vanilla dispersion trading strategy.

Also we tested the potential risk caused by the volatility smile. After assuming a downwards sloping volatility skew with respect to strike/spot price ratio the returns decreased by 0.47% to 10.97% and the Sharpe to 1.47 (in absence of transaction costs). The risk still remains that the assumption of the volatility skew was firstly not accurate and secondly, it might be very different for the individual stocks and the index itself. Some risk that volatility time skew could affect the efficiency of the strategy also remains.

Additionally, we showed that the more volatile the markets are, the more opportunities to initiate the dispersion strategy arise. Also we observed that the number of opportunities is positively influenced by the implied correlation of the index, but this correlation between potentially profitable opportunities and the implied correlation is statistically not significant at 10% level.

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Appendix.

In the calculations we use the implied volatilities of the of 1 month at-the-money option Therefore, the calculations are a little bit biased as volatility smile was not taken into account when the value of options was synthetically calculated using Black-Scholes formula. However, it is not a critical assumption since the negative influence is somehow reduced by the fact that the offsetting positions are taken in a trade. Hence, from a prospect of time skew in case the implied volatility of the index should increase when the 3rd Friday of the month (the expiry day of the options) is approaching due to the volatility smirk, we can reasonably assume that the implied volatilities of the individuals should also increase, so both (the long and short) positions are undervalued. In case the current volatility in the market is lower than historic one and as the result the implied volatility curve is upward sloping with respect to time to maturity, both positions will be overvalued in our calculations as the expiry date approaches. A more complicated problem is created by the upward sloping volatility smirk with respect to strike/spot price ratio. Although initially all the options are at the money, they move to different directions when maturity is approaching. Therefore, even when the option of the whole index goes out of the money (hence, such option in the calculations will be undervalued as we use each day's implied volatility of the ATM option), there will still be some individual at-the-money options, overvalued for the same reason. However, one can rely on the fact that the index is the weighted average of the stocks, so the mismatch in overpriced options (comparing to the real market price) will be not exactly, but reasonably accurately compensated by the mismatch in underpriced options.