TILBURG UNIVERSITY

Capital Structure Arbitrage

Ričardas Visockis

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Abstract

This paper examines the risk and return of the so-called "capital structure arbitrage", which exploits the mispricing between the company's debt and equity. Specifically, a structural model connects a company's equity price with its credit default swap (CDS) spread. Based on the deviation of CDS market spreads from their model predictions, a convergence type trading strategy is proposed and analyzed using daily CDS spreads on 419 North American obligors for the period of 2006-2010.

Table of Contents

1 Introduction

The first structural pricing model for credit risk was developed by Merton (1974), based on the results of Black and Scholes (1973), who values equity as a call option on the firms' assets. Following his work Black and Cox (1976) and Longstaff and Schwartz (1995) developed first-passage time models with an exogenous default barrier which also constitute the fundament for the CreditGrades model of Finger et al. (2002). While the asset value dynamics of firms in the CreditGrades model is based on diffusion process, Zhou (2001) additionally introduced jumps to account for sudden market changes. Although, Cserna and Imbierowitz (2008) show that the Zhou model is superior for capturing even slight inefficiencies, however, such additional return comes from more frequent and, therefore, more expensive trading in terms of commissions. In addition, the Zhou model is more sophisticated than the CreditGrades and the Merton models, what also makes it more difficult to implement and reduces its popularity among traders.

Since the early 2000's, capital structure arbitrage has become popular among hedge funds and bank proprietary trading desks. The arbitrageur may use a structural model to gauge the richness and cheapness of the credit default swap (CDS) spread. She can calculate the "fair" value of the CDS spread via some structural model based on a company's liability structure and its market value of equities. When the arbitrageur finds that the market CDS spread is substantially larger than the predicted spread, a number of possibilities can be entertained. The arbitrageur may think that either the equity market mis-prices equity value or the CDS market is "wrong". Luckily, both cases result in the exact same strategy: the arbitrageur should sell credit protection and delta-hedge it by selling short delta equities and expect convergence to occur. The logic is that if the CDS spread widens or if the equity price rises, the best one could hope for is that the theoretical relation between the CDS spread and the equity price would prevail, and the equity position can cushion the loss on the CDS position, and vice versa. The reverse is true when the market CDS spread is substantially lower than the predicted spread, when the arbitrageur should buy credit protection and hedge it by buying delta shares.

Although, such arbitrage strategy relaying on temporal mispricing of the market may look appealing at the first glance, but the "wrongfulness" of the market is questionable. In Currie and Morris (2002), traders are quoted saying that the average correlation between the CDS spread and the equity price is only on the order of 5% to 15%. Similar results are reported in Yu (2005), where correlation is between 1% and 6% among investment grade obligors and between 7% and 23% among speculative grade and not rated obligors. Therefore, the lack of a close correlation between the CDS and equity markets suggests that there can be prolonged periods when the two markets hold diverging views on an obligor. Irrational behavior of investors may also be an issue. In fact, prolonged periods of time before convergence may be partially caused by emotion driven or purely speculative trading in equity and/or the CDS markets. In other words, the divergence in short term and long term views of investors may cause drawdown on the capital structure arbitrageur position.

Furthermore, the arbitrageur is apt to imperfections of model implementation that can reduce the profitability of capital structure arbitrage. For example, the CDS market spread could be higher than the equity based model spread because of a sudden increase in asset volatility, newly issued debt or hidden liabilities that have come to light. These elements are omitted in a simple implementation of structural models using historical volatility and balance sheet information from Compustat quarterly data. In other words, one might enter into a trade when there are no profitable opportunities in either the CDS or the equity market.

In a first step, I calculate CDS premiums using two structural models, namely, the Merton model and the CreditGrades model. While the Merton model is very well known among academicals, the CreditGrades model was published in 2002 by affiliates of large investment banks and quickly became an industry benchmark according to Currie and Morris (2002) and Yu (2005). Further, I compare these model CDS spreads to market observations. For this purpose I employ a dataset which comprises stock prices, CDS spreads, balance sheet data and ratings for a total of 419 obligors in the period from January 2006 to December 2009. I find that predictions of the CreditGrades model follow market CDS spreads quite accurately whilst predictions of the Merton model are substantially underestimated comparing to the market CDS spreads. The deviations between market and model spreads serve as an indication of inaccurate market pricing and signal arbitrage opportunities in the firm's capital structure. Further, I use the assumption of F. Yu (2005) that the particular CDS is priced correctly if the deviation to the respective model spread amounts to less than 50 percent[1]. Therefore, if deviations from convergence can be observed, positions according to the principles of capital structure arbitrage are initiated. Accordingly, a portfolio is formed including debt (CDS) together with equity (stocks) as a hedging device. The individual positions are terminated if

a) market and model spreads converge
b) losses amount to 20 percent of initial capital
c) convergence did not occur during pre-specified holding period

My analysis of investment strategy follows the one presented in Yu (2005) and Duarte, Longstaff and Yu (2007), and Cserna and Imbierowicz (2008). In addition to a popular CreditGrades model, I also include the Merton model which is not used in any of previously mentioned studies. Moreover, this paper extends the analysis of Yu (2005) and Duarte, Longstaff and Yu (2007) by implementing a significantly larger database of North American obligors and taking the observation period from 2006 to 2010, which also extends the study of Cserna and Imbierowicz (2008). The analysis of Yu (2005) shows the riskiness of the arbitrage strategy for

[1] 100 percent and 200 percent deviations are also used in this research.

individual trades. However, when individual trades are aggregated into monthly capital structure arbitrage portfolio returns, the strategy appears to offer attractive Sharpe ratios ranging from 0.15 to 0.89 for different strategies. Furthermore, monthly index capital structure arbitrage returns cannot be explained by several well known equity and bond market risk factors. Cserna and Imbierowicz (2008) find that the capital structure arbitrage strategy produce significant positive average returns in the investigated period of 2002-2006. Moreover, they find that average monthly returns decline over time and, they provide empirical evidence that the CDS market becomes efficient after years 2004/2005. Also, Cserna and Imbierowicz (2008) show that the arbitrage strategy provides higher returns for speculative grade obligors than for investment grade obligors which is in line with standard investment theories. Additionally, they show that more complex models like the model of Zhou (2001) and Leland and Toft (1996) produce larger arbitrage returns than the CreditGrades model. Although, the study of Duarte, Longstaff and Yu (2006) show very attractive Sharpe ratios for capital structure arbitrage strategies ranging from 0.80 to 0.92, however, researchers emphasize riskiness of those strategies. The initial capital required to produce a 10 % annualized standard deviation of returns is between 45 % and 90 %, whereas other fixed income arbitrage strategies require only 30 % to 65 % of initial capital. Moreover, Duarte et al. (2007) find that capital structure arbitrage returns are related to factors that proxy for economy-wide financial distress. Also, neither of the strategies hold significantly positive alpha after accounting for fees.

The remainder of this paper is structured as follows. In the next section the anatomy of trading strategy is presented in depth, followed by dataset presentation and description of the Merton, the CreditGrades and the Zhou model. In the next part I present the hypothesis of this paper, followed by implementation and calibration of both models used in this study. Thereafter, I present the case study of Consolidation Coal Inc and overall results of the trading strategy for all obligors. Finally, I check the aggregate index returns for systematic equity market risk factors.

2 Anatomy of the Trading Strategy

This section analysis the anatomy of capital structure arbitrage. Since this is a model based strategy, I start with an introduction to CDS pricing, and then explore issues of implementation.

2.1 CDS Pricing

A credit default swap (CDS) is an insurance contract against credit events such as the default on a bond by a specific issuer (the obligor). The buyer pays a premium to the seller once a quarter until the maturity of the contract or the credit event, whichever occurs first. The seller is obligated to take delivery of the underlying bond from the buyer for face value should a credit event take place within the contract maturity. Although, this is the essence of the credit default swap contract, there are a number of practical issues. For example, if the credit event occurs between two payment dates, the buyer owes the seller the premiums that have accrued since the last settlement date. Another complicating issue is that the buyer usually has the option to substitute the underlying bond with other debt instruments of the obligor of equal priority. Therefore, the CDS spread has to account for the value of a cheapest-to-deliver option. For a simpler treatment of CDS pricing I assume continuous premium payments and ignore the embedded option[2].

First, the present value of the premium payments is equal to

$$E\left(c\int_0^T exp\left(-\int_0^s r_u du\right)1_{\{r>s\}}ds\right),$$

where c denotes the CDS spread, T is the CDS contract maturity, r the risk-free interest rate, and τ the default time of the obligor. Assuming independence between the default time and the risk-free interest rate, this can be written as

$$c\int_0^T P(0,s)q_0(s)ds,$$

where $P(0,s)$ is the price of a default-free zero-coupon bond with maturity s, and $q_0(s)$ is the risk-neutral survival probability of the obligor, $Pr(\tau > s)$, at $t = 0$.

[2] Duffie and Singleton (2003) show that the effect of accrued premiums on the CDS spread is typically small. For a discussion on cheapest-to-deliver option, see Berndt et al. (2004).

Second, the present value of the credit protection is equal to

$$E\left((1-R)exp\left(-\int_{0}^{\tau} r_u\,du\right)1_{\{\tau<T\}}\right),$$

where R measures the recovery of bond market value as a percentage of par in the event of default. Assuming a constant R, this can be written as

$$-(1-R)\int_{0}^{T} P(0,s)q_0'(s)ds,$$

where $-q_0'(t) = -dq_0(t)/dt$ is the probability density function of the default time. The CDS spread is then determined by setting the initial value of the contract to zero:

$$c = -\frac{(1-R)\int_0^T P(0,s)q_0'(s)ds}{\int_0^T P(0,s)q_0(s)ds}.$$

The preceding derives the CDS spread on a newly issued contract. If it is subsequently held, the relevant issue is the value of the contract as market conditions change. To someone who holds a long position from time 0 to t, this is equal to

$$\pi(t,T) = \big(c(t,T) - c(0,T)\big)\int_{t}^{T} P(t,s)q_t(s)ds, \tag{2.1}$$

where $c(t,T)$ is the CDS spread on a contract initiated at time t and with maturity date T, and $q_t(s)$ is the probability of survival through s at time t.

To compute the risk-neutral survival probability, I use the structural approach, which assumes that the default occurs when the firm's value falls below a certain default threshold. Since the equity is treated as the residual claim on the assets of the firm, the structural model can be estimated by fitting equity prices and equity volatilities. Because of the focus on trading strategies, the choice of a particular structural model should be less important. One model can produce slightly more unbiased spreads than another, but it is the daily change in CDS spread that is of principal concern here. At a daily frequency the only important contributor to the CDS spread is the equity price, as other inputs and parameter that vary from one model to another are essentially fixed. Another issue would be that different structural models produce different hedge ratios, that can impact trading profits. However, Schaefer and Strebulaev (2004) show that even a simple model such as Merton's (1974) produce hedge ratios for corporate bonds that cannot be rejected in empirical tests.

Consequently, I use the Merton's model and the CreditGrades model to implement the trading analysis. The CreditGrades model is jointly developed by RiskMetrics, JP Morgan, Goldman Sachs and Deutsche Bank. The latter model is based on the model of Black and Cox (1976), and contains the additional element of uncertain recovery. This latter feature helps to increase the short-term default probability, which is necessary to produce realistic levels of CDS spreads. Although, CreditGrades model is not very appealing on theoretical side due artificially increased short-term CDS spreads, however, on the practical side, the model provides closed-form solutions to the survival probability and the CDS spread. The CreditGrades model is also reputed as the model used by most capital structure arbitrage professionals. For completeness, sections 3.2 and 3.3 give an overview over of the Merton model and the CreditGrades model, respectively.

2.2 Implementation

Assume that one has available time series of observed CDS market spreads $c_t = c(t, t+T)$ on newly issued, fixed maturity contracts. Also, there is available time series of observed equity prices S_t and information about the capital structure of the obligor. Such dataset allows traders to calculate theoretical CDS spread based on the ones chosen structural model. Let the predicted CDS spread be c'_t and the difference between the two time series $e_t = c_t - c'_t$.

If the focus is on pricing, then one would like to attain the best possible fit to market data. However, what motivates capital structure arbitrage is the trader's belief that e_t will move predictably over time. Specifically, assume that the pricing error has a mean of $E(e)$ and a standard deviation of $\sigma(e)$. As mentioned before, one would not expect pricing error to be unbiased, however, say that there comes a point at which the deviation becomes unusually large, for example when $e_t > E(e) + 2\sigma(e)$. At this point trader sees an opportunity for trading. If he considers the CDS overpriced, then he should sell the credit protection. On the other hand, if he considers the equity to be overpriced, then he should sell the equity short. Either way trader's strategy is based on the assumption that convergence will occur.

To see this logic more clearly, assume that the theoretical pricing relation is given by $c'_t = f(S_t, R, \theta)$ where S_t is the equity price, R is the expected recovery on a specific class of a firm's debt[3] and θ denotes the other fixed parameters, such as asset volatility. The actual CDS spread is given by $c_t = f(S_t, R^{imp}, \theta)$ where R^{imp} is the implied recovery rate obtained by inverting the

[3] In our case, it is senior subordinated debt

pricing equation. When $c_t > c'_t$, it may be that the implied recovery rate R^{imp} is unreasonably high, for example close to 1, and shall decrease to a lower level. The correct strategy in this case is to sell CDS and sell equity as a hedge, which is similar to strategy involving stock options, when one would sell overpriced stock options and use delta hedging to neutralize the effect of changing stock price. Another possibility is that the CDS is priced fairly, but the equity price reacts too slowly to new information. In this case the equity is overpriced and one should short CDS as a hedge against shorting equity. Both cases involve the same trading strategy. Yet another possibility is that other parameters of the model, such as the debt per share are mis-measured. This can be a problem when using balance sheet variables from the financial statements that are infrequently updated. Moreover, the gap between the CDS market and model spreads could simply be due to model misspecification. It is possible to address the last two scenarios, for example, by calibrating the model with option implied volatility, carefully monitoring the changes in a firm's capital structure or simply trying alternative models.

The other important part of previously described trading technique is hedging. While delta hedging is typically invoked in this context, there are differences from the usual practice in trading equity options where the trader bets on volatility and uses hedging to neutralize the effect of equity price changes. Trading the CDS, the equity hedge is often static, meaning that the equity position stays fixed for entire duration of the strategy. Moreover, traders often modify the model based hedge ratio according to their own opinion of the particular type of convergence that is likely to occur, for example, trader may decide to underhedge if he feels confident about the CDS spread falling, or in reverse, he may decide to overhedge if he feels that it is more likely that the equity price will fall. However, some traders do not seem to use a model based hedge ratio at all. Instead, they identify the maximum loss that can occur should the obligor default, and shorts an equity position in order to break even. In this research, I use the model based hedge ratio when entering the trade and fix this hedge ratio throughout the trade.

The next important aspect of the trading strategy is the liquidation of the open position. I assume that the exit will occur on the following conditions:

1) The pricing error e_t reverts to its mean $E(e)$.
2) Convergence has not occurred by the end of pre-specified holding period or the sample period.

To summarize, the risk involved in capital structure arbitrage trading can be understood in terms of the subsequent movements of the CDS spread and the equity price. For example, after the arbitrageur has sold credit protection and sold equity short, four likely scenarios can happen:

1) $c_t \downarrow, S_t \downarrow$. This is the case of convergence, allowing the arbitrageur to profit from both positions.

2) $c_t \downarrow, S_t \uparrow$. The arbitrageur loses on the equity, but profits from the CDS. He will profit overall if the CDS spread falls more rapidly than the equity price rises, allowing convergence to take place partially.
3) $c_t \uparrow, S_t \downarrow$. The arbitrageur loses on his CDS position, but the equity position acts as a hedge against this loss. There will be overall profit if the equity price falls more rapidly than the CDS spread rises.
4) $c_t \uparrow, S_t \uparrow$. This is a sure case of divergence. The arbitrageur suffers losses from both positions regardless of the size of the equity hedge.

Clearly, the delta hedging is effective in the second and the third scenarios. The likelihood of the first scenario, however, is critical to the success of capital structure arbitrage.

2.3 Trading Returns

The main part of this paper is the analysis of the trading returns. At the initiation of the trading strategy, the credit default swap position has zero market value. Although, this may be possible theoretically, however, in practice traders must have a margin account. Therefore, trading returns can be calculated by assuming that the arbitrageur possesses certain level of initial capital, which is also used to finance initial equity hedge. The assumed amount of initial capital is not trivial, since some of the trades may have to be liquidated early due to a large drawdown.

After the initiation of trading strategy, the value of the CDS and the equity positions can change. While the latter is trivial, the value of the CDS position has to be calculated according to equation 2.1. However, implementation of equation 2.1 is not straightforward and several simplifying assumptions have to be made. First, equation 2.1 requires secondary market quotes on an existing contract, while the CDS market predominantly quotes spreads on freshly issued CDS contracts with a fixed maturity. Therefore, I use the approximation $c(t,T) \approx c(t,t+T)$. This assumption should not have high impact to trading returns as I use only a 30-day convergence window.

2.4 Trading Costs

Except in section 8, where I explore the impact of the transaction costs on trading returns, I assume a 5% bid/ask spread for trading CDS. This implies that for an obligor with 100 basis point spread, the buyer of the CDS will be paying 102.5 bp per annum while the seller will receive only 97.5 bp. Clearly, the level of CDS transaction costs was higher during the earlier phase of the market. However, Yu (2005), and Cserna and Imbierowicz (2008) motivates 5% bid/ask spread to be reasonable estimate in recent periods. In addition, since the CDS bid/ask spread probably the largest source of trading costs for capital structure arbitrage, I ignore the transaction costs on common stocks. This assumption should not distort the trading returns as I will use static equity hedging in the trading strategy.

3 Data

In this study I use daily observations of 5-year mid-market CDS spreads obtained from DataStream. The choice of 5-year CDS contracts on senior unsecured debt is motivated by the largest liquidity and in line with the literature. My sample contains data of US dollar denominated CDS contracts of North American obligors from 2006 through the end of 2009 covering the peak of financial crisis which is of the main interest of this research.

Firstly, in order to generate subset suitable for trading analysis I remove all cases with standard deviation of zero for more than five trading days. Furthermore, I exclude all obligors with less than one full year of daily observations. Although, such actions may cause survivorship bias, but they help to construct data sample suitable for trading analysis.

Secondly, I restrict CDS data sample to publicly traded companies as I need firm-specific information to employ structural credit risk models. However, financial companies are excluded from this study as it is difficult to interpret their capital structure. Consequently, I obtain daily stock quotes from CRSP and quarterly balance sheet data from COMPUSTAT. Additionally, S&P credit ratings are retrieved from COMPUSTAT ratings and 5-year constant maturity Treasury yields are obtained from DataStream. It is worth mentioning that quarterly balance sheet data is lagged by one month from the end of the quarter in order not to have look-ahead bias [4]. Further, I merge the CDS data with daily stock observations and balance sheet information. Additionally, I divide merged dataset into subsets accordingly to S&P credit ratings of the obligors. Consequently, data filtering yields final data sample of 419 obligors with overall 371321 daily CDS quotes.

Table 1: Number of observations per rating class and calendar year

The table displays the number of daily observations on CDS for all rating classes and non-rated issuers (NR) over the period 2006 until 2009 for individual years as well as the complete time horizon.

Rating	**2006**	**2007**	**2008**	**2009**	**2006-2009**
NR	3,402	3,060	2,620	2,886	11,968
D	134	176	104	183	597
B	6,501	6,254	5,941	6,243	24,939
BB	14,468	13,290	12,165	12,113	52,036
BBB	43,404	39,208	37,859	38,733	159,204
A	26,402	24,747	24,925	25,050	101,124
AA	3,952	3,878	4,044	4,085	15,959
AAA	1,672	1,447	1,367	1,008	5,494
Total	99,935	92,060	89,025	90,301	371,321

[4] The same method is used in F.Yu (2005)

Table 2: Summary statistics for the 419 obligors across credit rating

N is the number of obligors, SPD is the daily CDS spread in basis points, VOL is the 1000-day historical equity volatility, LEV is the leverage ratio defined as total liabilities divided by the sum of total liabilities and the equity market capitalization, and SIZE is equity market capitalization in millions USD.

Rating	**N**	**SPD**	**VOL**	**LEV**	**SIZE**
NR	15	398	0.44	0.44	10,342
D	1	1151	0.80	0.88	4,089
B	27	529	0.50	0.62	5,182
BB	62	362	0.44	0.49	6,519
BBB	179	173	0.35	0.44	12,477
A	111	113	0.35	0.37	29,207
AA	17	186	0.35	0.33	73,162
AAA	7	26	0.35	0.34	177,931

4 Empirical Analysis

4.1 Hypotheses

Prior findings of Yu (2006), Larsen and Bajlum (2007), and Cserna and Imbierowicz (2008) suggest that the CDS market has been inefficient. I am analyzing this in more detail by splitting the overall sample into investment grade and speculative grade obligors, also through the years of observation. Since CDS have not been issued before the mid-1990's, it can be assumed that the market did not always reflect accurate pricing and inefficiencies in general exist.

H1: The capital markets remain efficient throughout the observation period reflected in non-positive excess capital structure arbitrage returns.

While stocks are traded on an exchange, CDS are still traded in an over-the-counter market, resulting in more constrained trading. However, through time and experience investors should improve their pricing and trading knowledge and react faster and more accurate to changing market information. Therefore, I expect that the CDS market becomes more efficient over time, which would be reflected in declining capital structure arbitrage returns. If, on the other hand, strategy holding period returns remain stable or even increase through time, there could be two possible explanations: the market remains inefficient or the models used are misspecified.

H2: Capital markets are efficient during Credit Crunch reflected in non-positive capital structure arbitrage returns in the period of 2007-2008.

During the years of Credit Crunch there was a lot of human emotion such as fear and insecurity which led to sudden jumps in asset prices. Therefore, arbitrageurs should have had plenty of good opportunities to make money in the markets, since asset values returned to fundamentals, if, of course, had not defaulted. Thus, I expect somewhat higher capital structure arbitrage returns during 2007-2008 year period.

H3: The arbitrage returns will increase by declining credit quality of obligors.

The other part of my analysis concerns the difference in arbitrage returns over rating classes. Risk averse investors demand higher rewards for riskier positions. Accordingly, investment in higher rated obligors, which can be assumed to have a smaller default probability and less volatility in market spreads, should also produce lower returns in my arbitrage strategy compared to investments in lower rated obligors.

5 Model Implementation

5.1 Preliminaries

Let $V_t, S_{t,} D_t,$ and K_t denote the asset value, the market value of equity, the amount of total liabilities, and the default barrier of the associated firm at time t divided by the number of shares outstanding, respectively. Correspondingly, σ_V and σ_S are used to denote annualized asset and equity volatilities. R symbolizes the recovery rate on the senior unsecured debt underlying CDS contract initiated at time $t = 0$ with maturity $T = 5$. Considering recent Fitch overview of U.S. high yield bond recovery rates[5], results of Covitz and Han (2004), and Altman, Brady, Resti and Sironi (2005), R is set to 0.4, as it is a long term mean of U.S. corporate recovery rates over senior unsecured debt. Furthermore, $\Delta t = 1/252$, and r is five-year constant maturity Treasury yield used as a proxy of risk-free interest rate.

5.2 The Merton Model

Merton (1974) proposes a simple model of the firm that provides a way of relating credit risk to the capital structure of the firm. In this model the value of the firm's assets is assumed to follow a lognormal diffusion process with a constant volatility. The firm has issued two classes of securities: equity and debt. The equity receives no dividends whilst the debt is a pure discount bond where a payment is promised at time T.

If at time T the firm's asset value exceeds the promised payment, D, the lenders are paid the promised amount and the shareholders receive the residual asset value. On the other hand, if the asset value is less than the promised payment the firm defaults, the lenders receive a payment equal to the asset value, and the shareholders get nothing.

In this model a firm's asset value is modeled as a lognormal process and it is assumed that the firm would default if the asset value, V, falls below a certain default boundary X. Moreover, the default is allowed at only one point in time, T. The equity, E, of the firm is modeled as a call option on the underlying assets. The value of the equity is given as

$$E = V\Phi(d_1) - X_T exp(-rT)\Phi(d_2), \tag{5.1}$$

where

[5] For further details see Fitch Ratings, "U.S. High Yield Default and Recovery Rates 2009 Review and Outlook" (2010)

$$d_1 = \frac{\ln\left(\frac{V}{X}\right) + \left(\mu + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \tag{5.2}$$

$$d_2 = d_1 - \sigma\sqrt{T}, \tag{5.3}$$

and Φ represents the cumulative normal distribution function. The debt value, D, is then given by

$$D = V - E.$$

The CDS spread can be computed as[6]

$$s = -\frac{1}{T} ln\left(\Phi(d_2) + \frac{V}{X} exp(rT)\Phi(-d_1)\right),$$

where V is the initial asset value of the firm, X is the default barrier for the firm, i.e., if the firm's asset value V is below X at the terminal date T, then the firm is in default. μ is the drift of the asset return, and σ is the volatility of the asset returns.

Hedging costs are proportional to the size of Δ, which may be computed in closed-form as follows:

$$\begin{aligned}
\Delta &= \frac{\partial C}{\partial S} \\
&= \frac{\partial C}{\partial V} \cdot \frac{\partial V}{\partial S} \\
&= \frac{\partial}{\partial V} \cdot \Phi(-d_2) \cdot \frac{1}{\Phi(d_1)} \\
&= -\Phi(-d_2) \cdot \frac{\partial d_2}{\partial V} \cdot \frac{1}{\Phi(d_1)} \\
&= \frac{-\Phi(-d_2)}{\Phi(d_1)} \cdot \frac{1}{V\sigma\sqrt{T}}
\end{aligned}$$

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6 For further details refer to Navneet Arora et al, "Reduced Form vs. Structural Models of Credit Risk: A Case Study of Three Models" (2005)

5.3 The CreditGrades Model

In this model, V_t is assumed to follow a geometric Brownian motion with zero drift under the risk-neutral measure, namely

$$\frac{dV_t}{V_t} = (r - q)dt + \sigma_V \, dW_t,$$

where W_t is a Wiener process, r is a risk-free interest rate, q is dividend rate and $(r - q)$ is asset drift rate. In CreditGrades technical document it is assumed that over time a firm issues more debt to maintain a steady level of leverage, or else pays dividends so that the debt has the same drift as the stock price. Therefore, $(r - q) = 0$.Such assumption of zero drift rate results in stationary leverage ratios. According to Collin-Dufresne and Goldstein (2001) structural credit default models with stationary (mean-reverting) leverage ratios produce credit spreads that are more consistent with empirical findings compared to outputs of other structural credit default models. Therefore, the default barrier is defined by $K_t = L \cdot D_t$, where L is a log-normal random variable with $E(L) = \bar{L}$ and $var(\ln L) = \lambda^2$ revealed at the moment of default[7]. The uncertainty of the default barrier allows possibility that the firm's asset value may be closer to the default point than we may expect it to be according to past accounting data. Therefore, randomness of the default barrier results in higher short-term default probabilities. The current probability $q(t)$ that the asset value does not reach the default barrier before $t \in [0; T]$ is approximated using time-shifted Brownian motion, yielding the following result[8]:

$$q(t) = \Phi(a_t^+) - d \cdot \Phi(a_t^-), \tag{5.4}$$

where $\Phi(..)$ is the cumulative distribution function of the standard normal distribution and

$$a_t^{\pm} = -\frac{A_t}{2} \pm \ln\frac{d}{A_t}, \qquad A_t^2 = \sigma_V^2 \cdot t + \lambda^2, \qquad d = \frac{(S_t + \bar{L} \cdot D_t) \cdot exp(\lambda^2)}{\bar{L} \cdot D_t}.$$

Note that $q(0) \neq 1$ due to uncertainty in the default barrier at $t = 0$. Moreover, it implicitly includes the possibility of default in the period $(-t; 0]$. Although, this may be considered an issue in modeling assumptions, at the same time, this feature aids in obtaining a simple formula for survival probability and in producing reasonable spreads for short maturity instruments.

The value of the current credit default swap premium $c(0, T)$ initiated at time $t = 0$ with maturity T is expressed as

7 In this paper I keep R constant and use implied $\bar{L}$ of 10 first CDS quotes, as specified in F.Yu (2005)

8 The approximation assumes that W_t starts not at $t = 0$, but from an earlier time. As a result, even for very small t the default probability is not equal to 0. For more details refer to Lardy, Finkelstein, Khuong-Huu and Yang (2000).

$$c(0,T) = r \cdot (1-R) \cdot \frac{1 - q(0) + H(T)}{q(0) - exp(-r \cdot T) \cdot q(T) - H(T)}\,, \tag{5.5}$$

where

$$H(T) = exp(r \cdot \xi) \cdot \big(G(T+\xi) - G(\xi)\big),$$

$$G(T) = \ d^{\omega+1/2} \cdot \Phi(g_T^-) + d^{-\omega+1/2} \cdot \Phi(g_T^+),$$

$$g_T^{\pm} = \frac{-\ln d \ \pm \omega \cdot \sigma_V^2 \cdot T}{\sigma_V \cdot \sqrt{T}}, \qquad \xi = \frac{\lambda^2}{\sigma_V^2}, \qquad \omega^2 = \frac{1}{4} + 2 \cdot \frac{r}{\sigma_V^2}.$$

The asset volatility σ_V is approximated in Finger et al. (2002) as

$$\sigma_V = \sigma_S \cdot \frac{S_t}{S_t + \bar{L} \cdot D_t},$$

whereas σ_S is estimated from historical data. Specifically, in CreditGrades Technical Document it is shown that the most precise estimation window is 1000-day period.

Hedging costs are proportional to the size of δ, which may be computed through $\delta(t,T) = \frac{\partial \pi(t,T)}{\partial S_t}$ and equation (5.5):

$$\delta(0,T) = \frac{1}{r}\frac{\partial c(0,T)}{\partial S}\Big(q(0) - q(T)e^{-rT} - e^{r\xi}\big(G(T+\xi) - G(T)\big)\Big),$$

Because by definition c is numerically equal to $c(0,T)$, which corresponds to an equity price of S. I then differentiate $c(0,T)$ numerically with respect to S to complete the evaluation of δ.

5.4 The Model of Zhou

This model is the most sophisticated model among the three models described in this research. Although, the Zhou model is theoretically appealing and Czerna and Imbierowitz (2008) find that this model generates close to market CDS spreads, the Zhou model is not popular among traders due to its complexity. The application of this model consists of a parameter estimation which is followed by a Monte Carlo approach. Let X_t denote the asset value of the associated firm at time t relative to the default barrier, i.e. $X_t = V_t / K_t$. Here, it is assumed that $V_t = S_t + D_t$ and $K_t = D_t$. In Zhou (2001) X_t follows a jump-diffusion process given by

$$\frac{dX_t}{X_t} = (\mu - \lambda \cdot \upsilon)dt + \sigma dW_t + (\Pi - 1)dY_t\,, \tag{5.6}$$

where μ is a drift and σ is a volatility parameter, Y_t is a homogenous Poisson process with intensity λ, and Π is the log-normal jump-amplitude with $ln\Pi \sim N(\mu_\pi;\ \sigma_\pi^2)$. The parameter υ satisfies $\upsilon = E(\Pi - 1) = exp(\mu_\pi + \sigma_\pi^2/2) - 1$.

Note that W_t, Y_t and Π are assumed to be independent. According to Scherer (2005) and Zhou (2001), the specification in (5.6) allows for the possibility of sudden changes in firm's asset value resulting in higher short term spreads. Applying Ito's lemma Cserna and Imbierowicz (2008) obtain

$$dx_t = \left(\mu - \frac{\sigma^2}{2} - \lambda \cdot \upsilon\right) dt + \sigma dW_t + ln\Pi\ \mathrm{dY_t}. \tag{5.7}$$

Let $\theta = (\mu, \sigma, \lambda,\ \mu_\pi, \sigma_\pi)'$ denote the parameter vector to be estimated from observed sample of x_t with n observations. As shown in Wong (2006), the probability of observing more than one jump during a short period of time (i.e. between two sampling periods) is small enough to ignore. Hence, the corresponding likelihood function $L(\theta)$ is given by

$$L(\theta) = \prod_{i=2}^{n} g(x_i|x_{i-1}, \theta),$$

where $g(x_i|x_{i-1}, \theta)$ is the density function of x_i conditioning on x_{i-1} which is approximated via

$$\mathrm{g(x_i|x_{i-1}, \theta) = (1 - \lambda \cdot \Delta t) \cdot f_X(x_i|x_{i-1}, \theta) + \lambda \cdot \Delta t \cdot f_{XY}(x_i|x_{i-1}, \theta).}$$

The maximum likelihood estimation is performed numerically for each firm followed by a Monte Carlo approach, which incorporates the estimates of θ and is based on simulated samples of the discrete time version of x_t given in (5.7) under the risk-neutral measure. However, this procedure is complicated, since x_t itself is a non-linear function.

The simulated samples are generated for each observation of x_t. Let $\tilde{x}_{i,j}$ denote the *i*-th simulated observation in the *j*-th sample and let τ_j denote a hitting time satisfying $\tau_j = min\{i \,|\, \tilde{x}_{i,j} \leq 0\}$, where $i = 1, \dots, m$ and $j = 1, \dots, M$. The current credit default swap premium $c(0,T)$ is calculated by

$$c(0,T) = -T^{-1} \cdot ln\left(1 - \sum_{j=1}^{M} \frac{z_j}{M}\right),$$

where

$$z_j = \begin{cases} (1 - R) \cdot exp\left(\tilde{x}_{\tau_j,j}\right) & if \left\{\tau_j\right\} \neq \emptyset \\ 0 & otherwise. \end{cases}$$

6 Calibrating Model Parameters

6.1 The CreditGrades Model

First of all, in order to implement the survival probability formula (5.4), it is necessary to link the initial asset value V_0 and the asset volatility σ_V to market observables. It is accomplished by examining the boundary conditions[9]. The focus is on long-term tenors $(t > \lambda^2/\sigma^2)$, since the short-term default probability is mainly driven by the level of λ.

In general, the equity and asset volatilities are related through

$$\sigma_S = \sigma_V \frac{V}{S}\frac{\partial S}{\partial V},$$

where S is the firm's equity price and σ_S the equity volatility.

The first boundary condition is the behavior of V near the default threshold $L \cdot D$. The assumption is that as default approaches, the value of the equity approaches zero (that is, $V_{|S=0} = LD$). The second boundary condition is far from default barrier, where $S/V \rightarrow 1$.The simplest expression of the initial asset value V_0 that satisfies the near and far from default boundary conditions is

$$V_0 = S_0 + \bar{L}D, \tag{6.1}$$

where S_0 is the current stock price. This also gives the needed relation between market observables and asset value and asset volatility

$$\sigma_V = \sigma_S \frac{S}{S + \bar{L}D}.$$

In deriving (5.4), another assumption has been that the asset value has zero drift $\big((r - q) = 0\big)$. It is important to note that it is not the asset drift itself, but rather the drift of the asset value relative to the default boundary that is relevant in this situation. It is assumed that on average over time a firm issues more debt to maintain a stable level of leverage, or else pays dividends so that the debt has the same drift as the stock price. Therefore, given (6.1), to avoid arbitrage the same drift should be assigned to the asset value V, implying that the drift of the assets relative to the default barrier is indeed zero.

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[9] Finger et al. "CreditGrades Technical Document" (2002)

Although, Cao, Yu and Zhong (2007) and Benkert (2004) have shown that implied volatility derived from option markets dominates historical volatility in forecasting the future volatility on individual stocks, the latter one is used in this research due to data availability and accuracy. Therefore, 1000 days of equity returns standard deviation is used as a volatility estimate. This choice is well justified in CGTD (CreditGrades Technical Document) as it provides the most accurate estimates of the volatility implied by the 5-year CDS quotes.

The percentage standard deviation (λ) of the global recovery L is estimated using the Portfolio Management Data and Standard & Poor's database (Hu and Lawrence (2000)). Based on the results of the previously mentioned study λ is set equal to 30%. Further, using the recommendations given in CGTD the expected recovery on a specific class of firm's debt (R)[10] is set equal to 0.4. Although, all the previous aspects of CreditGrades model application were in line with the CGTD the implementation of the CreditGrades model here differs from that of CGTD in one crucial aspect. The CGTD assumes that $\bar{L} = 0.5$ and uses a bond-specific recovery rate R taken from a proprietary database from JP Morgan. In practice, traders usually leave R as a free parameter to fit the level of market spreads. In this research, however, R is fixed and $\bar{L}$ is left as a free parameter to fit actual credit default swap spreads. Specifically, I minimize the sum of squared residuals of ten first observations

$$\min_{\bar{L}} \sum_{t=1}^{10} e_t^2 = \sum_{t=1}^{10} (c_t - c_t')^2 , \tag{6.2}$$

where c_t is observed market spread at time t, c_t' is calculated model spread at time t.

Table 3: average CreditGrades model and market CDS spreads
N is the number of obligors, SPD is the daily CDS spread in basis points.

Rating	N	SPD	CreditGrades
NR	15	398	474
D	1	1151	1013
B	27	529	373
BB	62	362	334
BBB	179	173	204
A	111	113	147
AA	17	186	194
AAA	7	26	384

[10] The value of R is consistent with Moody's estimated historical recovery rate on senior unsecured debt.

6.2 The Merton Model

First of all, asset values and asset volatilities of the obligors are not directly observable and, therefore, have to be linked to market observables. All the companies are publicly traded, therefore, stock prices of the obligors can be observed on a daily basis. Another important variable, equity volatility, can be estimated from historical data. These two observables together with associated quarterly balance sheet data allow for estimation of asset value and asset volatility through equation (5.1) and

$$\sigma_E E_0 = \Phi(d_1)\sigma_V V_0,$$

where d_1 is from equation (5.2).

Further, using MS Excel Solver and VBA I solve for the values of asset volatility and asset value. Specifically, the values of parameters are found when

$$F^2(\theta) + G^2(\theta) = 0,$$

where
$\theta = (\sigma_V, V, \sigma_E, E, D, r, T)$,

$$F(\theta) = V\Phi(\mathrm{d}_1) - \mathrm{X}\mathrm{e}^{-\mathrm{rT}}\Phi(\mathrm{d}_2) - \mathrm{E}, \qquad G(\theta) = \Phi(d_1)\sigma_V V - \sigma_E E,$$

$$d_1 = \frac{\log\left(\frac{V}{X}\right) + \left(r + \frac{\sigma_V^2}{2}\right)T}{\sigma_\mathrm{V}\sqrt{\mathrm{T}}}, \qquad d_2 = d_1 - \sigma_V\sqrt{T}.$$

Therefore, having solved for asset value and asset volatility, which is considered to be static over the observation period, I can compute the theoretical CDS spreads for all obligors.

Table 4: average Merton model and market CDS spreads

N is the number of obligors, SPD is the daily CDS spread in basis points.

Rating	N	SPD	Merton
NR	15	398	102
D	1	1151	530
B	27	529	487
BB	62	362	247
BBB	179	173	99
A	111	113	65
AA	17	186	25
AAA	7	26	56

7 Case Study of Consolidation Coal Inc

7.1 Application of CreditGrades Model

In this section I use Consolidation Coal company as an example to illustrate the general procedure. In the first step, theoretical CDS spreads are computed from the CreditGrades model. As shown above, CreditGrades model requires the following inputs: the equity price S, the debt per share D, the mean global recovery rate $\bar{L}$, the standard deviation of the global recovery rate λ, the expected recovery rate on a specific class of a firm's debt - R, the equity volatility σ_S, and the risk-free interest rate r. Specifically, I use

σ_S = 1000-day historical equity volatility,
r = five-year constant maturity Treasury yield,
$\lambda = 0.3$,
$R = 0.4$,

$$Financial\_Debt = 0.5 * (Other\_ST\_Liabilities + Other\_LT\_liabilities) + \\ +ST\_Borrow + LT\_Borrow + \ 0 * (Acct\_Payable).$$

ST_Borrow and LT_Borrow are the short-term and long-term interest bearing financial obligations including bank overdrafts, bonds, loans, etc. $Other\_ST\_Liabilities$ and $Other\_LT\_Liabilities$ represent current and long-term obligations that do not bear explicit interest, such as tax liabilities and pension liabilities. 0.5 multiplier for $Other\_ST\_Liabilities$ and $Other\_LT\_Liabilities$, as some of these are similar to financial liabilities (pension liabilities, leases, etc.) while some of them are not (deferred taxes, provisions, etc.). 0 percent weight is used for accounts payable as they do not participate in the financial leverage of a company.

The liabilities of subsidiaries are consolidated at 100 percent on a consolidated balance sheet even though the parent company may not own 100 percent of the subsidiary. To adjust for this, in CreditGradets Technical Document it is assumed that the subsidiary has a debt-to-equity ratio of k. Thus,

$$Minority\_Debt = k * Minority\_Interest,$$

where $Minority\_Interest$ represents the portion of interest the parent company does not own in the subsidiary. The total debt used in debt per share calculation is then given by

$$Debt = Financial\_Debt - k * Minority\_Interest. \tag{7.1}$$

In the calculation it is assumed that $k = 1$ and $Minority\_Debt$ is limited to no more than half of $Financial\_Debt$.

The total number of shares is the sum of all classes common shares and preferred equity, namely

$$Number\_of\_Shares = Common\_Shares + PFD\_Shares. \tag{7.2}$$

$Common\_Shares$ is the number of shares outstanding and PFD_Shares is the number of preferred shares outstanding. Finally, debt per share (D) can be calculated from (7.1) and (7.2)

$$D = \frac{Debt}{Number\_of\_Shares}.$$

Although, I closely follow method specified in CreditGrades Technical Document, however, as mentioned in previous section, application of CreditGrades model here differs from the original one in specification of global recovery parameter. In CGTD $\bar{L}$ is set equal to 0.5 and R is taken from a proprietary database from JP Morgan. In this approach, however, R is fixed and $\bar{L}$ is fitted to data. Specifically, global recovery rate has to satisfy equation (6.2). In this case I find that implied $\bar{L}$ is equal to 0.87. Plugging this estimate along with the above assumed parameters in the CreditGrades model allows the theoretical CDS spread for the Consolidation Coal Inc to be computed.

Figure 1 compares the market and model CDS spreads for the Consolidation Coal Inc. For ease of comparison, it also shows the Consolidation Coal equity price and equity volatility during the same period. There are some observations that can be made from figure 1. First of all, comparing the market spread in the first panel and the equity price in the second panel, there appears to be a negative association between changes in the CDS spread and the equity price of Consolidation Coal company. In fact, a simple calculation confirms this observation, since correlation between the CDS spread and the equity price of Consolidation Coal Inc is -0.49. It can be seen from the first and second panels in figure 1 that major changes in the equity price are reflected in the changes of opposite direction of the market CDS spread. Secondly, despite calibrating the model using only the first ten observations, the market and the model CDS spreads stay quite close and roughly follow the same trend through almost all five years of observation. One key difference between the two, however, is that the model spread appears much more volatile.

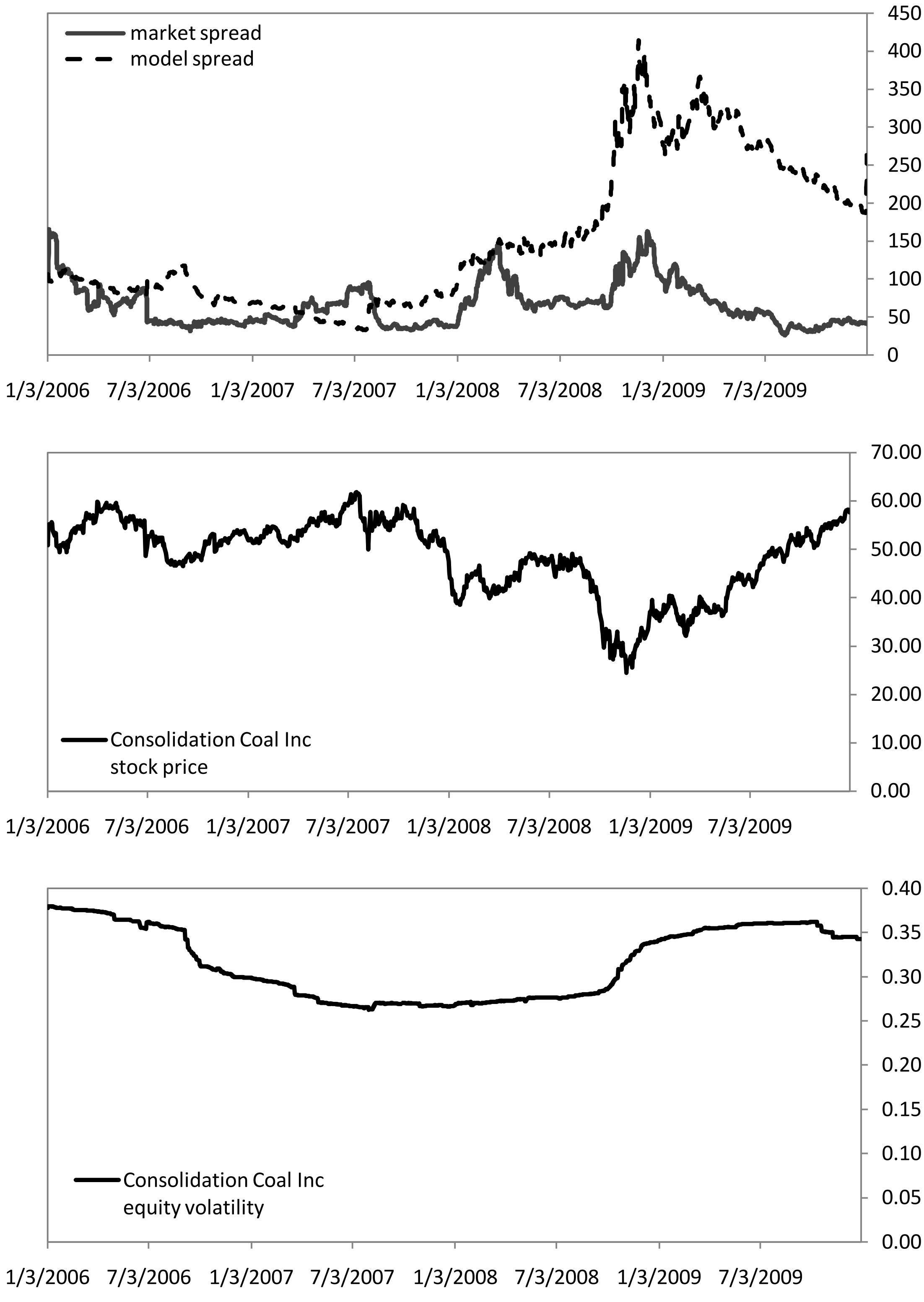


Figure 1: Time-series of CDS spreads, equity prices, and equity volatilities for the Consolidation Coal Inc

Next, I conduct a simulated trading exercise following the ideas laid out in section 2. For each of the 997 days in Consolidation Coal sample period, I check whether the market spread and the model spread differ by more than a threshold value. Specifically, the following conditions are verified

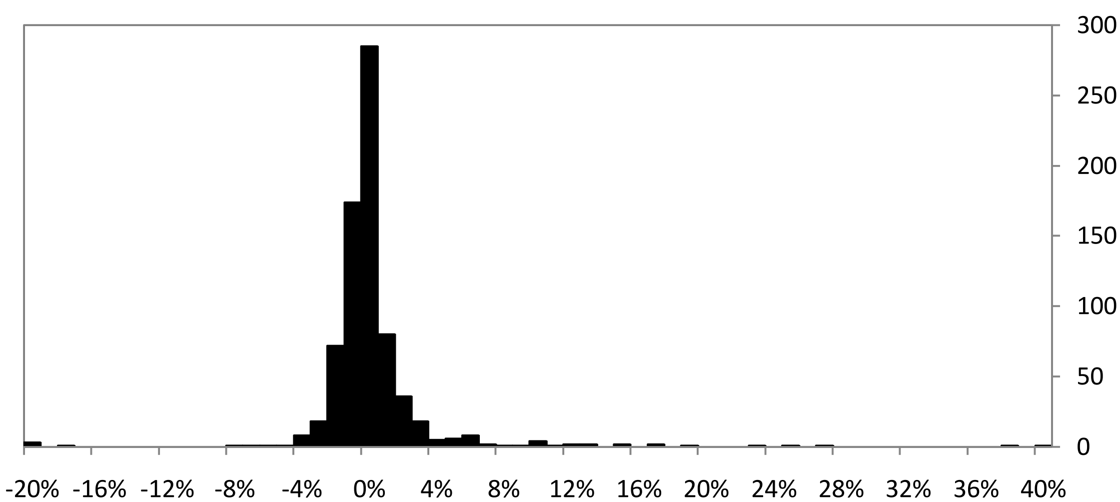
$$c_t > (1+\alpha)c_t' \text{ or } c_t' > (1+\alpha)c_t, \tag{7.3}$$

where α is a trading trigger. In this research $\alpha = 1, 2$ and 3. Recall that c_t and c_t' are the market and the model spread, respectively. In the first case, a CDS position with a notional amount of $1 and $-\delta_t$ shares of the common stock are shorted. In the second case, a unit-sized CDS position along with $-\delta_t$ shares is bought. These positions are held for a 30-day period or until convergence, where convergence is defined as $c_t = c_t'$.

Table 5: Summary statistics of holding period returns for the Consolidation Coal Inc

α	N	N_1	N_2	Mean	Min	Max	Sharpe
0.5	741	3	280	0.7%	-24.1%	46.8%	0.25
1.0	486	3	168	0.9%	-24.1%	38.8%	0.24
2.0	226	2	48	0.5%	-24.1%	12.5%	0.24

Table 5 presents the summary statistics on holding period returns. α is a trading trigger as described above. N is the total number of trades entered into. N_1 is the number of trades with drawdown exceeding 20%. N_3 is the number of trades with negative holding period returns. The mean, minimum and maximum of holding period returns are also presented in table 5.



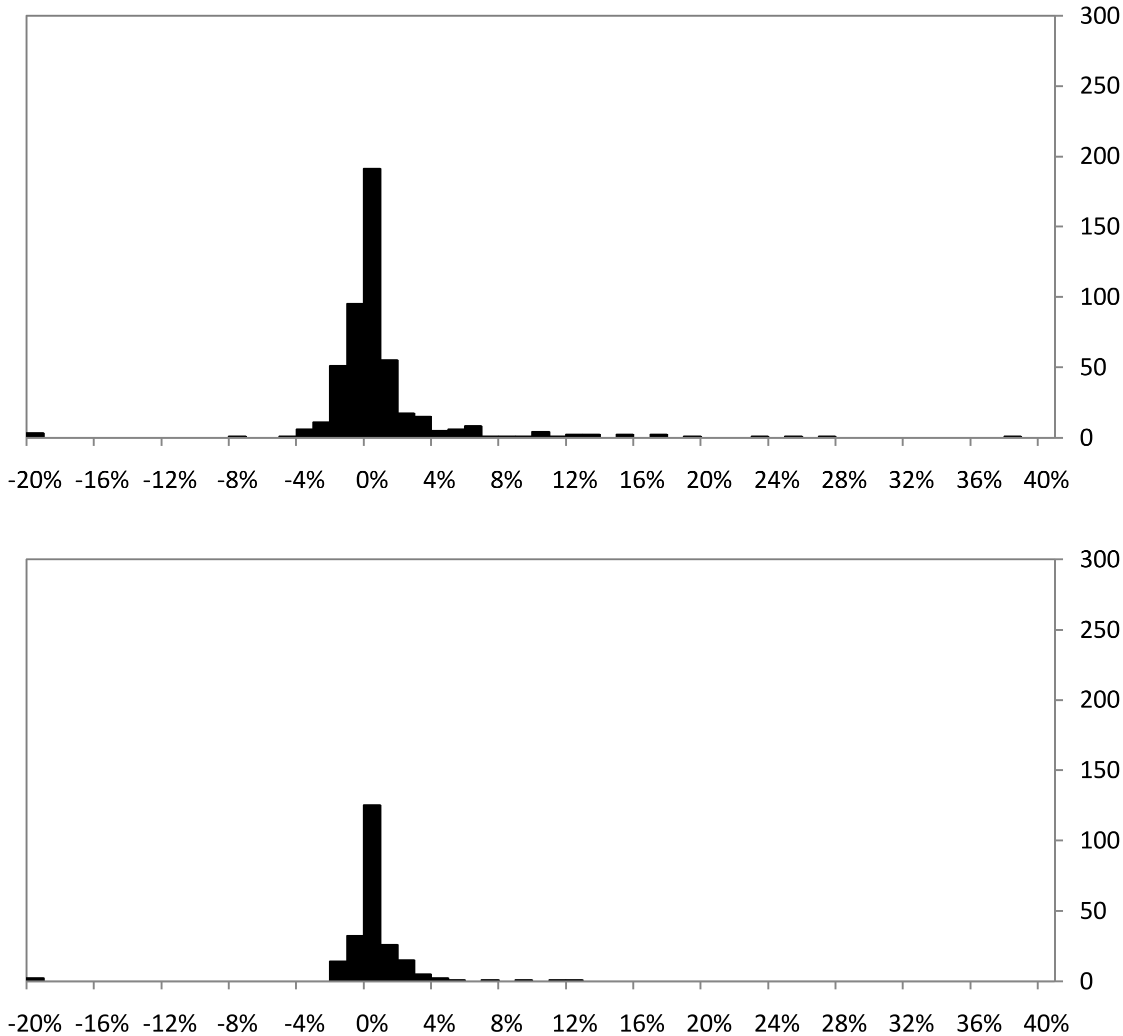


Figure 2: Histograms of holding period returns for the Consolidation Coal Inc

It is not difficult to see the overall risk of the strategy declining when the threshold for trading is raised. Not only does the distribution of holding period returns become seemingly tighter (Figure 2), the fraction of trades with negative returns declines as well. The mean returns increases when alpha is raised from 0.5 to 1.0. However, it declines again, when alpha is further raised to 2.0. This decline in returns may be explained by the size of equity hedge, since larger gap between market and model spreads requires larger equity hedge, which might reduce overall profitability.

7.2 Application of Merton Model

In this section I will illustrate the general procedure of application of Merton model for the same Consolidation Coal Inc. First of all, the CDS spreads are computed using Merton model.

Merton's model requires the following inputs: risk-free interest rate r, initial asset value V, the default boundary X, asset volatility σ_V, equity volatility σ_E, and the total debt D. Specifically, I use

r = five-year constant maturity Treasury yield,
D = total liabilities of the obligor,
σ_E =1000-day historical equity volatility.

The last parameters σ_V, V are calculated using Solver and MS Excel VBA as specified in section 5.2. The total debt of the obligor is used as a proxy of the default boundary[11]. Therefore, the output of the estimation process is asset volatility of 0.25, which is assumed constant, and asset value equal to \$ 14,2 billion.

Next, figure 3 compares the market and model CDS spreads for the Consolidation Coal Inc. Despite very limited opportunities to calibrate parameters of Merton model, the market and the model CDS spreads stay quite close and roughly follow the same trend through almost all five years of observation. One key difference between the two, however, is that the market spread appears much less volatile. The same property is observed in section figure 1 with CreditGrades model.

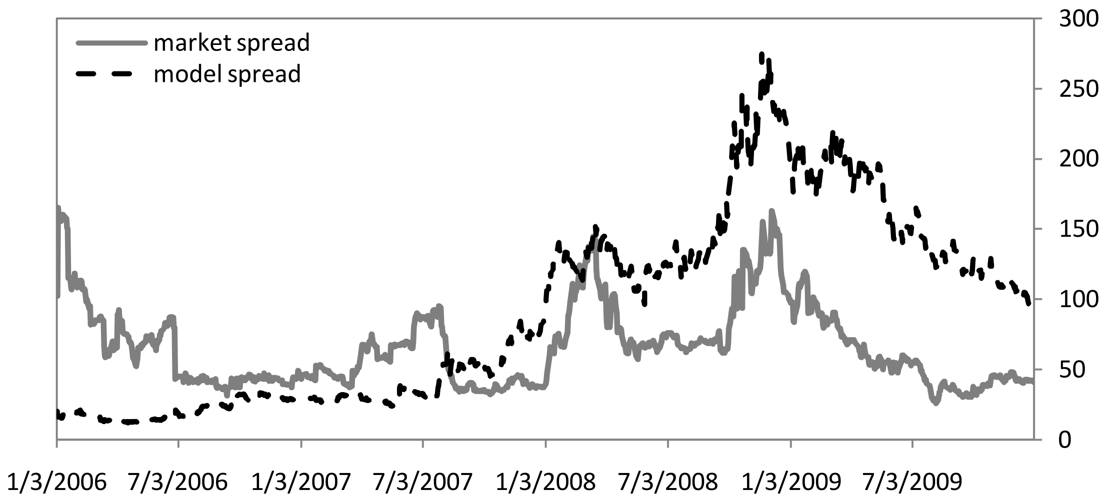


Figure 3: Time-series of CDS spreads for the Consolidation Coal Inc

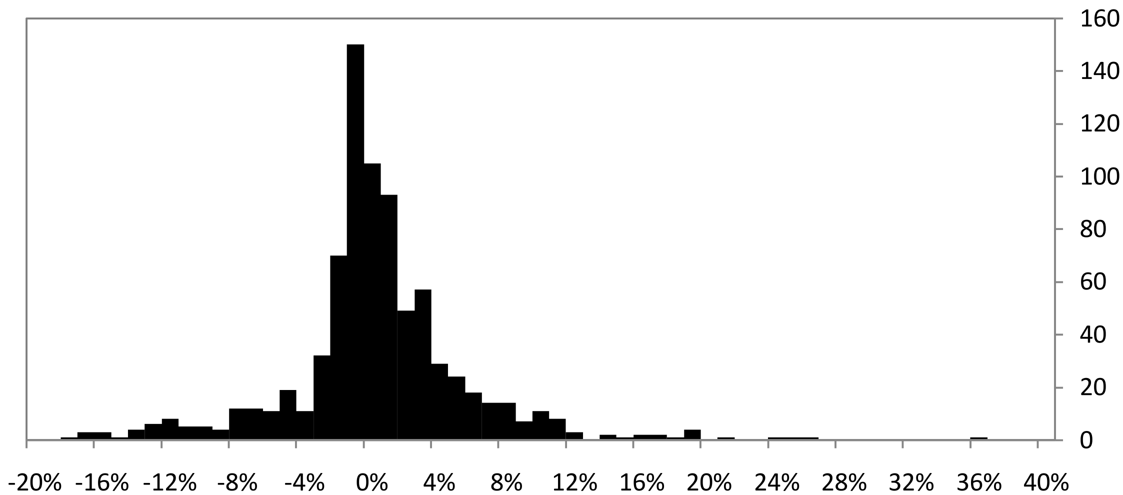
Further, I use the same trading strategy described in previous section, namely formula (7.3).

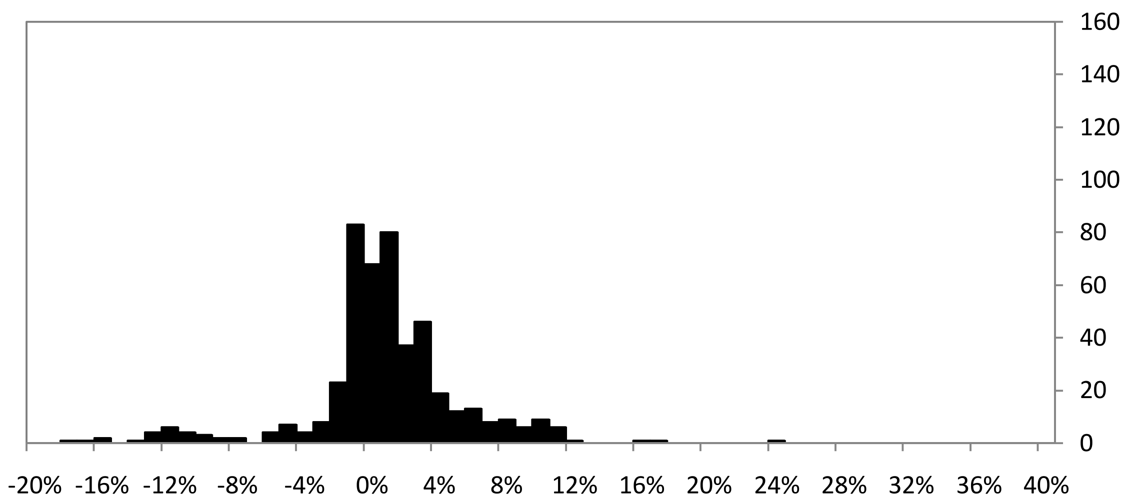
[11] "Quantitative Risk Management: Concepts, Techniques and Tools", McNeil, Frey and Embrechts (2005)

Table 6: Summary statistics of holding period returns for the Consolidation Coal Inc

α	N	N_1	N_2	Mean	Min	Max	Sharpe
0.5	806	0	357	0.8%	-17.2%	36.7%	0.16
1.0	472	0	155	1.3%	-17.2%	24.6%	0.24
2.0	230	0	60	2.0%	-1.9%	12.0%	0.36

Table 6 presents the summary statistics on holding period returns. α is a trading trigger as described above. N is the total number of trades entered into. N_1 is the number of trades with drawdown exceeding 20%. N_3 is the number of trades with negative holding period returns. The mean, minimum and maximum of holding period returns are also presented in table 6.





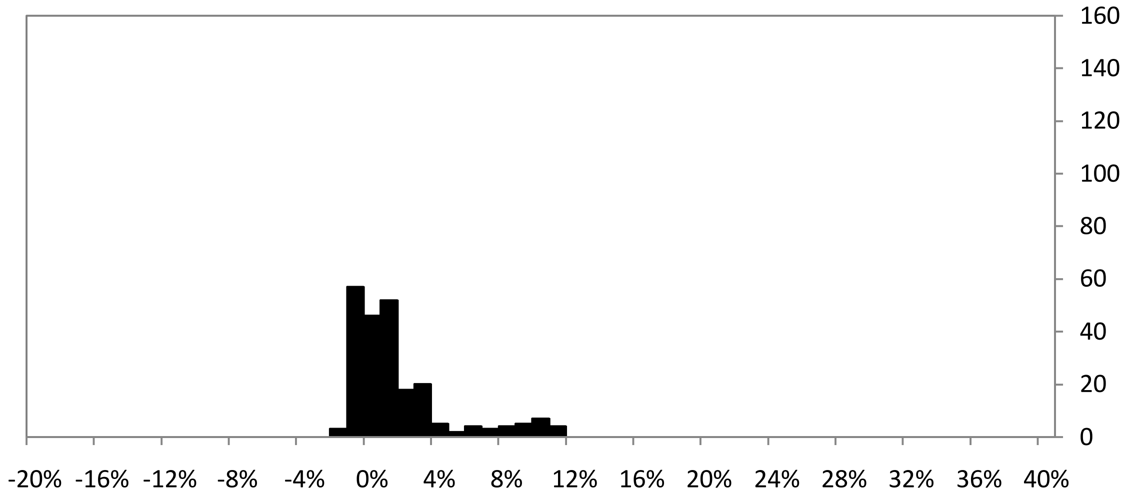


Figure 4: Histograms of holding period returns for the Consolidation Coal Inc

It is not difficult to see the overall risk of the strategy declining when the threshold for trading is raised. Not only does the distribution of holding period returns become seemingly tighter (Figure 4), the fraction of trades with negative returns declines as well. The mean of returns increases when alpha is raised from 0.5 to 2.0. In contrast to previous application of the CreditGrades model, the mean of returns does not decline when alpha is raised from 1.0 to 2.0.

8 Model Results

8.1 Analysis of Model Results

In this section I replicate the previously in detail described trading algorithm for all 419 obligors using the CreditGrades model and Merton model. Specifically, I set an initial capital of $0.5 for each trade which longs or shorts the CDS position with a $1 notional amount along with its equity hedge and use the same trading trigger as equation (7.3).

For the analysis of the capital structure arbitrage returns I split the sample into two rating sub-categories. In these sub-samples the firms are characterized as investment grade, defined for the rating interval BBB- to AAA, and speculative grade, ranging from rating class D to BB+. This allows me to analyze lower CDS spread and lower volatility investment grade obligors separately from higher CDS spread and higher volatility speculative grade obligors. Supposedly, speculative grade trading should result in higher capital structure arbitrage returns due to higher volatility. However, speculative grade obligors are more likely to default, which would hurt capital structure arbitrage returns.

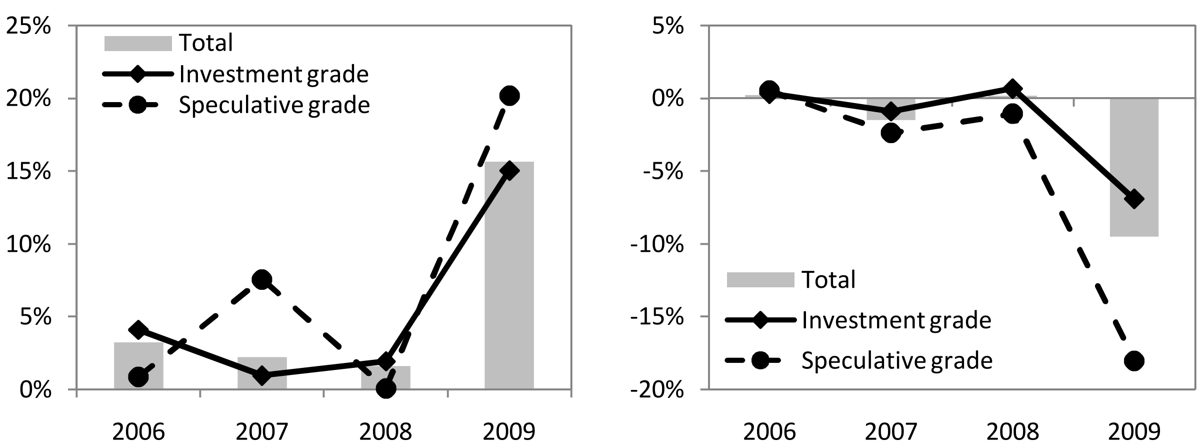


Figure 5: Average monthly capital structure arbitrage returns over the time period 2006-2009, the CreditGrades model results on the left, and Merton model results on the right hand side. Note: the graphs show returns with trading trigger $\boldsymbol{\alpha} = \mathbf{1}$.

In figure 5, in the graph of the CreditGrades model on the left hand side, it can be observed that the arbitrage strategy results in significant positive average returns, arguing with the first hypothesis. Although, the average returns consistently decline during the period of 2006-2008, but it spikes in 2009 with average monthly returns of around 15%. Interestingly, the total capital structure arbitrage returns of the CreditGrades model show smoothly declining pattern for the period 2006-2008 despite the financial crisis and emotion shaded trading in the financial markets, supporting conjecture that market participants learned about the market and its dynamics and adjusted their trading behavior and pricing over time. However, the result of 2009 is worth closer analysis, as the average monthly return of 15% is rather high in comparison to previous periods of observation.

In contrast, results of the Merton model suggest that markets were efficient during the years of 2006-2008, supporting the first hypothesis. However, it is interesting that in 2009, contrary to the same year result of the Creditgrades model, the Merton model produced significantly negative average monthly capital structure arbitrage returns.

Table 7: average monthly capital structure arbitrage returns through years and rating classes of the CreditGrades model

Rating Class: Low is rating B+ and lower, Intermediate is rating BB- to BBB+ and High is A- and higher. Investment grade includes companies of credit rating BBB- and higher, whereas speculative grade includes companies of BB+ and lower.
Note that non-rated companies are included only in Total* line.

alpha	*Rating Class*	*2006*	*2007*	*2008*	*2009*	*Total*	*Sharpe*
0.5	Low	2.0%	0.4%	-9.0%	36.1%	6.8%	0.21
	Intermediate	6.0%	3.5%	0.5%	24.1%	8.2%	0.43
	High	-0.3%	-1.2%	-0.2%	10.9%	2.1%	0.30
	Investment Grade	4.3%	1.0%	0.0%	18.7%	5.7%	0.23
	Speculative Grade	1.3%	5.3%	-0.7%	26.3%	7.7%	0.40
	Total*	3.5%	1.8%	-0.1%	19.5%	5.9%	0.22
1.0	Low	1.8%	3.0%	-8.8%	17.3%	3.0%	0.16
	Intermediate	5.3%	3.9%	3.1%	20.9%	8.0%	0.45
	High	0.4%	-1.0%	0.0%	6.8%	1.4%	0.26
	Investment Grade	4.1%	1.0%	1.9%	15.0%	5.3%	0.38
	Speculative Grade	0.9%	7.6%	0.1%	20.2%	6.9%	0.42
	Total*	3.2%	2.2%	1.6%	15.6%	5.5%	0.29
2.0	Low	0.2%	23.9%	-11.5%	13.2%	6.3%	0.21
	Intermediate	6.2%	4.9%	1.7%	16.0%	7.0%	0.46
	High	1.5%	0.4%	0.1%	2.8%	1.2%	0.30
	Investment Grade	5.3%	2.1%	1.3%	10.6%	4.7%	0.15
	Speculative Grade	0.7%	12.8%	-2.4%	14.9%	6.3%	0.29
	Total*	4.2%	4.1%	0.7%	11.1%	4.9%	0.18

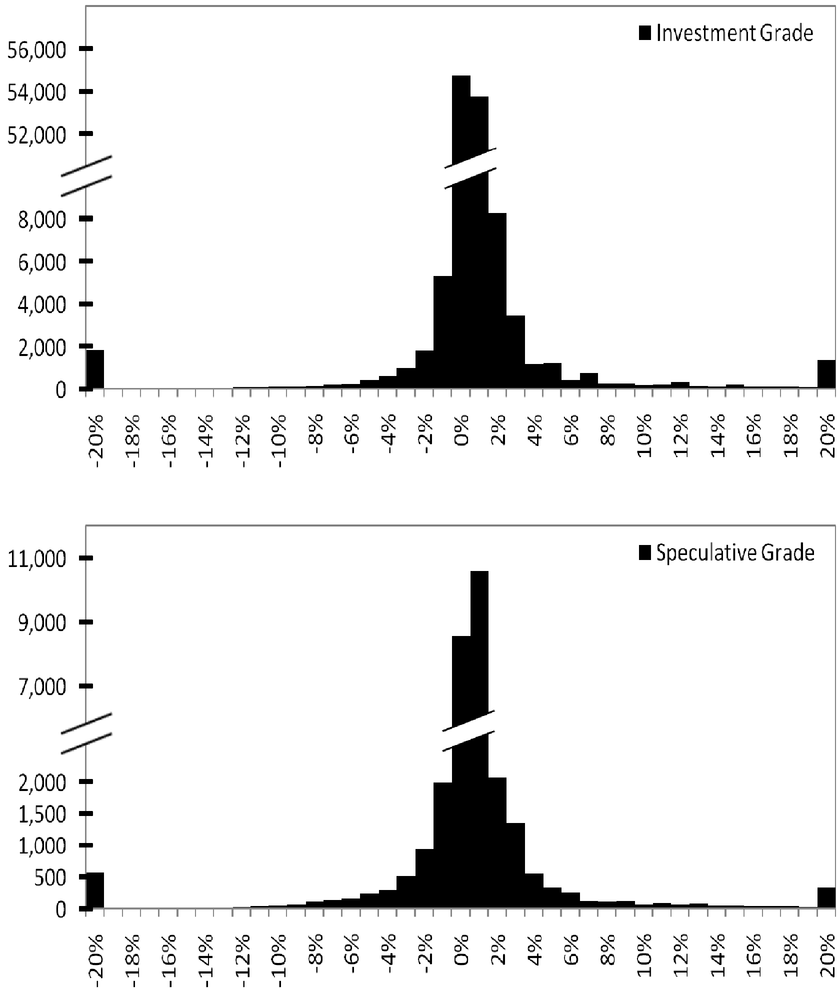


Figure 6: Histograms of holding period returns of the CreditGrades model with alpha equal to 1.0

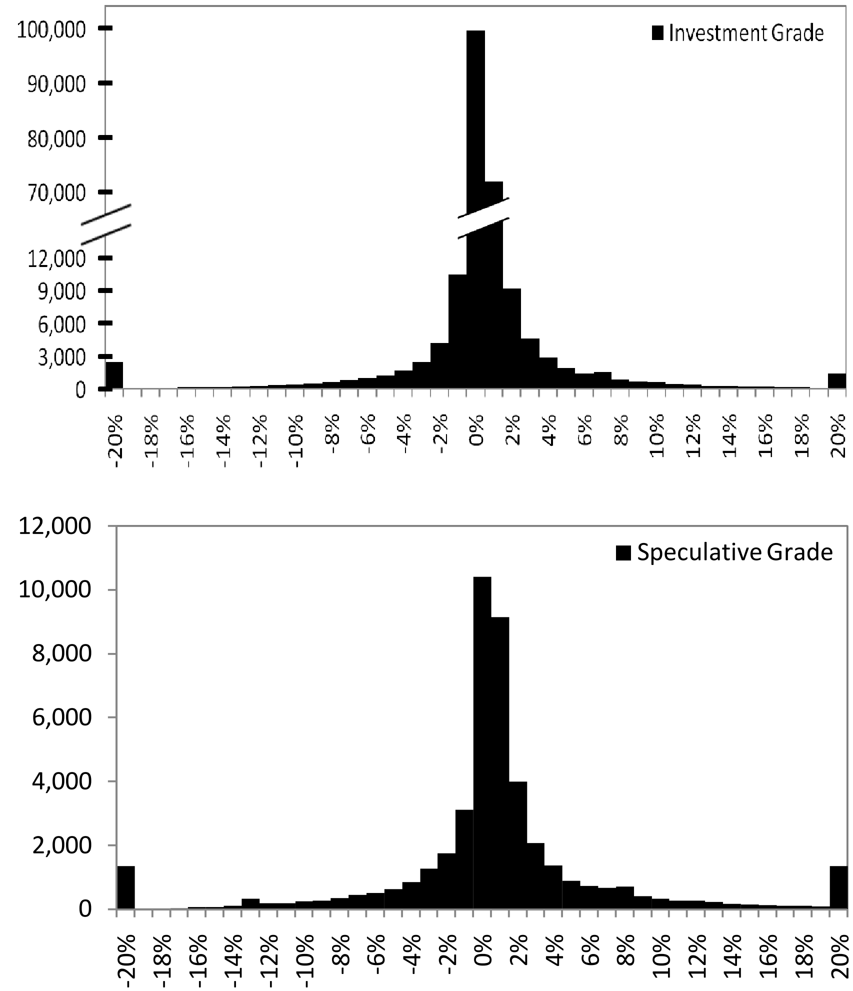


Figure 7: Histograms of holding period returns of the Merton model with alpha equal to 1.0

Table 8: average monthly capital structure arbitrage returns through years and rating classes of Merton model

Rating Class: Low is rating B+ and lower, Intermediate is rating BB- to BBB+ and High is A- and higher. Investment grade includes companies of credit rating BBB- and higher, whereas speculative grade includes companies of BB+ and lower.

Note that non-rated companies are included only in Total* line.

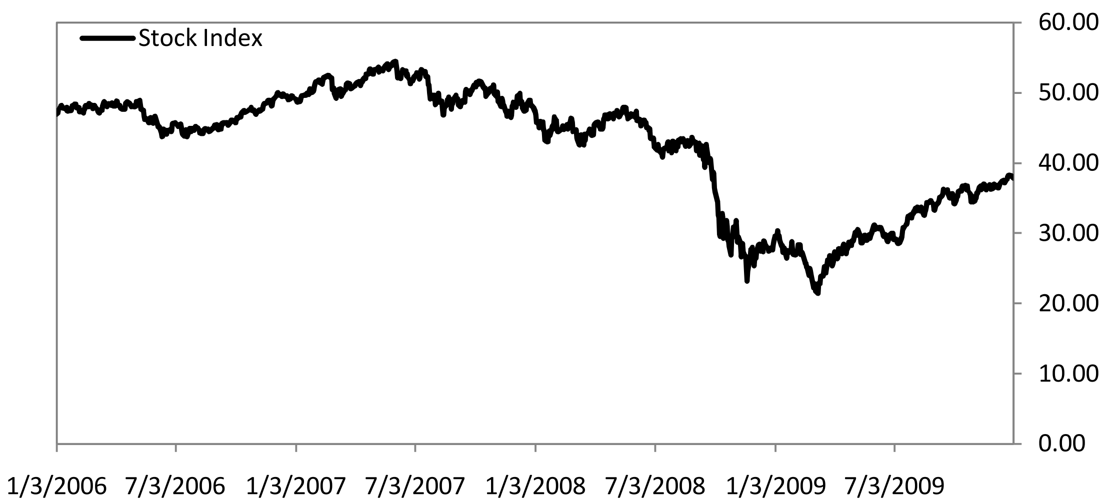
alpha	*Rating Class*	*2006*	*2007*	*2008*	*2009*	*Total*	*Sharpe*
0.5	Low	-11.1%	-6.0%	3.7%	-10.7%	-5.9%	-0.11
	Intermediate	0.3%	-2.8%	-0.4%	-12.2%	-3.6%	-0.20
	High	0.2%	0.4%	3.6%	-14.5%	-2.3%	-0.12
	Investment Grade	0.1%	-1.4%	0.6%	-9.2%	-2.3%	-0.63
	Speculative Grade	-1.8%	-4.1%	3.2%	-29.0%	-7.5%	-1.09
	Total*	-0.4%	-2.2%	1.0%	-13.3%	-3.5%	-0.46
1.0	Low	-1.5%	2.4%	-1.7%	5.8%	1.1%	0.03
	Intermediate	0.5%	-2.1%	-0.6%	-6.4%	-2.1%	-0.09
	High	0.2%	0.2%	2.4%	-15.6%	-2.9%	-0.12
	Investment Grade	0.3%	-0.9%	0.7%	-6.9%	-1.6%	-0.44
	Speculative Grade	0.5%	-2.4%	-1.1%	-18.0%	-5.0%	-0.88
	Total*	0.2%	-1.5%	0.1%	-9.5%	-2.5%	-0.33
2.0	Low	-1.1%	3.2%	2.5%	14.1%	4.5%	0.15
	Intermediate	1.2%	-0.7%	0.4%	0.0%	0.2%	0.01
	High	0.2%	-0.2%	4.2%	-10.1%	-1.3%	-0.05
	Investment Grade	0.6%	-0.7%	1.3%	-2.3%	-0.2%	-0.01
	Speculative Grade	1.7%	0.8%	3.7%	-5.0%	0.4%	0.09
	Total*	0.7%	-0.7%	1.5%	-3.5%	-0.4%	-0.03

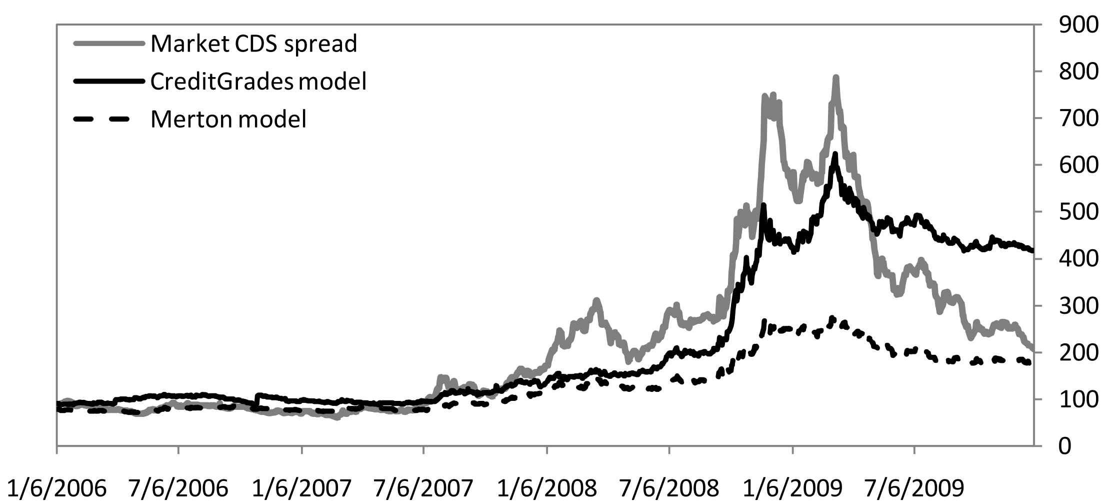
The first hypotheses does not seem to hold with the CreditGrades results. At the same time, the results of the Merton model suggest that the capital markets were efficient during the same period. The results of year 2009, however, require closer examination. Although, it was expected that the efficiency of capital markets decline during the period of 2007-2008, results of the CreditGrades model seem to reject this hypothesis. At the same time, both models show high average monthly returns in absolute values during year of 2009. This may be caused by model misspecification and/or increased inefficiency of the markets. Further, as it is seen in table 7, the third hypothesis is supported on average, since the capital structure arbitrage returns of speculative grade obligors are higher than the returns of investment grade obligors for the CreditGrades model for all levels of alpha. In contrast to the capital structure arbitrage returns of the Merton model (table 8), where riskier obligors on average show significantly worse results than the higher investment grade obligors.

The Sharpe ratios of applied strategies are difficult to interpret for the Merton model, since most of the ratios are negative, suggesting that riskless investment would be superior. The Sharpe ratios of strategies using the CreditGrades model, however, range from 0.16 to 0.46. It is important to note that the highest Sharpe ratios are generated for speculative grade obligors. The choice of trading trigger appears to be of secondary importance., however, the best Sharpe ratios seem to be generated with trading trigger $\alpha = 1$. In comparison to previous studies of Duarte et al. (2007) and Yu (2005), Sharpe ratios presented in this study are lower by 0.2 on average.

8.2 Analysis of Year 2009 Results

In this section I will closer analyze the causes of higher than normal average monthly capital structure arbitrage returns for both models. In order to facilitate the analysis, I have constructed equally weighted stock index as well as equally weighted CDS and delta indexes.





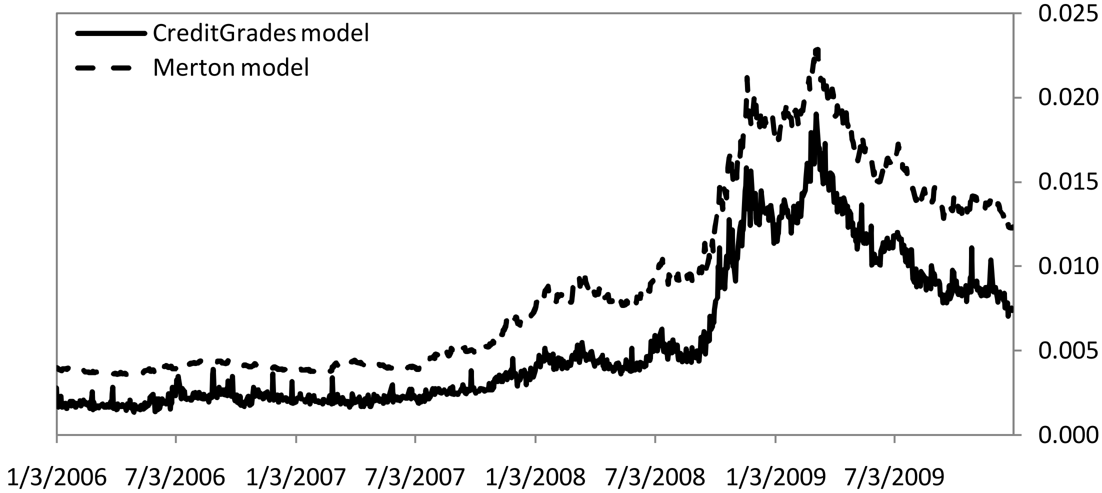


Figure 6: from top to bottom: average stock price, average market and model CDS spreads and the average delta values of the CreditGrades model and the Merton model for all obligors through the observation period.

As it can be seen in figure 6, on average the Merton model predicts market CDS spread very accurately till the beginning of 2008. However, afterwards market CDS spread and CDS spread predicted by the Merton model diverge in value until the end of observation period, when market and model CDS spreads converge again. The CreditGrades model, on the other hand, seems to follow the market CDS spread quite accurately until the middle of 2009, when it started to diverge from the market CDS spread. Interestingly, while the average market CDS spread was heavily falling during year 2009, the CreditGrades model predicted a steady CDS spread. On the other hand, it looks like the market CDS spread is converging to the value predicted by the Merton model.

The other important factor is the absolute value of model delta. Although, consistently decreasing, delta of both models remained quite large comparing to first couple of years of observation. It appears that the cause of high (in absolute value) average monthly capital structure arbitrage returns is over-hedging with stocks. The CreditGrades model produced a "buy" signal in the CDS market and, also, caused hedging by buying delta stocks in the market. As we can see in figure 6, such strategy on average resulted in loss on the CDS contract and gain on the stocks. However, due to heavy hedging, the overall value of this strategy was positive. The opposite is true for the Merton model during the same period, since Merton model produced a "sell" signal.

9 Robustness of the Results

As in any study of trading strategies, one needs to examine the robustness of the returns to a variety of parameters that can influence how the strategies are implemented. Table 9 and table 10 present average monthly capital structure arbitrage returns for the CreditGrades and the Merton model respectively having the CDS market bid/ask spread increased from 5 percent to 10 percent.

Table 9: Summary statistics of monthly excess returns for the CreditGrades model

Here alpha is 1.0 and bid-ask spread is increased to 10%, the difference column indicates difference between the mean returns having 5% bid/ask spread and 10% bid/ask spread.

Rating Class	*2006*	*2007*	*2008*	*2009*	*Total*	*Difference*
Low	1.8%	3.0%	-8.8%	17.3%	3.0%	0.00%
Intermediate	4.9%	3.9%	3.0%	20.6%	7.9%	-0.17%
High	0.4%	-1.0%	0.0%	6.8%	1.4%	0.00%
Investment Grade	3.9%	1.0%	1.9%	15.0%	5.2%	-0.06%
Speculative Grade	0.7%	7.4%	0.0%	19.2%	6.6%	-0.30%
Total	3.0%	2.2%	1.6%	15.5%	5.4%	-0.10%

Table 10: Summary statistics of monthly excess returns for the Merton model

Here alpha is 1.0 and bid-ask spread is increased to 10%, the difference column indicates difference between the mean returns having 5% bid/ask spread and 10% bid/ask spread.

Rating Class	*2006*	*2007*	*2008*	*2009*	*Total*	*Difference*
Low	2.0%	0.4%	4.9%	-4.1%	0.9%	-0.23%
Intermediate	0.5%	-2.1%	-0.6%	-6.4%	-2.1%	-0.01%
High	0.2%	0.2%	2.4%	-15.6%	-2.9%	0.00%
Investment Grade	0.3%	-0.9%	0.7%	-6.9%	-1.6%	0.00%
Speculative Grade	1.2%	-2.8%	0.3%	-20.1%	-5.0%	-0.05%
Total	0.4%	-1.5%	0.5%	-9.7%	-2.6%	-0.10%

As it can be seen in table 9, raising the CDS bid/ask spread to 10 percent does not eliminate the monthly capital structure arbitrage returns for the CreditGrades model. The mean returns, however, are decreased by up to 30 basis points. In the application of the Merton model (table 10), we can observe a decrease in total mean returns by up to 23 basis points.

Table 11: regression of capital structure arbitrage monthly index returns on common market factors

Abbreviations: Int. is intercept, Mkt-Rf is excess market return, SMB is “Small Minus Big” i.e., the return of a portfolio of small stocks in excess of the return on a portfolio of large stocks, HML is “High Minus Low” i.e., the return on a portfolio of stocks with a high book-to-market ratio in excess of the return on a portfolio of stocks with a low book-to-market ratio, UMD is “Up Minus Down” or momentum factor i.e., calculated as the return on a portfolio with an overweight of companies with the highest return and an underweight of companies with the lowest return.

Note: p-value is presented below estimates.

	Int.	Mkt-Rf	SMB	HML	UMD	R^2
Merton Model	-0.26	-0.15	-0.03	-0.09	0.07	0.78
	0.02	0.00	0.60	0.03	0.00	
CreditGrades Model	0.13	0.00	-0.02	0.00	0.04	0.07
	0.37	0.96	0.82	1.00	0.16	

Table 11 presents regression results of capital structure arbitrage monthly index returns on common market factors. Specifically, I use the Fama-French three-factor model with additional momentum factor to proxy for equity market risk. Table 11 shows that there is an apparent relation between capital structure arbitrage monthly index returns of the Merton model and common market factors with exception of SMB factor. R-squared in this case is also rather high – 0.78. As analyzed in section 7.2, this is caused by rather high delta used for delta-hedging with stocks and/or model mis-specification as seen from a large distance between market and model CDS spreads in figure 6. Therefore, we can see results of the Merton model to be highly dependent on equity market factors.

Capital structure arbitrage monthly index returns of the CreditGrades model, on the other hand, present completely different estimates. With R-squared being at only 0.07, none of the equity market factors is significant even at 10% level. Although, the intercept of the regression is quite close to the original level of the mean monthly capital structure arbitrage returns in table 7 (alpha=1), suggesting that none of the risk factors can bid away the “alpha” of the strategy, however, the “alpha” itself is not statistically significantly different from zero. Therefore, one cannot reject hypothesis that capital markets were efficient during the observation period, which is in line with results of Cserna and Imbierowicz (2008) saying that “the CDS market overcame its inefficiency in the years 2004/2005 consistent with soaring trading volumes and the introduction of CDS index trading at this time”.

10 Conclusion

This paper presents a popular trading strategy in which the arbitrageur takes advantage of the temporary divergence between the CDS market spreads and predicted spreads from a structural credit risk model. My simple implementation computes theoretical CDS spreads by calibrating the industry standard[12] CreditGrades model as well as basic Merton model, further, compares the time-series of market and model CDS spreads. A CDS position is entered into whenever the market and model CDS spreads diverge from each other by a fixed threshold level, with an accompanying equity position taken out as a hedge.

The results of the Merton model show that average monthly capital structure arbitrage returns are negative which may be caused by model mis-specification and/or calibration error. The results of the CreditGrades model, on the other hand, show that the capital structure arbitrage strategy produces significant positive average returns in the investigated period. However, the strategy does not produce a statistically significant "alpha", suggesting that positive average monthly arbitrage returns may be caused due to the short sample period and/or survivorship bias. Very similar results are met if higher transaction costs are incorporated. The decline of average returns over time using the CreditGrades model further indicates an improving CDS market as average monthly arbitrage returns are diminishing.

Partitioning the sample into rating intervals reveals that the arbitrage returns are much larger for high risk obligors which holds true for the results of both models. This result is in line with standard investment theories, also this adheres to the notion of risk averse investors who demand higher rewards for increasing risk. In addition, I find that the mean monthly capital structure arbitrage returns generated by the CreditGrades model cannot be explained by the well known equity market risk factors.

This study leaves some questions to future research, especially the use of more advanced structural credit risk models like Leland and Toft (1996) or Zhou (2001). These models could be able to better capture fast paced CDS and equity market movements during the credit crisis starting in 2007. Moreover, another important issue to check is the average monthly arbitrage returns during post credit crisis period.

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