The impact of using the full corporate bond yield curve on accounting for Defined Benefit Plans according to the International Accounting Standard 19

by

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Management Summary

Under International Accounting Standard (IAS) 19, an employer has to account for a Defined Benefit plan by recognizing a liability or asset on the balance sheet and a loss or gain in the profit and loss statement (P&L). To determine the amount that has to be recognized on the balance sheet and in the P&L, IAS 19 prescribes a method called the Projected Unit Credit Method (PUCM). This method calculates the present value of the Defined Benefit Obligation (DBO) based on a projection to the future. For these calculations, many assumptions have to be made and one of the most important assumption concerns the discount rate. According to IAS 19, the discount rate should be based on high quality corporate bond yields. These corporate bond yields again should be consistent with the term and currency of the Defined Benefit plan. This suggests that a yield curve should be used, that is derived from market yields, but IAS 19 allows for a simplification. IAS 19 also allows the use of a single discount rate, which should be derived from the high quality corporate bond yield curve. In practice, this simplification is used in nearly all IAS 19 calculations, but is it a necessary simplification? And what is the impact of applying that simplification?

To determine whether it is feasible to use a yield curve in the IAS 19 calculations, a standardized method to derive the yield curve from market data is desirable. Existing literature describes several models to derive a yield curve from market data. These models can be split up into four categories: the affine yield curve models, the arbitrage free yield curve models, the spline-based models and the parsimonious models. To find the appropriate model for deriving the high quality corporate bond yield curve, I assess the yield curve models on four criteria: smoothness, flexibility, stability and tractability. The affine models drop out due to a lack of flexibility. The arbitrage-free models exactly fit the data, which makes them very sensitive for outliers, therefore they are not appropriate to estimate the high quality corporate bond yield curve. Spline-based and parsimonious models both are smooth, flexible and stable, and therefore they are appropriate to estimate the high quality corporate bond yield curve. Compared to spline-based models, parsimonious models need less parameters and only use one formula to describe the complete yield curve. Parsimonious models are therefore more tractable and hence preferred to spline-based models. From the existing parsimonious models, the Nelson-Siegel (1987) model and the extended Nelson-Siegel model of Svensson (1994), are most frequently used. The model by Svensson is more flexible and therefore I prefer the Svensson model to the Nelson-Siegel model.

When an appropriate model is selected, the market data needs to be obtained. The data is selected based on credit ratings from Standard & Poor's (S&P), Moody's, and Fitch. High quality corporate bonds are defined as bonds issued by companies with an average rating comparable to S&P's AA- or better. Next an appropriate fitting procedure has to be chosen. There are two ways to estimate the yield curve, one estimation procedure fits the model yields to the observed yields, the other estimation procedure fits the model bond prices to the observed bond prices. Since it is the aim to obtain the yield curve, it would be best to choose an estimation procedure that fits the model to the observed yields. However, the methods available require a great deal of computational power. Fitting the model bond prices to the observed bond prices is much easier, but leads to overfitting the prices of the long-term bonds at the expense of the fit for the price of short-term bonds. This overfitting is caused by the fact that mispricing the yield curve has much more effect on the difference between the model bond price and the observed bond price of a long-term bond,

compared to a short-term bond. A solution to this problem was introduced by Bliss in 1997. The Bliss method fits the model bond prices to the observed bond prices, but weighs these differences between model and observed bond prices, based on the duration of the bonds.

Another issue in estimating the high quality corporate bond yield curve is the long end of the yield curve. Since there are very few observations beyond a maturity of 20 years and no observations with a maturity of 30 years or more, it is hard to determine the best way to estimate that part of the yield curve. The estimated yield curves show curves that keep increasing after 30 years, which results in relatively high yields for these maturities. Until now, no real solution to this problem exists. I suggest two methods that stop the curve increasing from a certain maturity on. One method is to simply fix the yield from a certain maturity on, the other method adds a constraint to one of the parameters of the model.

After estimating the high quality bond yield curve, I investigated the impact of the full yield curve on the outcomes of the IAS 19 calculations. To interpret the impact correctly, actuarial factors are introduced. An actuarial factor is an adjustment to the present value methodology and determines the current amount of money that is needed for a certain annuity. Actuarial factors differ for each age, and for men and women. I compared the actuarial factors based on the full yield curve to the factors based on several fixed discount rates. This comparison shows that the differences between factors based on the full yield curve and the factors based on relatively high fixed discount rates, i.e. 5.5 or 6.0 percent, are the smallest for young people. In contrast to that, the actuarial factors based on the full yield curve for older people are closer to the factors based on the relatively low fixed discount rates, i.e. 4.5 or 5.0 percent. This effect was expected, since the yield curve is up sloping and contains relatively low yields at the shorter maturities. This effect holds for both men and women and for both old-age and widower's pension.

The outcomes of the IAS 19 calculations, such as the DBO, show a similar impact. The DBO of a representative portfolio of participants, based on a fixed discount rate of 5.2 percent is closest to the DBO of that same portfolio, based on the full yield curve. A comparison of the aforementioned possible 'solutions' for the long end of the yield curve, show that the differences in the present value of the DBO can be large, up to 10 percent. The decisions concerning the long end of the yield curve are therefore very important. The impact on P&L entries differs from the impact on the DBO, which shows that the calculations using the full yield curve cannot be accurately approximated using a fixed discount rate. An issue in using the full yield curve is the determination of the Interest Cost (IC). The IC is the cost of the DBO getting one year closer to the actual pension payments. Usually the IC is calculated using the fixed discount rate, but in case of using the full yield curve it is not immediately clear which rate should be used. There are however two clear possibilities that can be used to determine the IC: the 1-year yield or an 'average yield' based on a fixed discount rate that approximates the DBO based on the full yield curve. The difference between the two methods is large, especially with the (very) low 1-year yield at the moment and future will show which yield should be used.

1. Introduction

The reason for writing this thesis is the continuous development of market valuation. Next to the regulatory authorities, market valuation is also finding its way into accounting. Market valuation however sometimes causes problems, which are often avoided using approximation methods. Especially in accounting, simplifications are used in case there are no material differences between an exact method and an approximation. This is also the case in accounting for employee benefits, especially for pension schemes.

In The Netherlands, nearly all employers offer their employees one or more pension schemes. These pension schemes, although placed outside the sponsoring company, contain obligations for the employer to the pension fund or insurer. These obligations usually contain extra premiums that have to be paid in case of underfunding, but other obligations are also possible. Due to these obligations, the sponsoring companies, the employers, have to meet certain accounting regulations. In case of a listed Dutch company, the company has to comply with the accounting principles which are described in the International Accounting Standard (IAS) 19. IAS 19 is also used widely around the globe and many accounting standards are derived from IAS 19.

Under IAS 19 employers are required to account for their pension plans in their annual statements. In the complex calculations, prescribed by IAS 19, they are required to discount their future pension liabilities using high quality corporate bonds. At first, this suggests the use of a yield curve, based on high quality corporate bonds traded in the market. However, IAS 19 allows for a simplification. Instead of using the full high quality corporate bond yield curve, IAS 19 allows the use of a single discount rate. This discount rate, however, should still be based on the high quality corporate bond yield curve, i.e. it should be a spot on that curve. This means that the high quality corporate bond yield curve, which I will call 'the yield curve' for the remainder of this thesis, still has to be obtained.

In practice the simplification of using a fixed discount rate is applied in almost every case. It is however not always clear how the discount rate is obtained and the discount rates that are used in the market differ widely, even for comparable pension plans. The discount rate is sometimes even based on the outcome of a bargain between the company and the auditor. Since it is the objective of an annual statement to give a good picture of the state of the company, these methods are debatable.

The research question in this thesis is twofold. The first question concerns the construction of a high quality corporate bond yield curve, and specifically the development of a standardized procedure of such a construction. The second question concerns the impact of using such a yield curve on the outcomes of the IAS 19 calculations, which are presented in the annual statement.

Before these research questions are answered, IAS 19 will be introduced in Chapter 2. Chapter 3 contains an overview of the yield curve models available in literature and a discussion of their advantages and disadvantages. Chapter 3 also contains some examples and states my choice for the Svensson yield curve model, which will be used in this thesis to construct the yield curve. A detailed description of a standardized procedure that can be used to construct the yield curve using the Svensson model follows in Chapter 4. It shows that deriving such a yield curve is not too complex, although the problem of absence of corporate bonds with long maturities will still be present. Next to this description, the empirical results and some issues will be discussed in Chapter 4. Chapter 5 describes how a yield curve can be incorporated in the IAS 19 calculations. Furthermore, the results of the IAS 19 calculations for the yield curve as at May 31, 2010 will be discussed and compared with the results using a single discount rate. Chapter 6 contains a short summary, my conclusions and recommendations for further research.

2. International Accounting Standard 19: Employee Benefits

Before diving deep into the technical matters, I will introduce the applicable accounting standard in this chapter. The requirements, methods and calculations in this chapter will be used in Chapter 5. Besides the use in Chapter 5, this chapter will also give insight in the background of Chapters 3 and 4 of this thesis in general.

The International Accounting Standard (IAS) 19 prescribes how employers should account and disclose for employee benefits. According to IAS 19, an entity (the employer) should recognize (International Accounting Standards Board, 2009):

a) A liability when an employee has provided service in exchange for employee benefits to be paid in the future; and
b) An expense when the entity consumes the economic benefit arising from service provided by an employee in exchange for employee benefits.

IAS 19 and other IASs are part of the International Financial Reporting Standards (IFRS), a collection of standards, interpretations and the framework.

2.1 International Financial Reporting Standard

IFRS is adopted by the International Accounting Standards Board (IASB), who took over from the International Accounting Standards Committee (IASC) in 2001. The IASC published standards (IASs) during the period from 1973 to 2001 and IASB continued this job from 2001 on by developing new standards called IFRSs. IFRS consists of the following parts:

- *International Accounting Standards (IASs)*, these standards are issued before 2001;
- *International Financial Reporting Standards (IFRSs)*, these standards are issued after 2001;
- *Interpretations from the Standing Interpretations Committee (SIC)*, these interpretations are issued before 2001; and
- *Interpretations from the International Financial Reporting Interpretations Committee (IFRIC)*, these interpretations are issued after 2001.

The IASB is supported and advised by the Standards Advisory Council (SAC), the IFRIC also supports the IASB and issues statements on certain subjects in case of uncertainty in the market.

2.2 Employee benefits

IAS 19 paragraph 4 divides employee benefits into four categories, namely:

a) *Short-term employee benefits*, e.g. salaries, paid leave and bonuses, but not termination benefits, which are payable within twelve months after the end of the period;
b) *Post-employment benefits*, e.g. pensions, post-employment (life) insurance or medical care, but not termination benefits;
c) *Other long-term employee benefits*, e.g. sabbatical leave, jubilee benefits or long-term disability benefits, but also profit-sharing and bonuses payable more than twelve months after the end of the period; and
d) *Termination benefits*.

Since these categories have different characteristics, IAS 19 contains different requirements for each of these four categories. Accounting for post-employment benefits is much more complex than accounting for the other three categories. Therefore IAS 19 focuses on accounting for post-employment benefits. I will now briefly discuss categories a, c and d, after which I will delve deeper into post-employment benefits.

In general, accounting for short-term employee benefits is straightforward, since no actuarial assumptions are needed to measure the cost or obligation. IAS 19 requires an entity to recognize these benefits in the same period as the employee has rendered service corresponding to those benefits.

IAS 19 requires a simplified method of accounting for other long-term employee benefits, since the measurement of these benefits is usually subject to less uncertainty and changes in these benefits usually do not cause significant past service cost.

Termination benefits are treated separately in IAS 19, due to the fact that the event of termination and not the employee service gives rise to an obligation.

Within post-employment benefits, IAS 19 distinguishes between defined contribution (DC) plans and defined benefit (DB) plans. IAS 19 paragraph IN5 defines DC plans as follows:

"Under defined contribution plans, an entity pays fixed contributions into a separate entity (a fund) and will have no legal or constructive obligation to pay further contributions if the fund does not hold sufficient assets to pay all employee benefits relating to employee service in the current and prior periods."

When a plan is not classified as a DC plan it automatically will be defined as a DB plan. Examples of DB plans are final pay and average pay schemes, while available premium schemes and savings systems are examples of DC plans. According to paragraph 43 of IAS 19, accounting for DC plans is straightforward. Because there are no future obligations, except for the fixed contributions, no actuarial assumptions are needed.

This means that, except for post-employment DB plans, the treatment of employee benefits is quite straightforward. Therefore I will focus on DB plans for the remainder of this thesis.

2.3 Accounting for Defined Benefit plans

In case entities offer their employees one or more pension schemes that are classified as DB plans, these entities have to account for that by recognizing a liability or asset on their balance sheet. The construction of this liability or asset is shown in Table 1. The resulting net liability or asset is recognized in the balance sheet. The definitions of the different components of this construction are discussed below.

Table 1: The construction of the net liability or asset that has to be recognized on the balance sheet.

Present value of the Defined Benefit Obligation	(…)
Fair Value of the Plan Assets	…
Funded status	…
Unrecognized actuarial (gains) / losses	(…)
Net (liability) / asset	…

The *present value of the Defined Benefit Obligation* (DBO), is the net present value, based on actuarial assumptions, of the expected future payments resulting from all DB post-employment benefits. The Projected Unit Credit Method (PUCM), which I will explain in Section 2.7, is required to obtain the present value of the DBO.

The term 'fair value' in the *Fair Value of Plan Assets* means that the value of the assets is measured using market prices. When market prices are not available, e.g. in case of private equity, the fair value of Plan Assets is estimated using for example a discounted cash flow method.

Actuarial gains/losses are gains or losses caused by changes in actuarial assumptions, e.g. differences between the assumptions on forehand and the realizations during the period. Another effect that causes actuarial gains or losses is the fact that new employees can enter the plan during the period. Unrecognized means that these gains or losses are not yet recognized in the Profit and Loss Statement (P&L). The (recognition of) actuarial gains and losses will be discussed in more detail later on.

The amount an entity has to recognize in the P&L is called the Employer Pension Expense (EPE). The construction of the EPE is shown in Table 2.

Table 2: The construction of the Employer Pension Expense, which has to be recognized in the P&L.

Service Cost	(…)
Interest Cost	(…)
Expected return on Plan Assets	…
Amortization of actuarial gains / (losses)	(…)
Employer Pension Expense	…

In Table 2 the *Service Cost* (SC) is the change in the present value of the DBO resulting from employee service during the period. It is the cost for purchasing post-employment benefits earned by the employee during the current period. Just like the DBO, the SC should be calculated using the PUCM. In this thesis SC will refer to current service cost as past service cost will be neglected for simplicity. The *Interest Cost* (IC) is the rise in DBO due to the fact that the benefits are one period closer to payment. It is usually obtained by multiplying the DBO plus the SC during the period with the discount rate that is used in the PUCM, other methods will be discussed in Section 5.5. The *Expected return on Plan Assets* is the expected return on the plan assets at the start of the period, based on market expectations. At the end of the period the difference between the expected and actual return on plan assets is treated as an actuarial gain or loss. The *Amortization of actuarial gains* *and losses* is the amount of actuarial gains and losses that is recognized during this period. Actuarial gains and losses are amortized using the corridor approach or via Other Comprehensive Income (OCI) which will be discussed in Section 2.4.

Apart from the effects mentioned above, there are other effects that influence the EPE, like curtailments, settlements, asset ceiling and aforementioned past service cost. Because these effects are more or less exceptions, they will not be discussed in this thesis.

2.4 Actuarial gains and losses

Due to the fact that the calculation of the DBO, and thus the net liability or asset, is based on several actuarial assumptions, it is subject to a significant degree of uncertainty. Take for example the expected return on Plan Assets which differs almost always from the actual return on Plan Assets. This uncertainty causes actuarial gains and losses which might offset one another. Because the DBO is a 'current best estimate' it suffices if it lies within a corridor around the current best estimate. To reduce the P&L effect of actuarial gains and losses, the *corridor approach* was introduced. This corridor approach implies that IAS 19 requires an entity to recognize, as a minimum, a portion of the net cumulative unrecognized actuarial gains/losses if it exceeds the maximum of the following two amounts:

a) 10% of the DBO;
b) 10% of the Fair Value of Plan Assets.

The minimum portion an entity has to recognize is the excess divided by the "expected average remaining working lives of the employees participating in the plan" (IAS 19, paragraph 93).

As pointed out, this is a minimum that has to be recognized. An entity is free to recognize faster, as long as it does this according to a systematic method and consistently from period to period. Another widely used method is that recognition of actuarial gains and losses takes place in the period in which they occur. According to paragraph 93A, an entity should then recognize all of its actuarial gains and losses (for all DB plans) in Other Comprehensive Income (OCI). OCI is a separate income statement which is used to account for changes in equity value outside the "regular" income statement. This is usually done because the actuarial gains or losses have very little to do with the ordinary business of the employer and can thus be confusing for investors if included in the regular income statement.

At the moment of writing this thesis, an Exposure Draft (ED) of IAS 19 is published by the IASB in which the 'corridor approach' is repealed. It is not known yet when and in which exact form this ED will be adopted in IAS 19, but it is likely that the 'corridor approach' will disappear. This means that actuarial gains and losses have to be recognized directly through OCI in the period they occur. As a result of the direct recognition, changes in the actuarial assumptions, such as the discount rate, will have a larger impact on the balance sheet provision since it is not smoothened by the 'corridor approach'.

2.5 Numerical example

In this section I will present a numerical example to illustrate the concepts introduced in the previous sections. In this example it is assumed that the fiscal year is equal to a calendar year.

Suppose the DBO is calculated using PUCM and equal to 1,020 at January 1st 2010 and the Fair Value of Plan Assets at that date is equal to 800. Furthermore, the cumulative unrecognized actuarial loss per January 1st 2010 is 122. The SC is calculated for 2010 and equal to 70. Assuming a discount rate of 5.5% and taking into account only the SC and DBO, the IC is equal to 5.5% of the sum of the SC and DBO at January 1st 2010 and thus the IC is calculated as 60. The expected return on Plan Assets (RA) is assumed to be 7.5%, which comes down to an expected return on Plan Assets of 60.

The expected contribution, paid by the participants of the plan (PPC) for 2010 is 10 and the total contributions (CT) paid to the pension fund or insurance company is equal to 50. Furthermore, the expected benefit payments (BP) for 2010 are 40 in total.

All ingredients are now available to calculate the EPE for 2010, the net liability per January 1st 2010 and the expected net liability per December 31st 2010. These are shown in Table 3.

Table 3: A numerical example of the implications of IAS 19 for a DB plan on the P&L and the balance sheet.

	Actual	**Movement**				**Estimated**
	01/01/2010					31/12/2010
		P&L		Cash		
DBO	(1,020)	SC	(70)	BP	40	(1,120)
		IC	(60)	PPC	(10)	
Plan Assets	800	RA	60	CT	50	870
				BP	(40)	
Funded Status	(220)		(70)		40	(250)
Unrecognized items	122		(2)		-	120
Net (liability) / asset	(98)	EPE	(72)		40	(130)

Table 3 shows that the entity has to recognize a net liability on the balance sheet per January 1st 2010 of 98. This is calculated as the DBO minus the Fair Value of Plan Assets minus the cumulative unrecognized actuarial losses. In these calculations, the corridor approach is used and the bound of the corridor is calculated as 10% * max (1,020, 800) = 102. Since the cumulative unrecognized actuarial losses are equal to 122, they fall outside the bounds of the corridor. Now assume the "expected average remaining working lives of the employees participating in the plan" to be equal to 10. This means that the amount of amortized unrecognized actuarial losses is equal to (122 – 102) / 10 = 2. The EPE is therefore equal to 72, namely the SC plus the IC minus the RA plus the amortized unrecognized actuarial losses.

Per December 31st, the estimated DBO is equal to the sum of the DBO per January 1st 2010, the SC, the IC and the PPC minus the BP. As shown in Table 3, this results in an estimate of 1,120. The estimated Fair Value of Plan Assets at the end of the period is equal to the value at the beginning of the period plus RA and CT minus BP, which results in an estimate of 870.

Table 3 also shows two ways of calculating the estimated balance sheet provision at the end of the period. The first one is shown in the last column, where the estimated liability is calculated by the estimated DBO minus the estimated Fair Value of the Plan Assets and the cumulative unrecognized actuarial losses. The second one is shown in the last row of Table 3, where the estimated liability is calculated by the actual net liability at the start of the period plus the estimated EPE minus the estimated cash flow.

2.6 Actuarial assumptions

In the calculations in the previous section, several actuarial assumptions had to be made. One can group these assumptions into two categories, namely financial assumptions and demographic assumptions. The latter category consists of assumptions about future characteristics of both current and former participants (employees) in the plan. The most important actuarial assumptions are listed below.

Financial assumptions:

a) Discount rate;
b) Expected rate of return on Plan Assets;
c) Benefit levels;
d) Future increases in salaries;
e) Indexation.

Demographic assumptions:

a) Mortality rates;
b) Employee turnover rates;
c) Disability rates;
d) Early retirement rates;
e) Proportion of plan members and dependants eligible for benefits.

In paragraph 72 of IAS 19 it is stated that actuarial assumptions shall be unbiased and mutually compatible. Paragraph 74 explains that with unbiased is meant that the actuarial assumptions should be neither imprudent nor excessively conservative. Paragraph 75 explains that the assumptions are mutually compatible if they reflect the economic relationships between factors such as inflation, rates of salary increase, the return on Plan Assets and discount rates.

No further guidelines are given for demographic assumptions, except that the best estimates available have to be used. Financial assumptions however in general should satisfy three conditions, according to paragraph 77. They have to be based on:

a) Market expectations;
b) At the balance sheet date; and
c) For the period over which the obligations are to be settled.

Special attention is given to the discount rate used. In paragraph 78 it is stated that the discount rate should be constructed by reference to market yields on high quality corporate bonds. In case there is no deep market in such bonds, market yields on government bonds should be used. Furthermore, the currency and term of the bonds that are used should be consistent with the currency and estimated term of the post-employment benefit obligations. This suggests the use of a term structure of discount rates, which is based on the full corporate bond yield curve. In paragraph 80 however, it is stated that, in practice, an entity often achieves this by applying a single weighted average discount rate that reflects the estimated timing and amount of benefit payments. This means that, in practice, an entity can use a single discount rate instead of a term structure. However, as Bader & Ma (1995) already discussed, it is better to use the full yield curve instead of using a single discount rate.

Reasons for applying a fixed discount rate are the complexity of calculations in PUCM when using the full yield curve, the complexity of deriving the full yield curve from market data and the absence of corporate bonds with long maturities in the market. However, there are also problems using a fixed discount rate, since this rate should also be based on similar yield curve. Next to that, the fixed discount rate should also be based on the duration of the pension plan (see also Sections 4.2 and 4.3), which again depends on the yield curve. Therefore it is my opinion that the full yield curve should be used in accounting for DB plans under IAS 19. In Chapters 3 and 4 of this thesis I will first show that the problems deriving the full yield curve are surmountable. In Chapter 5 I will show the impact of using the full yield curve instead of a fixed discount rate. Before diving deeper into the corporate bond yield curve, I will explain the concept of the PUCM.

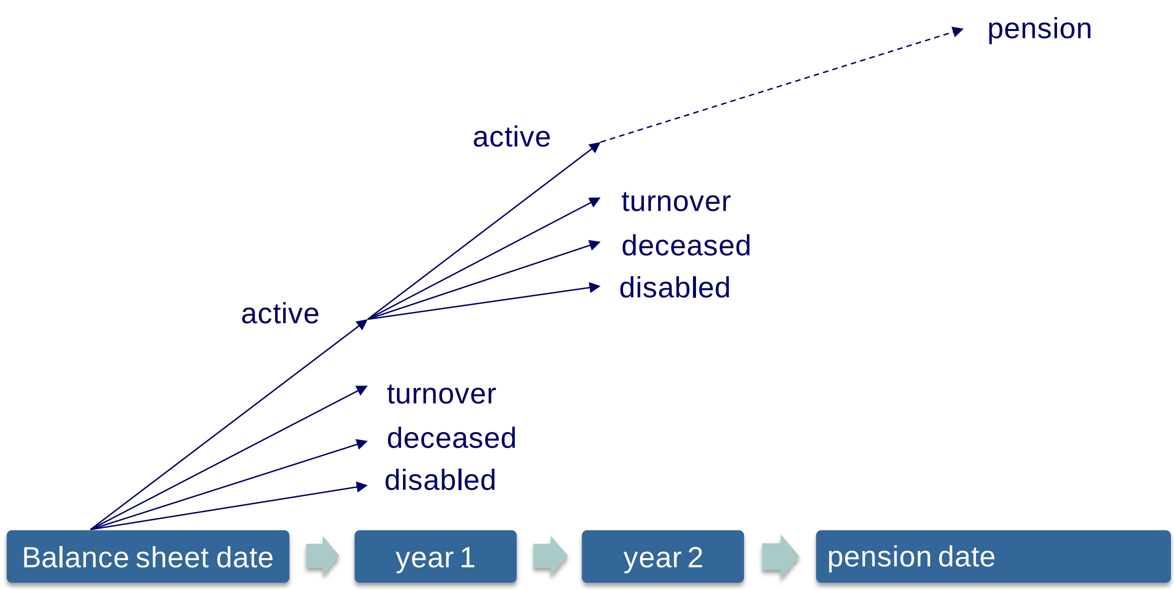
2.7 The Projected Unit Credit Method

The aforementioned Projected Unit Credit Method (PUCM) is used to calculate the present value of the DBO and current Service Cost. The method works as follows (IAS 19, paragraph 65):

a) Each period of service gives rise to an additional unit of benefit entitlement;
b) Each unit is measured separately to build up the final obligation;
c) The PUCM calculates the obligation in each future year, taking into account the actuarial assumptions corresponding to each possible scenario (see Figure 1);
d) The total obligation is equal to the sum of the obligations, multiplied with their corresponding chances.

The possible scenarios are illustrated in Figure 1.

Figure 1: An illustration of the possible scenarios each year. These scenarios are measured separately by the PUCM to build up the total obligation.



The additional amount of pension entitlement an employee is promised each period is called Coming Service (CS). The present value of the CS increases every period, assuming a constant pensionable base (salary minus social security offset), due to two reasons. The first is the time value of money, the employee is one period closer to retirement and the available capital thus has one period less to yield a profit. Second, the likeliness that the employee will die before retirement decreases because the employee survived the period. I will elaborate on these calculations in Section 5.1.

In practice, the pensionable base is not constant; it is usually increasing due to periodical salary increases. As a result, the CS is increasing as well from period to period. In final pay schemes, benefits earned in previous periods also increase due to the salary increase. This increase is called Back Service (BS). Entitled indexation has a similar effect in case of an average pay pension scheme. This is illustrated in Figure 2 for an average pay and in Figure 3 for a final pay pension scheme.

In case the accrual percentage is equal, the CS of an average and a final pay scheme is also equal. To show these concepts, a numerical example is useful. In Table 4, an example of the accrual of pension entitlements for a single employee is given. Note that the numbers in the table are nominal yearly pension entitlements; they are not present values of the pension entitlements.

In the example in Table 4, the pensionable base is equal to 15,000 at the start of the first period and increases with 5 percent every period. The accrual rate of both schemes is 2 percent, which means that after 35 periods (years), the employee should have accrued 70 percent of his or her respectively final or average pensionable base. The percentage of (unconditional) indexation is set at 2 percent.

The calculation of the CS is very straightforward; it is 2 percent of the pensionable base. The BS is calculated as the difference between the CS in the current period minus the CS in the previous period multiplied with the number of service years before the current period. If I take period 3 as an example, I see that the CS in the current period is 331 and in the previous period 315, a difference of 16. Since there are two past service years, the BS is 16 x 2 = 32. The indexation is obtained by summing up the entitlements accrued in the previous periods and multiplying the outcome with the indexation fraction (2 percent in this example). If I take period 3 again, I see that the accrued entitlements sum up to 621 and 2 percent of 621 is 12.42.

It may appear that a final pay scheme is more expensive than an average pay scheme, since that is the case in this example. In general however, this does not need to be the case, depending on several parameters like salary increases, indexation percentages and accrual rates.

Figure 2: Coming Service and indexation for an average pay pension scheme.

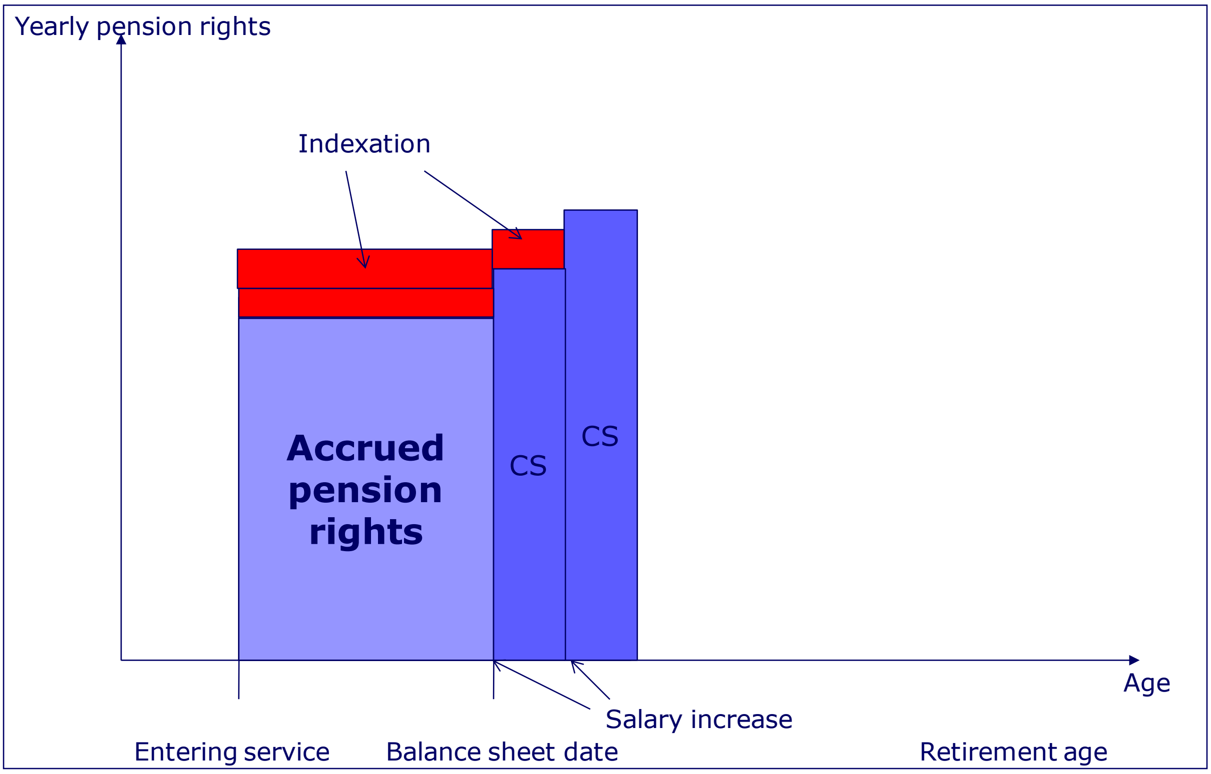


Figure 3: Coming Service and Back Service for a final pay pension scheme.

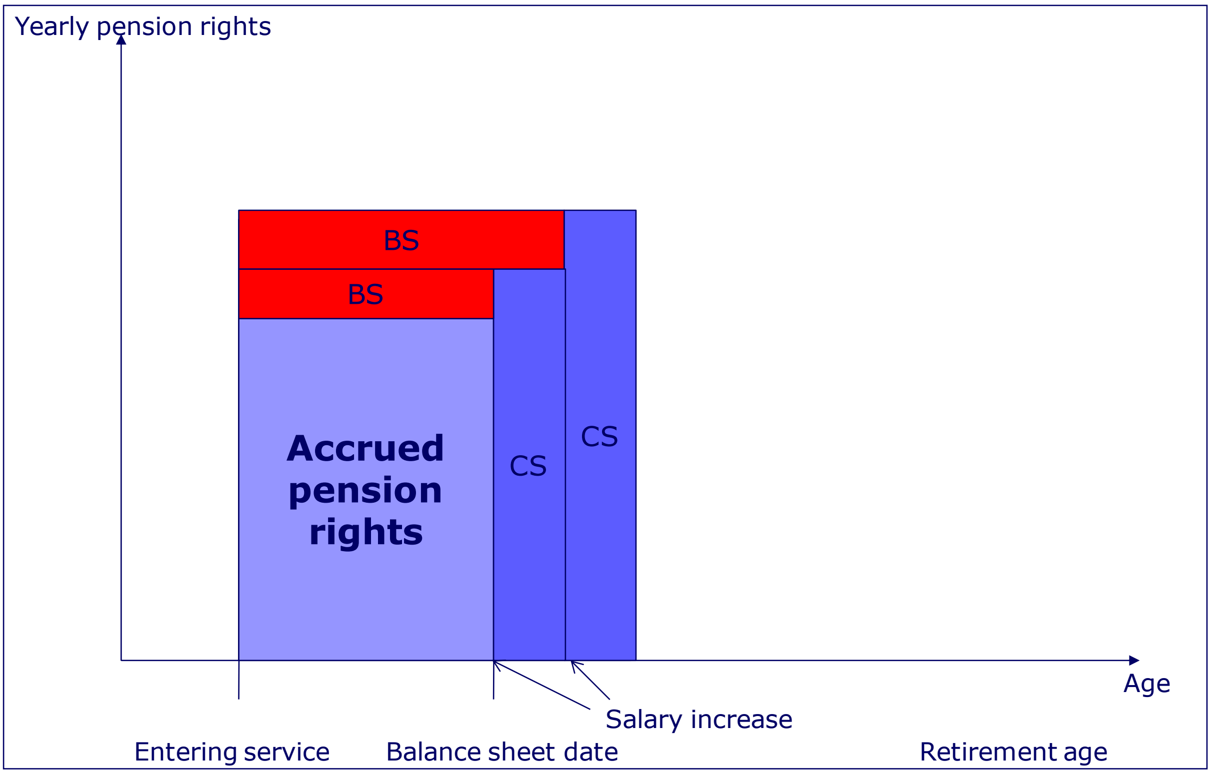


Table 4: A numerical example of the Coming Service, the Back Service and indexation for an average pay pension scheme and a final pay pension scheme.

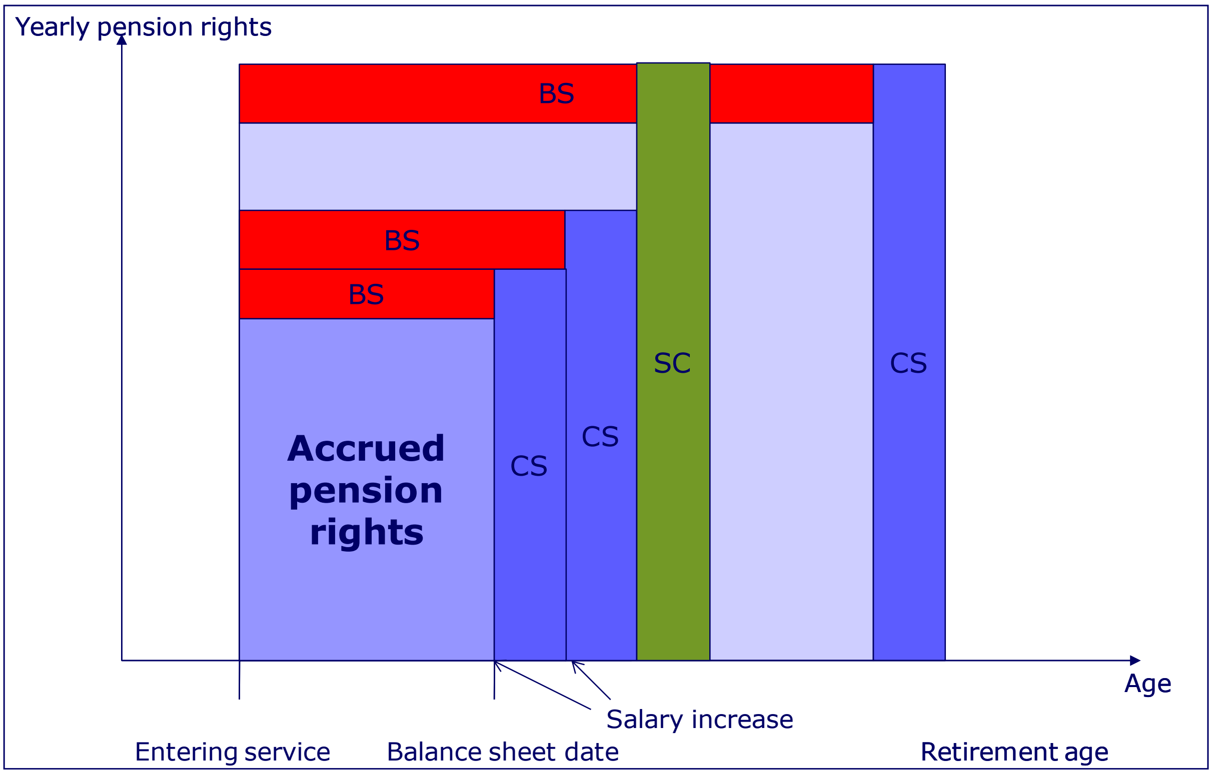
Period		**1**	**2**	**3**	**4**	**Total**
Pensionable base		15,000	15,750	16,538	17,364	
	CS	300	315	331	347	**1,293**
Final pay scheme	BS	-	15	32	50	**96**
	Total	300	330	362	397	**1,389**
	CS	300	315	331	347	**1,293**
Average pay scheme	Indexation	-	6	12	19	**38**
with indexation	Total	300	321	343	367	**1,331**

What the example also shows is that in employee's service is future years can cause higher benefits from current year's service, due to BS or unconditional indexation. IAS 19 prescribes in paragraph 67 that an entity should attribute benefits on a straight-line basis. This should result in a net present value of the SC that is constant from period to period. This is illustrated in Figure 4.

In case of a young employee, the cash cost for an entity are relatively low and for an older employee relatively high. Since the SC is constant from period to period, besides fluctuations in actuarial assumptions, the costs are spread equally over the periods an employee is in service. In the earlier years of an employee's service, the contributions paid by the employer are usually lower than the SC, which causes a liability on the balance sheet. In the later years of an employee's service exactly the opposite occurs and the liability on the balance sheet is reduced again. This first is shown in the example in Section 2.5, where the contribution is lower than the SC, which causes the liability on the balance sheet to rise. The employer includes the SC in its P&L via the EPE.

An advantage of this method is that the different components of the EPE can be calculated at the beginning of the period and will not change over the period. Changes in the different components are considered as actuarial gains or losses at the end of the period. These actuarial gains or losses can be either recognized through OCI or through the balance sheet provision via the unrecognized actuarial gains/losses.

Figure 4: The differences between the cash cost, which consists of the Coming Service and the Back Service, and the Service Cost in case of a final pay scheme.



2.8 Post-employment benefits in other accounting standards

IFRS is not only used by listed companies in the EU, it is widely used around the globe. Besides that, there are many local Generally Accepted Accounting Principles (GAAP) that are based on or similar to the IFRS, although there can be (minor) textual differences. It can also be the case that there is a time lag between changes in the IFRS and the adoption of these changes in local GAAP.

Next to IFRS there are two major international accounting standards, namely the Financial Reporting Standards (FRS) from the United Kingdom (UK) and Financial Accounting Standards (FAS) from the United States of America (USA). These standards are also applied in The Netherlands and have many similarities with IAS 19 (Dietvorst, Dilling, & Stevens, 2009).

FAS is published by the Financial Accounting Standards Board (FASB) and the accounting for pensions is treated in standard 87, *Employers' Accounting for Pensions* (FAS 87). Other employee benefits are treated in standards FAS 88, FAS 106, FAS 112 and FAS 146. These accounting standards apply to American companies, their foreign subsidiaries and also to non-American companies listed on a American stock exchange. In 2006 there have been some adjustments to the standards FAS 87, FAS 88 and FAS 106. These adjustments are put together in the new standard FAS 158.

FRS is published by the Financial Reporting Council (FRC), and applies to British companies, their foreign subsidiaries and also to non-British companies that are listed on a British stock exchange. Post-employment benefits are treated in FRS 17 *Retirement Benefits.*

The differences between FAS 87, FRS 17 and IAS 19 will not be treated in this thesis, since they have the same essence.

A local GAAP that deviates significantly from IAS 19 in their 'pension paragraph' is the Dutch GAAP that is published by the Dutch Accounting Standards Board (Raad voor de Jaarverslaggeving (RJ)). The RJ accounting standards apply to non-listed companies in The Netherlands. The standard that treats employee benefits, standard 271 (RJ 271) was based on IAS 19 until 2009. In 2009 RJ 271 was changed dramatically. Since then RJ 271, in essence, only requires an entity to account for the actual premium that has to be paid during the period in the P&L.

Next to these existing accounting standards, a new accounting standard was introduced by the IASB in 2009: *IFRS for Small and Medium Enterprises* (IFRS for SMEs). This standard could apply to non-listed companies that do not have any public importance like banks and insurance companies. Since there is not much jurisprudence yet, it is not known whether this set of accounting standards will be applied widely in The Netherlands, the EU and/or around the globe. The paragraph that deals with Employee Benefits, paragraph 28, is based on IAS 19 but allows for some simplifications which can lead to significant differences (Delsman & Spieker, 2009).

3. Modeling the yield curve

As discussed in Section 2.6, the discount factor that is used to obtain the present value of the DBO should be estimated using the yield on high quality corporate bonds. This means that a yield curve, based on high quality corporate bonds, has to be derived. Since it is not possible to derive the yield curve from market data directly, which I will explain later, it is necessary to use an appropriate yield curve model. To choose the appropriate yield curve model, I will use four criteria:

1. *Smoothness*: The model should provide smooth yield curves, it must not try to fit every data point;
2. *Flexibility*: The model should be sufficiently flexible to describe adequately the yield curve in any economic situation;
3. *Stability*: The resulting estimates from the model should not be too sensitive to changes in single observations, e.g. in one of the rare bonds with a long maturity;
4. *Tractability*: Since the model has to be used in practice, it has to be easy to use.

In general, four different types of models can be found in literature and in practice: affine models, no-arbitrage models, spline-based models and parsimonious models. In this chapter I will first elaborate on yield curves in general. Next, to make an adequate choice for a yield curve model, I will discuss the four different types of yield curve models in Sections 3.2 to 3.5. I will discuss their advantages and disadvantages and give some examples. I will conclude this chapter with Section 3.6, the model choice.

3.1 Yield curve representations

A *yield curve* or *term structure of interest rates* is based on the assumption that a functional relationship exists between either forward rates, discount factors, par yields or spot rates on the one hand and corresponding maturities on the other hand. It represents the yields or interest rates for a number of terms and can be represented in different ways, e.g. by a par yield (also called yield to maturity) curve, discount factors or a zero coupon yield curve. A par yield curve represents the yields for which the price of the bond is equal to its face (or par) value. The zero coupon yield curve represents the yields for a bond that pays 1 at maturity. These different yield curves contain the same information and can be derived from each other.

Since a cash flow model is used to obtain the present value of the DBO, eventually the discount function $d_{t,m}$ is of my interest. The discount function $d_{t,m}$ returns the discount factors at time t for every maturity m. Yields on bonds which do not pay any coupon (zero-coupon) are called spot rates. Assuming continuous compounding, these spot rates $s_{t,m}$ are directly related to the discount factor:

$$d_{t,m} = e^{-s_{t,m}m}, \qquad [3.1]$$

Spot rates are depending on the maturity, therefore forward rates, $f_{t,m}$, can be defined as follows:

$$s_{t,m} = \frac{1}{m}\int_0^m f_{t,u}du, \qquad [3.2]$$

where the spot rates are the forward rates continuously compounded up to maturity or, equivalently, the forward rates are the rates for which the difference between maturity and settlement approaches zero. A direct expression can also be derived:

$$f_{t,m} = \frac{1}{m}\int_0^m f_{t,u}du + \; m\left(\frac{-1}{m^2}\int_0^m f_{t,u}du + \frac{1}{m}f_{t,m}\right) = s_{t,m} + m\frac{\delta}{\delta m}s_{t,m} \qquad [3.3]$$

or, equivalently:

$$f_{t,m} = -\frac{1}{\exp(-\int_0^m f_{t,u}du)}\left(-\exp\left(-\int_0^m f_{t,u}du\right)f_{t,m}\right) = -\frac{1}{d_{t,m}}\frac{\delta}{\delta m}d_{t,m} \qquad [3.4]$$

Note that I assume continuous compounding to simplify calculations. To calculate the present value of the DBO however, I will need rates based on annual compounding. The continuously compounding spot rate can be easily converted to the annual compounding spot rate using the following formula:

$$s_{t,m}^{annual} = \exp\left(s_{t,m}\right) - 1. \qquad [3.5]$$

In general, bonds that can be used as a discount bond directly are not traded and therefore zero-coupon rates cannot be observed directly in the market. To extract these zero-coupon rates from bonds that are traded in the market, several techniques are available. In the next sections I will discuss the different types of yield curve models.

3.2 Affine yield curve models

The affine yield curve models are present in the literature for quite some time now and there are several definitions known. In this thesis the class of affine yield curve models is defined as follows (Piazzesi, 2009): an affine yield curve model is any arbitrage-free model in which bond yields are affine (constant-plus-linear) functions of some state vector x. These models can thus be written as follows:

$$y^{(m)} = A(m) + B'(m)x, \qquad [3.6]$$

where $y^{(m)}$ represents the yield for a m-period bond and where coefficients $A(m)$ and $B'(m)$ depend on time to maturity m. The functions $A(m)$ and $B'(m)$ provide these yield equations with consistency with the state dynamics and for each other for different maturities, i.e. freedom of arbitrage.

The major advantages of these affine models are their tractability and closed-form solutions. The literature on affine models started with the equilibrium models of Vasicek (1977), and Cox, Ingersoll & Ross (1985). These equilibrium models are based on a process for the short (or instantaneous) rate. This short rate, r, is the yield $R(t, t+m)$ at time t with time to maturity $m \downarrow 0$. The process is described by the following Itô process:

$$dr_t = \mu(r_t)dt + \sigma(r_t)dW_t, \qquad [3.7]$$

where $\mu(r_t)$ represents the instantaneous drift, $\sigma(r_t)$ the instantaneous standard deviation and dW_t the Wiener process. Both $\mu(r_t)$ and $\sigma(r_t)$ are assumed to be a function of r, but independent of time. Furthermore, an efficient market and no-arbitrage is assumed. In general the short rate follows a risk-neutral process, which is a process in which it is assumed that the current value is equal to the expected value, i.e. $\mu = 0$.

As mentioned before, an important example of an equilibrium model is the model that was introduced by Vasicek in 1977. In this model r follows an Ornstein-Uhlenbeck process of the following form (Vasicek, 1977):

$$dr_t = a(b - r_t)dt + \sigma dW_t, \quad [3.8]$$

where dW_t again represents a Wiener process and a, b and σ are constant. Equations [3.9] shows that the Vasicek model belongs to the affine yield curve class. For $a > 0$ the Ornstein-Uhlenbeck process has the mean reversion property. This means that the yields have the tendency to fluctuate around a certain long-term average, b in this case. This is caused by the drift term $a(b - r_t)$ which pulls the yield back to the long-term average, the fluctuations are caused by the stochastic element σdW_t.

One of the advantages of affine yield curve models is that there exists a closed-form solution. This is also the case for the Vasicek model, from which the following expression can be derived for the price $P(t, t+m)$ of a zero-coupon bond at time t, that pays 1 at maturity $T(= t+m)$ (Rebonato, 1998):

$$P(t, t+m) = A(t, t+m)e^{-B(t,t+m)r_t}, \quad [3.9]$$

where

$$B(t, t+m) = \frac{1-e^{-am}}{a}, \quad [3.10]$$

$$A(t, t+m) = \exp\left(\frac{(B(t,t+m)-m)\left(a^2 b - \frac{\sigma^2}{2}\right)}{a^2} - \frac{\sigma^2 B(t,t+m)^2}{4a}\right). \quad [3.11]$$

If one defines $R(t, t+m)$ as the continuous compounded interest rate, implied by $P(t, t+m)$, one can obtain $R(t, t+m)$ from $P(t, t+m)$ using the following expression:

$$e^{R(t,t+m)m} P(t, t+m) = 1, \quad [3.12]$$

which has the following solution

$$R(t, t+m) = -\frac{\ln\left(P(t,t+m)\right)}{m}. \quad [3.13]$$

For the Vasicek model the expression becomes:

$$R(t, t+m) = -\frac{1}{m}\left(\ln\left(A(t, t+m)\right) - B(t, t+m)r_t\right). \quad [3.14]$$

This expression is used to construct the yield curve by fixing t and varying m. Note that once the parameters of the Vasicek model are chosen, the complete yield curve follows from the short rate r_t. An important drawback of the Vasicek model is that it allows for negative interest rates, due to the stochastic term. Cox, Ingersoll and Ross (1985) solved this problem by introducing a slightly different model where the process for the yield r is given by:

$$dr_t = a(b - r_t)dt + \sigma\sqrt{r_t}dW_t, \quad [3.15]$$

in which only the stochastic term differs from the Vasicek model. It can be easily verified that dr_t is positive when $r_t = 0$ in case that $a > 0$ and $b > 0$. Like in the Vasicek model, a direct expression exists for the price of a zero-coupon bond:

$$P(t, t+m) = A(t, t+m)e^{-B(t,t+m)r_t}, \tag{3.16}$$

where now (Rebonato, 1998)

$$A(t, t+m) = \left(\frac{2\gamma e^{\frac{(a+\lambda+\gamma)(m)}{2}}}{(\gamma+a+\lambda)(e^{\gamma(m)}-1)+2\gamma}\right)^{\frac{2ab}{\sigma^2}}, \tag{3.17}$$

$$B(t, t+m) = \frac{2(e^{\gamma(m)}-1)}{(\gamma+a+\lambda)(e^{\gamma(m)}-1)+2\gamma}, \text{ and} \tag{3.18}$$

$$\gamma \equiv \sqrt{(a+\lambda)^2 + 2\sigma^2}. \tag{3.19}$$

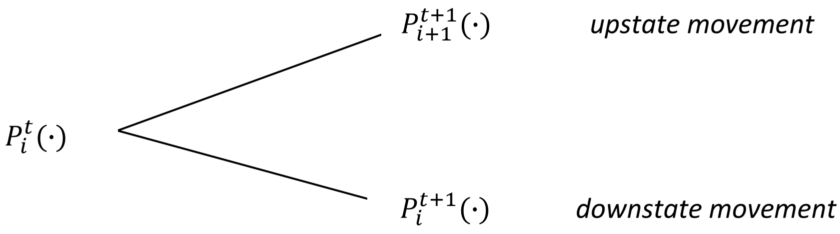
The parameter λ represents the market price of risk and is positive in case investors are risk-averse and non-positive in case investors are risk seeking or risk neutral. In case it is positive, λ represents the excess return per unit percentage of volatility over the risk-free rate.

As mentioned before, the advantage of these affine models is their tractability and closed-form solutions. The fourth criterion I defined before, tractability, is thus satisfied. The same holds for the smoothness criterion. A criterion that clearly not holds is the flexibility criterion, since the curve can only adopt three different shapes: a downward or upward sloping yield curve or a flat yield curve. Therefore I have to look for more flexible models, for example the class of no-arbitrage models.

3.3 No-arbitrage yield curve models

One of the major disadvantages of the equilibrium models mentioned above is that often the fit of the actual yield curve is poor. Since my goal is to fit the actual yield curve, these models are inappropriate. To tackle this problem, a new class of models was introduced by Ho & Lee in 1986. This new class of models, known as the arbitrage-free models, is created such that it exactly fits the actual yield curve. In general there are two approaches within no-arbitrage yield curve modeling. The first approach is introduced by Ho & Lee (1986) and can be described as a yield curve consistent approach, since it is consistent with the current yield curve. The second approach by Duffie & Kan (1996) is a multifactor model, where every yield is described as an affine combination of certain 'basis' yields that depend on the time to maturity. In no-arbitrage models the actual yield curve is an input and thus endogenous whereas it is exogenous in equilibrium models. This means that no-arbitrage models fully incorporate the information of the actual yield curve.

As mentioned above, the first no-arbitrage model was introduced by Ho & Lee. The Ho & Lee model is a discrete time model that describes the evolutions of the yield curve using a binomial tree. They define the price of a zero-coupon bond $P_i^t(\cdot)$ for state i at time t with time to maturity m. The price of the zero-coupon bond contains all the information of the yield curve in state i at time t. The process starts with an initial state, $P_0^0(\cdot)$and can go up or down at every point. This means that at $t = 1$ there are two possible prices for the zero-coupon bond, namely $P_1^1(\cdot)$ and $P_0^1(\cdot)$, where the number of upstate movements is denoted in the subscript. This process continues for subsequent periods and the price of the zero-coupon bond only depends on the total number of upstate movements, not on the actual sequence in which they occurred. Below the binomial process at time t is illustrated (Ho & Lee, 1986):



The binomial tree is formed by defining a price for a zero-coupon bond in every state i and every time m. Ho & Lee specify the upstate and downstate movements by two perturbation functions $h(t+m)$ and $h^*(t+m)$ which are both positive:

$$P_{i+1}^{t+1}(t+m) = \frac{P_i^t(t+m+1)}{P_i^t(1)} h(t+m), \quad [3.20]$$

in case at time $t+1$ an upstate occurs, and

$$P_i^{t+1}(t+m) = \frac{P_i^t(t+m+1)}{P_i^t(1)} h^*(t+m), \quad [3.21]$$

in case at time $t+1$ a downstate occurs. These perturbation functions specify the difference between downstate prices and upstate prices in the next period. To exclude arbitrage opportunities, the perturbation functions are restricted by:

$$\pi h(t+m) + (1-\pi)h^*(t+m) = 1, \quad [3.22]$$

where the implied (risk-neutral) binomial probability is denoted by π. As mentioned before Ho and Lee assume path independency, which means that the development of the prices from state to state does not depend on the order but only on the number of upward movements. This path independency condition implies that:

$$h(t+m+1)h^*(t+m)h^*(1) = h^*(t+m+1)h(t+m)h(1). \quad [3.23]$$

That implies the following two conditions:

$$h(t+m) = \frac{1}{\pi+(1-\pi)\delta^{t+m}}, \text{ and} \quad [3.24]$$

$$h^*(t+m) = \frac{\delta^{t+m}}{\pi+(1-\pi)\delta^{t+m}}, \quad [3.25]$$

with some constant δ, such that

$$h(1) = \frac{1}{\pi+(1-\pi)\delta}. \quad [3.26]$$

This means that the two perturbation functions only depend on two constant parameters, namely π and δ. Given the parameters π and δ the Ho & Lee model is defined by equations [3.24] to [3.26]. It can be shown that the continuous-time equivalent of the Ho & Lee model is given by (Dybvig, 1997):

$$dr_t = \theta_t dt + \sigma dW_t, \quad [3.27]$$

where the instantaneous standard deviation σ is constant. The model fits the initial yield curve by a function of time θ_t, independent of r, that defines the movement of r at time t. Parameter θ_t can be interpreted as the average direction of r at time t. It can be verified that the Ho-Lee model belongs to the affine class. Important advantages of the Ho & Lee model are the fact that it matches the current yield curve exactly and that it is easy to apply. The Ho & Lee model however does not incorporate mean reversion, like in the Vasicek (1977) model, which is a desirable property for a yield curve model in dynamic form. I am however not interested in the dynamics, since I want to derive the yield curve at a certain point in time.

In 1990 Hull and White presented an extension of the models of Cox, Ingersoll & Ross (1985) and Vasicek (1977) and showed that their extension is consistent with the current yield curve. Note that the Vasicek model and the model by Cox, Ingersoll & Ross together can be written in the following form:

$$dr_t = a(b - r_t)dt + \sigma r_t^{\beta} dW_t, \quad [3.28]$$

which represents the Vasicek model in case $\beta = 0$ and the model by Cox, Ingersoll & Ross in case $\beta = \frac{1}{2}$. Hull & White extended this model by allowing both a and σ to be functions of time and adding a drift term θ_t that is also a function of time. This leads to the following model (Hull & White, 1990):

$$dr_t = \big(\theta_t + a_t(b - r_t)\big)dt + \sigma_t r_t^{\beta} dW_t, \quad [3.29]$$

which can be rewritten to obtain the following expression:

$$dr_t = a_t\left(\frac{\theta_t}{a_t} + b - r_t\right)dt + \sigma_t r_t^{\beta} dW_t. \quad [3.30]$$

This shows that the mean reversion level is a function of time given by $\frac{\theta_t}{a_t} + b$. Based on this general form, Hull and White present two models, namely the extended Vasicek model:

$$dr_t = \big(\theta_t + a_t(b - r_t)\big)dt + \sigma_t dW_t, \quad [3.31]$$

and the extended Cox, Ingersoll & Ross model:

$$dr_t = \big(\theta_t + a_t(b - r_t)\big)dt + \sigma_t \sqrt{r_t} dW_t. \quad [3.32]$$

Again one can show that these models belong to the affine class. Moreover, Hull and White show that for the extended Vasicek model both the prices for a European bond option and the process for the short-rate can be derived analytically, which makes the model tractable.

Flexibility however is still an issue in these models. These models cannot adopt very steep yield curves or yield curves that contain humps. This issue can be solved by using the aforementioned multi-factor model, introduced by Duffie & Kan in 1996. Their model can be seen as a multivariate version of the single-factor model by Cox, Ingersoll & Ross (Duffie & Kan, 1996). These exogenous factor, F, which are certain 'basis' yields. In the model by Duffie & Kan, the price of a zero-coupon bond at time t with time to maturity m is given by the following exponential affine form:

$$P(t, t+m) = \exp\big(A(m) - B(m)'F(t)\big), \qquad [3.33]$$

where $A(m)$ and $B(m)$ satisfy certain ordinary differential equations (Brandt & Chapman, 2002).

Although these models seem to satisfy all criteria I defined at the beginning of this chapter, there is an important drawback. The no-arbitrage property of these models may result in fitting parts of the yield curve that are not structural and may not occur again, i.e. overfitting the yield curve. This has negative consequences for the stability of the yield curve and makes these arbitrage-free models are not appropriate (Ioannides, 2003). There are two approaches that might be appropriate to estimate the current yield curve, namely the spline-based models and parsimonious models. I will discuss these models in sections 3.4 and 3.5. Note that these models are not arbitrage-free, the estimation results can however be used as input for the no-arbitrage models described above.

3.4 Spline-based yield curve models

Spline-based models do not define a single function that covers the full maturity range. These models try to fit the yield curve using a piecewise polynomial, the so-called spline-function. The different segments are combined smoothly at so-called knot points. Since higher-order polynomials can cause smoothing problems, most spline functions are based on lower-order, quadratic or cubic, polynomials. A cubic spline for example is a piecewise cubic polynomial that is restricted at the knot points in such a way that the first two derivatives and the levels are equal. Each parameter of a spline-based model corresponds to a knot in the spline. Spline-based models can be categorised into parametric and non-parametric splines.

In 1971 McCulloch introduced a parametric cubic spline method, which he improved in 1975. In this model, the discount function $d(m)$ is defined as a collection of k continuously differentiable functions $\varphi_j(m)$ (McCulloch, 1971):

$$d(m) = 1 + \sum_{j=1}^{k} \alpha_j \varphi_j(m), \qquad [3.34]$$

where α_j is the coefficient that should be estimated via linear least squares regression and where m again denotes the time to maturity. The value of k and the form of the functions $\varphi_j(m)$ is very important, the performance of the model (the fit) highly depends on it. The value of k determines the number of knot points and it is clear that if the number of knot points is too low, a poor fit will be the result. Too many knot points however may result in overfitting and has consequences for the smoothness of the curve. I refer to McCulloch (1971 and 1975) for examples of the functions $\varphi_j(m)$.

An example of a non-parametric spline-based model is the *Smoothing Splines* method, developed by Fisher et al. (1995). This method starts with an over-parameterised model, where the flexibility throughout the spline is guaranteed by a large amount of knot points. The ratio between a goodness-of-fit measure and the number of parameters is then minimized to obtain the optimal number of parameters (knot points).

First, assume the following transformation holds:

$$g(h(\cdot), m) \equiv d(m), \qquad [3.35]$$

where $h(m)$ denotes the function that is being splined and $d(m)$ represents the discount function. Fisher et al. (1995) define the following penalty function:

$$\lambda \int_0^T h''(m)^2 dm, \qquad [3.36]$$

which is the integral of the squared second derivative of $h(m)$ multiplied with a constant λ. The minimization problem now comes down to the following function which consists of the residual sums of squares and the penalty function [3.36] (Fischer, Nychka, & Zervos, 1995):

$$\min_{h(m)\in\mathcal{H}} \left[\sum_{k=1}^{n}\left(P_k - \hat{P}_k\right)^2 + \lambda \int_0^T h''(m)^2 dm\right], \qquad [3.37]$$

with $\hat{P}_k = C_k \tilde{g}(h(\cdot), m_k),$ [3.38]

where $\mathcal{H}$ denotes the space of all possible functions that are defined on $\mathbb{R}^+$ whose squared second derivatives integrate to a value that is finite. The price and coupon payments of bond k are denoted by respectively P_k and C_k. The minimiser of equation [3.37] is then found using nonlinear least squares.

Many spline-based models are based on this Smoothing Splines method but differ in application of the smoothing criteria in order to get a better fit. Improvement of the Smoothing Splines method was also proved necessary by Bliss (1997), who even called it "a dubious choice for estimating term structure", due to the poor in-sample and out-of-sample performance. There are however spline-based methods that provide a very good fit, sometimes superior to the parsimonious methods I will discuss next. An important issue however can be overfitting or overparameterization. Fisher et al. give an example where the yield curve is estimated using 150 securities. In this example the McCulloch (1971) cubic spline method needs 18 parameters whereas Nelson & Siegel (1987) only needs four. Bliss (1997), who uses an Extended Nelson & Siegel model with five parameters, finds however that these two methods perform comparably, so the need for more parameters is not always clear. Since tractability is also an important property for the model I want to choose, I prefer a model with a small number of parameters.

3.5 Parsimonious yield curve models

The idea behind the parsimonious yield curve models is, in contrast to spline-based models, to specify a single function of a small number of parameters that covers the complete maturity range. The general approach for estimating the parameter values is to minimize the squared deviations of theoretical prices, obtained from the model, from observed prices. Another approach is to minimize the squared deviations of theoretical yield to maturities from quoted yield to maturities. The approach that should be used depends on whether one wants to fit the bond prices or the yields. A model that is widely used in both practice and literature is developed by Nelson & Siegel (1987). The Nelson-Siegel model tries to estimate the relationship between the instantaneous forward rates $f_{t,m}$ and the maturity m at time t by the following function (Nelson & Siegel, 1987):

$$f_{t,m} = \beta_{t,0} + \beta_{t,1}\exp\left(\frac{-m}{\tau_{t,1}}\right) + \beta_{t,2}\frac{m}{\tau_{t,1}}\exp\left(\frac{-m}{\tau_{t,1}}\right), \qquad [3.39]$$

where the parameters $\beta_{t,0}$, $\beta_{t,1}$, $\beta_{t,2}$ and $\tau_{t,1}$ are to be estimated. In the remainder of this chapter I will drop the index t to simplify notation. As mentioned before, the spot rates can be derived from the forward rates using equation [3.3] which results in the following expression:

$$s_m = \beta_0 + \beta_1\left(1-\exp\left(-\frac{m}{\tau_1}\right)\right)\left(\frac{m}{\tau_1}\right)^{-1} + \beta_2\left(\left(1-\exp\left(-\frac{m}{\tau_1}\right)\right)\left(\frac{m}{\tau_1}\right)^{-1} - \exp\left(-\frac{m}{\tau_1}\right)\right), \qquad [3.40]$$

which can be rewritten as follows:

$$s_m = \beta_0 + (\beta_1 + \beta_2)\frac{\tau_1}{m}\left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right) - \beta_2 \exp\left(-\frac{m}{\tau_1}\right). \qquad [3.41]$$

The parameter β_0 must be positive since the forward and spot rate approach β_0 as maturity increases and negative rates have to be avoided. The value of the function at maturity zero, $(\beta_0 + \beta_1)$ must also be positive. This also shows that β_1 represents the initial deviation from β_0. Parameters β_2 and τ_1 together create the so-called hump in the yield curve. The sign and absolute size of β_2 determine respectively the direction and size of the hump, where a positive β_2 represents a hump and a negative β_2 a U-shape. Parameter τ_1 must be positive and represents the position of the U-shape or hump on the yield curve.

In 1994 Lars Svensson extended the Nelson-Siegel model by adding an extra term to the initial function. Due to this extra term, the Svensson model allows for a second hump which results in a better fit of the yield curve in case of unusual yield curve shapes. The better fit comes at the cost of two extra parameters which have to be estimated: β_3 and τ_2. The function for the forward rate in the Svensson model is defined as follows (Svensson, 1994):

$$f_m = \beta_0 + \beta_1 \exp\left(\frac{-m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1}\exp\left(\frac{-m}{\tau_1}\right) + \beta_3 \frac{m}{\tau_2}\exp\left(\frac{-m}{\tau_2}\right), \qquad [3.42]$$

where β_3 and τ_2 are similar to β_2 and τ_1 and determine together the second hump or U-shape. The spot rates can again be obtained using equation [3.3], which results in:

$$s_m = \beta_0 + \beta_1\left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right)\left(\frac{m}{\tau_1}\right)^{-1} + \beta_2\left(\left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right)\left(\frac{m}{\tau_1}\right)^{-1} - \exp\left(-\frac{m}{\tau_1}\right)\right) + \beta_3\left(\left(1 - \exp\left(-\frac{m}{\tau_2}\right)\right)\left(\frac{m}{\tau_2}\right)^{-1} - \exp\left(-\frac{m}{\tau_2}\right)\right). \qquad [3.43]$$

The parameter restrictions mentioned earlier can be summarized as:

$\beta_0 > 0$, $\tau_1 > 0$, $\tau_2 > 0$, and $\beta_0 + \beta_1 > 0$. [3.44]

3.6 Model choice

To choose an appropriate model for estimating the yield curve that is used to obtain the DBO under IAS 19, I defined some criteria at the beginning of this chapter. These criteria are smoothness, flexibility, stability and tractability. Since the affine models of Vasicek and Cox, Ingersoll & Ross quite often result in poor fits of the actual yield curve due to a lack of flexibility, they are the first to drop out. The no-arbitrage models, like Ho-Lee and Hull-White match the actual yield curve exactly. However since they exactly fit the yield curve, they are also very sensitive for outliers and might be overfitting the data and thus do not satisfy the smoothness criterion. The risk of overfitting is also present within the spline-based models, which makes choosing the right number of knot points complex. The spline-based models also tend to over fit the long end of the yield curve (Ioannides, 2003).

The parsimonious models, like Nelson-Siegel and Svensson, may not possess the desirable no-arbitrage property, they do fit the actual yield curve quite well (Ioannides, 2003). Since I am not pricing any derivatives, the no-arbitrage property is no strict requirement for the yield curve model.

Parsimonious models are also more stable than spline-based modes, since there is no risk of overfitting. The ability of parsimonious models to estimate the yield curve from a small set of market data is very important to estimate the corporate bond yield curve. The parsimonious models are smooth, a single function describes the complete curve, and they are flexible enough to adopt most yield curve shapes. Furthermore, if the data are chosen well, they are robust for small changes in the data and one of the major advantages is their tractability. This means that all my criteria are met and therefore I choose to use a parsimonious model to estimate the corporate bond yield curve. Within the parsimonious models, I prefer the Svensson model since it is more flexible than the Nelson-Siegel model and adds only little complexity.

4. Empirical results for the high quality corporate bond yield curve

In Chapter 2, I discussed the need of using the full yield curve in calculating the DBO and in Chapter 3 I discussed the available yield curve models. In this chapter I will discuss the estimation procedure of the high quality corporate yield curve using the Svensson model and the empirical results. I will start with discussing the data selection problem, next I will discuss some estimation methods for the Svensson model and motivate my choice for the Bliss method. After choosing the estimation method I will elaborate on the estimation results and I will conclude this chapter with a discussion on the issue related to the longer maturities.

4.1 Data selection

To estimate the corporate yield curve that is to be used for IAS 19 discounting, the data has to meet some requirements. Most important requirement is that IAS 19 requires the discount rate to be based on the yield of high quality corporate bonds, which are considered to be corporate bonds with at least an AA credit rating from Standard & Poor's or Fitch or a, corresponding, Aa credit rating from Moody's. Furthermore, the term of the bonds and currency in which the bonds are denominated must be equal to the term and the currency, in this case Euros, of the pension liabilities. Since the term of pension liabilities can range up to 100 years[1], there are no bonds available that equal the term of the pension liabilities at longer maturities than approximately 30 years. I will discuss this problem in Section 4.5. I will further require these bonds to be liquid to assure that they are correctly priced. Due to all these requirements, selecting bonds can be quite involving. There are, however, benchmark indices published by International Index Company Limited, which are very useful. This collection of indices, called the Markit iBoxx EUR index family (Markit Group Limited, 2010), contains, among others, the *iBoxx Corporates AA* and the *iBoxx Corporates AAA* indices and is available on a monthly basis. These indices consist of bonds with the required currency, terms, credit rating and liquidity.

Besides the involvement of the selection of bonds it is also important to standardize the selection criteria and procedure to avoid wide discretion. Therefore it is very useful to use these publicly available indices to select the bonds that are used to obtain the yield curve. The iBoxx indices are widely used as a benchmark, are published monthly and the bond selection procedure is also published (Markit Group Limited, 2010) which makes them very suitable to solve the bond selection issue.

These indices are used for bond selection only. I use the ISIN numbers of the bonds that are included in the index to retrieve the necessary information, such as maturity date, bid/ask price and coupon rate, from Bloomberg. Since monthly data is available, I collected data over 18 months, from December 2008 to May 2010. To compare my results with yield curves that are used in the market, the bond prices at the last day of each month are used, which is usually the point in time at which the DBO is calculated. This is also the reason why I incorporated December 2008.

After selecting the appropriate bonds, the following data has to be obtained for every bond:

[1] In the calculation of the DBO usually a 'maximum age' Ω, which is typically equal to 120, is used. Assuming the youngest participant in the plan to be 20, the last possible cash flow is 100 years from now.

- Coupon type, rate, frequency, and payment date;
- Maturity date;
- Clean bid and ask prices at settlement date;
- Day count convention;
- Credit rating of Standard & Poor's, Fitch, and Moody's;
- Par yield.

Using this information, the dirty bid, ask, and mid prices (including accrued interest since last coupon payment) and the time to maturity can be calculated. In case any of this information, except the par yield, is not available on Bloomberg, the bond is excluded from the sample. Using this data and a yield curve, a bond price can be calculated, as will be shown in the next section.

4.2 Estimation methods

The objective of the estimation method is to find a set of values for the parameters of the Svensson model, such that the errors for all bonds in the dataset are minimized. Most important is the decision whether to minimize the (sum of squared) yield or price errors. In case one is interested in fitting the yields, as is the case in this thesis, it is obvious that one wants to minimize the sum of squared yield errors. Generally this goal can be achieved in three ways: the first is to modify the problem such that it is linear and apply ordinary least squares (OLS) on the spot rates, the second is to use a non-linear optimization technique such as non-linear least squares, maximum likelihood or the generalized method of moments to minimize the yield to maturity errors. The third method is to minimize price errors that are weighted based on the duration of the corresponding bond. I will discuss these three methods below.

The first technique is proposed by Diebold et al. (2006). They introduced a new interpretation of the Nelson-Siegel model and argued that Svensson added only little flexibility to the Nelson-Siegel model (Diebold & Li, 2006). They fix τ_1 and rewrite the model such that it is linear and the factors have an economic interpretation: level, slope and curvature. They rewrite formula [3.41] by taking $\lambda = \frac{1}{\tau_1}$ to obtain the following Nelson-Siegel formula for the spot rates:

$$s_m = \beta_0 + \beta_1 \left(\frac{1-e^{-\lambda m}}{\lambda m}\right) + \beta_2 \left(\frac{1-e^{-\lambda m}}{\lambda m} - e^{-\lambda m}\right), \qquad [4.1]$$

where again m is the time to maturity. Recall that τ_1 and thus λ is fixed which means that the corresponding spot rate curve now becomes a linear function. Diebold and Li (2006) show that the factors β_0, β_1 and β_2 can be viewed as respectively the level, slope and curvature factors. Diebold and Li use a bootstrap method to obtain the zero coupon yields, s_m, from observed bond prices and estimate this model using ordinary least squares (OLS). Diebold et al. (2006) tested their model and find that it performs well in both in-sample fitting and in out-of-sample forecasting. They also test whether the model belongs to the affine model class and find that it does not belong to that class.

The second method is based on the fact that for bonds that pay (fixed) coupons, sometimes the *par yield* or *yield to maturity* is quoted. The par yield is the yield for which the price of the bond is equal to its face (or par) value. The yield to maturity is also called the internal rate of return and is equal to the constant yield r_k of bond k such that the present value of all the cash flows is equal to the price P_k of the bond. These cash flows consist of both the coupon payments C as well as the final payment of the face value of the bond V. In mathematical terms:

$$P_k = \sum_{i=1}^{m} C \exp(-r_k i) + V \exp(-r_k m), \quad [4.2]$$

where m again represents the time to maturity of bond k. Index t is again suppressed just like in the latter sections of Chapter 3, however now it is not only suppressed to simplify notation, but also because I am not interested in the dynamics of the yield curve, only in the yield curve at a specific point in time. The price of bond k can also be expressed in terms of the spot rates, the (fixed) coupon payments C and the repayment of the face value V:

$$P_k = \sum_{i=1}^{m} C \exp(-s_i i) + V \exp(-s_m m). \quad [4.3]$$

The yield to maturity can therefore also be seen as an average of the spot rates and varies, in general, with the time to maturity.

In case the par yield is quoted, the Svensson model can be estimated in two steps. In the first step of this method the estimated bond prices are computed using spot rates $s_{t,m}$ and equation [4.3]. These spot rates are computed using the Svensson model with certain starting values for its parameters. In the second step, the estimated bond prices, $\hat{P}_k$, are used to obtain the corresponding estimated yield to maturity $\hat{r}_k$ by solving the following equation for every bond k,:

$$\hat{P}_k = \sum_{i=1}^{m} C \exp(-\hat{r}_k i) + V \exp(-\hat{r}_k m). \quad [4.4]$$

It is the objective to minimize the sum of the squared par yield errors. To achieve this goal, one runs an iterative process until convergence is reached. In every iteration, the values of the parameters of the Svensson model are changed and the two steps are repeated.

I found that for most bonds in the dataset, the par yield was not available. Furthermore, it is clear that it takes less effort to minimize the price errors, since this only requires a solution for the first step. However, this can lead to very large yield errors in cases with a relatively short term to maturity. This is also intuitively very clear if one recalls the duration concept. Assuming a parallel shift of the yield curve, changes in the yield have a larger effect on bonds with a longer term to maturity than on bonds with a short term to maturity. This means that in the optimization method with the differences in bond prices as the objective function, the penalty for a yield error is very small in case of bonds with a short term to maturity. This leads to overfitting the prices of the long-term bonds at the expense of the fit for the price of short-term bonds. As will be shown in Section 5.3 and 5.4, the shorter end of the yield curve can have significant influence on the outcomes of the PUCM calculations.

There is however a solution to this problem which was introduced by Bliss in 1997, which is the third method I will discuss. The method, which I will call the 'Bliss method' from now on, weighs the pricing errors based on the duration of the corresponding bonds. I will discuss this method in the next section.

4.3 The Bliss method

As mentions before, the 'Bliss method' is an alternative to minimizing the yield errors. Bliss (1997) minimizes the sum of the squared weighted price errors, where the weights are based on the inverse of the Macaulay duration of the bond. This results in higher weights for short term bonds and lower weights for long term bonds. These weights reduce the relative overfitting of the longer maturity bond prices.

In case of the Svensson model the parameter set ϕ I want to estimate is defined as $\phi \equiv [\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2]$. Equation [3.42] is estimated by the following non-linear, constrained optimization, estimation procedure (Bliss, 1997) for N bonds:

$$\min_\phi \sum_{i=1}^{N} (\omega_i \varepsilon_i)^2, \qquad [4.5]$$

where the error process is defined as follows:

$$\varepsilon_i = P_i - \hat{P}_i, \qquad \forall\, i = 1, \dots, N, \qquad [4.6]$$

with P_i is the average of the observed bid and ask price of bond i, and $\hat{P}_i$ is the estimated price of the bond i, based on the Svensson model. Note that the error term differs from the one in Bliss (1997). In his article, Bliss defines the error term as zero if the estimated price is within the bid-ask spread and as the difference between the estimated price and respectively the bid price or the ask price if the estimated price is lower than the bid price or higher that the ask price. Because the bid-ask spread is relatively large for corporate bonds, I found several solutions for which the objective function [4.5] equaled zero. Therefore I choose to define the error term as in equation [4.6]. The weights ω_i in equation [4.5] are a function of the Macaulay duration D_i:

$$\omega_i = \frac{1/D_i}{\sum_{j=1}^{N} 1/D_j}, \qquad [4.7]$$

where the Macaulay duration D_i is defined as follows (assuming one coupon payment per year):

$$D_i = \sum_{j=1}^{m} \frac{PV_{i,j}\, j}{\sum_{k=1}^{m} PV_{i,k}}, \qquad [4.8]$$

where j identifies the cash flow of which $PV_{i,j}$ is the present value and m the time to maturity of bond i. Note that $PV_{i,j}$ in equation [4.8] depends on the yield curve I want to estimate. Since the optimization method I use to estimate the yield curve is an iterative process, I first have to choose starting values for ϕ. Just like Skinner & Ioannides (2005), I found that the choice for certain starting values is not critical although it can speed up convergence.

Furthermore equation [3.39] is subject to the following three constraints, additionally to the constraints mentioned in [3.44]:

1. The spot rate is non-negative when the time to maturity approaches zero, i.e. $0 \leq s_{m_{min}}$;
2. The spot rate is non-negative when the time to maturity goes to infinity, i.e. $0 \leq s_\infty$;
3. The forward rates are non-negative, i.e. the discount function is non-increasing: $\exp(-s_{m_k} m_k) \geq \exp(-s_{m_{k+1}} m_{k+1})$, for all $m_k < m_{max}$.

In his paper Bliss states that only the second constraint is binding in rare cases, whereas I find in Section 4.4 that the first constraint is binding quite often. This is caused by the current shape of the yield curve, which is very steep at the short maturities. Bliss finds that this method performs very well and combined with the computational advantages this is also a suitable model to construct the yield curve.

To solve the optimization problem, I use the standard Solver tool in Microsoft Excel. This Solver tool uses a nonlinear optimization technique that is developed by Leon Lasdon from the University of

Texas and Allan Waren from Cleveland State University and is called the Generalized Reduced Gradient (GRG2) algorithm. The use of other optimization techniques could improve the fitting of the Svensson model (Gilli, Grosse, & Schumann, 2010). This falls however outside the scope of my thesis and therefore a recommendation for further research.

4.4 Empirical results

As mentioned before, I collected data over 18 months, from December 2008 to May 2010. I used the *iBoxx Corporates AA* and the *iBoxx Corporates AAA* indices to create a gross sample at the last trading day of each month. Using this sample, I collected the necessary data, such as bid and ask prices, maturity dates and credit ratings at Bloomberg. After collecting these data, I excluded bonds with variable coupons and bonds for which the prices were not available and created two samples: one example where I excluded bonds with an average rating of AA- and one in which these bonds are still included. This resulted in the sample sizes displayed in Table 5.

The reason I constructed these two samples is that it is debated whether one should consider bonds with an AA- rating as high quality bonds. Some say that some of these bonds are close to transition to an A+ rating and therefore cannot be seen as high quality bonds. An important fact however is that including these bonds approximately doubles the sample size, which could increase the quality of the estimated yield curves. The latter is also illustrated by Table 6, which shows that at September 30, 2009, more AA- rated bonds are present in the sample than bonds with a higher credit rating. I see the same effect for other dates in my data set.

The resulting yield curve estimates at September 30, 2009 are shown in Figure 5. It shows that the difference is especially at the longer maturities where the yield curve including AA- bonds lies below the one excluding AA- bonds. This is counter intuitive, since one would expect it to be the other way around, due to a larger credit spread on AA- bonds. An explanation for this result can be that the model is less stable in case the AA- bonds are excluded, due to a smaller sample size. The latter can result in a larger influence of the (rare) bonds with long maturities, especially when AA- bonds are excluded.

Table 5: This table shows the sample sizes for respectively the gross sample from the iBoxx Corporate AA and iBoxx Corporate AAA indices, the sample after excluding bonds with variable coupons and bonds for which not all data was available, and the sample after excluding bonds with an average AA- rating.

Sample size			
	Gross	Net including AA-	Net excluding AA-
December 30, 2008	312	151	77
January 30, 2009	287	156	79
February 27, 2009	266	163	83
March 31, 2009	247	168	88
April 30, 2009	238	174	89
May 29, 2009	226	177	91
June 30, 2009	241	191	98
July 31, 2009	245	197	99
August 31, 2009	245	197	100
September 30, 2009	250	202	99
October 30, 2009	252	208	102
November 30, 2009	254	209	102
December 31, 2009	244	211	103
January 29, 2010	243	216	106
February 26, 2010	244	218	106
March 31, 2010	245	221	107
April 30, 2010	246	224	108
May 31, 2010	233	218	104

Table 6: Sample size for September 30, 2009 per maturity bucket and sizes for respectively the gross sample from the iBoxx Corporate AA and iBoxx Corporate AAA indices, the sample after excluding bonds with variable coupons and bonds for which not all data was available, and the sample after excluding bonds with an average AA- rating.

Sample sizes per maturity bucket September 30, 2009							
Maturity bucket	1 to 3 years	3 to 5 years	5 to 7 years	7 to 10 years	10 to 15 years	15 to 25 years	25+ years
Number in gross sample	47	63	60	49	20	8	2
Number in sample including AA-	42	53	52	35	14	4	2
Number in sample excluding AA-	23	31	25	11	6	2	1

Table 7: Sample size for December 31, 2009 per maturity bucket and sizes for respectively the gross sample from the iBoxx Corporate AA and iBoxx Corporate AAA indices, the sample after excluding bonds with variable coupons and bonds for which not all data was available, and the sample after excluding bonds with an average AA- rating.

Sample sizes per maturity bucket December 31, 2009							
Maturity bucket	1 to 3 years	3 to 5 years	5 to 7 years	7 to 10 years	10 to 15 years	15 to 25 years	25+ years
Number in gross sample	43	70	51	47	21	8	2
Number in sample including AA-	40	64	45	40	15	5	2
Number in sample excluding AA-	23	38	20	12	6	3	1

Figure 5: Comparison between the estimates of the yield curve at September 30, 2009 using a data set respectively including and excluding corporate bonds with an average credit rating of AA-.

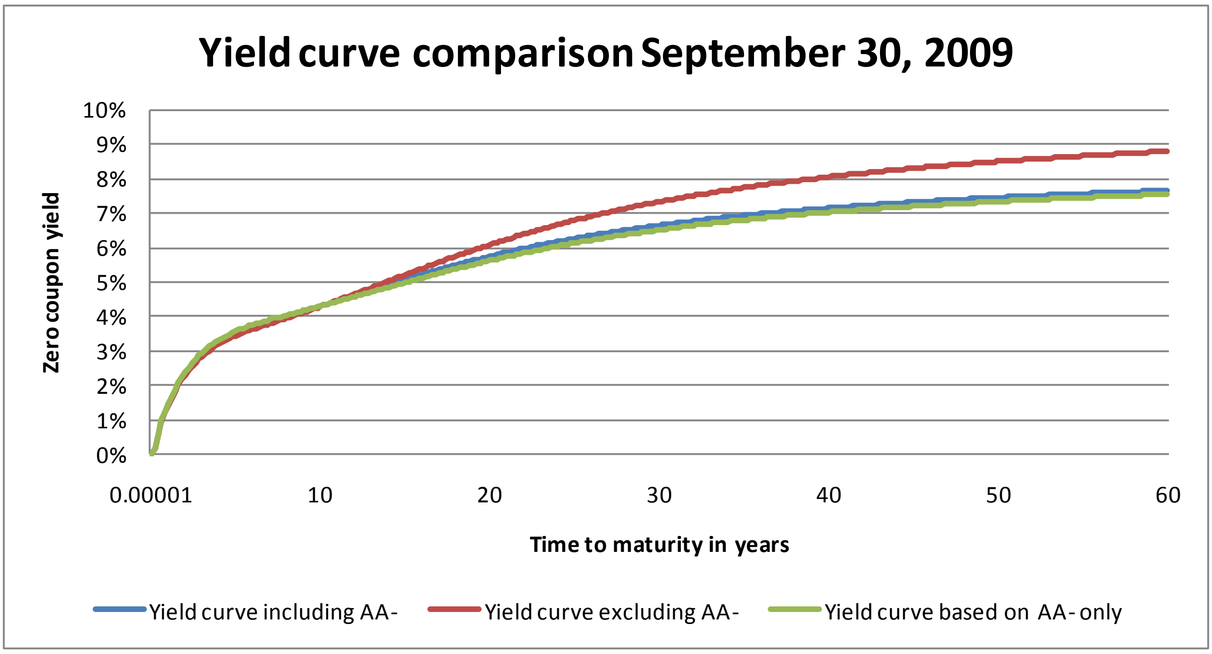
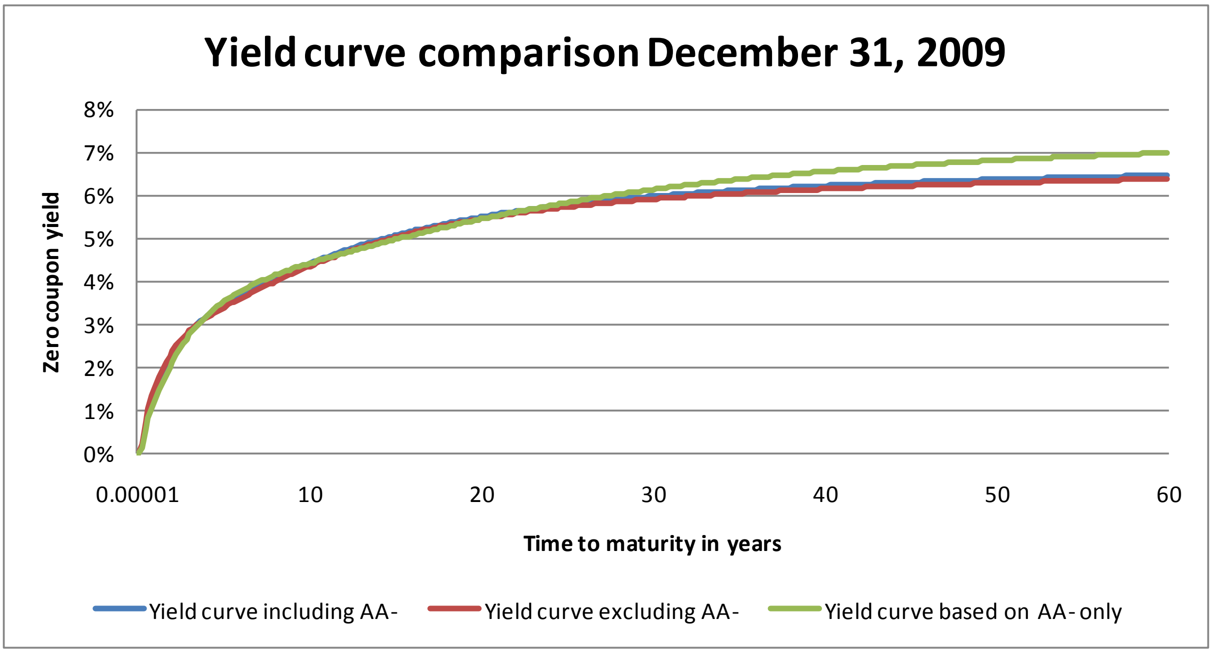


Figure 6: Comparison between the estimates of the yield curve at December 31, 2009 using a data set respectively including and excluding corporate bonds with an average credit rating of AA-.



To see whether this also holds for other dates, I made a similar overview of the sample size at December 31, 2009 Table 7 and estimated the corresponding yield curve which is shown in Figure 6. It is clear that the opposite effect appears at December 31, 2009. Therefore I conclude that the differences at the long end is caused by a lack of data points in the cases where AA- or AA and higher rated bonds are excluded and that it is best to include the AA- rated bonds. Similar results can be found in literature (Diaz & Skinner, 2001).

4.5 How to deal with the longer maturities?

Figures 5 and 6 showed yield curves that converged quickly to a certain long-term yield. This desirable situation does not always show up. I also found yield curves that increased to yields up to around 10% or higher as maturity increases, an example is the yield curve estimate at June 30, 2009. To solve the estimation problem on the long end of the maturity spectrum (i.e. 25 years and longer), in general, two methods are used in practice. The first method is to keep the spot rate constant from a certain maturity on. The second is to estimate the yield curve using government bond yields and extrapolate the corporate bond yield curve using that government bond yield curve and a constant credit spread.

The second method has an important disadvantage, namely the assumption that the credit spread is constant. The government yield often slightly decreases after a certain maturity (e.g. 15 or 20 years), due to the large demand for long-term bonds by, for example, pension funds. Pension funds often use these long-term government bonds to increase the duration of their fixed income portfolio to match the duration of their liabilities. If one keeps the credit spread constant in that maturity range, the corporate bond yield curve will also decrease. This is counter intuitive, since one would not expect the yield on long-term corporate bonds to decrease because of the increasing default risk. Moreover, many models, including the Svensson model I use, show an increasing yield curve at the longer maturities. Since this increase is caused by a combination of the shorter maturity bonds and the explicit formula in the Svensson model, one could ask whether that is the right situation.

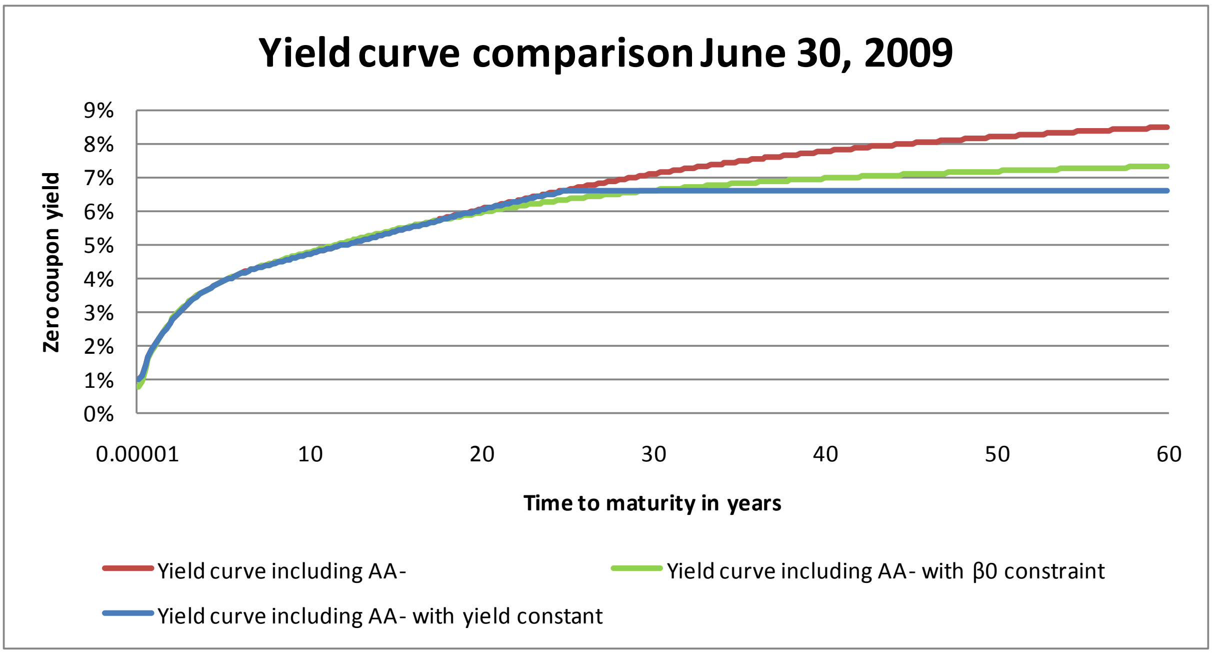
The first option of keeping the yield constant from a certain maturity, might be a better solution, but how to decide from which maturity? That decision can be based on the longest maturities in the data set, e.g. the average maturity of the five bonds with the longest maturity. This method usually causes a kink in the yield curve, i.e. the yield curve is not smooth at the crossover point.

A third method, which is especially suitable for the Nelson & Siegel and Svensson model, is to specify a certain (maximum) long rate to which the yield curve converges. This long rate is the β_0 parameter in the Svensson (and Nelson & Siegel) model. Especially in steep yield curves, such as appeared recently, the yield curves following from the Svensson model tend to keep increasing at the longer maturities. When the yield curve is upward sloping, the constraint on β_0 serves as a maximum for the long-term yield. One could base the choice for a constraint on the (variable) yield of perpetual bonds, if traded. In the data set of June 30, 2009, there is one perpetual with an average rating of AA- which paid an annual yield of 7.78% and whose mid dirty price was around 78% of its face value. I can use this information and maximize β_0 at, for example, 8%. The result is shown in Figure 7 and compared to the first method, where the yield is kept constant from the average maturity of the five bonds with the longest maturity on.

Figure 7 shows that, although β_0 is constrained, the curve follows the ‘standard’ curve closely up to around 20 years. At longer maturities, the constrained yield curve lies below the ‘standard’ curve. This method seems a nice and smooth method to combine the Svensson model and an assumption about the long-term yields.

Although I do not think there is one best solution for the ‘longer maturities problem’, I think either keeping the yield constant from a certain maturity on or putting a constraint on β_0 are superior to keeping the credit spread constant over the government yield curve. I base my opinion on the fact that the credit spread is not constant over the maturities where it can be measured and the aforementioned fact that long term government bonds are popular under pension funds, which puts pressure on the long-term government bond yields.

Figure 7: Comparison between the estimates of the yield curve at June 30, 2009 using a data set including corporate bonds with an average credit rating of AA-. The red curve shows the 'standard' curve without additional constraints, the blue curve shows the curve that is kept constant from maturity 24.9 that is defined as the average maturity of the five bonds in the data set with the longest maturity. The green curve shows the curve where β_0 is maximized at 8%.



5. Impact of the corporate bond yield curve on accounting

In Section 2.6 I argued that one should use the full yield curve in the Projected Unit Credit Method (PUCM) to calculate the Defined Benefit Obligation (DBO) and Service Cost. In Chapter 4 I showed how one could construct the applicable yield curve using market data. In this chapter I will discuss the impact of using the full yield curve on the DBO and Service Cost compared to using a single discount rate, i.e. applying IAS 19 paragraph 80. Next to the impact on the DBO and Service Cost I will discuss the impact on the Interest Cost and the Employer Pension Expense. I will also discuss the impact of the proposed abolition of the corridor method.

As discussed in Section 2.6, several assumptions, other than the discount rate, have to be made in order to calculate the DBO and Service Cost. These assumptions will be discussed briefly in Section 5.2. I will start this chapter by discussing the concept of actuarial factors, since these will be needed in the calculations later on in this chapter.

5.1 The concept of actuarial factors

In order to calculate the net present value of the DBO, next to the discount rate, also survival probabilities have to be taken into account. Therefore it is useful to obtain the current value of 1 euro entitlement of some type of pension. Examples of pension types are old-age pension, widower's pension and disability pension. The current value of 1 euro pension entitlement is called an actuarial factor since it is used to calculate the current value by multiplying the factor with the pension entitlement. I will discuss the actuarial factors for old-age pensions first.

In this chapter I will assume annual compounding, unless stated otherwise. Assume that the applicable yield curve is known and let s_m denote the annual compounding spot rates for every maturity m. From these spot rates, one can easily derive the corresponding annual compounded forward rates, where the τ-year forward rate for maturity m is denoted as $s_{\tau,m}$, by solving the following equation for $s_{\tau,m}$:

$$\left(1+s_{\tau,m}\right)^{m-\tau}\left(1+s_{\tau}\right)^{\tau}=\left(1+s_{\tau+m}\right)^{\tau+m},\quad m=1,\dots,\infty,\quad \tau=1,\dots,\infty. \qquad [5.1]$$

The discount factors are now defined as follows:

$$d_{\tau,m}=\left(\frac{1}{1+s_{\tau,m}}\right)^{m},\qquad m=1,\dots,\infty,\quad \tau=1,\dots,\infty. \qquad [5.2]$$

In case $\tau=0$, I will drop the τ index. Now consider a perpetual stream of annual payments of 1. This perpetual stream of annual payments is called a *perpetuity-due* if the first payment occurs at the beginning of the year. In case of an *immediate perpetuity* the first payment occurs at the end of the year. The current value of a perpetuity-due is denoted by $\ddot{a}_{\overline{\infty}|}$, where (Gerber, 1997):

$$\ddot{a}_{\overline{\infty}|}=1+d_1+d_2+\cdots. \qquad [5.3]$$

Clearly, the current value of the immediate perpetuity $a_{\overline{\infty}|}$ is equal to $\ddot{a}_{\overline{\infty}|}-1$. In case the number of annual payments n is finite it is called an *annuity-due*, denoted by $\ddot{a}_{\overline{n}|}$, where:

$$\ddot{a}_{\overline{n}|}=1+d_1+d_2+\cdots+d_{n-1},\qquad n\geq 2. \qquad [5.4]$$

In case of an old-age pension, the number of payments is unknown on forehand. Consider a person, x years of age, and denote the future lifetime of this person as $T(x)$, so $x + T(x)$ will be the age at which the person dies. It is clear that T is a random variable. Now define K as $K = \lfloor T \rfloor$, the number of complete future years lived by person x, so K is an integer valued random variable. Define S as the fraction of a year during which person x is alive in the year of death, i.e. $S = T - K$. Since K is a random variable, $\ddot{a}_{\overline{K+1}|}$ is a random variable too. To obtain the expected value of $\ddot{a}_{\overline{K+1}|}$, denoted as $\ddot{a}_x$, I need the probability distribution of $\ddot{a}_{\overline{K+1}|}$, which is given by (Gerber, 1997):

$$\Pr\left(\ddot{a}_{\overline{K+1}|} = \ddot{a}_{\overline{k+1}|}\right) = \Pr(K = k) = {}_kp_x * q_{x+k}. \quad [5.5]$$

In formula [5.5], ${}_kp_x$ denotes the probability that an x year old person will survive at least k years and q_{x+k} denotes the probability that an $x + k$ year old person dies within one year. The product ${}_kp_x q_{x+k}$ is thus equal to the probability that an x year old person dies in the period between k and $k + 1$ years from now. Now the net single premium $\ddot{a}_x$, the expected value of $\ddot{a}_{\overline{K+1}|}$, can be calculated as follows:

$$\ddot{a}_x = \sum_{k=0}^{\infty}\left(\ddot{a}_{\overline{k+1}|} * {}_kp_x * q_{x+k}\right). \quad [5.6]$$

Note that in practice instead of ∞, a 'maximum age', Ω, is used which is usually equal to 120 in The Netherlands. I am also interested in the current value of a deferred whole life annuity-due, since I need this to calculate the current value of the pension entitlements of active participants and early leaver, who left the company but did not transfer their pension entitlements, i.e. all participants who have not yet retired. Therefore I will first introduce the value of an annuity-due in p years, denoted as ${}_p\ddot{a}_{\overline{n}|}$ and defined as:

$${}_p\ddot{a}_{\overline{n}|} = 1 + d_{p,1} + d_{p,2} + \cdots + d_{p,n-1}, \quad [5.7]$$

where $d_{p,m}$ is the discount factor for maturity m, based on p-year forward curve (see formula [5.2] and Appendix B). Formula [5.6] can now be generalized to:

$${}_p\ddot{a}_x = \sum_{k=0}^{\infty}\left({}_p\ddot{a}_{\overline{k+1}|} * {}_kp_{x+p} * q_{x+p+k}\right). \quad [5.8]$$

Again, in case $p = 0$, I will drop index p. Consider the current value of a *whole life annuity-due*, deferred for τ years, denoted as ${}_{\tau|}\ddot{a}_x$:

$${}_{\tau|}\ddot{a}_x = {}_\tau p_x * d_\tau * {}_\tau\ddot{a}_x, \qquad \tau \geq 0. \quad [5.9]$$

Now denote the pensionable age, which is typically 65 years in The Netherlands, as x_{pa}. Using formulas [5.8] and [5.9], the actuarial factor for old-age pension for a person aged x, which will be denoted as OP_x, can be derived as follows (Bouwman, 2008):

$$OP_x = \begin{cases} {}_{x_{pa}-x}p_x * d_{x_{pa}-x} * {}_{x_{pa}-x}\ddot{a}_{x_{pa}} & x < x_{pa} \\ \ddot{a}_x & x \geq x_{pa} \end{cases}. \quad [5.10]$$

For the projections in the PUCM, as described in Section 2.7, I will also need the future OP_x factors for every projection year p, which will be denoted as OP_x^p. Since the projection continues until the pensionable age, I will only need the OP_x^p factors for $x + p < x_{pa}$. Using forward rates and equations [5.8] and [5.10] the formula for OP_x^p becomes:

$$OP_x^p = {}_{x_{pa}-(x+p)}p_{x+p} * d_{p,x_{pa}-(x+p)} * {}_{x_{pa}-(x+p)}\ddot{a}_{x_{pa}}, \qquad x + p < x_{pa}. \qquad [5.11]$$

In practice survival probabilities are derived from a mortality table, which consists of the observations of a (theoretical) cohort of lives over time. Such a mortality table gives the number of survivors at each age (in years), starting at 0 with typically 10 million lives. Now denote the number of survivors at age x as l_x, the probabilities ${}_kp_x$ and ${}_kq_x$ are then given by the following formulas:

$${}_kp_x = \frac{l_{x+k}}{l_x}, \text{ and} \qquad [5.12]$$

$${}_kq_x = \frac{l_x - l_{x+k}}{l_x}. \qquad [5.13]$$

This result can be used to simplify the calculation of actuarial factors by using so-called commutation functions (Gerber, 1997). I will immediately generalize these commutation functions for all projection years p. First consider the so-called *discounted number of survivors,* discounted m years, for projection year p and current age x, denoted as $D_{x,p,m}$, obtained by discounting l_{x+p+m}:

$$D_{x,p,m} = d_{p,m} l_{x+p+m}. \qquad [5.14]$$

Next $N_{x,p,m}$ is defined as follows:

$$N_{x,p,m} = D_{x,p,m} + D_{x,p,m+1} + D_{x,p,m+2} + \cdots. \qquad [5.15]$$

Note that equation [5.8] can be rewritten as follows (Gerber, 1997):

$${}_p\ddot{a}_x = \sum_{k=0}^{\infty} (d_{p,k} * {}_kp_{x+p}). \qquad [5.16]$$

A result of [5.16] is that ${}_p\ddot{a}_x$ can be written as a function of only $D_{x,p,0}$ and $N_{x,p,0}$:

$${}_p\ddot{a}_x = \sum_{k=0}^{\infty} (d_{p,k} * {}_kp_{x+p}) = \frac{d_{p,0} l_{x+p} + d_{p,1} l_{x+p+1} + d_{p,2} l_{x+p+2} + \cdots}{l_{x+p}} =$$
$$\frac{d_{p,0} d_{p,0} l_{x+p} + d_{p,0} d_{p,1} l_{x+p+1} + d_{p,0} d_{p,2} l_{x+p+2} + \cdots}{d_{p,0} l_{x+p}} = \frac{N_{x,p,0}}{D_{x,p,0}}, \qquad [5.17]$$

since $d_{p,0} = 1$.

In practice old-age pensions are usually not paid annual, but for example monthly. Therefore the actuarial factor derived above is biased. Although it is relatively easy to obtain the correct actuarial factor for each cash flow, this would increase the number of calculations. In case of monthly payments, twelve times more calculations are required. In practice these calculations are simplified by assuming a single annual payment, which is made halfway through the year. This results in $\bar{N}_{\tau,x}$, defined as follows:

$$\bar{N}_{x,p,m} = \frac{N_{x,p,m} + N_{x+1,p,m}}{2}. \qquad [5.18]$$

Now formula [5.17] can be rewritten for ${}_p\bar{a}_x$ as follows:

$$_p\bar{a}_x = \frac{\bar{N}_{x,p,0}}{D_{x,p,0}}. \quad [5.19]$$

Next to the old-age pension, there is another type of pension that frequently occurs and can lead to a significant amount of pension provision on the balance sheet, namely the widower's pension. A widower's pension is paid to the surviving person to whom the pension plan participant is legally married on the day of the participant's death. Typically the widower's pension is a fraction of the (projected) old-age, e.g. 50 or 70 percent. An important difference between the old-age pension and the widower's pension is that the latter depends on two lives, whereas the old-age pension depends on one. Now consider a joint probability distribution for two lives, x and y, denoted as $_kp_{x:y}$. I will now make a simplifying assumption, namely that the time of death of the two persons, $T(x)$ and $T(y)$, are independent. The expression for the joint probability distribution can then be derived as follows (Gerber, 1997):

$$_kp_{x:y} = \Pr\big(\min\big(T(x), T(y)\big) > k\big) = \Pr(T(x) > k, T(y) > k)$$
$$= \Pr(T(x) > k) * \Pr(T(y) > k) = {}_kp_x * {}_kp_y. \quad [5.20]$$

Based on [5.20] and the single life annuity-due, the *joint-life annuity-due* is introduced. A joint-life annuity-due pays only as long as both x and y are still alive. The current value of this annuity, in projection year p, is denoted as $_p\ddot{a}_{x:y}$ and defined as:

$$_p\ddot{a}_{x:y} = \sum_{k=0}^{\infty}\big(d_{p,k} * {}_kp_{x+p:y+p}\big). \quad [5.21]$$

I am, however, interested in an annuity that starts paying when x dies and stops when y dies. The current value of such an annuity is denoted as $_p\ddot{a}_{x/y}$ and can be defined in terms of $_p\ddot{a}_{x:y}$ and $_p\ddot{a}_y$. These terms are already known, so $_p\ddot{a}_{x/y}$ can be defined as follows:

$$_p\ddot{a}_{x/y} = {}_p\ddot{a}_y - {}_p\ddot{a}_{x:y}. \quad [5.22]$$

It is clear that, from a computational point of view, the widower's pension is much more difficult. Many combinations are possible; consider for example a switch of partner or an accrued widower's pension for a participant who does not have a partner (yet). Especially in the latter case, but sometimes also in general, there are general assumptions made concerning the age and gender of the participant's partners. These assumptions are typically that the participant has a gender that differs from his or her partner and a standard age difference of three years between men and women. Note that if these general assumptions are applied, the characteristics of the widower's pension depend on the characteristics of the participant only.

These general assumptions are also important in case the widower's pension must be exchangeable for old-age pension (e.g. in The Netherlands). This means that, at retirement, the participant can choose whether he or she wants to exchange the accrued widower's pension for additional old-age pension or vice versa. In case the participant does not have a partner, the choice is obvious. An actuarial factor for widower's pension, will be denoted as NP_x, is formulated as follows if x is already retired, i.e. $x \geq x_{pa}$:

$$NP_x = \begin{cases} \ddot{a}_y - \ddot{a}_{x:y} & \text{when } x \text{ is still alive} \\ \ddot{a}_y & \text{when } x \text{ is already dead} \end{cases}, \quad [5.23]$$

where the fictitious partner of x is denoted as y. Note that in equation [5.23] $p = 0$, and thus omitted, because x is already retired. In case $x < x_{pa}$, the actuarial factor becomes more complicated. Assumptions have to be made regarding the marital frequency of the participants. Similar to mortality tables there exist marital frequency tables, which can be used for these calculations. Such a mortality frequency is denoted as h_x. To obtain the actuarial factor for widower's pension in case $x < x_{pa}$, one can split the calculations in two parts: the actuarial factor for the widower's pension at retirement date discounted to the current point in time and the sum of all one-year risk factors for widower's pension from now up to the retirement date. In mathematical notation this comes down to:

$$NP_x = {}_{x_{pa}-x}p_x d_{x_{pa}-x} \left({}_{x_{pa}-x}\ddot{a}_{y_{pa}} - {}_{x_{pa}-x}\ddot{a}_{x_{pa}:y_{pa}} \right) + \sum_{k=0}^{x_{pa}-x} \left({}_k p_x d_k q_{x+k} h_{x+k} \frac{N_{y,k+1,0}}{D_{y,k,0}} \right). \qquad [5.24]$$

Note that, where one would expect $\bar{N}_{y,k,0}$ from equation [5.17], in equation [5.24] $N_{y,k+1,0}$ is used. The payments of widower's pension however starts at $k+1$ instead of $\bar{k}$. I will omit the derivation of NP_x^p, which is analogous to the derivation of OP_x^p.

In a similar way, one can construct formulas for the current value of different types of pensions. The current value of an orphan's pension is for example similar to that of a widower's pension. Although some of these types of pension are included in the calculation of the DBO and SC later on in this chapter, I will not discuss the derivation of the actuarial factors here. For more information on this subject I refer to Gerber (1997), although Gerber uses a single discount rate, his book discusses many types of pensions and other life insurance types.

5.2 Actuarial assumptions and the pension plan

As discussed in Section 2.6, several assumptions have to be made to calculate the DBO for a certain pension plan. In this section I will discuss the assumptions I will make for the remainder of this chapter, unless stated otherwise. Next to the assumptions from Section 2.6, I will make assumptions concerning the (fictitious) pension plan participants and the pension plan itself. These assumptions will be based on information from, among others, the Dutch Central Bank (De Nederlandsche Bank (DNB)) and Statistics Netherlands (Centraal Bureau voor de Statistiek (CBS)). I will start this section with the financial assumptions discussed in Section 2.6.

The discount rate I use in the calculations is based on the corporate bond yield curve as at May 31, 2010 and obtained using the method I described in Chapter 4. For comparison I will also use several single discount rates, which give results close the ones using the full yield curve. More information on this specific yield curve, such as the parameter values for the Svensson model, can be found in Appendix B.

Some important assumptions that are related to each other are price inflation, indexation of pension entitlements, yearly wage increases and the yearly increase of the social security offset. Although the Consumer Price Index (CPI), calculated by the CBS, over 2009 was only 1.2 percent in The Netherlands, I assume the average future price inflation to be 2 percent, which is the target of the European Central Bank (ECB). I directly link the yearly increase of the social security offset, directly linked to the state pension (AOW), to this inflation. Due to this inflation, the real value of the pension entitlements decreases over time. To correct for the erosion of pension entitlements due to inflation a, usually conditional, indexation on pension entitlements is given in The Netherlands. The indexation is typically based on the CPI or wage inflation. in case of active participants. and conditional on the funding ratio of the pension fund. Indexation is not incorporated in my calculations since it makes it harder to interpret the results. The assumption concerning the yearly wage increase is usually split in two parts: a general wage increase and an individual wage increase depending on the age of the participant. These numbers can be based on company specific data, which is however not always available. Therefore I will be using an often used, general table for the individual wage increases, specified in Appendix A. The first part, the general wage increase, is set equal to the price inflation, i.e. 2 percent.

For the projections of the future service of the plan participants in the PUCM, which I described in Section 2.7, assumptions concerning turnover and disabilities need to be made. Again both these rates can differ widely between companies and often there is no data available to estimate these rates. Therefore a standard set of rates exists for both turnover and disability, depending on the age of the participant. These standard sets are used very often and I will use them in my calculations too.

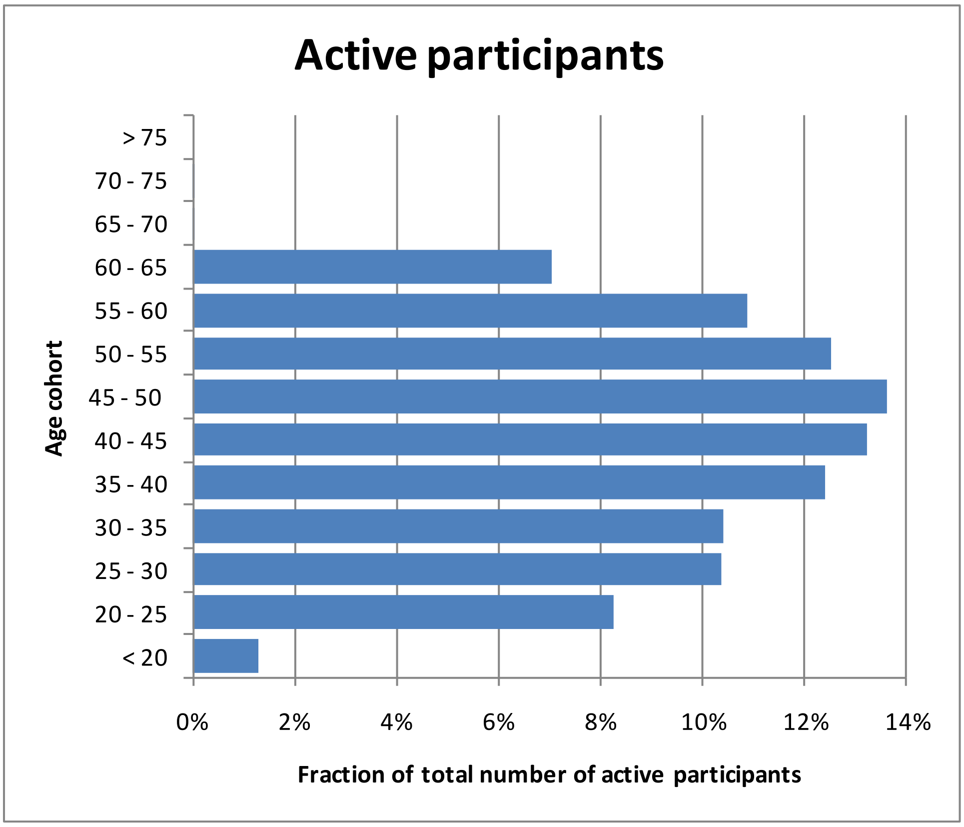
The accrual of pension entitlement is typically based on the average salary minus the social security offset, which is called the pensionable base. This means that I will need to make assumptions concerning the salaries of the participants, the social security offset and the pension accrual percentage. I will base my salary estimate on the Dutch standard income, published by the Netherlands Bureau for Economic Policy Analysis (Centraal Planbureau (CPB, 2010)), which is equal to 32,500 Euros for 2010. Since I will discuss the impact on DBO in terms of percentages, the exact height of the salaries is not that important. Concerning the social security offset and the accrual percentage, I assume the most common figures according to DNB. The latter comes down to an average pay scheme with an accrual percentage of 2 percent. The widower's pension accrual percentage is assumed to be 1.4 percent.

Next I will make some demographic assumptions. The most important demographic assumption is the mortality table that is to be used. In general there are two types of mortality tables: the 'ordinary' mortality table and a projection table. A projection table consists of multiple mortality tables, one for every year of birth. It takes into account (expected) future development of mortality rates, whereas an ordinary mortality table only consists of the current mortality rates. The latter might be corrected by applying age corrections, i.e. assuming participants to be 1 or more years younger than they really are. For more information on mortality tables I refer to Bouwman (2008), who also described the effects of projections tables on accounting Defined Benefit plans under IAS 19. Due to the multiple mortality tables involved in a projection table, a projection table requires more computational power than an ordinary mortality table. Since the calculations using the full yield curve already require a lot of computational power, I will use an ordinary mortality table from the Dutch Actuarial Society (Actuarieel Genootschap (AG)) with an age correction factor of -1 for men and -2 for women. Furthermore, assumptions concerning marriage frequencies have to be made. Again I will use a recent table from the AG. Furthermore I will assume the partner to have a different gender than the participant.

Since I want to create a fictitious pension plan, I need to create fictitious participants. Next to the salary, which I discussed above, the three most important characteristics are age, gender and the number of past service years. Regarding to the age of the participants I used a distribution similar to the total distribution of participant in Dutch pension funds, which is given in Figure 8 (DNB, 2010).

Additionally I assume the distribution to be uniform within an age cohort and the minimum age for entering the pension plan to be 18. Concerning the gender of the participants, I will distinguish three different sets of plan participants, namely a plan with only male participants, a plan with only female participants and a mixed plan with 50 percent male and 50 percent female participants. The number past service years obviously depends, among others, on the age of the participant, but many assumptions are possible. I assume that the number of past service years is equal to the age of the participant, minus the minimum entering age, divided by 2.

Figure 8: The distribution of active participants within Dutch pension funds (DNB, 2010).



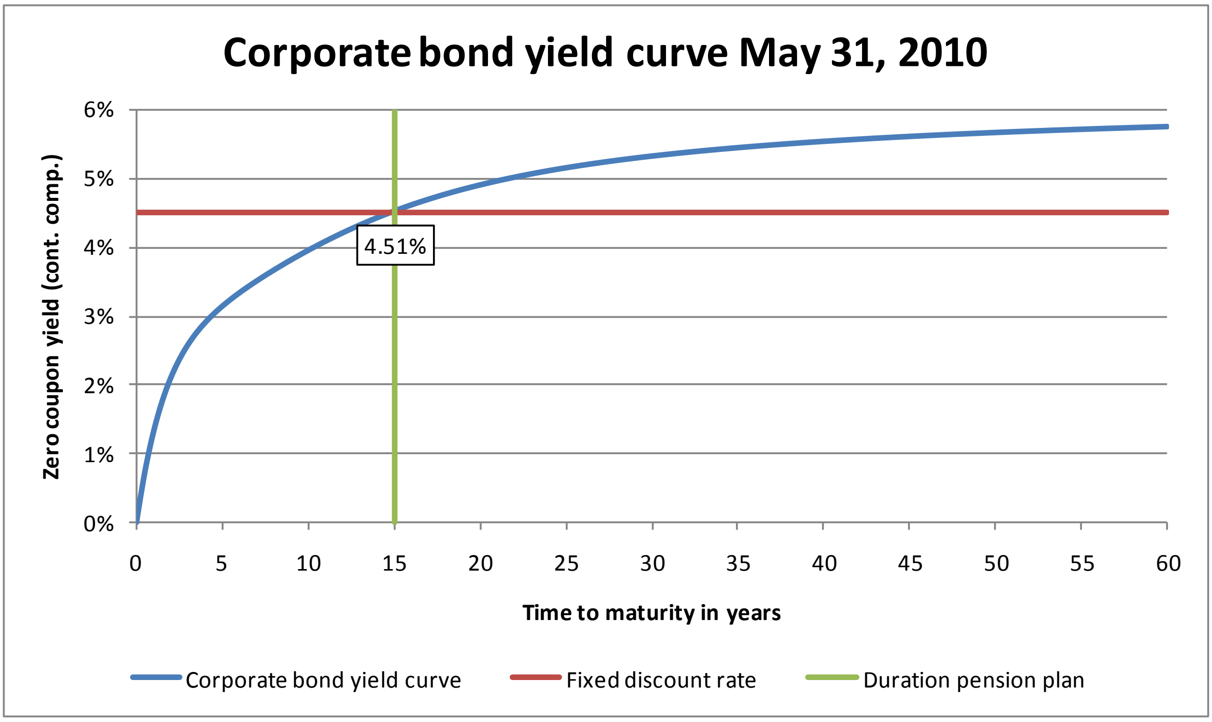
5.3 Impact of the yield curve on actuarial factors

The first effect of applying the full yield curve I will look into is the impact on the actuarial factor, discussed in Section 5.1. I will compare the actuarial factors that are calculated using the full yield curve with the ones that are calculated using a fixed discount rate.

To interpret the impact on the actuarial factors, I will first have a look at the shape of the yield curve as at May 31, 2010 itself. Figure 9 shows that the yield curve is very steep at the beginning of the curve. The (very) short yields in the May 31, 2010 yield curve are really low (around 1 percent for the 1 year rate), whereas the longer rates converge to a stable long rate of around 6 percent. Now assume the duration of a certain pension plan is around 15, which means that in case a fixed discount rate is used, one would use the 15-year rate from the yield curve. This is also shown in Figure 9. Since the yield curve is increasing over the complete maturity spectrum, the rates for cash flows that occur more than 15 years from now are higher than the fixed rate. This implicates that the corresponding discount factor, and thus the corresponding actuarial factor, will be lower. The opposite holds for cash flows that occur less than 15 years from now.

Since the cash flows for young (< 50) participants will only occur after 15 years or more from now, it is expected that the actuarial factor for these participants based on the full yield curve will be lower than the one based on a fixed discount rate. Again, the opposite holds for the older participants.

Figure 9: The high quality corporate bond yield curve, compared to the fixed discount rate of a pension plan with duration 15.



I will now first have a look at the actuarial factors itself. In Figure 10 and Figure 11 I compared respectively the male and female OP_x factors using the full yield curve with the OP_x factors based on a range of fixed discount rates.

Figure 10 and 11 show that the differences are almost equal for male and female OP_x factors. The difference between these two graphs is that the latter is slightly shifted to the right compared to the first. This is caused by the higher life expectancy for women. The graphs also show that the impact is often largest for the younger ages, which can be explained by the fact that the duration of pension liabilities are much larger for young participant than they are for older participants. In case the fixed discount rate is equal to 6 percent (annually compounded the impact for the youngest ages is relatively small because that rate is close to the longer rates on the curve (see also Figure 9).

Figure 10: Comparison of the male OP_x factors, the actuarial factors for an old age pension starting at 65. The graph shows the difference between the factor based on the full yield curve and the factor based on four different fixed discount rates in terms of percentages of the factor based on the corresponding fixed discount rate.

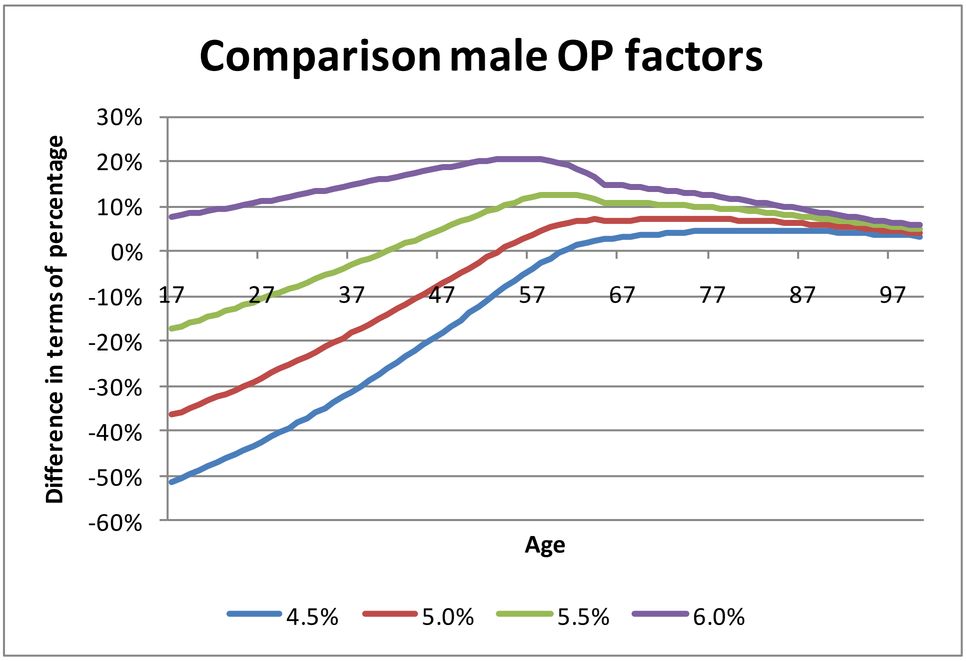
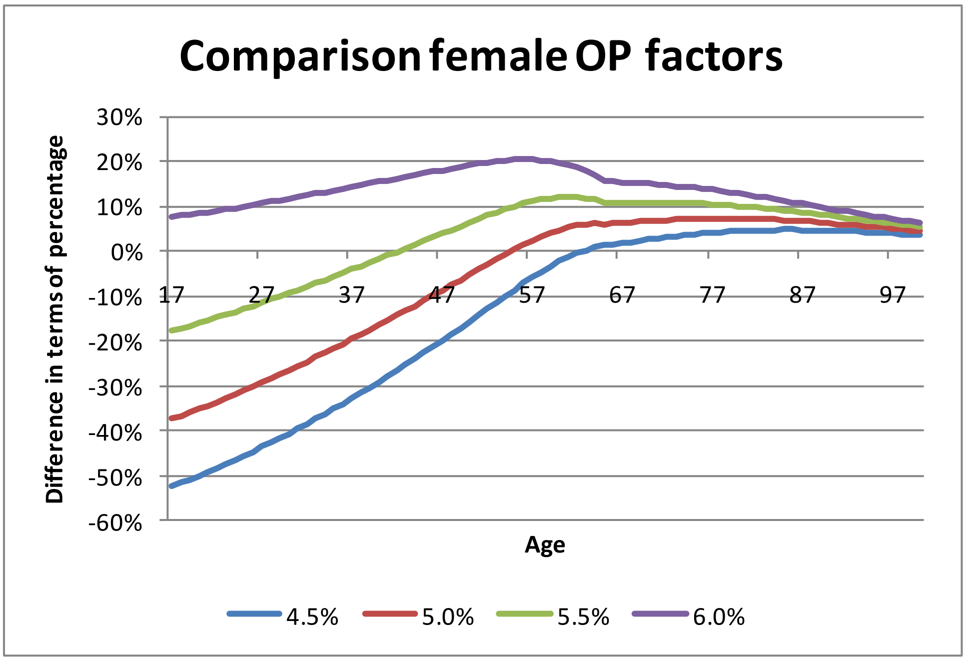


Figure 11: Comparison of the female OP_x factors, the actuarial factors for an old age pension starting at 65. The graph shows the difference between the factor based on the full yield curve and the factor based on four different fixed discount rates in terms of percentages of the factor based on the corresponding fixed discount rate.



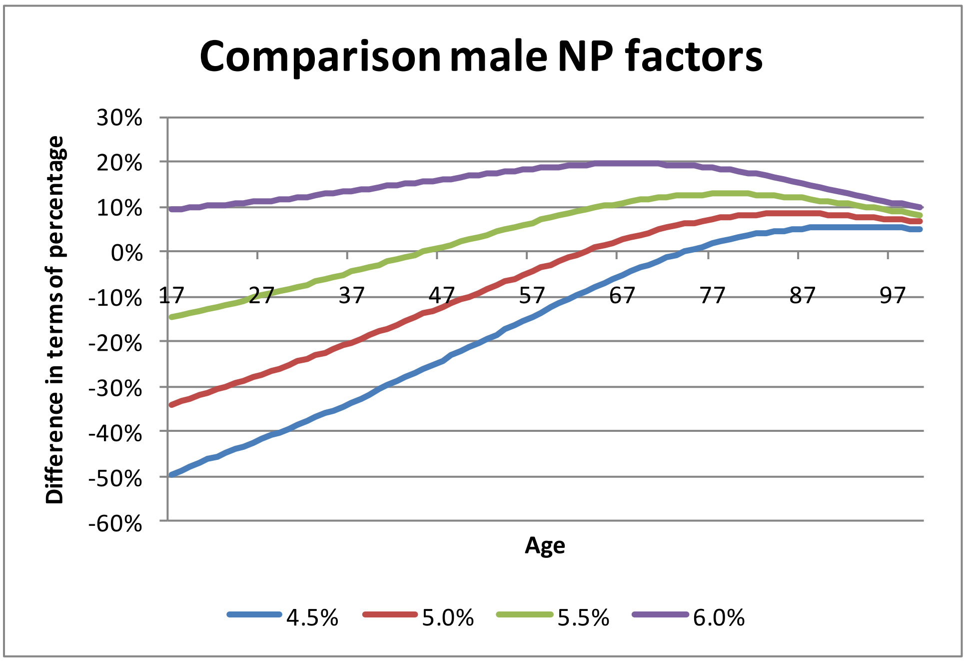
Another notable effect is that the factors at the very old ages (> 90) are positive and around 4 to 8 percent. This is caused by the low yields for the shorter maturities (around 1 to 2 percent) in the yield curve as at May 31, 2010, which are much lower than the fixed rates, which increases the actuarial factor. Since at those ages not many pension payments are expected anymore, the difference for those ages is not too large. The latter also explains why the differences seem to converge at the older ages.

The kink in the graph around age 65 is also caused by the steepness of the yield curve at the shorter maturities. These yields for shorter maturities start to play a role when a person's age gets close to retirement, because the pension payments are then in the near future. Due to the relatively low yields at the shorter maturities, the 'average discount rate' for the pension payments decreases rapidly, which has a positive impact on the actuarial factor. This effect is most obvious in the difference with the high 6 percent fixed discount rate. This difference decreases rapidly as age increases around age 60 because the duration decreases, around 65 however, the low yields kick in, which moderates this decrease in difference.

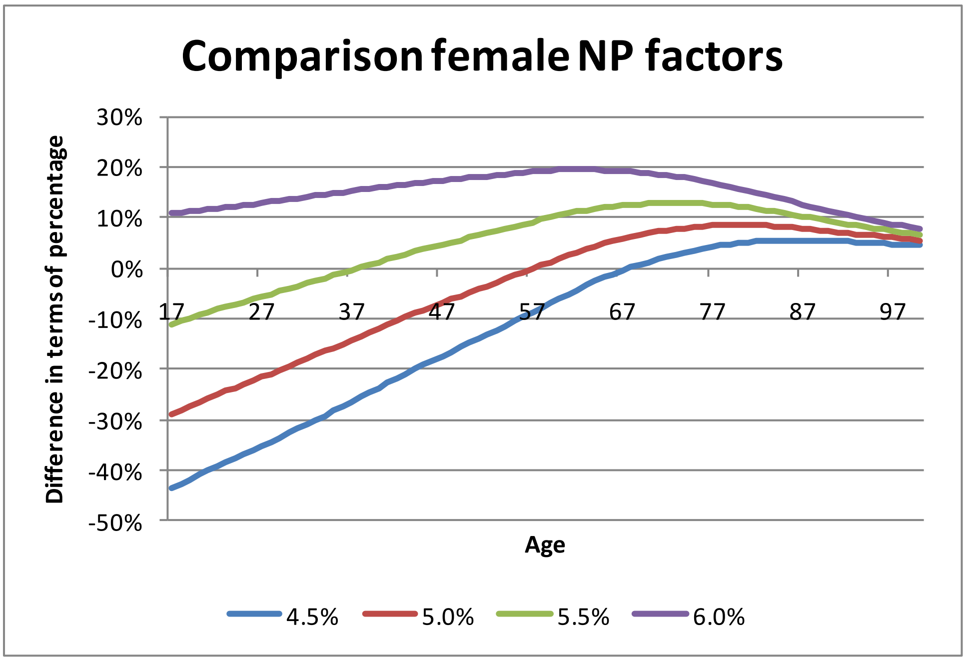
The impact on the NP_x factors is less straightforward since they depend on two lives. The impact for respectively men and women is shown in Figures 12 and 13. As with the OP_x factors, the graphs for men and women look very similar. In the case of NP_x factors however, the differences in female factors are slightly shifted to the left compared to the differences in the male factors, whereas they shifted to the right in the OP_x factors case. Next to the opposite direction, the difference between the male and female graphs is also larger than in the OP_x factors case. This larger shift to the left has three causes:

- The assumption that the participant and partner have the opposite gender;
- The assumption that there is an age difference of 3 years between a man and a woman in a relationship; and
- Again the higher life expectancy for women.

Figure 12: Comparison of the male NP_x factors, the actuarial factors for a widower's pension. The graph shows the difference between the factor based on the full yield curve and the factor based on four different fixed discount rates in terms of percentages of the factor based on the corresponding fixed discount rate.



Figuur 13: Comparison of the female NP_x factors, the actuarial factors for a widower's pension. The graph shows the difference between the factor based on the full yield curve and the factor based on four different fixed discount rates in terms of percentages of the factor based on the corresponding fixed discount rate.

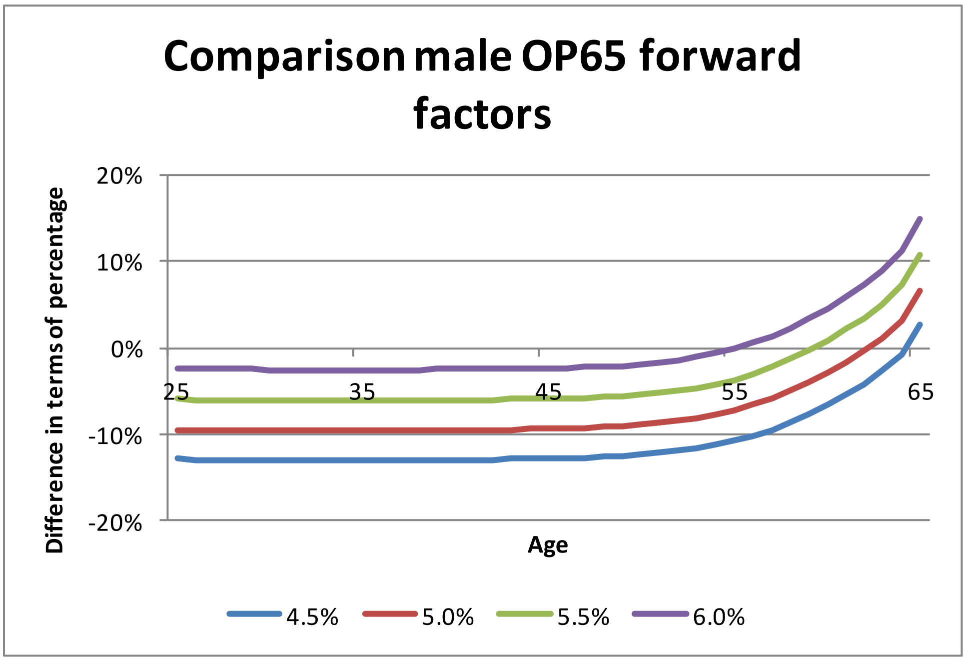


The impact of using the yield, which is the point of interest for this thesis, is again similar for male and female actuarial factors. Comparing Figures 12 and 13 to Figures 10 and 11, the overall impact on the NP_x factors looks very similar to the impact on the OP_x factors. Again the differences more or less converge as age increases, due to the fact that the duration decreases. The kink I observed in Figures 10 and 11 disappeared in Figures 12 and 13. The disappearance of the kink can be explained by the fact that the pension payments do not start at a specific age, they start when the participant dies. This has a smoothening effect on the differences displayed in the graphs.

There is an important difference between actuarial factors based on a fixed discount rate and actuarial factors based on a full yield curve. In case of a fixed discount curve, the OP_x factor does not depend on the current age x of the participant, i.e. the OP_{65} factor is equal to, for example, OP_{25}^{40} or OP_{35}^{30}. In case of using a full yield curve, forward rates will be applied in determining the latter, which will cause differences unless the yield curve is completely flat. In Figure 14 I compared the male OP_x^p factors for different x and p such that $x + p = 65$. Note that when a fixed discount rate is used, all OP_x^p factors for which $x + p = 65$ are equal, which explains the parallel lines in Figure 14.

The graph in Figure 14 also shows that the factors based on the full yield curve stay approximately constant until age 50, at which point they start to increase. This effect can be explained by the shape of the yield curve. The yield curve is relatively flat at the long end, which causes stable factors for the younger ages. Due to the up sloping yield curve, the forward rates lie above the spot rates and seem to be close to 6 percent (see Appendix B). From age 50 on, p gets below 15 and the yields for the shorter maturities start to play a role causing the forward rates to drop. These dropping forward rates cause the factors to increases and since the factor based on the fixed discount rate is constant, the difference also increases.

Figure 14: Comparison of male OP_x^p factors for different x (ages) and p such that $x + p = 65$. The graph shows the difference between the factor based on the full yield curve and the factor based on four different fixed discount rates in terms of percentages of the factor based on the corresponding fixed discount rate.



Note that, since the impact is very similar, I do not describe the impact on the female and widower's pension factors.

5.4 Impact of the yield curve on Defined Benefit Obligation and Service Cost

To calculate the impact of using the full yield curve on the DBO and the SC, I have to distinguish between active participants and inactive participants. For an active participant, the DBO is equal to the number of years of past service times the SC, which I showed in Figure 4. Since the DBO for a plan is the sum of the DBOs of the participants the impact in terms of percentages will be exactly equal for the DBO and the SC. The impact on the DBO of inactive participants however, is different. In case of an inactive participant, there is no (current) SC, since it is defined as cost "resulting from employee service in the current period" (IAS 19). Since an inactive participant is not in service, there is no SC. The DBO of an inactive participant is calculated as the accrued pension entitlements time the corresponding actuarial factor, e.g. OP_x in case of old-age pension. The impact on the DBO of inactive participant, in terms of percentages, will therefore be equal to the impact discussed in Section 5.3 and shown in Figures 10 to 13. In the remainder of this section, I will only discuss the impact on the DBO and SC for active participants.

In practice, most participants have already accrued some pension entitlements and will also accrue pension entitlements in the future. There is however a difference between already accrued pension entitlements, based on past service years, and pension entitlements that will be accrued in the future. To show the difference, I will first investigate the case where a participant without pension entitlements, but who will accrue pension entitlements in the future based on a yearly salary of €32,500. Second I will investigate the case where a participant has already accrued old-age pension and widower's pension in a 100:70 ratio, but who has a salary of €0.

Figure 15: Comparison of the DBO of a male participant, age x, without accrued pension entitlements and a yearly salary of €32,500. The graph shows the difference between the DBO based on the full yield curve and the DBO based on four different fixed discount rates in terms of percentages of the DBO based on the corresponding fixed discount rate.

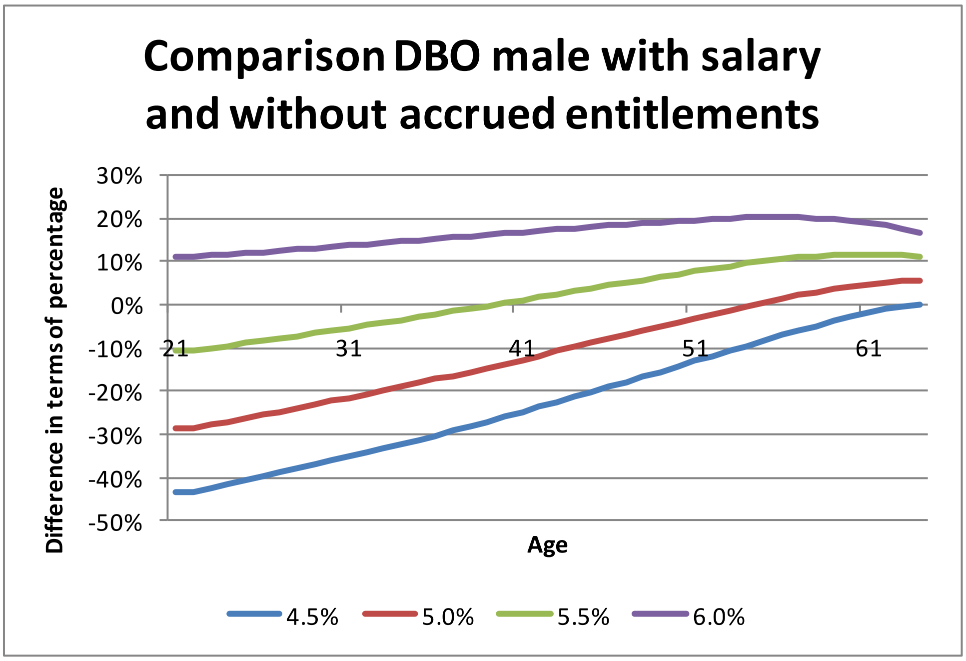
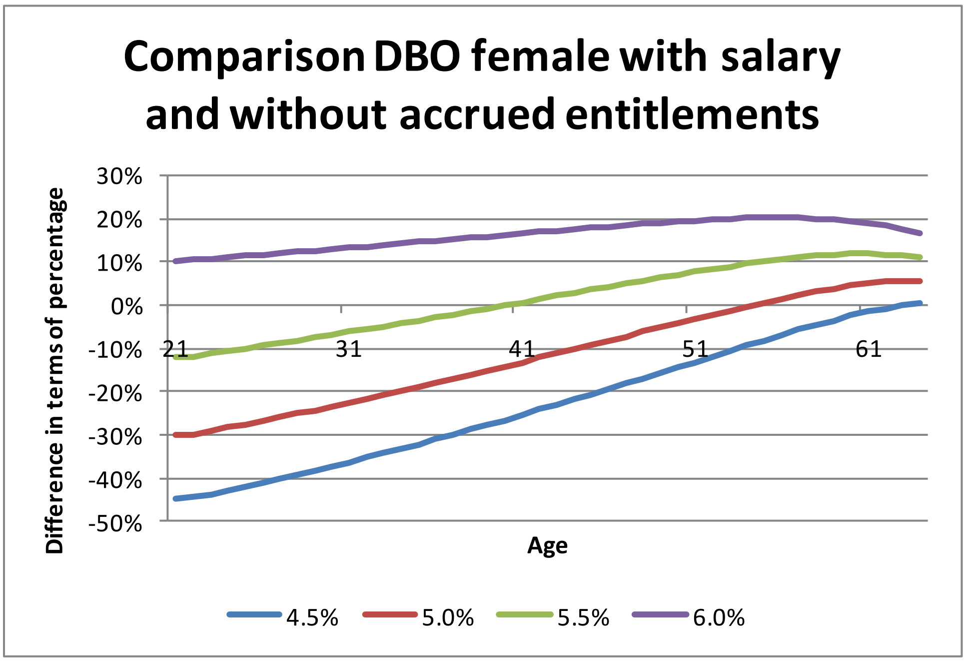


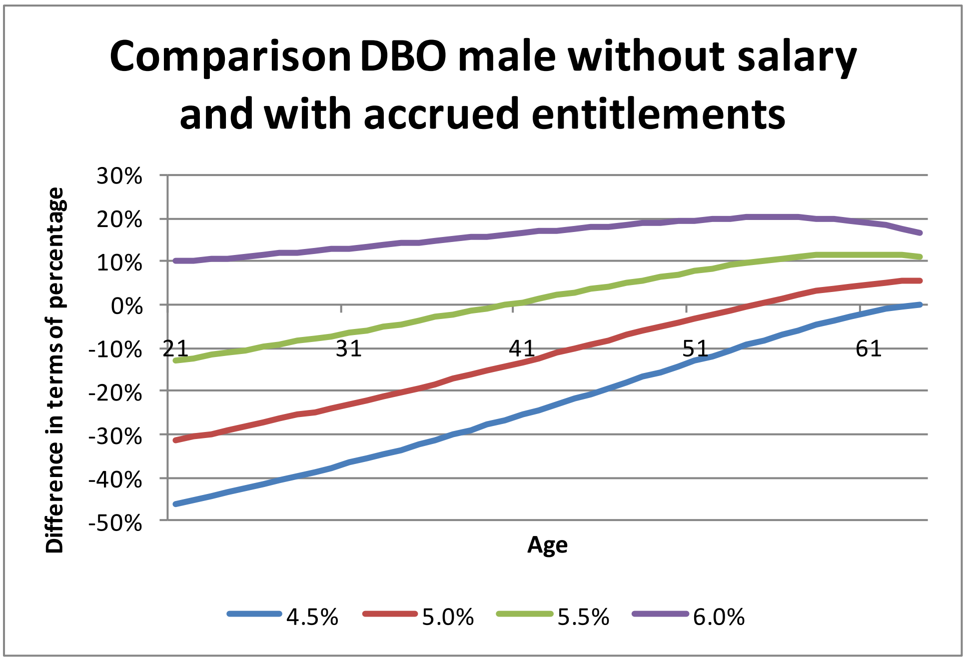
Figure 16: Comparison of the DBO of a female participant, age x, without accrued pension entitlements and a yearly salary of €32,500. The graph shows the difference between the DBO based on the full yield curve and the DBO based on four different fixed discount rates in terms of percentages of the DBO based on the corresponding fixed discount rate.



In Figure 15 and Figure 16, the impact of using the full yield curve on the DBO for a participant without accrued pension entitlements and a salary of €32,500 is shown. The graph shows that for a 41 year old the DBO based on a fixed discount rate of 5.5 percent is close to the DBO based on the full yield curve. For a 54 year old this holds for a fixed discount rate of 5.0 percent and for a 63 year old even 4.5 percent. This has obviously a lot to do with the current shape of the yield curve. As mentioned before, the yields for the shorter maturities are (much) lower than those for the longer maturities. The graph also shows that if one chooses for a fixed discount rate, the choice should heavily depend on the composition of the plan participants. Figure 15 and Figure 16 also show that there is no noteworthy difference between the impact on the DBO of male and female participants.

Although there is a difference in calculating the DBO for already accrued pension entitlements and the DBO of future accrual of pension entitlements, the impact of using the full yield curve is the same. This is shown in Figure 17, which shows the differences for a male participant with accrued pension entitlements, but without salary and thus no future accrual. I will omit the graph for the female participant, since there is again no noteworthy difference between male and female participants.

Figure 17: Comparison of the DBO of a male participant, age x, with accrued pension entitlements and a yearly salary of €0. The graph shows the difference between the DBO based on the full yield curve and the DBO based on four different fixed discount rates in terms of percentages of the DBO based on the corresponding fixed discount rate.



As mentioned in Section 5.2, I also constructed a set of plan participant that reflects the pension plans in The Netherlands. As discussed in Section 5.2, I choose to let the number of past service years depend on the age of the participant. This means that in my participant set, the older participants have more accrued pension entitlements than the younger ones. This means that the DBO for older participants will be higher, which means that they will have a larger impact on the total DBO. This will also mean that the differences will not be equal for the DBO and the SC. As mentioned before, I will omit inactive participants, since they only influence the DBO and not the SC.

I calculated the DBO and SC for three different portfolios of participants, one with only male participants, one with only female participants and one with 50 percent male and 50 percent female participants. The results are shown in Table 8. The table shows that, as expected, the impact on the SC differs from the impact on the DBO. As I mentioned before, the older participants have a relatively high influence on the DBO, which results in a smaller impact on the DBO, relative to the impact on the SC.

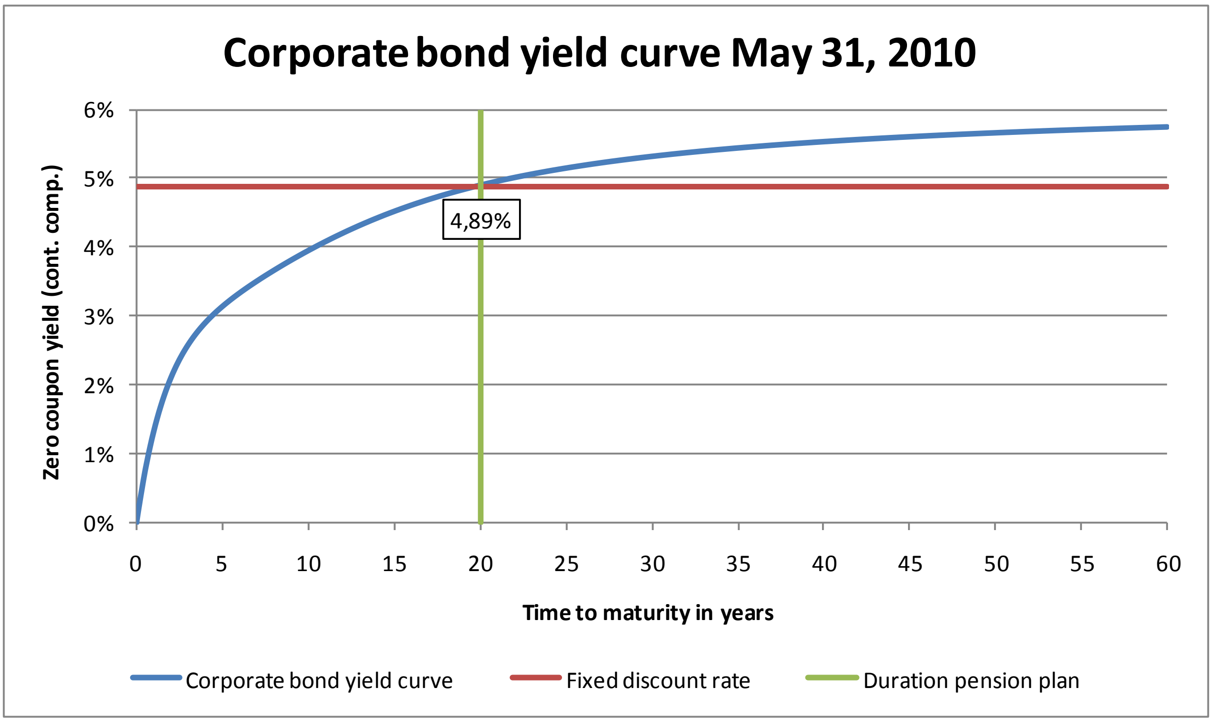
Table 8: Comparison of the DBO, SC and duration of a portfolio of 500 active participants, which reflects the pension plans in The Netherlands. There are three portfolios, one with only male participants, one with only female participants and one with 50 percent females and 50 percent males. The table denotes the difference between the DBO or SC based on the full yield curve and the DBO or SC based on a fixed discount rate. The difference is given in terms of percentage, relative to the DBO or SC based on the corresponding fixed discount rate.

Male Portfolio					
	Curve	4.5%	5.0%	5.5%	6.0%
Difference in DBO	-	-13.4%	-3.6%	6.9%	18.1%
Difference in SC	-	-18.3%	-7.4%	4.5%	17.5%
Duration	19.4	21.4	20.7	20.0	19.4

Female Portfolio					
	Curve	4.5%	5.0%	5.5%	6.0%
Difference in DBO	-	-13.0%	-3.4%	7.0%	18.1%
Difference in SC	-	-18.1%	-7.2%	4.6%	17.5%
Duration	19.2	21.1	20.5	19.8	19.2

Male/Female Portfolio					
	Curve	4.5%	5.0%	5.5%	6.0%
Difference in DBO	-	-13.2%	-3.5%	6.9%	18.1%
Difference in SC	-	-18.2%	-7.3%	4.6%	17.5%
Duration	19.3	21.3	20.6	19.9	19.3

Figure 18: The corporate bond yield curve, compared to the fixed discount rate of a pension plan with duration 20.



The differences are the smallest for the 5.0 and 5.5 percent fixed discount rate. Taking into account the duration of the mixed pension plan of around 20, the yield curve suggests a discount rate of around 5.0 percent annually compounded (approximately 4.89 percent continuously compounded, see also Appendix B), as is shown in Figure 18. Using a 5.0 percent fixed discount rate, however, results in a significant difference in DBO and especially SC, as Table 8 shows. Using a fixed discount rate of 5.2 percent reduces the difference in DBO for the mixed portfolio to 0.6 percent, but still leaves a difference in SC of -2.7 percent. The difference in SC is smallest, -0.3 percent, for a fixed discount rate of 5.3 percent, but leaves a difference of 2.7 percent in DBO. This means that it is hard to approximate the results using a full yield curve with a fixed discount rate, due to the different impact on DBO and SC. What Table 8 also shows is the large impact of the discount rate on the DBO. In practice discount rates for the same maturity often differs tithes of a percent, sometimes even 0.5 percent. Table 8 indicates that the impact on the DBO can then easily be around 10 percent. This is also the impact one would expect based on duration analysis: 0.5% x 20 = 10%.

In Section 4.5 the longer maturities were discussed. Two different methods were suggested to deal with the continuously up sloping yield curve. One method put a constrained on the β_0 parameter, which is the yield if the maturity goes to infinity. The other method keeps the yield constant from a certain maturity, e.g. the average of the five longest maturities in the data set, on. I constructed these two methods for the yield curve as at May 31, 2010. I put the β_0 constrained at 5.1 percent (continuously compounded, 5.23 percent annually compounded), which is equal to the yield corresponding to a maturity of 24 years. The maturity of 24 years is equal to the average maturity of the five longest bonds in the data set at May 31, 2010. This is also the maturity from which the second method keeps the yield constant. These two alternatives of the standard yield curve are illustrated in Appendix B. The comparison of the results of the IAS 19 calculations is shown in Table 9.

Table 9: Comparison of the DBO, SC and duration of a portfolio of 500 active participants, which reflects the pension plans in The Netherlands. There are three portfolios, one with only male participants, one with only female participants and one with 50 percent females and 50 percent males. The table denotes the difference between the DBO or SC based on the standard full yield curve and two alternatives which deal with the longer maturities. The first alternative is the yield curve with a constrained on the β_0 parameter, the second keeps the yield curve constant, i.e. flattens the yield curve, from maturity 24 on. The difference is given in terms of percentage, relative to the DBO or SC based on the corresponding alternative yield curve.

Male Portfolio			
	Curve	Constrained	Flattened
Difference in DBO	-	-10.0%	-3.5%
Difference in SC	-	-12.9%	-5.7%
Duration	19.4	20.6	20.0

Female Portfolio			
	Curve	Constrained	Flattened
Difference in DBO	-	-9.8%	-3.3%
Difference in SC	-	-12.8%	-5.5%
Duration	19.2	20.3	19.8

Male/Female Portfolio			
	Curve	Constrained	Flattened
Difference in DBO	-	-9.9%	-3.4%
Difference in SC	-	-12.9%	-5.6%
Duration	19.3	20.4	19.9

Table 9 shows that the impact of these alternatives is large, especially for the constrained alternative. This shows that the choice for an alternative is very important.

5.5 Impact of the yield curve on Employer Pension Expense

Whereas the impact on the net liability or asset on the balance sheet, according to Table 1, is quite clear, the impact on the EPE, and thus the P&L, is less clear. The EPE depends on the SC, the IC, the Expected return on Plan Assets and the Amortization of actuarial gains or losses. Especially interesting is the calculation of the Interest Cost. According to IAS 19, the Interest Cost is defined as: "the increase during a period in the present value of a defined benefit obligation which arises because the benefits are one period closer to settlement". In Section 2.3 I argued that the I is obtained by multiplying the DBO plus the SC with the discount rate. The question arises which discount rate to use when applying the full yield curve?

Assuming the 'period' mentioned in IAS 19 to be 1 year, which it usually is, one could argue that it is the 1-year yield that should be used. Given the very low 1-year yield at this time (1.3 percent), this would decrease the IC enormously. The future cash flows however, except the ones next year, are discounted using a much higher yield, so intuitively using the 1-year yield is not right. Another option would be to use a sort of average discount rate. To obtain such a discount rate, the DBO is recalculated using a fixed discount rates. The fixed discount rate that results in a DBO that is closest to the DBO based on the full yield curve then should be used to calculate the IC. In the example of the mixed portfolio in Section 5.4 this method would result in a discount rate of 5.2 percent. This approximation might also be used for disclosure under IAS 19, since disclosing the complete yield curve may be problematic. An comparison of the two options is given in Table 10.

Table 10: Comparison of the IC as a percentage of the DBO plus SC, obtained using two different methods: the IC is equal to the 1-year annualized yield from the high quality corporate bond yield curve and the IC is based on a fixed discount rate that approximates the DBO based on the full yield curve. The IC of these two methods is based on the mixed portfolio from Section 5.4 and the high quality corporate bond yield curve as at May 31, 2010.

	IC equal to 1-year yield	**IC based on 'average' yield**
IC in percentages of the DBO + SC	1.3%	5.2%

Table 10 shows that there is a large difference between the two methods. A result of using the first two methods is that, with an up sloping yield curve, the IC, ceteris paribus, will increase every year. The question is whether an increasing IC is a desirable result from an accounting perspective. One could argue that the IC, just like the SC, should be spread equally over the periods an employee is in service. In that case, the 'average' yield would be more appropriate. This method, however, can also be seen as too prudent. Besides that, it will also generate actuarial gains every period, which is not desirable.

6. Conclusions and recommendations for further research

The research question in this thesis is twofold. The first question concerns the development of a standardized construction of a yield curve, based on market prices. In Chapter 3, different types of yield curve models were investigated and compared. Based on four criteria, the Svensson yield curve model was chosen. The Svensson model is able to describe yield curves in many different economic situations and provides smooth yield curves. Combined with the tractability, due to a single function for the complete yield curve, the Svensson model is an appropriate model to construct the yield curve.

During the construction of the yield curve, I came across a few issues. First a choice had to be made whether to estimate the yield curve by fitting the yields or by fitting the bond prices. Since discount factors, and thus yields are needed for the IAS calculations, fitting the yields would be best. However, that requires a great deal of computational power and the necessary data is not (always) available. Fitting bond prices is much easier and using the Bliss method, described in Section 4.3, the result is approximately similar to fitting the yields. A second problem was to determine what high quality bonds are, or more specifically: should AA- bonds be included? I conclude that they should be included because rating agencies describe AA- bonds as 'high quality'. Furthermore these AA- bonds extend the sample, which improves estimation results. A third problem is the absence of (high quality) corporate bonds with long maturities, which makes it impossible to estimate the long end of the yield curve appropriately. Several extrapolation methods are used in practice and there is no general best method. I suggested two methods, in one method the curve becomes flat from a certain maturity on, in the other method a constraint is put on the parameter in the Svensson model that determines the yield for the very long maturities.

What eventually matters is of course the outcomes of the IAS 19 calculations and the impact of the yield curve on these outcomes. To investigate the effect I introduced the concept of actuarial factors, which are used in the IAS 19 calculations. Before examining the impact on the outcomes of the IAS 19 calculations, based on the yield curve as at May 31, 2010, I examined the impact on the actuarial factors. This showed that the actuarial factors, based on the full yield curve, for younger participants are more or less similar to the actuarial factors based on the higher, i.e. 5.5 or 6.0 percent, fixed discount rates. The actuarial factors for older participants however, showed similarities with lower, i.e. 4.5 or 5.0 percent, fixed discount rates I investigated. These facts showed that if one uses a fixed discount rate, this rate should heavily depend on the composition of the pension plan. I did not find any significant differences for male and female participants.

The comparison of the outcomes of the IAS 19 calculations is split op in a comparison of the DBO and SC for an individual participant and a comparison for a representative pension plan. The impact of the high quality bond yield curve on the DBO and SC of the individual participants is quite similar to the impact on the actuarial factors. For young participants the difference between the DBO or SC based on the full yield curve and the DBO or SC based on the higher fixed discount rates, i.e. 5.5 or 6.0 percent, was smaller. For the older participants the difference was smaller for the lower, i.e. 4.5 or 5.0 percent, fixed discount rates.

A maybe more interesting comparison in DBO and SC is that of a representative pension plan, which composition is based on the composition of all pension plans in The Netherlands with respect to the age of the participants. Using this composition, three fictitious pension plans were created, one with only male participants, one with only female participants and one with an equal number of male and female participants. The results of the comparison showed that the difference in DBO based on the full yield curve and 5.2 percent fixed discount rate is the smallest. The difference in SC, however, was smallest when a fixed discount rate of 5.3 percent is used. This showed that it is hard to approximate the results using the full yield curve with a fixed discount rate. Again the difference between male and female participants turned out to be small.

An issue when using the full yield curve under IAS 19 is how to determine the IC, the cost for getting one year closer to the actual pension payments. In case a fixed discount rate is used, the IC is simply the discount rate multiplied with the sum of the DBO and SC. Mathematically one could argue that the 1-year yield should be used to calculate the IC. This would lower the IC dramatically, due to the low 1-year yield nowadays. This would however also imply a rising IC over the future periods. If it is the aim to spread the IC equally over the periods, using the 1-year yield might not be the right method. In that case, the aforementioned approximation of the DBO with a fixed discount rate could be used. The fixed discount rate that results in a DBO closest to the DBO based on the full yield curve then can be used to calculate the IC. Both methods have their advantages and disadvantages, in my opinion there is no best method.

One of the aims of this thesis is to show that there is a relatively easy way to construct a high quality corporate bond yield curve. A possible method is shown in Chapter 4, however, there is always room for improvement, especially at the long end of the curve. This thesis also shows that the results of the IAS calculations heavily depend on the assumptions made concerning the discount rate or yield curve. For some other issues, like the issue concerning the IC, future will show which method will be used.

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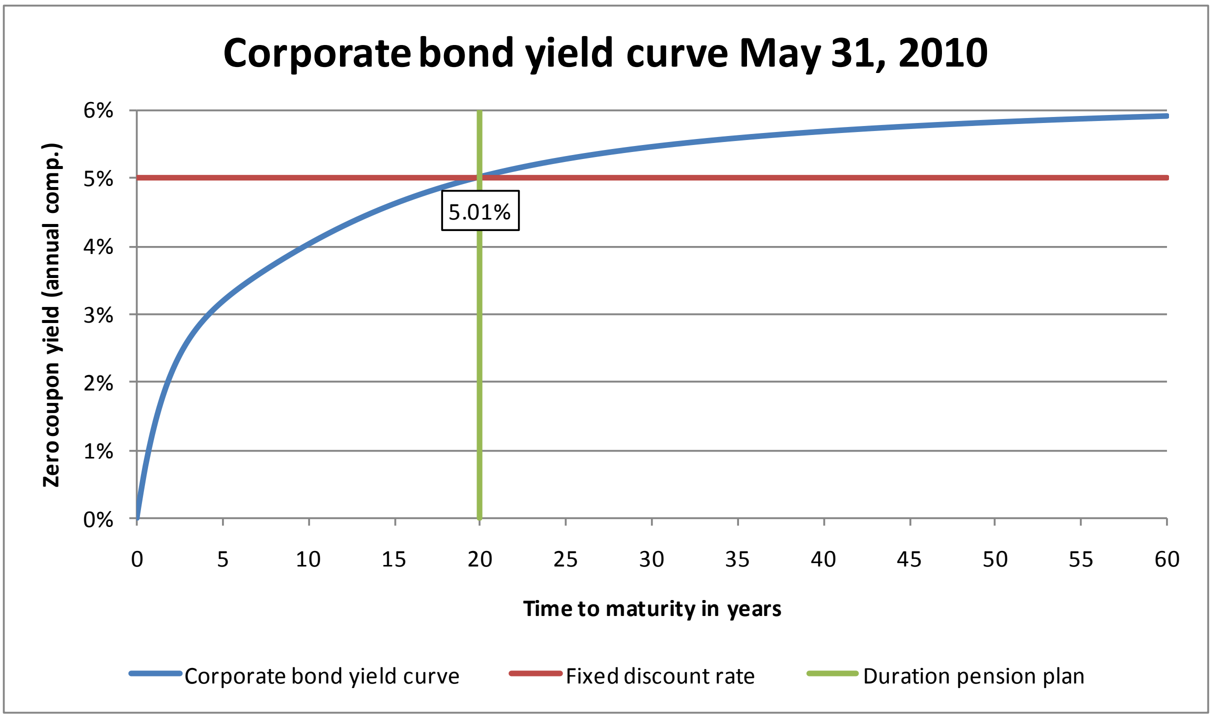
Appendix A: Actuarial assumptions in IAS 19 calculations

Assumptions as used in IAS 19 calculations			
Macro ecomomic assumptions			
Wage inflation			2.00%
Price inflation			2%
Standard salary			€ 32,500
Specific assumptions			
Old-age pension accrual percentage			2%
Widower's pension accrual percentage			1.40%
Indexation accrued pension entitlements			0%
Social security offset			€ 12,465
Type of pension scheme			Average pay
	age	*male*	*female*
Individual salary increase rate	18-24	2.50%	2.50%
	25-29	2.50%	2.50%
	30-34	2.50%	2.50%
	35-39	2.00%	2.00%
	40-44	2.00%	2.00%
	45-49	1.50%	1.50%
	50-54	1.50%	1.50%
	55-59	0.50%	0.50%
	60-64	0.50%	0.50%
Individual turnover rates	18-24	8.00%	8.00%
	25-29	8.00%	8.00%
	30-34	8.00%	8.00%
	35-39	6.00%	6.00%
	40-44	6.00%	6.00%
	45-49	4.00%	4.00%
	50-54	4.00%	4.00%
	55-59	1.00%	1.00%
	60-64	1.00%	1.00%
Disability rates	18-24	0.06%	0.06%
	25-29	0.12%	0.12%
(recovery chances are not taken into account)	30-34	0.17%	0.17%
	35-39	0.22%	0.22%
	40-44	0.28%	0.28%
	45-49	0.35%	0.35%
	50-54	0.45%	0.45%
	55-59	0.57%	0.57%
	60-64	0.72%	0.72%
Other (actuarial) assumptions	Mortality table	2000-2005	
	Marriage frequency	2000-2005	
	Age correction	-1	-2
	Age difference	3	

Appendix B: Corporate Bond Yield Curve as at May 31, 2010

More details on the high quality bond yield curve, applied in Chapter 5, can be found here.

Figure 19: The annually compounding high quality corporate bond yield curve as at May 31, 2010, compared to the fixed discount rate of a pension plan with duration 20.



The May 31, 2010 high quality yield curve follows formula [3.43] with the following parameter values:

Table 11: Svensson parameter values of the high quality bond yield curve as at May 31, 2010. The first column contains the parameter values of the yield curve estimated without an upper limit for β_0. The second column contains the parameter values of the yield curve in case the constraint $\beta_0 < 5.10\%$ is added.

Corporate bond yield curve May 31, 2010		
Parameter	**Value**	
	β_0 unconstrained	**β_0 constrained**
β_0	6.16%	5.10%
β_1	-6.16%	-5.10%
β_2	-25.19%	-31.57%
β_3	24.59%	179.01%
τ_1	2.62	1.71
τ_2	2.30	0.31

Figure 20: The annually compounding high quality corporate bond yield curve as at May 31, 2010, compared to the corresponding forward curves 1 to 10 years in the future.

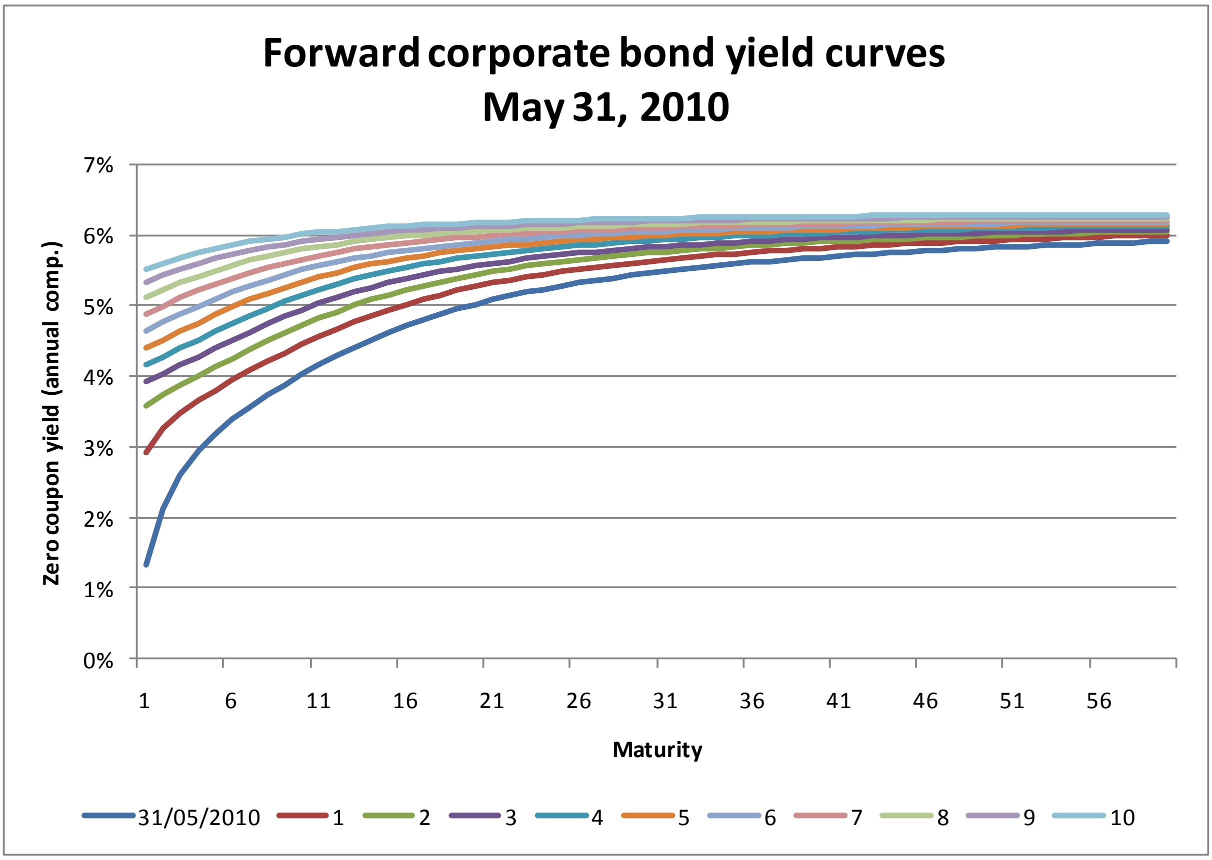


Figure 21: The annually compounding high quality corporate bond yield curve as at May 31, 2010, compared to the constrained high quality corporate bond yield curve as at May 31, 2010. The flattened high quality corporate bond yield curve is a combination of the standard curve (in blue, until maturity 24) and the fixed discount rate (in red, fixed at maturity 24).