Intergenerational risk-sharing

by

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Chapter 1

Abstract

In this thesis, we study the concept of intergenerational risk-sharing. Intergenerational risk-sharing is the concept in which the young generation partially compensates for low returns on the assets of the generation which is almost going to retire. We are first going to simulate a model in which there is no risk-sharing at all. We investigate how much should be invested in risky assets in order to get the highest expected utility. We estimate that it is optimal to invest about one-third of the wealth into risky assets. Afterwards we develop and simulate a model in which risk-sharing is included. In this case, we estimate that it is optimal to invest 50% of your wealth into risky assets. Besides this, shareholders receive a return of about 10%.
Chapter 2

Introduction

In The Netherlands there exists a pension system which consists of three pillars. The first pillar provides a basic level of income for every inhabitant. It is introduced after the Second World War, as a national insurance. The aim of this pillar is to prevent poverty of elderly people. In The Netherlands the first pillar is called the ’AOW’ (Algemene Ouderdomswet) and it is financed through the PAYG system. In this system the ’AOW’ of the current retirees is financed through the ’AOW’-premiums paid by the current working generation. It is noteworthy to mention that the level of ’AOW’ received after retirement is independent of the salary obtained before retirement. So the level of ’AOW’ is the same for every inhabitant.

The second pillar of the Dutch pension system is the part of the pension which is obtained during the time in which someone is active on the labor market. The aim of this pillar is to provide a reasonable level of pension benefits. It is obligated to participate in this pillar of the pension system. There are different kinds of pension plans which belong to the second pillar, namely the Defined Benefit (DB) plan and the Defined Contribution (DC) plan. In a Defined Benefit plan the amount of money you will receive after retirement is known in advance. The amount of money depends either on the final salary (final salary pension plan) or the average salary received during the working life (average salary pension plan). Recently, the average salary pension plans have gradually replaced the final salary pension plans since the pension based on the final salary is more expensive for a pension fund. In a Defined Contribution plan not the pension benefits are known in advance but the contributions to be paid. Every month the premium is paid into a pension fund which invests the premiums in assets. At the age of 65, it is known how much the returns on the assets were and which level of pension is obtained. An important difference between the DC and DB plan is that in the DC plan the risk is carried by the participant, while in the DB plan the risk is carried by the executor of the pension plan. The reason for this is that in a DB plan benefits are guaranteed no matter what the returns on the assets were, while in a DC plan nothing is guaranteed at all. These plans are examples of funded plans in which every generation saves for its own retirement. We would like to note that premiums are invested in both risky and riskless assets. It is necessary to invest in risky assets to obtain a reasonable level of pension since risky assets have a higher expected return.
The third pillar exists of individual savings. It is voluntary to participate in this pillar. Thus everybody can decide for themselves if they want to save for extra income. This can be done for example by annuities or saving on a bank account.

As we already mentioned before, in a funded system every generation saves for its own pension benefits. If for some reason (for example due to the credit crunch of last year) the market collapses, the pension benefits of all generations shrink. Generations which have some years to go until retirement, still have a chance that the loss is compensated by better returns in the future. Generations which participate in a DC plan and who are going to retire soon, have a huge problem. Their assets are not worth that much anymore and their pension benefits are lower than expected. This is when intergenerational risk-sharing comes in. In case of a particular generation having bad returns, the young generation compensates for the losses such that the generation with the bad returns still has a reasonable pension benefit. So, there is a shift of capital from the young generation to the old generation. The young generation has a longer horizon until retirement, so they have a longer horizon to recover from the losses. For the same reason, the younger generation can take more risk than the old generation and have a higher expected return.

Intergenerational risk-sharing is important since if the risk is shared between different generations, every generation can take more risk themselves. Due to this, the expected return is higher and as a result leads to higher pension benefits. If there is a low return on the assets of a particular generation, the risk is shared between different generations so they all take a part of the loss. Now, you might think why this also is attractive for the young generation. The reason for this is that young generations do not have much pension rights yet. They are willing to accept a lower return on their low pension rights since they hope that younger generations will also help to cover bad returns on their assets when they are old themselves. When they are older, they have much more pension rights and that is why it is more important to have reasonable returns. It is crucial that participation to a pension fund is obligated since otherwise the younger generation can decide not to enter the pension fund when the returns on the assets are low. When the returns are low, they have to pay to the older generations. They can decide to participate later when they are older, such that they can gain from other generations while they did not pay to other generations themselves. When this happens, the whole phenomenon of intergenerational risk-sharing is disturbed.

The approach taken in this thesis is that we first model the case when there is no intergenerational risk-sharing at all. We use the results of this model as a benchmark. Afterwards, we are going to model the case in which there is risk-sharing between different generations. We are going to investigate how much capital should be transferred from the young to the old generation in order to get the optimal risk-sharing. We also study how much every generation gains from this. Later on, we will investigate if the optimal strategy depends much on the utility function which is used.

In chapter 3, we give a summary of the most important findings of others in the field of intergenerational risk-sharing. In chapter 4 we start modeling the case in which there is...
no risk-sharing at all. In this chapter we show how much capital should be invested in risky assets in order to obtain the highest expected utility. In chapter 5 the phenomenon of intergenerational risk-sharing is included. In chapter 6 we investigate the sensitivity of the parameters used in the model. Finally, the conclusions and recommendations for further research are stated in chapter 7.
Chapter 3

Literature study

There already is a lot of literature available about intergenerational risk-sharing.

Teulings and de Vries (2006) argue for a system of generational accounts (GA). In this system everybody pays contribution to the same pension fund, but every generation gets its own generation account and all contributions of that generation are paid into their own account. This is important to do, since the optimal investment strategy is different for every generation. If the value of the assets drop, the consumption level of every generation will drop with the same percentage. So, the risk is shared between different generations. This is an improvement compared to the current Defined Benefit plans, since in these kind of plans the working generation pays for the retired generation. In DB plans, the benefits of the retirees are fixed in advantage. Teulings and de Vries emphasize the importance of administration. All generations should have a separate administration since it is very important to have insight in the investment returns per generation. Otherwise it would be impossible to see which generation does not have enough equity and needs additional payments from other generations. Furthermore, they plead that participation should be mandatory. Otherwise, young generations only enter the pension fund when the returns are higher than the expected return. When there are bad results, entering is not very tempting for the young ones since they have to pay for the older ones. Another interesting feature of their paper is about investment strategies. It is well known that in general younger people take more risk in their investment strategies. The standard argument for this is that younger people have an higher expectancy of remaining lifetime so they have a longer period to recover from any losses. However, Teulings and de Vries argue that the real argument is that young people have much more wealth than only their financial capital when they enter the labor market. They namely also have human capital. This human capital works as a counterbalance. Since young people have most human capital, they can bear most risks.

Demange (2001) makes use of a two period OLG model (overlapping generations) to model the problem. In an OLG model every generation lives for a fixed number of years and in every period a new generation is born. Demange focuses on the Pareto concept. In this concept something is a Pareto improvement if there is at least one
person who is better off without making someone else worse off. In his thesis, he investigates two kinds of optimality’s; interim optimality and ex ante optimality. In interim optimality, decisions are based on the information which is available at birth of an agent. Thus, it is based on the utility level of an agent which is calculated at birth. In interim optimality the option of insurance is excluded. This seems quite logical since it is not possible for any agent to trade before the agent is born. The difference between interim optimality and ex ante optimality is that in ex ante optimality there are also possibilities to be insured. Another difference is that ex ante optimality is measured at the initial date. Demange distinguishes these two cases since utility is dependent on time. When there is new information available, this changes the utility level and this can lead to different strategies. Demange also argues for government intervention. The role of the government is to share risk between the old and the young.

Thøgersen (1998) makes use of yet another system: the pay-as-you-go (PAYGO) Social Security system. In this system, the pensions of the retirees are financed through contributions of the current working generation. So, in this system there is only a transfer from the working generation to the retired generation. This system requires a lot of solidarity between generations since the retirees are exposed to any shocks in income of the working generation. Thøgersen investigates two pension programs, namely fixed replacement rate and fixed tax rate. Every generation has a pension benefit and has to pay tax to the government in order to support the older generation in their pension income. In the fixed replacement rate the pension benefits are a fixed percentage of the earnings earlier in life. There also has to be paid tax to support older generations in their income. The tax burden in this case is a function of the income of the older generation. Thøgersen proves in his paper that this leads to an overall increase in income risk since every generation is also exposed to any shocks in income of the older generation. In the fixed tax rate there is, as it name already reveals, a constant tax rate. In this system the tax burden is a fixed percentage of its own income. The amount of pension is a function of the income of the younger generation and fluctuates with the change of income of the working generation. Thøgersen proves that the fixed tax rate system will lead to a smaller variance and, contrary to the fixed replacement rate, the overall welfare will increase. Recently, there is a lot to do in the news about nominal and real pension payments. When it would be the case that pensions are indexed, the pension of a retiree is dependent of the income of the current working generation. Thus, in that case we are dealing with a fixed replacement rate. When pension promises are nominal, we do not cover indexation and the income of the retirees is independent of the income of the working generation so we are dealing with a fixed replacement rate system.

Gordon and Varian (1988) argue for the use of both the Government debt-finance and the Social Security transfer risk. The Social Security is a system which transfers from the young participants to the retirees by use of tax. Government debt-finance is a system which spreads large debts, made for example by waging war, between different generations. So this is a transfer from the old to the young ones. In the model of Gordon and Varian, there is a two period overlapping generation model. In their model there are always two generations; a working generation and a retired generation. At the same time, the retired generation dies, there is a new generation born. The assumption
is made that the outcomes of the returns are revealed at this moment in time. They also explain why risk-sharing via markets fail to exist. The reason for this is, according to Gordon and Varian, that it is impossible for two generations to be alive before the reveal of the returns and after the reveal of the return. Furthermore, generations cannot participate in the security market before birth. To solve this problem, they assume that the government can precommit generations which are not born yet. Gordon and Varian also prove in their paper that the expected utility of all generations will raise if the government transfers income between generations. However, it is possible that unborn generations are worse off in this system then they would be in case of autarky. When risk is shared between many different generations, social risk turns into idiosyncratic risk. Idiosyncratic risk is risk which can be eliminated by diversification. This confirms our intuition that we have to share risk between many generations and as a consequence, pass most of the risk to other generations.

Hemert (2005) developed a model, just like Thøgersen, based on the PAYG system. He models a two-period overlapping generations model where there is a central planner which can transfer money from workers to retirees. In this system the workers and retirees both have financial risk. The workers bear risk, since their labor income is stochastic and the retirees bear risk since the return on their savings is stochastic. It is important to note that the risk of the retirees is larger than the risk of an income shock. Van Hemert explains that the size of a transfer depends on affordability and desirability. So, it depends on the availability of money and the desired level of consumption of the retirees. The desirability is determined by the level of consumption the retirees had in their working life. Thus it is dependent on the level of income they had, the return on their own savings and transfer which they paid themselves to older generations. One of the outcomes of van Hemert is that the transfer from the working generation to the retirees is low when the income of the working generation is low. When the working generation has less to spend, for example due to a decrease in income or a large transfer to the retirees, the young generation also consumes less. This seems quite intuitive. Another outcome is that when the retirees have bad luck and their return is low, there is always a transfer from the working generation to the retirees which is larger than zero. He also shows that the largest transfer occurs when the working generation has high income and the return on the capital is low. When the working generation has to transfer a large amount of money to the retirees, the transfer scheme does not improve their welfare. However, if the returns are low this system is welfare improving since they do not have to transfer money to the old, but they still have the security to receive money when they have any low returns at retirement.

Blake (1998) explains in his article how we can see Defined Benefit (DB), Defined Contribution (DC) and Target Money Purchase (TMP) plans as options. A DB plan is a pension plan in which the pension benefits are known before retirement. For every working year, the employee receives entitlements which are most of the time between 1.75 and 2.25 percent of the pensionable salary. There are two kinds of DB plans, namely the final pay system and the average pay system. In the final pay system, the size of the pension is only dependent of the last pensionable salary before retirement. In the average pay system, the size of the pension is calculated by use of the size of
the average received pensionable salary of the employee. Blake explains that we can replicate a DB plan by making use of a long put option and a short call option. The exercise price of both options should be equal. The trick is that the sponsor writes the put option while it is held by the employee and the employee writes a call option which is held by the sponsor. So, when the call option is in the money, the sponsor makes a profit. However, if the call option is out of the money, the put option is in the money and the sponsor makes a loss. Thus, in this system there is no market risk for the employee. This makes this plan perfect for employees who are risk averse. As a consequence, the expected benefits are also lower. In a DC plan the contributions are known beforehand but the pension benefits are unknown. Every employee has a separate account on which the contributions are paid into. The contributions are invested and at the age of retirement you see how much your return is and how much pension benefits you receive. So, this is a system in which there is only invested in the underlying asset. This system has a high expected return but employees have to bear quite some risk and this system is therefore appropriate for employees who are substantial risk takers. A system which is appropriate for moderate risk takers is the TMP. A TMP system is a system which guarantees a minimum pension return but it does not provide a maximum. Since an employee has to buy a protective put option, the expected return in this system is lower than the expected return in a DC plan.

The main finding in the paper of Gollier (2008) is the fact that the benefit of inter-generational risk-sharing is not to reduce risk, but that it does lead to a higher expected payoff of the investments. The reason for this is that people in general take more risk when there is an option to diversify risk. According to Gollier, this is also socially efficient to do. Gollier developed a model in which he modeled a collective DC fund. He thinks this model is much more efficient than the commonly used PAYG system since the PAYG system has a relatively low return. In this thesis, we build a model which is based on the model of Gollier.
Chapter 4

Modeling autarky

First we are going to model the benefits for a generation when there is no intergenerational solidarity at all. In this case every generation just saves for its own pension benefit. This is called autarky. The model we developed is based on the model of Gollier (2008).

In this model we assume that there are \( n \) working generations. When one generation retires, a new one is going to be active on the labor market. As a consequence, there are always \( n \) generations active on the labor market and every generation works for \( n \) years.

Also, we assume that all generations participate in a Defined Contribution pension plan. Thus, their own pension benefit is dependent on the money they put in and the return they have on their investments. Every generation has to pay a certain contribution, \( y \), to the pension fund at the beginning of every year. We assume \( y \) is equal to 1.

Furthermore, a generation can have a financial reserve at the end of the year. The flow from the financial reserve from year \( t - 1 \) to year \( t \) is denoted by \( w_t \). The total money which is available in the pension fund can be invested in two different assets. Namely, a risk free asset and a risky asset. In this model, \( \alpha_t \) is the percentage of the total money available which is invested in the risky asset at time \( t \). The return on the risk free asset is fixed and equal to 2% per year. The return on the risky asset, \( x \), is based on the AEX stock index.

We used the data of the AEX stock index between 1983 (year of establishment of the index) and 2009 (see figure 4.1) to determine the average return and the standard deviation of the last 27 years. The average return on the AEX index of the last 27 years is equal to \( \mu = 11.56\% \) while the standard deviation is equal to \( \sigma = 27.96\% \). We would like to note that the value of \( \sigma \) is quite high. This can be explained by the fact that there was a big crash in 1987 as well as 2009. This can also be seen in figure 4.1. These two crashes took place in a relatively short period and that is why \( \sigma \) is quite high.

To calculate the return on the risky asset, we assume that the AEX stock index is
Development of the AEX index per year (ex. dividend)

<table>
<thead>
<tr>
<th>Date</th>
<th>AEX index</th>
<th>Annual Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>31-12-1983</td>
<td>73.21</td>
<td>0.61</td>
</tr>
<tr>
<td>31-12-1984</td>
<td>85.75</td>
<td>0.17</td>
</tr>
<tr>
<td>31-12-1985</td>
<td>121.50</td>
<td>0.42</td>
</tr>
<tr>
<td>31-12-1986</td>
<td>114.69</td>
<td>-0.06</td>
</tr>
<tr>
<td>31-12-1987</td>
<td>77.87</td>
<td>-0.32</td>
</tr>
<tr>
<td>31-12-1988</td>
<td>117.68</td>
<td>0.51</td>
</tr>
<tr>
<td>31-12-1989</td>
<td>136.59</td>
<td>0.16</td>
</tr>
<tr>
<td>31-12-1990</td>
<td>104.01</td>
<td>-0.24</td>
</tr>
<tr>
<td>31-12-1991</td>
<td>125.72</td>
<td>0.21</td>
</tr>
<tr>
<td>31-12-1992</td>
<td>129.71</td>
<td>0.03</td>
</tr>
<tr>
<td>31-12-1993</td>
<td>187.99</td>
<td>0.45</td>
</tr>
<tr>
<td>31-12-1994</td>
<td>186.08</td>
<td>0.00</td>
</tr>
<tr>
<td>31-12-1995</td>
<td>220.24</td>
<td>0.17</td>
</tr>
<tr>
<td>31-12-1996</td>
<td>294.16</td>
<td>0.34</td>
</tr>
<tr>
<td>31-12-1997</td>
<td>414.61</td>
<td>0.41</td>
</tr>
<tr>
<td>31-12-1998</td>
<td>536.36</td>
<td>0.30</td>
</tr>
<tr>
<td>31-12-1999</td>
<td>671.41</td>
<td>0.25</td>
</tr>
<tr>
<td>31-12-2000</td>
<td>637.60</td>
<td>-0.05</td>
</tr>
<tr>
<td>31-12-2001</td>
<td>506.78</td>
<td>-0.21</td>
</tr>
<tr>
<td>31-12-2002</td>
<td>322.73</td>
<td>-0.36</td>
</tr>
<tr>
<td>31-12-2003</td>
<td>337.65</td>
<td>0.05</td>
</tr>
<tr>
<td>31-12-2004</td>
<td>348.08</td>
<td>0.03</td>
</tr>
<tr>
<td>31-12-2005</td>
<td>436.78</td>
<td>0.25</td>
</tr>
<tr>
<td>31-12-2006</td>
<td>495.34</td>
<td>0.13</td>
</tr>
<tr>
<td>31-12-2007</td>
<td>515.77</td>
<td>0.04</td>
</tr>
<tr>
<td>31-12-2008</td>
<td>245.94</td>
<td>-0.52</td>
</tr>
<tr>
<td>31-12-2009</td>
<td>335.33</td>
<td>0.35</td>
</tr>
</tbody>
</table>

| mean        | 11.56%   |
| standard deviation | 27.96%   |

Figure 4.1: Development AEX index per year ex. dividend

lognormal distributed with $\mu_2 = \mu - \frac{1}{2}\sigma^2 \approx 7.65\%$ and $\sigma = 27.96\%$. The reason why we use a lognormal distribution in stead of just a normal distribution is that we want the values to be positive. The return at time $t$ is determined by a random number drawn out a lognormal distribution with $\mu_2$ and $\sigma$ as stated above.
Just like Gollier (2008), we use a CRRA utility function which is equal to
\[ u(b) = \frac{b^{1-\gamma}}{1-\gamma} \] (4.1)

In this utility function \( b \) is the pension benefit of the generation and \( \gamma \) is the risk aversion parameter. We assume that the generations are moderate risk averse, so \( \gamma = 2 \).

The problem of maximizing utility under autarky can be stated as:
\[
\begin{align*}
u &= \max_{\alpha_1, \ldots, \alpha_n} Eu(b) \\
\text{s.t } w_1 &= 0; \\
w_{t+1} &= (e^R)(w_t + y) + \alpha_t(w_t + y)(e^{x_t} - e^R), \ t = 1, \ldots, n; \\
b &= w_{n+1}; \\
\alpha_t &\leq 1. 
\end{align*}
\] (4.2-4.6)

Of course, every generation wants to maximize its own expected utility obtained by retirement benefit.

The first constraint states that there is no financial reserve before a generation starts working.

The second constraint tell us how we can calculate the financial reserve for the coming year. In this model, \((w_t + y)\) is the total amount of money in the pension fund at time \(t\). When everything would be invested in riskless assets, at the end of the year the total amount of money in the pension fund will be equal to \((R + 1)(w_t + y)\). However, there can also be additional returns from investing in risky assets. When \(\alpha_t\) is invested in the risky asset at time \(t\), at the end of the year the return would be equal to \(\alpha_t(w_t + y)(x_t + 1)\). Since the first part of the constraint assumes that everything is invested in riskless asset and thus already guarantees a return of 2\%, we have to use excess returns in stead of just the returns. The excess returns in this model are equal to \(x_t - R\).

Since every generation is working for \(n\) years, they are going to retire after \(n + 1\) years. \(w_{n+1}\) is the financial reserve flow from year \(n\) to year \(n + 1\). This is exactly the amount of money which is equal to the pension benefit. That is why the third constraint is added to the model.

The last constraint assumes that it is not possible to go short.

This maximization problem can be solved by Matlab. In this thesis, we concentrate on solving this problem for \(n = 2\). The reason for this is that the calculation time is quite long and the calculation steps for \(n > 2\) are trivial. So our two period maximiza-
tion problem then becomes:

\begin{align}
U &= \max_{\alpha_1, \alpha_2} Eu(b) \\
\text{s.t} \quad & w_1 = 0; \quad (4.7) \\
& w_2 = (e^R)(w_1 + y) + \alpha_1(w_1 + y)(e^{x_1} - e^R); \quad (4.8) \\
& w_3 = (e^R)(w_2 + y) + \alpha_2(w_2 + y)(e^{x_2} - e^R); \quad (4.9) \\
& b = w_3; \quad (4.10) \\
& \alpha_1 \leq 1; \quad (4.11) \\
& \alpha_2 \leq 1. \quad (4.12)
\end{align}

This can be solved by the matlab code which can be found in Appendix B. The outcome of the model is that the optimal value of \( \alpha_1 \) is equal to 35% and \( \alpha_2 \) is equal to 34%. This is also illustrated in figure 4.2.

![Figure 4.2: Optimal \( \alpha_1 \) and \( \alpha_2 \)](image)

So, it is optimal to invest about a third of your wealth into risky assets and about two
third in riskless assets.

Since there is no risk-sharing in this model at all, we developed a model in which risk-sharing is included.
Chapter 5

Modeling risk-sharing

In order to receive a reasonable level of pension, it is necessary to invest a part of your paid premiums into risky assets since risky assets in general have an higher expected return than riskless assets. A big drawback of investing in risky assets is that you can loose a lot of money as well. This is what happened in the credit crunch in 2008. The value of the assets which where bought by pension fund investors dropped tremendously. If there is no intergenerational solidarity at all, people who retire just after the credit crunch (and participate in a Defined Contribution scheme) have a very low pension. This is the case since in a Defined Contribution scheme you build up a capital and at the retirement age you buy a lifetime annuity income. So in this case, people carry a lot of risk because the level of pension is very much dependent on the value of the assets at retirement. In order to prevent people from poverty and to share this risk, the concept of intergenerational solidarity was introduced. In this chapter we extend the model of the previous chapter and introduce the concept of intergenerational solidarity to make the model more realistic. So, the model is extended to a collective defined contribution scheme.

First of all we need to determine how much risk a particular generation has to bear. Gollier (2008) developed a formula in his paper which you can use to calculate this number $m$. This formula is formulated as follows:

$$m = 1 - (\beta R^{1-\gamma} E[1 + \alpha x]^{-\gamma})^{1/\gamma}$$

(5.1)

In this formula $R$ is just as in chapter 4 the return on the riskless asset which is 2%. The $\alpha$ and $x$ which are necessary are the $\alpha$ and $x$ which were developed in the autarky model. So in our case $\alpha = 35\%$ and is a vector which is drawn out a lognormal distribution with the $\mu$ and $\sigma$ as before. In chapter 4 we chose $\gamma$ to be equal to 2 and we also stick to this assumption. The only parameter which is still unknown is $\beta$. This parameter has to make sure that (in expectation) every generation gets the same utility of their pension. Gollier (2008) also developed a formula to determine this $\beta$:

$$\beta = R^{-1}[E(1 + \alpha x)^{-\gamma}]^{\frac{1}{\gamma - 1}}$$

(5.2)
In appendix C the Matlab code can be found which is used to calculate the value of $m$ and $\beta$. In our case we have that $m = 3.58\%$ and $\beta = 0.9642$.

Now we know how much risk every generation bears, we can formulate our model:

$$U = \max_{\alpha, b, d} \mathbb{E} \left[ \sum_{t=1}^{\infty} \beta^t u(b_t) \right]$$

(5.3)

s.t.

$$v_1 \leq \mathbb{E} \left[ \sum_{t=1}^{\infty} \beta^t u(d_t ny) \right];$$

(5.4)

$$w_1 = y_1 + e_1;$$

(5.5)

$$w_{t+1} = (e^R)(w_t - b_t - d_t ny + ny) + \alpha_t (w_t - b_t - d_t ny + ny)(e^{x_t} - e^R), \forall t \geq 1;$$

(5.6)

$$\alpha_t \leq 1, \forall t \geq 1;$$

(5.7)

$$d_t \leq 1, \forall t \geq 1;$$

(5.8)

$$b \geq b_{\text{min}};$$

(5.9)

$$E(w) \geq w_{\text{min}}.$$  

(5.10)

(5.11)

In this model there are some parameters which we didn’t define yet. We will give an overview of the parameters and variables (see table 6.1) and afterwards we will explain in words what the constraints actually mean.

The utility reservation level is the level of utility which can be obtained by investing the initial equity of the investors directly into the market. This value can be calculated by the following formula:

$$v_1 = \frac{m - \gamma e_1}{1 - \gamma}$$

(5.12)

When we fill in the numbers, we see that $v_1$ equals -2.0479.

In the model we see that the benefit $b$ should be larger or equal to $b_{\text{min}}$. This means that every participant is guaranteed a minimum level. This minimum level is equal to the sum of all contributions (thus $ny$). So the minimum guaranteed return on their premiums is equal to

Also can be seen that $w$ should be larger of equal to $w_{\text{min}}$. The pension fund needs a minimum level of wealth. In our model, we set $w_{\text{min}}$ equal to the initial wealth $w_1$. In this way we have that the expected wealth is not lower than the initial wealth we have. This makes sure that the money of the pension fund in expectation does not drop.

Now the only variables left in the model are $b$, $\alpha$ and $d$. In this model, $d$ is the level of dividends which is paid to the shareholders. So this $d$ is a percentage of the yearly contribution. Just as in chapter 4, $\alpha$ is the percentage which is invested in risky assets. It is
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>returns on risky assets</td>
<td>lognormal distributed (0.1156,0.2796)</td>
</tr>
<tr>
<td>$w_t$</td>
<td>total wealth at time $t$</td>
<td></td>
</tr>
<tr>
<td>$y$</td>
<td>yearly contribution per generation</td>
<td>1</td>
</tr>
<tr>
<td>$n$</td>
<td>number of generations and working years</td>
<td>40</td>
</tr>
<tr>
<td>$R$</td>
<td>return on riskless assets</td>
<td>0.02</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>risk aversion parameter</td>
<td>2</td>
</tr>
<tr>
<td>$\beta$</td>
<td>fair utility across generations</td>
<td>0.9642</td>
</tr>
<tr>
<td>$m$</td>
<td>risk per generation</td>
<td>3.58%</td>
</tr>
<tr>
<td>$v_1$</td>
<td>utility reservation level</td>
<td>-2.0479</td>
</tr>
<tr>
<td>$y_1$</td>
<td>initial contribution</td>
<td>1638y</td>
</tr>
<tr>
<td>$e_1$</td>
<td>initial equity pension fund</td>
<td>381y</td>
</tr>
</tbody>
</table>

Table 5.1: Overview parameters model

important to mention that in chapter 4 this $\alpha$ is a percentage of the total wealth available while in this chapter it is a percentage of the total wealth available after paying the dividends and the pension of the generation which retires that year. The meaning of $b$, the benefit obtained, is the same as in chapter 4. Finally, we would like to note that the values for $y_1$ and $e_1$ are based on the paper of Gollier (2008).

Of course, the goal of the maximization problem is to maximize the utility which can be obtained by the benefit $b$. Since we have introduced a parameter to make sure that every generation obtains in expectation the same level of utility, we have to 'discount' the utility function with $\beta$.

Since the investors would have made money themselves if they would have had invested their money directly on the market, they want to be compensated for their investment. Since the generations wants to obtain as much utility for themselves, they need an incentive to give dividends to the investors. This is what is happening in the first constraint. The minimum level of utility obtained (in expectation) by the dividends should be equal to the minimum reservation level.
The second constraint determines the initial wealth available. Since the pension fund already has a level of equity and at the beginning all generations need to pay an initial contribution, we take the initial wealth available as the sum of those two.

Constraint 5.6 tells us how we can calculate the wealth of the coming year. The money available is equal to the money we already had, minus the money which must be given to the generation which is going to retire this year, minus the dividends paid to the investors, plus the contributions of all generations. We first assume that all money available is invested in riskless assets. So, we already assume an return of 2%. In the second part of the equation we assume that \( \alpha \% \) is invested in risky assets which have a return of \( x_t \). Due to the fact that we already guaranteed a return of 2%, we have to subtract that 2% again.

Constraint four and five are added since we assume that we cannot go short and thus cannot invest more than 100% in risky assets and we cannot give more than 100% dividend to the investors.

The last two constraints state that the minimum return on the premiums paid is 0% and that the expected wealth in the pension fund should be larger or equal to the initial wealth available respectively.

Just like we did with the autarky model, we take only solve this problem for two periods. The reason for this is that the calculation time is quite long and the calculation steps for the other periods are trivial.
For those two periods the maximization problem is equal to:

\[
U = \max_{\alpha, b, d} E \left[ \sum_{t=1}^{2} \beta^t u(b_t) \right] \tag{5.13}
\]

s.t. \( v_1 \leq E \left[ \sum_{t=1}^{2} \beta^t u(d_t ny) \right] ; \tag{5.14} \)

\[
w_1 = y_1 + e_1; \tag{5.15}
\]

\[
w_2 = (e^R)(w_1 - b_1 - d_1 ny + ny) + \alpha_1 (w_1 - b_1 - d_1 ny + ny)(e^{x_1} - e^R); \tag{5.16}
\]

\[
w_3 = (e^R)(w_2 - b_2 - d_2 ny + ny) + \alpha_2 (w_2 - b_2 - d_2 ny + ny)(e^{x_2} - e^R); \tag{5.17}
\]

\[
\alpha_1 \leq 1; \tag{5.18}
\]

\[
\alpha_2 \leq 1; \tag{5.19}
\]

\[
d_1 \leq 1; \tag{5.20}
\]

\[
d_2 \leq 1; \tag{5.21}
\]

\[
b_1 \geq b_{\text{min}}; \tag{5.22}
\]

\[
b_2 \geq b_{\text{min}}; \tag{5.23}
\]

\[
E(w_1) \geq w_{\text{min}}; \tag{5.24}
\]

\[
E(w_2) \geq w_{\text{min}}. \tag{5.25}
\]

We programmed this model in Matlab (see Appendix D). The outcome of this matlab program is that it is optimal to invest 50% of your wealth into risky assets. Besides this, in both periods, the optimal dividends you pay is 10% and the pension benefits for the generation which is going to retire in the first period is equal to 190 and the generation which retires a year afterwards receive 240.

When we compare those results with the results we had in our autarky model, we see that the result of risk-sharing is that there will be more invested in risky assets. This seems quite logical since in this case bad results can be partially compensated by other generations. This is the reason why people are willing to take more risk.
Chapter 6

Sensitivity analysis

In our model we used some input variables. It is quite crucial to know whether or not those variables are sensitive to changes. This is why we performed a sensitivity analysis.

6.1 Contributions

The first input variable is $y$, which is the contribution paid by the participants. When we run our model with different values for $y$ we get the graph which can be seen in figure 6.1.

![Graph showing outcome of the model for different values of $y$.]({image-url})

Figure 6.1: Outcome of the model for different values of $y$
From this graph we can conclude that the outcome of our model is independent of $y$. The reason for this is that we are using the power utility function. In our model we maximize the expectation of the discounted utility which is obtained by the benefit $b$. So what we actually have is:

\[
\max_{\alpha,b,d} E[\beta * u(b)]
\]

\[
= \max_{\alpha,b,d} E[\beta * u((w - dny + ny) * R + \alpha(w - dny + ny)(e^{z_1} - e^R))] 
\]

\[
= \max_{\alpha,b,d} E[\beta * u((2190y - dny + ny) * R + \alpha(2190y - dny + ny)(e^{z_1} - e^R))] 
\]

\[
= \max_{\alpha,b,d} E[\beta * u((2190 - dn + n) * R + \alpha(2190 - dn + n)(e^{z_1} - e^R))] 
\]

\[
= \max_{\alpha,b,d} E[\beta * ((382 - dn + n) * R + \alpha(2190 - dn + n)(e^{z_1} - e^R))]^{-1} 
\]

\[
= -\frac{1}{y} \max_{\alpha,b,d} E[\beta * ((382 - dn + n) * R + \alpha(2190 - dn + n)(e^{z_1} - e^R))]^{-1} 
\]

This also underlines our findings that the model is independent of $y$.

### 6.2 Gamma

In chapter 4, we denoted the participants to be moderate risk takers and for that reason we set the risk aversion parameter, $\gamma$, equal to two. In figure 6.2 we show the results of the sensitivity analysis for $\gamma$.

When we look at this figure we see that the value for $\gamma$ has much influence on the level of risky investments. The higher $\gamma$, the lower the level of investments in risky assets. This result seems logical, because when the risk aversion parameter is higher, people are less willing to take risk and as a result they will invest less in risky assets. We can also conclude from figure 6.2 that the higher $\gamma$, the higher the dividends the investors receive. This can be explained by the fact that $\gamma$ has an influence on $\beta$ and $m$.

Due to this the value for $v_1$ will rise and as a consequence, the investors need to receive an higher level of dividend. This can also intuitively be explained. When $\gamma$ raises, the investors will be more risk-averse and as a consequence they will ask an higher return on their investment. Finally, the outcome of the sensitivity analysis shows that the level of pension benefit is independent on the level of $\gamma$. 

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6.3 Return on risk free assets

In both models we set the return on the risky assets, $R$, equal to 1.02. We also ran our model with some different values for $R$. The results can be seen in figure 6.3.

In this graph we see that a change the return on the risk free assets has a big influence on the outcome of the model. The higher the return on the risk free assets, the lower the part of the wealth which is invested in risky assets. This can be explained by the fact that it is not very lucrative anymore to take more risk and invest in risky assets since the return on the riskless assets is guaranteed and already quite high. The dividends which have to be paid are also quite sensitive for changes in the return on the riskfree asset. The reason for this is in the same line as the explanation we had for a change in $\gamma$. When the riskfree return raises, the values for $m$ and $\beta$ change and as a consequence $v_1$ will raise as well. Since the expected utility obtained by the dividends should be larger than $v_1$, the dividends should also raise. Intuitively this is also plausible. When the risk free rate is higher, the investors would have had an higher return when they would invest their money into the financial market, so they would like an higher return from the pension fund as well. The level of pension benefit in also this case is independent of $R$. 

Figure 6.2: Outcome of the model for different values of $\gamma$
6.4 Initial equity

The outcome of the model for different values of the initial equity, $e_0$, can be found in figure 6.4. As we can see in graph e0, the initial equity has no influence on the percentage which is invested in risky assets. We can see as well that it does have influence on the level of dividends. The more the initial equity, the higher the dividends. The reason for this is that $v_1$ raises when $e_1$ raises. Due to that, the dividends which have to be paid are higher as well.

In contradiction to the previous input variables, the level of initial equity does have an influence on the level of pension benefit. This can graphically be seen in figure 6.5. This also seems quite plausible. The more money a pension fund has, the more money it has to give to the participants. So we can conclude that it is not the contribution which plays a role in the level of pension benefit, but it is the initial equity that determines the level of pension benefit.
Finally, we tested the sensitivity of the number of generations and working years. These results are summarized in figure 6.6. In our model we set the number of generations and working years equal to 40. We see in figure 6.6 that when the number of generations and working years is decreased or increased, the same percentage is still invested in risky assets, the dividends which are paid to the investors remain the same and the
pension benefits also remain the same. So our model is not sensitive for changes in $n$. 

Figure 6.6: Outcome of the model for different values of $n$
Chapter 7

Conclusions and Recommendations

In this thesis, we developed a model so that we can predict how much money should optimally be invested in risky assets and how much should be shared with other generations. We wanted to know what the implications of risk sharing are, so we first developed a model in which there no risk sharing at all. We found out that in that case is it optimal to invest about one third of your wealth into risky assets. Later on we introduced the concept of intergenerational risk-sharing. In this case we still used the same assumptions but we introduced a collective defined contribution scheme. The outcome of this model was that is it optimal to invest about 50% of your wealth into risky assets and besides this, to pay 10% dividend. The level of pension benefit is about 200. When we compare this result to the result we had in our autarky model, we can conclude that intergenerational risk-sharing causes an increase in investments in risky assets. Any bad results can be (partially) be compensated by other generations such that people are less risk averse.

On the 4th of June the employers and unions have made an agreement about the AOW. They propose that the AOW age in 2020 will be increased to the age of 66 and to 67 in 2025. From 2020 on every five years there will be investigated whether or not the life expectation raised. If this is the case, the AOW age for ten years later will be based on that life table. So, the AOW age can change every 5 years. When the AOW age is raises, it is very likely that the pensionable age will also raise. This is something which also can be integrated in our model. The agreement causes that there are cohorts of people and every cohort has another AOW/pensionable age so they all work for another number of years.

Another improvement which can be done is about the contributions paid. Most of the time the contribution which has to be paid is a percentage of the pensionable salary. Most likely, people who start working earn much less money than people who are almost going to retire. So there can be a big difference between the contributions paid.
by the young ones and the old ones. In our model we use one level of contributions for everybody since we used the power utility function and in that case the model is independent of the contributions paid. So, in case one wants to make this model more realistic, one needs to use another utility function.
Chapter 8

Appendix A

In the literature there are many utility functions used. One of the most commonly used classes of utility functions is CRRA. CRRA is an abbreviation for Constant Relative Risk Aversion. It is a class of utility functions. CRRA functions always have the same isoelastic form:

\[ u(x) = \frac{x^{1-c}}{1-c} \]  

(8.1)

In this utility function, c is constant and it should be unequal to one. L’Hôpital’s rule states that

\[ \lim_{x \to 0} \frac{b^x - 1}{x} = \lim_{x \to 0} \frac{b^x \ln(b)}{1} = \ln(b) \lim_{x \to 0} b^x = \ln(b) \]  

(8.2)

When we use this rule we see that

\[ \lim_{c \to 1} u(x) = \lim_{c \to 1} \frac{x^{1-c}}{1-c} = \ln(x) \]  

(8.3)

Often, when the constant c is almost equal to one, the utility function \( u(x) = \ln(x) \) is used.

The name Constant Relative Risk Aversion implies that the coefficient of relative risk aversion should be constant. The definition of the coefficient of relative risk aversion is stated as follows:

\[ -\frac{u''(x)x}{u'(x)} \]

(8.4)

Now, we are able to prove that functions of the form \( u(x) = \frac{x^{1-c}}{1-c} \) are indeed CRRA functions. The first derivative of \( u(x) \) with respect to \( x \) is equal to
The second derivative to $u(x)$ with respect to $x$ is equal to
\[ u''(x) = -cx^{c-1} \] (8.6)

So the relative risk aversion is
\[ \frac{-u''(x)x}{u'(x)} = \frac{cx^{-c}}{x^{-c}} = c \] (8.7)

Since the relative risk aversion is constant, this function is a member of the CRRA class.

The power utility is often used to model saving behaviour. The reason for this is that the second derivative is positive which means that saving is encouraged. This utility function is not suitable to use when one studies the behaviour around $c = 1$. When we do this the utility explodes and the outcomes are not precise anymore. So when one is dealing with a subject in which precise answers are needed since otherwise the consequences are dramatic, one can better use another utility function. In our case, we do not study the behaviour around $c = 1$ and a little error does not have dramatic consequences. Besides this, the power utility is easy to work with so we used the power utility function to study intergenerational risk-sharing.
Chapter 9

Appendix B

clear all

sigma=0.2796;
var=sigma*sigma;
mu=0.1156;
eenheid=ones(1000,1);
R=1.02*eendheid;
y=1+eendheid;
gamma=2;
random=randn(1000,1);
x1=random*sigma+mu-0.5*var;
random2=randn(1000,1);
x2=random2*sigma+mu-0.5*var;
for i=1:100
    alfa1(i)=i/100;
end
for j=1:100
    alfa2(j)=j/100;
end
for i=1:100
    for j=1:100
        w2(:,i)=y.*(exp(log(R)))+alfa1(i).*y.*(exp(x1)-exp(log(R)));
        w3(:,i,j)=(exp(log(R))).*(w2(:,i)+y)+alfa2(j).*
        (exp(x2)-exp(log(R))).*(w2(:,i)+y);
        utility(:,i,j)=w3(:,i,j).^(1-gamma)/(1-gamma);
    end
    expectedutility(i,j)=mean(utility(:,i,j));
end
maximum=max(max(expectedutility));
[I,J] = find(expectedutility == maximum);
optimal_percentage_alfa1=I/100;
\texttt{optimal\_percentage\_alfa2=J/100;}

\texttt{plot3(alfa1,alfa2,expectedutility)}

Matlab code autarky model (Chapter 4)
Chapter 10

Appendix C

Matlab code to determine m (Chapter 5)

```matlab
% MatLab code to determine m (Chapter 5)

clear all
sigma=0.2796;
var=sigma*sigma;
mu=0.1156;
random=randn(1000,1);
x1=random*sigma+mu-0.5*var;
eenheid=ones(1000,1);
R=1.02;
gamma=2;
alfa=0.35.*eenheid;
hulp=mean(((alfa.*x1)+1.*eentheid).^(-1.*gamma));
hulp2=hulp.^(1/(gamma-1));
b=(R.^-1).*hulp2;
hulp3=(b.*R.^(1-gamma).*hulp).^(1/gamma);
m=1-hulp3;
```
Chapter 11

Appendix D

clear all

sigma=0.2796;
var=sigma*sigma;
mu=0.1156;
eenheid=ones(1000,1);
R=1.02*eenheid;
y=1*eenheid;
gamma=2;
e0=381*y;
y0=1638*y;
w1=y0+e0;
onthouden=-999999999999;
m=0.0358;
b=0.9642;
v=(m.^(1-gamma)*e0.^(1-gamma))/(1-gamma);
v1=mean(v);
n=40;
random=randn(1000,1);
x1=random*sigma+mu-0.5*var;
random2=randn(1000,1);
x2=random2*sigma+mu-0.5*var;
i=1:2:21;
for j=1:11
    alfa1(j)=i(j)/20;
end
for j=1:11
    alfa2(j)=i(j)/20;
end
for j=1:11
d1(j)=i(j)/20;
end
for j=1:11
d2(j)=i(j)/20;
end
f=40:50:540;
for j=1:11
b1(j)=f(j);
end
for j=1:11
b2(j)=f(j);
end
for i=1:11 %a1
  for j=1:11 %a2
    for k=1:11 %d1
      for l=1:11 %d2
        for m=1:11 %b1
          for n=1:11 %b2
            w2=(w1-d1(k).*n.*y+n.*y-b1(m)).*(exp(log(R)))+alfa1(i).*
            (w1-d1(k).*n.*y+n.*y-b1(m)).*(exp(x1)-exp(log(R)));
            w22=mean(w2);
            utility2(:,1)=b1(m).ˆ(1-gamma)/(1-gamma);
          end
        end
      end
    end
  end
end
utility3(:,1)=((d1(k)-0.05).*n.*y).ˆ(1-gamma)/(1-gamma);
utility4(:,1)=((d2(l)-0.05).*n.*y).ˆ(1-gamma)/(1-gamma);
hulp=b*utility3+b.ˆ2*utility4;
expectedutility2=mean(hulp);
if ((expectedutility2>v1) && (expectedutility>onthouden) &&
  (w22>2019) && (w33>2019))
  onthouden=expectedutility;
optimala1=i/20-0.05;
optimala2=j/20-0.05;
optimald1=k/20-0.05;
optimald2=l/20-0.05;
optimalb1=b1(m);
optimalb2=b2(n);
end
end
end
end
end
end
Matlab code model including risk-sharing (Chapter 5)
Bibliography


